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VERTICAL AGREEMENTS BETWEEN AIRPORTS AND CARRIERS[1]

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Abstract. This paper investigates vertical contracts between airports and airlines, in the context of two competing facilities and three different types of agreements. The downstream market consists in a route operated by one leader and *n-1* follower competing *à la* Stackelberg in each airport. In this sense, the paper adds to literature as it considers the issue of vertical contracts both in the airports competition and airlines competition. We develop a multistage facility-rivalry game where each airport and the respective dominant airline decide whether to enter into a contract and, if so, which one to engage in.

In this framework, we investigate the Nash equilibrium to analyse the incentives for vertical contracts: we find that the airport and the dominant airline have incentive to collude in each facility. Nevertheless, the equilibrium is not efficient in terms of social welfare, so that there is a misalignment between private and social incentives.

Key words: vertical contracts, airports competition, airlines competition

[1] An early version of this paper has been presented at the 14th Annual Congress of the Air Transport Research Society in Porto, 6-9 July 2010.

1. Introduction

Liberalization has led to radical changes in the competitive structure of the aviation industry: after the initial acts of deregulation (Airline Deregulation Act, 1978), which has seen the entry of several carriers into the market, the persistence of structural, strategic and regulatory barriers created the basis for exerting market power in an oligopoly centred around hub and spoke air transport arrangements. Therefore, recent dynamics in the industry have been outlining an increase in the degree of concentration in the supply of air services, and a market polarization all around few carries with a relevant market share, challenged by smaller competitors (Alderighi et al, 2005). As a consequence, dominance allows a carrier to achieve higher bargaining power and to turn the airport-airline relation into a bilateral-monopoly (monopoly-monopsony).

On the other hand, when an airport faces competition from other airports, either an adjacent airport sharing the same catchment area, or another major airport competing for connecting traffic, it is in each airport's interest to ally with one airline, normally the dominant carrier (Oum et Fu, 2008). Actually, competition between airports is increasing, particularly in the case they are located in different metropolitan areas, but share, at least in part, the same catching area (e.g. the case of major hub-and-spoke airports: Fiumicino in Rome and Malpensa in Milan, the airports of Barcelona and Madrid, Brussels and Amsterdam or Brussels and Paris); moreover, even if they are located in the same metropolitan area, and are managed by the same company (notably, Paris ADP airports, London BAA airports, Rome ADR airports, Milan SEA Airports), some competitive issues may arise, due to possible cross-subsidies and the ensuing distortions.

In this scenario, vertical relations in the aviation industry are of increasing concern and source of debate for both academic and practitioners, constituting a fundamental issue because of its implications for the operation of the industry and the ensuing regulatory requirements. Indeed, evidence suggests that there may be strong incentives for airports and their respective dominant airlines to vertical cooperation and they need to be analyzed: (i) airports can obtain financial support and secure business volume, which are important for daily operation as well as for long term expansion; (ii) airlines can secure key airport facilities on favorable terms, which are essential conditions for them to make long term commitment/investment at an airport; (iii) since concession revenues are increasingly important, airports and airlines now use various agreements to internalize the positive demand externality between aviation services and concession services and, with growing pressures for airports to improve their financial performances, this is a crucial issue.

On the other hand, such airline-airport cooperation raises anticompetitive concerns. Vertical restraints may harm competition in the downstream airline market: such a dominance of one airline at an airport allows the airline to obtain a substantial "hub premium", even more evident for flights connecting two hubs of the same carriers. An airline with 50% of the traffic at each endpoint of a route is estimated to charge high-end prices about 12% above those of a competitor with 10% of the traffic at each endpoint (Oum et Fu, 2008). Moreover, the dominant airline's control over key airport facilities, such as slots and gates, is likely to impose significant entry barriers to other potential competitors, especially at congested airports.

Different forms and types of agreement have been observed in practice. For example: (i) master use-and-lease agreements, where airlines become guarantors of the airport's finance; in return, they are given varying degrees of influence over airport planning and operations (i.e. terminal usage); (ii) concession revenue sharing agreement, where the sharing airline can internalize positive demand externality, and benefits from its competitors' output expansion in terms of getting more concession revenue. In many cases it occurs when airports allow airlines to hold shares or control airport facilities; Tampa International Airport, as of 2005, shares 20% of its net revenue with the signatory airline, i.e. Continental Airlines, Inc. who continued to operate in the facility under an amended lease that expired on September 30, 2009; (iii) airlines own or control airport facilities (i.e. Terminal 2 of Munich airport is a joint investment by FMG (60%) and Lufthansa (40%); Lufthansa has also invested in Frankfurt airport, and holds a 29% share of Shanghai Airport Cargo Terminal); (iv) long term usage contract, as service guarantee and usage commitment (i.e. in 2002 Melbourne airport and Virgin Blue reached a 10-year agreement for the airline to operate from the former Ansett Domestic Terminal; (v) airport revenue bond, where airports retain asset ownership but transfer the right for exclusive usage to the bondholders airlines under long-term lease agreements.

Nevertheless, vertical relations between airports and airlines have received little attention in the literature so far, probably due to the fact that price discrimination on aviation services is prohibited by IATA and EU rules: an airport is required to charge all airlines the same price for identical services. (EU Directive 2009/12/EC-Art.3, EEC Treaty-Art.87/88, EEC Council Regulation No 95/93). Such a restriction, together with the historical public utility status of most airports, has often excluded airports from the lists of anti-trust investigation until the recent privatization wave. Therefore, research documented in the literature appears to lack maturity in this direction.

Pels et al. (2003) analyse the correlation in the dimensions of passengers choice: access mode, airport and airline. They find that the set of "airport and airline" is considered, not only the facility alone, but they don't investigate the issue of vertical relations between the carriers and the airport. Basso and Zhang (2007) focus on both airport rivalry and airline competition, but with respect to the issue of congestion delays. Basso (2008) considers the issue of facility rivalry and finds increased cooperation between airports and airlines, in the form of maximization of joint profits, provides some improvements, even if the resulting airport pricing strategy (two part tariff) leads to a downstream airline cartel. Nevertheless, he does not analyse other different forms of vertical relations. Starkie (2008), Oum and Fu (2008) give an overview of airport-airline vertical relationships and policy implications, but they do not build a model to analyse different types of contracts or the effects in terms of competitiveness, social welfare and consumer surplus. Barbot (2009a) focuses on the issue of facility rivalry: she analyses the incentives to vertical collusion for an airport-dominant airline system if the other airport and dominant airline also engage in agreement, finding that they exist when airports and airlines have different market sizes or, in some cases, when there is a secondary airport and LCC carriers. Nevertheless, she does not analyse the issue of airlines competition within each airport. Barbot (2009b) develops an airport-airlines model to examine the effects of three types of contracts, according to Starkie (2008): the European case, the Australian case and the US case. The European case, namely “Vertical Collusion”, depicts the case of a negotiated fare between the airport and the dominant airline, depending on their bargaining power (i.e. Charleroi – Ryanair, Finnish or Portuguese airports contracts). The airport and the leader airline collude and maximize their joint profits: the negotiation aims at both partners obtaining the highest joint profits and the solution is the same of a vertical merger. The other airlines will pay a higher facility charge. In the Australian case, i.e. “Airlines in the upstream market”, long term leases on terminals are analyzed (i.e. Sydney, Melbourne, Dallas Forth Worth). The airport operates the runway for all airlines, while the leader airline leases and operates the terminal, using it and selling it to the followers. Finally, the US case depicts the case of “Price discrimination” (i.e. Atlanta, Orlando): the leader airline pay the airport the variable cost of its facility plus a part, which is agreed between the two partners, of its fixed costs. Specifically, the competitive pressures in the airlines market over the incentives to the three types of vertical contracts are analysed and it is found that: (i) two of them are anti-competitive and (2) in all of them consumers are better-off. Nevertheless, in this context, facility rivalry is not investigated. Zhang and Fu (2010) deal with the issue of both airports and airlines competition, but with respect to the case of a single type of contract: concession revenue sharing. They find that: (i) the degree of revenue sharing will be affected by how airlines’ services are related to each other; (ii) whether an airport is subject to competition is critical to the welfare consequences of alternative revenue sharing arrangements.

In this paper, the three types of vertical contracts analysed in Barbot (2009b) are considered in the context of two competing facilities: in this sense, the paper adds to literature as it considers the issue of vertical alliances with respect to both airports competition and airlines competition. Specifically, we develop a multistage facility-rivalry game and we investigate the subgame perfect Nash equilibria to analyse the incentives for vertical contracts and the effects in terms of welfare, consumer surplus and pro-competitiveness.

The paper is organized as follows. Section 2 sets up the model; section 3 describes the different cases according to the different types of vertical agreements between airports and airlines; in section 4, we find the airports and airlines optimal strategies for each case; section 5 contains the concluding remarks.

2. The model

We assume there is an infinite linear city[2] where potential consumers are uniformly distributed with a density of one consumer per unit length. There are two airports, A_0 and A_1, respectively located at 0 and 1. The locations of the facilities are exogenous and there are consumers also beyond the airports: facility 0 also captures the consumers at its immediate left side and facility 1 those at its immediate right side. In each airport the downstream market consists in a route operated by one leader and *n-1* followers, which offer a homogeneous good/service, the flight. Let L^h and F_i^h stand for the leader and the followers, respectively, which operate in airport A^h, with h = 0, 1 and i = 1, …, n-1. The number of carriers at each facility is exogenous. The airlines cannot choose which facility they operate, but they are bound to a certain airport: they do not compete for the airport to operate. Hence, each follower-leader set of airlines will supply the demand for only one airport, which they do not choose. In this sense, airports do not compete for the airlines but compete through airlines to get passengers: each airport gets the number of passengers the carriers are able to capture.

In each airport the leader and the followers compete in *à la* Stackelberg. The leader L_0 competes in quantities with the leader L_1 in a Cournot fashion; the followers compete in quantities with the followers at the same facility and with the followers at the other facility in a Cournot fashion. Therefore the followers take into account the strategic choices of the leader at the same airport and the ones of the leader at the other airport.

In the downstream market, the only cost both the leader and the followers meet is the airport aeronautical fare, varying with respect to the type of contract the leader and the airport have enter in. Other variable airline costs are constant and normalized to zero. The airports have a constant marginal operating cost per flight, c, and a fixed cost F_h, with h = 0,1. Airlines sell tickets directly to consumers, at prices-per-passenger p_0 and p_1. Demands at each facility, Q_0 and Q_1 respectively, are measured by the total number of flights offered.

The vertical structure of airport-airlines behavior is represented by a multistage game: in the first stage, the airport A_0 and its dominant carrier L_0 decide, simultaneously with the airport A_1 and its dominant carrier L_1, whether to enter into a contract and, if so, which one to engage in; prices for the input to be used by carriers are decided; in the second stage, L_0 compete with L_1 in the output market choosing quantities; in the third stage, F_i^0 compete in quantities with F_j^0, $j \neq i$, at the same facility, and F_i^1, at the other facility, with i = 1, …, n-1 and j = 1, …, n-1; finally, passengers decide whether to fly or not and if so which facility to go.

According to Barbot (2009b), the application of Stackelberg quantity leadership to the downstream market seems realistic: the dominant carriers may be considered as quantity leaders because, as first comers, they choose the quantity and left the remaining slots for other carriers. Table 1 shows a high concentration of airlines' flights in the 20 largest airports in Europe, for 2009, and high

[2] In this sense we consider a framework similar to Basso and Zhang's (2007).

shares belonging to flag carriers (i.e. Air France at Paris CDG, Alitalia at Rome Fiumicino, SN Brussels Airlines at Brussels National) or to carriers that established their bases at particular airports (i.e. Lufthansa at Frankfurt and München F.J. Strauss or Spanair at Barcelona).

ASK (%) of the top five carriers in the 20 biggest European airports							
Rank	**Airport**	**First carrier**	**ASK (%) First Carrier**	**ASK (%) Top Two Carriers**	**ASK (%) Top Three Carriers**	**ASK (%) Top Four Carriers**	**ASK (%) Top Five Carriers**
1	Roma Fiumicino	Alitalia	37,2%	42,2%	47,0%	49,5%	51,9%
2	Parigi Charles De Gaulle	Air France	54,6%	57,4%	59,9%	62,3%	64,3%
3	Francoforte	Lufthansa	53,3%	59,2%	62,0%	64,6%	67,1%
4	Londra Heathrow	British Airways	40,1%	45,5%	50,9%	54,4%	57,6%
5	Milano Malpensa	Alitalia	31,4%	37,7%	43,3%	47,6%	51,5%
6	Amsterdam-Schiphol	KLM	49,6%	62,3%	66,5%	69,7%	72,1%
7	Madrid Barajas	Iberia	51,8%	57,3%	62,8%	65,9%	68,8%
8	München F.J. Strauss	Lufthansa	56,1%	62,1%	67,0%	70,2%	73,0%
9	Barcellona	Spanair	13,7%	23,7%	32,3%	39,5%	45,4%
10	Londra Gatwick	British Airways	26,4%	43,3%	50,7%	56,2%	61,4%
11	Atene Eleftherios	Olympic Airlines	30,9%	41,6%	47,1%	51,6%	55,3%
12	Brussels National	SN Brussels Airlines	24,7%	41,5%	51,3%	55,8%	60,2%
13	Zurigo	SWISS	58,0%	64,0%	67,3%	70,5%	73,6%
14	Vienna	Austrian	59,4%	63,7%	68,0%	71,3%	73,6%
15	Manchester	Emirates	9,0%	17,5%	25,1%	31,8%	37,0%
16	Copenhagen	SAS	51,7%	58,2%	61,2%	64,0%	66,6%
17	Geneva-Cointrin	Easyjet Switzerland	14,4%	28,5%	36,8%	44,9%	50,9%
18	Stoccolma-Arlanda	SAS	39,9%	48,0%	52,8%	57,6%	62,2%
19	Dusseldorf	Air Berlin	22,6%	44,3%	63,3%	68,5%	73,6%
20	Malaga	easyJet	21,3%	28,6%	34,8%	40,7%	45,5%

Table 1. ASK (%) of the top five carriers in the 20 biggest European airports. Data related to 2009. Source: ICCSAI-Fact Book 2010.

With respect to assuming that airlines cannot choose which facility they operate, but they are bound to a certain airport, it can be argued that airlines could, actually, decamp all or part of their operations to an alternative site: in particular, this is evident once we take into account non-networked air services operated by charter or low cost carriers which have more scope for switching operations between airports in order to reduce costs[3].

Nevertheless, when air services are concentrated at a transfer point, i.e. at a hub airport, the significance of the agglomeration economies/network externalities may be such that they tie the individual dominant airline to the hub airport. In this case, for a scheduled carrier, with a high level of transfer passengers to and from other airlines, to choose to forego the revenue and cost advantages of the hub by substituting a proximate, even adjacent, alternative airport would seem most unlikely (Starkie 2002). British Airways or British Midland at Heathrow, Air France at Paris Charle De Gaulle or Alitalia at Rome Fiumicino provide an example in this sense. Moreover, some airlines own or control airport facilities: Lufthansa has invested in Frankfurt airport and Munich airport; Latvia's Riga Airport has offered a contract to the national airline Air Baltic to build and operate a 92 euro million terminal for seven million passengers per annum by 2014. This means that the costs of switching airport are higher for the dominant airlines, which is an essential condition for them to make long term commitment to the airport itself.

Finally, the assumption of each follower-leader set of airlines supplying the demand for only one airport seems also reasonable once the presence of followers aligned with the leaders in co-sharing or alliance agreements, within a given airport, is taken into account. Actually, this occurs in particular with respect to the case of regional subsidiary carriers (Table 2).

Alliance	First Carrier	Consociate	Hub
SkyTeam	Air France	Brit Air Régional	Paris-Orly Lion-Saint Exupéry
	KLM	KLM Cityhopper	Amsterdam-Schipol
	Alitalia	Airone CAI-Second (i.e.Volare Airlines)	Milano Malpensa
		Airone Cityliner CAI-First (i.e. Alitalia Express)	Rome Fiumicino
Star Alliance	Lufthansa	Lufthansa Regional	Frankfurt München F.J. Strauss
Oneworld	Iberia	Iberia Regional Vueling ClickAir	Madrid Barajas

Table 2. Alliance agreements between national and regional consociate carriers within some European hubs. Data related to 2010. Source: www.skyteam.com ; www.staralliance.com ; http://www.oneworld.com.

[3] Competition between Luton and Stansted in the early 1990s for the custom of Ryanair provides an example in this sense.

We investigate the subgame perfect Nash equilibria of the game. For this purpose, we first focus on airline's demand. Potential consumers have unit demand for flights and they care for their "full price". Indeed, passengers may not necessarily choose the airport with cheaper fare, but may go to an airport that is nearer and has a shorter total travel time. Therefore, the full price is the sum of the ticket price and the travel cost to the facility.

For a consumer located at $0 \leq z \leq 1$ and who goes to facility 0, the full price is given by:

$$p_0 + 4tz \qquad (1)$$

where $4t$ is a parameter capturing consumers' transportation cost, assumed to be positive[4]. If the consumer decide to fly she derives a net utility:

$$U_0 = U - p_0 - 4tz$$

where U denoting the gross benefit. Similarly, if the consumers goes to facility 1, then she derives a net benefit:

$$U_1 = U - p_1 - 4t(1 - z)$$

Assuming that everyone in the [0,1] interval decides to fly and both airports receive consumers from [0,1], then the indifferent consumer $\tilde{z} \in (0,1)$ is determined by $U_0 = U_1$, or

$$\tilde{z} = \frac{1}{2} + \frac{p_1 - p_0}{8t}$$

Thus the number of [0,1] consumers going to facility 0 increases in p_1 and decreases in p_0. Let z^l be the last consumer on the left side of the city, who decides to fly and goes to facility 0, and z^r the last consumer on the right side of the city, who decides to fly and goes to facility 1. Given the uniformity and unit density of consumers, z^l is determined by $U_0 = 0$ and $z^l < 0$, or:

$$z^l = -\frac{U - p_0}{4t}$$

Similarly z^r is determined by $U_1 = 0$ and $z^r > 1$, or:

$$z^r = 1 + \frac{U - p_1}{4t}$$

The points z^l, z^r and $\tilde{z}$ define the catchment areas of each airport as shown in Fig. 1.

Hence, the demands for flight at each facility are given by $Q_0 = \tilde{z} + |z^l|$ and $Q_1 = (1 - \tilde{z}) + (z^r - 1)$, or:

$$Q_0 = \frac{1}{2} + \frac{2U - 3p_0 + p_1}{8t}$$
$$Q_1 = \frac{1}{2} + \frac{2U - 3p_1 + p_0}{8t} \qquad (2)$$

In order to have everyone in the [0,1] interval decides to fly we need $U_0 \geq 0$ and $U_1 \geq 0$ or:

$$2U \geq p_0 + p_1 + 4t$$

[4] The parameter *4t* is chosen to simplify equations in the model.

Similarly to have both airports receive consumers from [0,1] or, in other words, to have at least one consumer in both of the two airports, we need $0 \leq \tilde{z} \leq 1$ or:

$$|p_{1-}p_0| < 4t$$

which remain maintained assumptions.

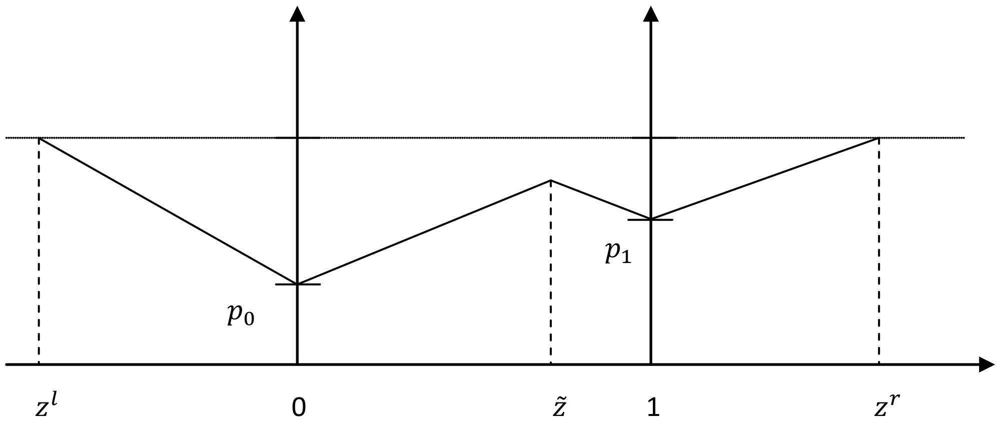

Fig 1. Consumer distribution and facilities' catchment areas.

Inverting the demand system (2) in $(p_{0,} p_1)$ we obtain the inverse demand functions faced by carriers at each airport:

$$p_o = U + 2t - 3tQ_o - tQ_1$$
$$p_1 = U + 2t - 3tQ_1 - tQ_0 \quad (3)$$

Hence, in the output market the demands depend on both Q_0 and Q_1: each carrier faces direct competition from the others carriers at the same airport and indirect competition from the airlines in the other one. To save notations we shall, in what follows, simply use $p_0$and p_1, for $p_0(Q_0, Q_1)$and $p_1(Q_0, Q_1)$ respectively. Given the structure of the downstream market, the total demand for flight at facility h can be rewritten as:

$$Q_h = q_L^h + \sum_{i=1}^{n-1} q_i^h \quad (4)$$

where q_L^h is the demand for flights faced by the leader and q_i^h is the demand for flights faced by the *i-th* follower, with $i = 1, \dots, n-1$ and $h = 0,1$. In considering the choices of carriers at facility h, we shall use q_L^{-h} and q_i^{-h} to indicate the demand for flights faced by the leader and the *i-th* follower, respectively, at the other facility.

In order to analyse the effects in terms of welfare and consumer surplus we specify the two functions. Given the uniformity and unit density of consumers, (see Fig. 1), the consumers' surplus is given by:

$$CS = \int_0^{|z^l|} [U - p_0 - 4tz]dz + \int_0^{\tilde{z}} [U - p_0 - 4tz]dz + \int_0^{1-\tilde{z}} [U - p_1 - 4tz]dz + \int_0^{z^r - 1} [U - p_1 - 4tz]dz$$

Using (3) to replace p_0 and p_1 both in the integrands and in z^l, z^r and $\tilde{z}$, and solving the integrals we get:

$$CS = \frac{(-4 + 3Q_0^2 + 2Q_0Q_1 + 3Q_1^2)t}{2} \tag{5}$$

With this specification, the welfare function is given by:

$$W = CS + \sum_{h=0}^{1} \pi_A^h + \sum_{h=0}^{1} \pi_L^h + \sum_{i=1}^{n-1} \pi_i^h \tag{6}$$

We find the subgame perfect Nash equilibria using the software Mathematica (Wolfram Research Inc., 2007), to analyse the incentives for vertical contracts and the effects in terms of welfare, consumer surplus and pro-competitiveness.

3. Analysis of the different types of vertical agreements

In this section we analyse both the symmetric cases and the asymmetric cases, according to the different choices of the two airport – dominant airline systems. In section 3.1 we analyse the symmetric cases, that is, the choice of the airport and its respective leader airline at facility 0 is the same of that at facility 1. We refers to these cases with the wording “two sided”. In section 3.2 we specify the asymmetric cases, that is, the choice of the airport and its respective dominant airline at facility 0 is different from that at facility 1.

3.1 Symmetric cases

In section 3.1.1 we specify the basic case in which no agreement occurs in both the two facilities; then, in sections 3.1.2, 3.1.3 and 3.1.4, we analyse the cases in which at each facility the airport and the respective dominant airline both sign the same type of contract.

3.1.1 *“Two sided No-Agreement”.* The airport and the leader airline do not sign any type of contract. Both the leader and the followers will pay the facility charge T_h, an input price, at facility *h*. Each follower competes in quantities with the followers at the same airport and with the followers at the other airport. The profit function for follower *i* at facility *h*, can be written as:

$$\pi_i^h = (p_h - T_h)q_i^h \tag{7}$$

for $h = 0,1$ and $i = 1, \dots, n-1$, where p_h is given by (3) and (4). The equilibrium is characterized by $2(n-1)$ first-order conditions[5]:

$$\frac{\delta \pi_i^h}{\delta q_i^h} = U + 2t - 3tq_L^h - tq_L^{-h} - 3t(n-2)q_i^h - 6tq_i^h - t(n-1)q_i^{-h} - T_h = 0$$

We derive the best reply functions (BRF) of the followers, i.e. $q_i^h(q_L^h, q_L^{-h}, T_h, T_{-h})$. The leader L_0 competes in quantities with the leader L_1. They maximize simultaneously their profit:

$$\pi_L^h = (p_h - T_h)q_L^h \tag{8}$$

for $h = 0,1$, where p_h, again, is given by (3) and (4). Substituting the followers' BRF into (4) and solving the 2-first order conditions system, we derive $q_L^h(T_0, T_1)$. and so the quantities $q_i^h(T_0, T_1)$ of the followers. In the first stage, the airports compete choosing the input prices, T_h. The profit function for airport h can be written as:

$$\pi_A^h = (T_h - c)Q_h - F_h \tag{9}$$

for $h = 0,1$, where Q_h is given by (4). Substituting $q_i^h(T_0, T_1)$ and $q_L^h(T_0, T_1)$ into (9), we solve the 2-first order conditions system finding solutions for all variables.

Analytical results for facility $h = 0, 1$ and i = 1, …, n-1 are shown in the appendix in section A.1.1, as a function of parameters depending on *n*, so on the number of followers in the downstream market. The parameters are defined in section A.2.1 and the superscript NA will be used to denote them.

3.1.2 "Two sided Vertical Collusion". At each facility, the airport and the leader airline negotiate the aeronautical fare $T_{L,h}$ depending on their bargaining power: the two partners maximize their joint profits and both of them, through the negotiation, obtain the highest joint profit so that the outcome is the same of a vertical merger[6]. The other *n-1* followers will pay the facility charge T_h, with $T_{L,h} < T_h$. Furthermore, we assume that there are no transaction costs of colluding.

Given the structure of the downstream market, the total demand for flight at facility h can be rewritten now in the form:

$$Q_h = q_C^h + \sum_{i=1}^{n-1} q_i^h \tag{10}$$

where q_C^h is the demand for flights faced by the colluded firm and q_i^h is the demand for flights faced by the *i-th* follower, with $i = 1, \dots, n-1$ and $h = 0,1$.

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5 As costs are identical $q_i^h = q_j^h$

6 For our purposes, it does not matter which will be the negotiated fare $T_{L,h}$. The market solution for $T_{L,h}$ depends on the bargaining power of each contracting party, thus, within our framework, it is impossible to know if either the airport or the leader airline alone has an incentive for collusion: the only possibility is to consider the incentive of the two partners together.

Each follower competes in quantities with the followers at the same airport and with the followers at the other airport. The profit function for follower *i* at facility *h*, can be written as:

$$\pi_i^h = (p_h - T_h)q_i^h \tag{11}$$

for $h = 0{,}1$ and $i = 1, \dots, n-1$, where p_h is given now by (3) and (10). The equilibrium is characterized by $2(n-1)$ first-order conditions:

$$\frac{\delta \pi_i^h}{\delta q_i^h} = U + 2t - 3tq_C^h - tq_C^{-h} - 3t(n-2)q_i^h - 6tq_i^h - t(n-1)q_i^{-h} - T_h = 0$$

We derive the best reply functions (BRF) of the followers, i.e. $q_i^h(q_L^h, q_L^{-h}, T_h, T_{-h})$. The colluded firm at facility 0 compete with the colluded firm at facility 1; they choose q_C^h and T_h maximizing simultaneously their profit:

$$\pi_C^h = (p_h - c)q_C^h + (T_h - c)(n-1)q_i^h - F_h \tag{12}$$

for $h = 0{,}1$, where p_h, again, is given by (3) and (10). Substituting the followers' BRF into (12) and solving the 2-first order conditions system, we find solutions for all variables.

Analytical results for facility $h = 0, 1$ and $i = 1, \dots, n-1$ are shown in the appendix in section A.1.2, as a function of parameters depending on *n*, so on the number of followers in the downstream market. The parameters are defined in section A.2.2 and the superscript C is used to denote them.

3.1.3 "Two sided Airlines in the Upstream Market". The airport *h* operates the runways for all airlines, both the leader and the followers, at a price T_h^r; the leader airline operates and leases the terminal, using it at the marginal cost and selling it to the followers at a price T_h^t. Terminals have a constant marginal cost of *tm*, and runways a constant marginal cost of *r*. Furthermore, we assume that there are no transaction costs of signing this type of contract.

Each follower competes in quantities with the followers at the same airport and with the followers at the other airport. The profit function for follower *i* at facility *h*, can be written as:

$$\pi_i^h = (p_h - T_h^r - T_h^t)q_i^h \tag{13}$$

for $h = 0{,}1$ and $i = 1, \dots, n-1$, where p_h is given by (3) and (4). The equilibrium is characterized by $2(n-1)$ first-order conditions:

$$\frac{\delta \pi_i^h}{\delta q_i^h} = U + 2t - 3tq_L^h - tq_L^{-h} - 3t(n-2)q_i^h - 6tq_i^h - t(n-1)q_i^{-h} - T_h^r - T_h^t = 0$$

We derive the best reply functions (BRF) of the followers, i.e. $q_i^h(q_L^h, q_L^{-h}, T_h^r, T_h^t, T_{-h}^r, T_{-h}^t)$. The two leaders compete choosing q_L^h and T_h^t, the terminal charge. They maximize simultaneously their profit:

$$\pi_L^h = (p_h - T_h^r - tm)q_L^h + (T_h^t - tm)(n-1)q_i^h \tag{14}$$

for $h = 0{,}1$, where p_h, again, is given by (3) and (4). Substituting the followers' BRF into (14) and solving the 2-first order conditions system, we derive T_0^t, T_1^t, $q_L^h(T_0^r, T_1^r)$ and so the quantities $q_i^h(T_0^r, T_1^r)$ of the followers. In the first stage, the airports compete choosing the runways charge T_h^r. The profit function for airport *h* can be written as:

$$\pi_A^h = (T_h^r - r)Q_h - F_h \tag{15}$$

for $h = 0{,}1$, where Q_h is given by (4). Substituting $q_i^h(T_0^r, T_1^r)$ and $q_L^h(T_0^r, T_1^r)$ into (15), we solve the 2-first order conditions system finding solutions for all variables.

Results for facility $h = 0{,}1$ and $i = 1, \dots, n-1$ are shown in the appendix in section A.1.3, as a function of parameters depending on the number of followers in the downstream market. The parameters are defined in section A.2.3 and the superscript AUM will be used to denote them.

3.1.4 "Two sided Price Discrimination". The leader airline pays the airport the variable cost of its facility, *c*, plus a part *k*, which is agreed between the two partners, of its fixed costs. This situation depicts the case of a two-part tariff. The other $n-1$ followers will pay the facility charge T_h. Furthermore, we assume that there are no transaction costs of signing this type of contract.

With these specifications, each follower competes in quantities with the followers at the same airport and with the followers at the other airport. The profit function for follower *i* at facility *h*, can be written as:

$$\pi_i^h = (p_h - T_h)q_i^h \tag{16}$$

for $h = 0{,}1$ and $i = 1, \dots, n-1$, where p_h is given by (3) and (4). The equilibrium is characterized by $2(n-1)$ first-order conditions:

$$\frac{\delta \pi_i^h}{\delta q_i^h} = U + 2t - 3tq_L^h - tq_L^{-h} - 3t(n-2)q_i^h - 6tq_i^h - t(n-1)q_i^{-h} - T_h = 0$$

We derive the best reply functions (BRF), i.e. $q_i^h(q_L^h, q_L^{-h}, T_h, T_{-h})$. The leader L_0 competes in quantities with the leader L_1. They maximize simultaneously their profit:

$$\pi_L^h = (p_h - c)q_L^h - kF_h \tag{17}$$

for $h = 0{,}1$, where p_h, again, is given by (3) and (4). Substituting the followers' BRF into (17) and solving the 2-first order conditions system, we derive $q_L^h(T_0, T_1)$ and so the quantities $q_i^h(T_0, T_1)$ of the followers. In the first stage, the airports compete choosing the input prices, T_h. The profit function for airport *h* can be written as:

$$\pi_A^h = (T_h - c)(n-1)q_i^h - (1-k)F_h \tag{18}$$

for $h = 0{,}1$. Substituting $q_i^h(T_0, T_1)$ into (18), we solve the 2-first order conditions system finding solutions for all variables.

Analytical results for facility $h = 0{,}1$ and $i = 1, \dots, n-1$ are shown in the appendix in section A.1.4 as a function of parameters depending on the number of followers in the downstream market. The parameters are defined in section A.2.4 and the superscript PD will be used to denote them.

Two types of agreements are anti-competitive: (i) "Vertical Collusion", where the merger implies a downstream market foreclosures by making $p_h = T_h$, i.e., price-squeeze; (ii) "Airlines in the upstream market", where $q_i^h = 0$ as well and the followers are driven out of the market. With respect to the case of "Price Discrimination", the airport will never make $p_h = T_h$, or it would lose all revenues except kF_h, which only covers part of the fixed costs and is not relevant for the determination of T_h. Therefore, this type of contract does not foreclosure the downstream market.

With respect to the case of "Airlines in the upstream market", it is possible to find that, if the leader does not improve efficiency in the airport facilities it operates, the agreement may only be interesting for both partners if the leader airline pays a rent to the airport that compensates it for its losses: there is an interval in which values for this rent exist. Moreover, with respect to the case of "Price Discrimination", it is also possible to find that there are no incentives for airports and airlines to sign it: there is not a value for the rent the leader airline pays to the airport that is interesting for both parties.

With respect to the symmetric cases, in each scenario we find that the input charges increase with the marginal cost of the facilities, namely c in the cases of "No – Agreement", "Vertical Collusion" and "Airlines in the upstream market", or $(r + tm)$ in the case of "Airlines in the upstream market". Specifically, we find $T_h^C < T_h^{NA}$, for h = 0,1, that is at each facility the input charge in the case of "Two sided Vertical Collusion" is smaller than the input charge in the case of "Two sided No Agreement". Indeed, $Q_h^C \geq Q_h^{NA}$ and $p_h^C \leq p_h^{NA}$, because of the internalization of vertical externalities due to a double-marginalization effect; therefore, a smaller value for T_h^C is sufficient for the colluded firm to engage in price squeezing. For a similar reason, even in the case of "Two sided Price Discrimination" we find $T_h^{PD} \leq T_h^{NA}$: the internalization of vertical externalities occurs since the leader airline pays the airport a part kF_h of its fixed costs and the variable cost of its facility, i.e. two-part tariff.

Final prices for consumers, p_h, increase with the marginal cost of the facilities, as well as with the transportation cost t and the gross benefit U of consumers; the quantities of carriers, both the leader and the followers, decrease with the marginal cost of the facilities; moreover, they increase with the gross benefit U and decrease with the transportation cost t, when $c - U < 0$, i.e. when the consumers' willingness to pay is greater than the airport marginal cost.

Finally, with respect to the issue of airlines competition, we find an increase in the number of followers in the downstream market leads to a decrease in the equilibrium prices at each facility. Demands, measured by the total number of flights offered, increase at each facility as a consequence of the decreasing prices. Both consumer surplus and welfare increase with an increase in the number of followers: competitiveness in the airlines market has positive effects in social terms.

3.2 Asymmetric cases

In this section we specify six cases with respect to the choices of the airport and its dominant airline at facility 0: (i) "No Agreement" – "Vertical Collusion"; (ii) "No Agreement" – "Airlines in the upstream market"; (iii) "No Agreement" – "Price Discrimination"; (iv) "Vertical Collusion" –

“Airlines in the upstream market”; (v) “Vertical Collusion” – “Price Discrimination”; (vi) “Airlines in the upstream market” – “Price Discrimination”. Specifically, in each case, the first choice refers to the one of the airport and its dominant airline at facility 0; the second choice refers to the one at facility 1.

In each case, the profit functions of the airports, the dominant airline and the followers are defined in the previous sections, according to the different choices in the two facilities. Backward induction is used to find solutions for all variables, as in the previous section.

Analytical results are presented in the appendix in sections A.1.5 – A.1.10, as a function of parameters depending on the number of followers in the downstream market, defined in sections A.2.5 – A.2.10. Results with respect to the choices of the airport and its dominant airline at facility 1 are symmetric to those obtained with respect to the choices at facility 0. Superscripts NA, C, AUM and PD will be used to denote the parameters according to the different choices in the two facilities. Specifically, in each case, the first superscript refers to the one of the airport and its dominant airline at facility 0, the second to the one at facility 1.

4. The optimal strategies of airports and airlines

We find the Nash equilibrium of the game where airports and their dominant airline decide whether to enter into a contract and, if so, which one to engage in among the three different types of agreements analyzed in the previous section.

In the cases of “No – Agreement” (NA), “Airlines in the upstream market” (AUM) and “Price Discrimination” (PD) we consider the sum of the airport’s and leader airline’s profits and we compare it with the profit of the merged firm in the case of “Vertical Collusion” (C)[7]. With respect to “Airlines in the upstream market”, we suppose that $c = r + tm$, i.e., the leader airline does not improve efficiency in the airport it operates. Hence, a direct comparison of the profits obtained in all the cases is possible. In particular, we find that given n > 1, i.e., at least one follower is present in the downstream market, it is:

$$\pi_h(C, x_{-h}) > \pi_h(x_h, x_{-h}) \qquad \forall\, h = 0{,}1 \qquad \forall x_h \in X_h \qquad \forall x_{-h} \in X_{-h}$$

where $X_h = \{NA, C, AUM, PD\}$ is the action set of the player h, namely the airport – dominant airline

[7] Such a framework implies a perfect alignment between the interests of the two agents, namely the airport and the dominant airline in each facility. This is the case, because we assume there are no transaction costs of colluding or signing any other type of contract. Clearly, if the agents are subject to transaction costs, if they can benefit from informational advantages, or if there are situations in which irreversible investments must be made, then it is reasonable to expect that a perfect alignment between the interest of the two parts does not occur and the equilibrium of the game may change: a contract economics approach would be more suitable to evaluate if each part alone has an incentive for vertical collusion.

system in the facility h, and $\pi_h(x_h, x_{-h})$ is the payoff [8] of the system h when its choice is x_h and the choice of the other system is x_{-h} .

Therefore, an iterated dominant strategy equilibrium exists, $s^* = (C, C)$, that is, in each facility the airport and the dominant airline have incentive to collude. The result can be summarized as follows:

In the context of an infinite linear city and two competing facilities, if both the two airport-leader airline systems share the same market and anticipate that her rival plays the best strategy, both of them have incentive to collude, given at least a follower is present in the downstream market.

The equilibrium input charges, quantities, final prices, payoffs, consumer surplus and welfare are:

$$T_h = \frac{12nc + (1+8n)(2t+U)}{1+20n} \qquad p_h = \frac{12nc + (1+8n)(2t+U)}{1+20n}$$

$$q_i^h = 0 \qquad q_h^C = \frac{3n(2t+U-c)}{(1+20n)t}$$

$$\pi_i^h = 0 \qquad \pi_h^C = \frac{3n(1+8n)(2t+U-c)^2}{(1+20n)^2 t} - F_h$$

$$CS = \frac{1}{2}t\left(-4 + \frac{72n^2(2t+U-c)^2}{(1+20n)^2t^2}\right) \qquad W = \frac{(6n+84n^2)(2t+U-c)^2}{(1+20n)^2 t} - 2t - F_0 - F_1$$

The results hold under our maintained assumptions, that is $2U \geq p_0 + p_1 + 4t$, i.e. everyone in the [0,1] interval decides to fly, and $|p_1 - p_0| < 4t$, i.e. both airports receive consumers from [0,1]. Substituting the equilibrium final prices, we derive[9]:

$$U - c - 5t \geq 0$$

The result differs from the findings of Barbot (2009a), where there are no incentives for collusive agreements when both pairs of firms share the same market. Our results depend on the hypothesis that in the model, i.e. infinite linear city, there would be some consumers for whom the sum of the flight's price plus the total transportation costs would exceed their reservation price: in other words, we do not assume that the market is covered, or that $Q_0 + Q_1 = 1$, as in the usual Hotelling address model. In the case of a one sided vertical collusion, i.e. when only a pair of firms decide to engage in vertical collusion, the colluded firm's demand, Q_1 (or Q_2) increase by a larger amount and the left-alone firm's demand, Q_2 (or Q_1) also increase, depending on the price elasticities of demands. The same applies to

[8] Equal to the colluded firm's profit when "Vertical Collusion" is signed, while to the sum of the airport's profit and the dominant airline's profit when no agreement or any other type of agreement is signed by the two partners.

[9] We obtain $U - c - \frac{1+14n}{3n}t \geq 0$: given $-\frac{1+14n}{3n}$ is an increasing function of n, with $n>0$, if the relationships is satisfied for n=1, then it is $\forall$ $n>0$.

the case of a "Two sided Vertical Collusion", with both merged firms disputing in identical conditions the demand from the consumers that did not fly before the collusion.

With respect to the consumer surplus and to the social welfare, given n > 1 we find that:

$$\frac{{\delta^{PD}}^2}{2{\theta^{PD}}^2} > \frac{{\delta^{C}}^2}{{\theta^{C}}^2} \qquad \text{and} \qquad \frac{\phi^{PD}}{{\theta^{PD}}^2} > \frac{\phi^{C}}{{\theta^{C}}^2}$$

which allow to conclude that:

$CS^{PD} > CS^{C}$ and $W^{PD} > W^{C}$

Hence, the Nash equilibrium is not efficient in social terms: indeed consumer surplus and social welfare are maximized at $s' = (PD, PD)$, namely in the case of "Two sided Price Discrimination". Indeed, as we noted previously, internalization of vertical externalities occurs since the leader airline pays the airport a part kF_h of its fixed costs and the variable cost of its facility, i.e. two-part tariff. Nevertheless, the result of the "Two sided Vertical Collusion" case is not perfectly repeated here: in the case of "Two sided Price Discrimination", the airport will never make $p_h = T_h$, or it would lose all revenues except kF_h, which only covers part of the fixed costs. Therefore, this type of contract does not foreclosure the downstream market, i.e. $q_i^h > 0$ and $Q_h^{PD} \geq Q_h^C$ or $p_h^{PD} \leq p_h^C$.

However, there are no incentives for airports and airlines to sign it; therefore there is a misalignment between private and social incentives.

5. Concluding remarks

In this paper, vertical contracts between airports and airlines in the context of two competing facilities and three different types of agreements have been considered. Specifically, we have developed a multistage facility-rivalry game and we have investigated the Nash equilibrium to analyse the incentives for vertical contracts and the effects in terms of welfare, consumer surplus and pro-competitiveness. The paper adds to existing literature as it considers the issue of vertical contracts both in the airport competition and airline competition: indeed, we have analysed the case of a leader and *n-1* followers at each facility. Moreover, airports do not compete for the airlines but compete through airlines to get passengers.

With respect to the issue of airlines competition, results show that with an increase in the number of followers in the downstream market there is a decrease in the equilibrium prices at each facility. The total number of flights offered increase at each facility as a consequence of the decreasing prices. Both consumer surplus and welfare increase with an increase in the number of followers: competitiveness in the airlines market has positive effects in social terms. With respect to the issue of airports competition, we have found that the airport and the dominant airline at each facility may have incentive to collude. The result differs from the findings of Barbot (2009a), where there are no incentives for collusive agreements when both pairs of firms share the same market. Our findings depend on the hypothesis that in the model, i.e., infinite linear city, there would be some consumers for whom the sum of the flight's price plus the total transportation costs would exceed their reservation price: in other words, we do not assume that the market is covered.

The results raise some policy issues and avenues for future research. In particular, the merger implies a downstream market foreclosure through a price-squeeze strategy: the follower airlines are driven out of the market and the equilibrium is anti-competitive. On the other hand, consumers' surplus and welfare increase with respect to the case in which no agreement occurs: indeed final quantities increase and final prices for consumers decrease because of the internalization of vertical externalities due to a double-marginalization effect. Therefore, the agreement exhibits a trade-off between competitiveness and welfare. In addition, the equilibrium is not efficient in social terms: consumer surplus and social welfare, though increasing with respect to the case in which no-agreement occurs, are maximized in the case of another type of agreement, that is "Price Discrimination". The problem is that there are no incentives for airports and airlines to sign it: therefore there is a misalignment between private and social incentive.

In this sense, the problem of vertical relations constitutes a fundamental issue because of the ensuing regulatory requirements and further developments of the present work may go along two directions within the scope of policy implications: on one hand, how regulation might balance the trade-off raised by the vertical collusive agreement, by giving room for the merger, so leaving consumers better-off, but not for market foreclosure; on the other hand, how regulation could provide incentives, both to airports and dominant airline, for other types of agreements, namely those that maximize social welfare, (i.e. "Price Discrimination" in this framework).

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Appendix A

A.1 Solutions for variables

A.1.1 "Two sided No Agreement"

$$T_h = \frac{\alpha^{NA}c + \beta^{NA}(2t+U)}{\gamma^{NA}} \qquad p_h = \frac{\delta^{NA}c + (\theta^{NA} - \delta^{NA})(2t+U)}{\theta^{NA}}$$

$$q_L^h = \frac{3\varphi^{NA}(2t+U-c)}{\theta^{NA}t} \qquad q_i^h = \frac{3\sigma^{NA}(2t+U-c)}{\theta^{NA}t}$$

$$\pi_i^h = \frac{27{\sigma^{NA}}^2(2t+U-c)^2}{{\theta^{NA}}^2 t} \qquad \pi_L^h = \frac{\varepsilon^{NA}(2t+U-c)^2}{{\theta^{NA}}^2 t} \qquad \pi_A^h = \frac{\eta^{NA}(2t+U-c)^2}{{\theta^{NA}}^2 t} - F_h$$

$$CS = \frac{1}{2}t\left(-4 + \frac{{\delta^{NA}}^2(2t+U-c)^2}{2{\theta^{NA}}^2 t^2}\right) \qquad W = \frac{\phi^{NA}(2t+U-c)^2}{{\theta^{NA}}^2 t} - 2t - F_0 - F_1$$

A.1.2 "Two sided Vertical Collusion"

$$T_h = \frac{\alpha^C c \; + \beta^C(2t+U)}{\theta^C} \qquad p_h = \frac{\alpha^C c \; + \beta^C(2t+U)}{\theta^C}$$

$$q_i^h = 0 \qquad q_C^h = \frac{3\varphi^C(2t+U-c)}{\theta^C t}$$

$$\pi_i^h = 0 \qquad \pi_C^h = \frac{\eta^C(2t+U-c)^2}{{\theta^C}^2 t} - F_h$$

$$CS = \frac{1}{2}t\left(-4 + \frac{{\delta^C}^2(2t+U-c)^2}{{\theta^C}^2 t^2}\right) \qquad W = \frac{\phi^C(2t+U-c)^2}{{\theta^C}^2 t} - 2t - F_0 - F_1$$

A.1.3 "Two sided Airlines in the upstream market"

$$T_h^r = \frac{\alpha_r^{AUM} r + \beta_r^{AUM}(2t+U-tm)}{\gamma^{AUM}} \qquad T_h^t = \frac{\alpha_t^{AUM} tm + \beta_t^{AUM}(2t+U-r)}{\theta^{AUM}}$$

$$p_h = \frac{\delta^{AUM}(r+tm) + (\theta^{AUM} - \delta^{AUM})(2t+U)}{\theta^{AUM}}$$

$$q_i^h = 0 \qquad q_L^h = \frac{3\varphi^{AUM}(2t+U-r-tm)}{\theta^{AUM}t}$$

$$\pi_i^h = 0 \quad \pi_L^h = \frac{\varepsilon^{AUM}(2t+U-r-tm)^2}{{\theta^{AUM}}^2 t} \qquad \pi_A^h = \frac{\eta^{AUM}(2t+U-r-tm)^2}{{\theta^{AUM}}^2 t} - F_h$$

$$CS = \frac{1}{2}t\left(-4 + \frac{{\delta^{AUM}}^2(2t+U-r-tm)^2}{2{\theta^{AUM}}^2 t^2}\right) \qquad W = \frac{\phi^{AUM}(2t+U-r-tm)^2}{{\theta^{AUM}}^2 t} - 2t - F_0 - F_1$$

A.1.4 "Two sided Price Discrimination"

$$T_h = \frac{\alpha^{PD} c + \beta^{PD}(2t+U)}{\gamma^{PD}} \qquad p_h = \frac{\delta^{PD} c + (\theta^{PD} - \delta^{PD})(2t+U)}{\theta^{PD}}$$

$$q_L^h = \frac{3\varphi^{PD}(2t+U-c)}{\theta^{PD} t} \qquad q_i^h = \frac{\sigma^{PD}(2t+U-c)}{\theta^{PD} t}$$

$$\pi_i^h = \frac{3{\sigma^{PD}}^2(2t+U-c)^2}{{\theta^{PD}}^2 t} \qquad \pi_L^h = \frac{\varepsilon^{PD}(2t+U-c)^2}{{\theta^{PD}}^2 t} \qquad \pi_A^h = \frac{\eta^{PD}(2t+U-c)^2}{{\theta^{PD}}^2 t} - F_h$$

$$CS = \frac{1}{2} t \left(-4 + \frac{{\delta^{PD}}^2(2t+U-c)^2}{2{\theta^{PD}}^2 t^2} \right) \qquad W = \frac{\phi^{PD}(2t+U-c)^2}{{\theta^{PD}}^2 t} - 2t - F_0 - F_1$$

A.1.5 "No Agreement" – "Vertical Collusion"

$$T_0 = \frac{\alpha^{NA,C} c + \beta^{NA,C}(2t+U)}{\gamma^{NA,C}} \qquad T_1 = \frac{\delta_1^{NA,C} c + \rho_1^{NA,C}(2t+U)}{\theta^{NA,C}}$$

$$p_0 = \frac{\delta_0^{NA,C} c + \rho_0^{NA,C}(2t+U)}{\theta^{NA,C}} \qquad p_1 = \frac{\delta_1^{NA,C} c + \rho_1^{NA,C}(2t+U)}{\theta^{NA,C}}$$

$$q_i^0 = \frac{\sigma^{NA,C}(2t+U-c)}{\theta^{NA,C} t} \qquad q_i^1 = 0$$

$$q_L^0 = \frac{\varphi^{NA,C}(2t+U-c)}{\theta^{NA,C} t} \qquad q_{VM}^1 = \frac{\lambda^{NA,C}(2t+U-c)}{\theta^{NA,C} t}$$

$$\pi_i^0 = \frac{3{\sigma^{NA,C}}^2(2t+U-c)^2}{{\theta^{NA,C}}^2 t} \qquad \pi_i^1 = 0$$

$$\pi_L^0 = \frac{\varepsilon^{NA,C}(2t+U-c)^2}{{\theta^{NA,C}}^2 t}$$

$$\pi_A^0 = \frac{\eta^{NA,C}(2t+U-c)^2}{{\theta^{NA,C}}^2 t} - F_0 \qquad \pi_{VM}^1 = \frac{\mu^{NA,C}(2t+U-c)^2}{{\theta^{NA,C}}^2 t} - F_1$$

$$CS = \frac{1}{2} t \left(-4 + \frac{\xi^{NA,C}(2t+U-c)^2}{{\theta^{NA}}^2 t^2} \right) \qquad W = \frac{\phi^{NA,C}(2t+U-c)^2}{{\theta^{NA,C}}^2 t} - 2t - F_0 - F_1$$

A.1.6 "No Agreement" – "Airlines in the upstream market" [10]

$$T_0 = \frac{\alpha^{NA,AUM} c + \beta^{NA,AUM}(2t+U) + \tilde{\alpha}^{NA,AUM}(r+tm)}{\gamma^{NA,AUM}}$$

$$T_1^r = \frac{\tilde{\alpha}_r^{NA,AUM} c + \tilde{\beta}_r^{NA,AUM}(2t+U) + \hat{\alpha}_r^{NA,AUM} r + \hat{\beta}_r^{NA,AUM} tm}{3\gamma^{NA,AUM}}$$

$$T_1^t = \frac{\tilde{\alpha}_t^{NA,AUM} c + \tilde{\beta}_t^{NA,AUM}(2t+U) + \hat{\alpha}_t^{NA,AUM} r + \hat{\beta}_t^{NA,AUM} tm}{3\theta^{NA,AUM}}$$

$$p_0 = \frac{\delta_0^{NA,AUM} c + \tilde{\delta}_0^{NA,AUM}(r+tm) + \rho_0^{NA,AUM}(U+2t)}{\theta^{NA,AUM}}$$

[10] Closed form solutions for consumer surplus and welfare are available from the author by request.

$$p_1 = \frac{\delta_1^{NA,AUM} c + \tilde{\delta}_1^{NA,AUM}(r + tm) + \rho_1^{NA,AUM}(U + 2t)}{3\theta^{NA,AUM}}$$

$$q_i^0 = \frac{\sigma_0^{NA,AUM} c + \tilde{\sigma}_0^{NA,AUM}(r + tm) + \hat{\sigma}_0^{NA,AUM}(U + 2t)}{3\theta^{NA,AUM}}$$

$$q_i^1 = 0$$

$$q_L^0 = \frac{\varphi_0^{NA,AUM} c + \tilde{\varphi}_0^{NA,AUM}(r + tm) + \hat{\varphi}_0^{NA,AUM}(U + 2t)}{\theta^{NA,AUM}}$$

$$q_L^1 = \frac{\varphi_1^{NA,AUM} c + \tilde{\varphi}_1^{NA,AUM}(2t + U) + \hat{\varphi}_{0,r}^{NA,AUM} r + \hat{\varphi}_{0,t}^{NA,AUM} tm}{\theta^{NA,AUM}}$$

$$\pi_i^0 = \frac{\left(\zeta_0^{NA,AUM} c + \tilde{\zeta}_0^{NA,AUM}(r + tm) + \hat{\zeta}_0^{NA,AUM}(U + 2t)\right)^2}{3\theta^{NA,AUM^2} t}$$

$$\pi_i^1 = 0$$

$$\pi_L^0 = \frac{\left(\varepsilon_0^{NA,AUM} c + \tilde{\varepsilon}_0^{NA,AUM}(r + tm) + \hat{\varepsilon}_0^{NA,AUM}(U + 2t)\right)^2}{3\theta^{NA,AUM^2} t}$$

$$\pi_L^1 = \frac{\left(\varepsilon_1^{NA,AUM} c + \tilde{\varepsilon}_1^{NA,AUM}(2t + U) + \hat{\varepsilon}_{0,r}^{NA,AUM} r + \hat{\varepsilon}_{0,t}^{NA,AUM} tm\right)^2}{3\theta^{NA,AUM^2} t}$$

$$\pi_A^0 = \frac{\left(\eta_0^{NA,AUM} c + \tilde{\eta}_0^{NA,AUM}(r + tm) + \hat{\eta}_0^{NA,AUM}(U + 2t)\right)^2}{3\theta^{NA,AUM^2} t} - F_0$$

$$\pi_A^1 = \frac{\left(\eta_1^{NA,AUM} c + \tilde{\eta}_1^{NA,AUM}(r + tm) + \hat{\eta}_1^{NA,AUM}(U + 2t)\right)^2}{3\theta^{NA,AUM^2} t} - F_1$$

A.1.7 "No Agreement" – "Price Discrimination"

$$T_0 = \frac{\alpha_0^{NA,PD} c + \beta_0^{NA,PD}(2t + U)}{3\gamma^{NA,PD}} \qquad T_1 = \frac{\alpha_1^{NA,PD} c + \beta_1^{NA,PD}(2t + U)}{\gamma^{NA,PD}}$$

$$p_0 = \frac{\delta_0^{NA,PD} c + \rho_0^{NA,PD}(2t + U)}{\theta^{NA,PD}} \qquad p_1 = \frac{\delta_1^{NA,PD} c + \rho_1^{NA,PD}(2t + U)}{\theta^{NA,PD}}$$

$$q_i^0 = \frac{\sigma_0^{NA,PD}(2t + U - c)}{\theta^{NA,PD} t} \qquad q_i^1 = \frac{\sigma_1^{NA,PD}(2t + U - c)}{3\theta^{NA,PD} t}$$

$$q_L^0 = \frac{\varphi_0^{NA,PD}(2t + U - c)}{\theta^{NA,PD} t} \qquad q_L^1 = \frac{\varphi_1^{NA,PD}(2t + U - c)}{\theta^{NA,PD} t}$$

$$\pi_i^0 = \frac{\sigma_0^{NA,PD^2}(2t + U - c)^2}{\theta^{NA,PD^2} t} \qquad \pi_i^1 = \frac{\sigma_1^{NA,PD^2}(2t + U - c)^2}{\theta^{NA,PD^2} t}$$

$$\pi_L^0 = \frac{\varepsilon_0^{NA,PD}(2t + U - c)^2}{\theta^{NA,PD^2} t} \qquad \pi_L^1 = \frac{\varepsilon_1^{NA,PD}(2t + U - c)^2}{\theta^{NA,PD^2} t} - kF_1$$

$$\pi_A^0 = \frac{\eta_0^{NA,PD}(2t + U - c)^2}{\theta^{NA,PD} t} - F_0 \qquad \pi_A^1 = \frac{\eta_0^{NA,PD}(2t + U - c)^2}{\theta^{NA,PD} t} - (1 - k)F_1$$

$$CS = \frac{1}{2} t \left(-4 + \frac{\xi^{NA,PD}(2t + U - c)^2}{\theta^{NA,PD^2} t^2}\right) \qquad W = \frac{\phi^{NA,PD}(2t + U - c)^2}{\theta^{NA,PD^2} t} - 2t - F_0 - F_1$$

A.1.8 "Vertical Collusion" – "Airlines in the upstream market" [11]

$$T_0 = \frac{\alpha^{C,AUM}c + \beta^{C,AUM}(2t+U) + \tilde{\alpha}^{C,AUM}(r+tm)}{\theta^{C,AUM}}$$

$$T_1^r = \frac{\tilde{\alpha}_r^{C,AUM}c + \tilde{\beta}_r^{C,AUM}(2t+U) + \hat{\alpha}_r^{C,AUM}r + \hat{\beta}_r^{C,AUM}tm}{\gamma^{C,AUM}}$$

$$T_1^t = \frac{\tilde{\alpha}_t^{C,AUM}c + \tilde{\beta}_t^{C,AUM}(2t+U) + \hat{\alpha}_t^{C,AUM}r + \hat{\beta}_t^{C,AUM}tm}{\theta^{C,AUM}}$$

$$p_0 = \frac{\delta_0^{C,AUM}c + \tilde{\delta}_0^{C,AUM}(r+tm) + \rho_0^{C,AUM}(U+2t)}{\theta^{C,AUM}}$$

$$p_1 = \frac{\delta_1^{C,AUM}c + \tilde{\delta}_1^{C,AUM}(r+tm) + \rho_1^{C,AUM}(U+2t)}{\theta^{C,AUM}}$$

$$q_i^0 = 0$$

$$q_i^1 = 0$$

$$q_{VM}^0 = \frac{\varphi_0^{C,AUM}c + \tilde{\varphi}_0^{C,AUM}(r+tm) + \hat{\varphi}_0^{C,AUM}(U+2t)}{\theta^{C,AUM}}$$

$$q_L^1 = \frac{\varphi_1^{C,AUM}c + \tilde{\varphi}_1^{C,AUM}(r+tm) + \hat{\varphi}_1^{C,AUM}(2t+U)}{\theta^{C,AUM}}$$

$$\pi_i^0 = 0$$

$$\pi_i^1 = 0$$

$$\pi_{VM}^0 = \frac{\left(\varepsilon_0^{C,AUM}c + \tilde{\varepsilon}_0^{C,AUM}(r+tm) + \hat{\varepsilon}_0^{C,AUM}(U+2t)\right)^2}{\theta^{C,AUM^2}t} - F_0$$

$$\pi_L^1 = \frac{\left(\varepsilon_1^{C,AUM}c + \tilde{\varepsilon}_1^{C,AUM}(r+tm) + \hat{\varepsilon}_1^{C,AUM}(2t+U)\right)^2}{\theta^{C,AUM^2}t}$$

$$\pi_A^1 = \frac{\left(\eta_1^{C,AUM}c + \tilde{\eta}_1^{C,AUM}(r+tm) + \hat{\eta}_1^{C,AUM}(U+2t)\right)^2}{\theta^{C,AUM^2}t} - F_1$$

A.1.9 "Vertical Collusion" – "Price Discrimination"

$$T_0 = \frac{\alpha_0^{C,PD}c + \beta_0^{C,PD}(2t+U)}{\theta^{C,PD}} \qquad T_1 = \frac{\alpha_1^{C,PD}c + \beta_1^{C,PD}(2t+U)}{\gamma^{C,PD}}$$

$$p_0 = \frac{\alpha_0^{C,PD}c + \beta_0^{C,PD}(2t+U)}{\theta^{C,PD}} \qquad p_1 = \frac{\delta^{C,PD}c + \rho^{C,PD}(2t+U)}{\theta^{C,PD}}$$

$$q_i^0 = 0 \qquad q_i^1 = \frac{\sigma^{C,PD}(2t+U-c)}{\theta^{C,PD}t}$$

$$q_{VM}^0 = \frac{\lambda^{C,PD}(2t+U-c)}{\theta^{C,PD}t} \qquad q_L^1 = \frac{\varphi^{C,PD}(2t+U-c)}{\theta^{C,PD}t}$$

$$\pi_i^0 = 0 \qquad \pi_i^1 = \frac{3\sigma^{C,PD^2}(2t+U-c)^2}{\theta^{C,PD^2}t}$$

[11] Closed form solutions for consumer surplus and welfare are available from the author by request.

$$\pi_L^1 = \frac{\varepsilon^{C,PD}(2t+U-c)^2}{\theta^{C,PD^2}t} - kF_1$$

$$\pi_A^1 = \frac{\eta^{C,PD}(2t+U-c)^2}{\theta^{C,PD}t} - (1-k)F_1 \qquad \pi_{VM}^0 = \frac{\mu^{C,PD}(2t+U-c)^2}{\theta^{C,PD^2}t} - F_0$$

$$CS = \frac{1}{2}t\left(-4 + \frac{\xi^{C,PD}(2t+U-c)^2}{\theta^{C,PD^2}t^2}\right) \qquad W = \frac{\phi^{C,PD}(2t+U-c)^2}{\theta^{C,PD^2}t} - 2t - F_0 - F_1$$

A.1.10 "Airlines in the upstream market" – "Price Discrimination" [12]

$$T_0^r = \frac{\tilde{\alpha}_r^{AUM,PD}c + \tilde{\beta}_r^{AUM,PD}(2t+U) + \hat{\alpha}_r^{AUM,PD}r + \hat{\beta}_r^{AUM,PD}tm}{\gamma_r^{AUM,PD}}$$

$$T_0^t = \frac{\tilde{\alpha}_t^{AUM,PD}c + \tilde{\beta}_t^{AUM,PD}(2t+U) + \hat{\alpha}_t^{AUM,PD}r + \hat{\beta}_t^{AUM,PD}tm}{3\gamma_t^{AUM,PD}}$$

$$T_1 = \frac{\alpha^{AUM,PD}c + \beta^{AUM,PD}(2t+U) + \tilde{\alpha}^{AUM,PD}(r+tm)}{\gamma^{AUM,PD}}$$

$$p_0 = \frac{\delta_0^{AUM,PD}c + \tilde{\delta}_0^{AUM,PD}(r+tm) + \rho_0^{AUM,PD}(U+2t)}{\theta^{AUM,PD}}$$

$$p_1 = \frac{\delta_1^{AUM,PD}c + \tilde{\delta}_1^{AUM,PD}(r+tm) + \rho_1^{AUM,PD}(U+2t)}{\theta^{NA,AUM}}$$

$$q_i^0 = 0$$

$$q_i^1 = \frac{\sigma_1^{AUM,PD}c + \tilde{\sigma}_1^{AUM,PD}(r+tm) + \hat{\sigma}_1^{AUM,PD}(U+2t)}{\theta^{AUM,PD}}$$

$$q_L^0 = \frac{\varphi_0^{AUM,PD}c + \tilde{\varphi}_0^{AUM,PD}(r+tm) + \hat{\varphi}_0^{AUM,PD}(U+2t)}{\theta^{AUM,PD}}$$

$$q_L^1 = \frac{\varphi_1^{AUM,PD}c + \tilde{\varphi}_1^{AUM,PD}(r+tm) + \hat{\varphi}_1^{AUM,PD}(2t+U)}{\theta^{NA,AUM}}$$

$$\pi_i^0 = 0$$

$$\pi_i^1 = \frac{\left(\zeta_1^{AUM,PD}c + \tilde{\zeta}_1^{AUM,PD}(r+tm) + \hat{\zeta}_1^{AUM,PD}(U+2t)\right)^2}{\theta^{AUM,PD^2}t}$$

$$\pi_L^0 = \frac{\left(\varepsilon_0^{AUM,PD}c + \tilde{\varepsilon}_0^{AUM,PD}(r+tm) + \hat{\varepsilon}_0^{AUM,PD}(U+2t)\right)^2}{\theta^{AUM,PD^2}t}$$

$$\pi_L^1 = \frac{\left(\varepsilon_1^{AUM,PD}c + \tilde{\varepsilon}_1^{AUM,PD}(r+tm) + \hat{\varepsilon}_1^{AUM,PD}(U+2t)\right)^2}{\theta^{AUM,PD^2}t} - kF_1$$

$$\pi_A^0 = \frac{\left(\eta_0^{AUM,PD}c + \tilde{\eta}_0^{AUM,PD}(r+tm) + \hat{\eta}_0^{AUM,PD}(U+2t)\right)^2}{\theta^{AUM,PD^2}t} - F_0$$

$$\pi_A^1 = \frac{\left(\eta_1^{AUM,PD}c + \tilde{\eta}_1^{AUM,PD}(r+tm) + \hat{\eta}_1^{AUM,PD}(U+2t)\right)^2}{\theta^{AUM,PD^2}t} - (1-k)F_1$$

[12] Closed form solutions for consumer surplus and welfare are available from the author by request.

A.2 Values of parameters

A.2.1 “Two sided No Agreement”

$\alpha^{NA} \coloneqq -3 + 9n + 48n^2$

$\beta^{NA} \coloneqq -1 + 14n + 32n^2$

$\gamma^{NA} \coloneqq -4 + 23n + 80n^2$

$\delta^{NA} \coloneqq 12(-2 - 5n + 16n^2)(-1 + 3n + 16n^2)$

$\theta^{NA} \coloneqq (-1 + 4n)(5 + 16n)(-4 + 23n + 80n^2)$

$\varphi^{NA} \coloneqq (-1 + 4n)(-1 + n + 22n^2 + 32n^3)$

$\sigma^{NA} \coloneqq (1 + 8n)(-1 + 3n + 16n^2)$

$\varepsilon^{NA} \coloneqq 27(1 + 2n)(-1 + 4n)(1 + 8n)(-1 + 3n + 16n^2)^2$

$\eta^{NA} \coloneqq 3(1 + 2n)(-1 + 4n)(5 + 16n)(-1 + 16n)(-2 - 5n + 16n^2)(-1 + 3n + 16n^2)$

$\phi^{NA} \coloneqq 2(n-1)27{\sigma^{NA}}^2 + 2\varepsilon^{NA} + 2\eta^{NA} + 1/4\,{\delta^{NA}}^2$

A.2.2 “Two sided Vertical Collusion”

$\alpha^{C} \coloneqq 12n$

$\beta^{C} \coloneqq 1 + 8n$

$\theta^{C} \coloneqq 1 + 20n$

$\varphi^{C} \coloneqq n$

$\eta^{C} \coloneqq 3n(1 + 8n)$

$\delta^{C} \coloneqq \sqrt{72}n$

$\phi^{C} \coloneqq 6n + 84n^2$

A.2.3 “Two sided Airlines in the upstream market”

$\alpha_r^{AUM} \coloneqq (1 + 17n)$

$\beta_r^{AUM} \coloneqq (1 + 14n)$

$\alpha_t^{AUM} \coloneqq 1 + 46n + 484n^2$

$\beta_t^{AUM} \coloneqq 1 + 25n + 136n^2$

$\gamma^{AUM} \coloneqq 2 + 31n$

$\delta^{AUM} \coloneqq 12n(1 + 17n)$

$\theta^{AUM} \coloneqq (2 + 31n)(1 + 20n)$

$\varphi^{AUM} \coloneqq n(1 + 17n)$

$\varepsilon^{AUM} \coloneqq 3n(1 + 8n)(1 + 17n)^2$

$\eta^{AUM} \coloneqq 3n(1 + 14n)(1 + 17n)(1 + 20n)$

$\phi^{AUM} \coloneqq 2\varepsilon^{AUM} + 2\eta^{AUM} + 1/4\,{\delta^{AUM}}^2$

A.2.4 “Two sided Price Discrimination”

$\alpha^{PD} \coloneqq 2 - 155n + 192n^2 + 2368n^3 + 2048n^4$

$\beta^{PD} \coloneqq 3(1+2n)(1+8n)(-1+16n)$

$\gamma^{PD} \coloneqq -1 - 137n + 624n^2 + 3136n^3 + 2048n^4$

$\delta^{PD} \coloneqq -4 + 1104n + 12n^2 - 31744n^3 - 31488n^4 + 135168n^5 + 131072n^6$

$\theta^{PD} \coloneqq (-1+4n)(5+16n)(-1 - 137n + 624n^2 + 3136n^3 + 2048n^4)$

$\varphi^{PD} \coloneqq (-1+4n)(1+2n)(-2 - 91n + 240n^2 + 1664n^3 + 1024n^4)$

$\sigma^{PD} \coloneqq (1+8n)(-2 - 91n + 240n^2 + 1664n^3 + 1024n^4)$

$\varepsilon^{PD} \coloneqq 27(1+2n)(-1+4n)(1+8n)\,(1 - 55n + 24n^2 + 896n^3 + 1024n^4)^2$

$$\eta^{PD} \coloneqq 3(-1+n)(1+2n)(-1+4n)(-1+16n)(5+16n)(1+8n)^2(-2 - 91n + 240n^2 + 1664n^3 + 1024n^4)$$

$\phi^{NA} \coloneqq 2(n-1)3{\sigma^{PD}}^2 + 2\varepsilon^{PD} + 2\eta^{PD} + 1/4\,{\delta^{PD}}^2$

A.2.5 “No Agreement” – “Vertical Collusion”

$\alpha^{NA,C} = 1 + 20n$

$\beta^{NA,C} = 1 + 14n$

$\gamma^{NA,C} = 2(1+17n)$

$\theta^{NA,C} = 3\,(-1 + 4\,n\,(-3 + 2\,n\,(9 + 32\,n)))$

$\delta_0^{NA,C} = -6 + 3n\left(-71 + 2n\left(-327\big(8n(-13+320n)\big)\right)\right)/(1+17n)$

$\rho_0^{NA,C} = n(1+14n)(13 + 208n + 256n^2)$

$\delta_1^{NA,C} = -2 + n\left(-103 + 2n\big(-483 + 16n(115 + 488n)\big)\right)$

$\rho_1^{NA,C} = (1+8n)\left(-4 + n\big(-39 + 2n(243 + 656n)\big)\right)$

$\sigma^{NA,C} = (1+8n)(1+14n)$

$\varphi^{NA,C} = (1+14n)(-1 + 2n + 8n^2)$

$\lambda^{NA,C} = 3n\left(-4 + n\big(-39 + 2n(243 + 656n)\big)\right)/(1+17n)$

$\varepsilon^{NA,C} = 3(1+8n)(1+14n)^2(-1 + 2n + 8n^2)$

$\eta^{NA,C} = (1+14n)^2(-2 - 5n + 16n^2)/12(1+17n)$

$\mu^{NA,C} = 3(1+8n)\left(-4 + n\big(-39 + 2n(243 + 656n)\big)\right)^2/4(1+17n)^2$

$$\xi^{NA,C} = 3\Big((1+17n)^2(1+14n)^2(-2 - 5n + 16n^2) + 2n(1+17n)(1+14n)(-2 - 5n + 16n^2)\left(-4 + n\big(-39 + 2n(243 + 656n)\big)\right) + 9n^2\left(-4 + n\big(-39 + 2n(243 + 656n)\big)\right)\Big)/(1+17n)^2$$

$\phi^{NA,C} = (n-1)\sigma^{NA,C} + \varepsilon^{NA,C} + \eta^{NA,C} + \mu^{NA,C} + (1/2)\xi^{NA,C}$

A.2.6 "No Agreement" – "Airlines in the upstream market"

$\alpha^{NA,AUM} = 2(1+17n)(-1+3n+16n^2)$

$\beta^{NA,AUM} = 8+23n+130n^2$

$\tilde{\alpha}^{NA,AUM} = 3n(-1+3n+16n^2)$

$\gamma^{NA,AUM} = -4+n(-54+n(273+1072n))$

$\tilde{\alpha}_r^{NA,AUM} = (1+17n)(-2-5n+16n^2)$

$\tilde{\beta}_r^{NA,AUM} = 4+n(-39+2n(243+656n))$

$\hat{\alpha}_r^{NA,AUM} = 6(-1+n(-14+n(67+272n)))$

$\hat{\beta}_r^{NA,AUM} = 6-78n-3n^2(139+528n)$

$\tilde{\alpha}_t^{NA,AUM} = (1+8n)(1+17n)(-2-5n+16n^2)(-1+3n+16n^2)$

$\tilde{\beta}_t^{NA,AUM} = 4+n\left(59+n\left(-451+2n(-2907+8n(493+256n(28+41n)))\right)\right)$

$\hat{\alpha}_t^{NA,AUM} = -3(1+8n)(-1+3n+16n^2)(-2+n(-26+n(139+528n)))$

$\hat{\beta}_t^{NA,AUM} = 6+3n\left(66+n\left(176+n(-6741+32n(-207+4n(845+1616n)))\right)\right)$

$\theta^{NA,AUM} = (-1+4n(-3+2n(9+32n)))(-4+n(-54+n(273+1072n)))$

$\delta_0^{NA,AUM} = (1+17n)(-2-5n+16n^2)(-2+n(-1+8n)(25+64n))$

$\tilde{\delta}_0^{NA,AUM} = n(-3+9n+48n^2)$

$\rho_0^{NA,AUM} = (-2-23n+130n^2+480n^3)(n(13+16n(13+16n))$

$\delta_1^{NA,AUM} = (1+17n)(-2-5n+16n^2)(-2+n(-17+16n(7+24n)))$

$\tilde{\delta}_1^{NA,AUM} = 3n(-1+n(3+16n))(-16+5n(-39+8n(33+112n)))$

$\rho_1^{NA,AUM} = 8+n(146+n\left(-757+2n\left(-8395+128n(67+n(1303+1968n))\right)\right))$

$\sigma_0^{NA,AUM} = (1+8n)(1+17n)(2+n(26-n(139+528n)))$

$\tilde{\sigma}_0^{NA,AUM} = 3n(1+8n)(1+17n)(-1+3n+16n^2)$

$\hat{\sigma}_0^{NA,AUM} = (1+8n)(1+17n)(-2-23n+130n^2+480n^2)$

$\varphi_0^{NA,AUM} = (1+17n)(-1+2n+8n^2)(2+n(26-n(139+528n)))$

$\tilde{\varphi}_0^{NA,AUM} = 3n(1+17n)(-1+2n+8n^2)(-1+3n+16n^2)$

$\hat{\varphi}_0^{NA,AUM} = (1+17n)(-1+2n+8n^2)(-2-23n+130n^2+480n^2)$

$\varphi_1^{NA,AUM} = (-1+3n+16n^2)\,(1+17n)(-2-5n+16n^2)$

$\hat{\varphi}_1^{NA,AUM} = (-1+3n+16n^2)\,(4+n(-39+2n(243+656n)))$

$\tilde{\varphi}_1^{NA,AUM} = (-1+3n+16n^2)\,(6-78n-3n^2(139+528n))$

$\zeta_0^{NA,AUM} = (1+8n)(1+17n)(2+n(26-n(139+528n)))$

$\tilde{\zeta}_0^{NA,AUM} = (1+8n)(1+17n)(3n(-1+3n+16n^2))$

$\hat{\zeta}_0^{NA,AUM} = (1+8n)(1+17n)(-2-23n+130n^2+480n^2)$

$\varepsilon_0^{NA,AUM} = (1+17n)^2(1+8n)(-1+2n+8n^2)(2+n(26-n(139+528n)))$

$\tilde{\varepsilon}_0^{NA,AUM} = (1+17n)^2 3n(1+8n)(-1+2n+8n^2)(-1+3n+16n^2)$

$\hat{\varepsilon}_0^{NA,AUM} = (1+17n)^2(1+8n)(-1+2n+8n^2)(-2-23n+130n^2+480n^2)$
$\varepsilon_1^{NA,AUM} = (n(1+8n))^{1/2}(-1+3n+16n^2)\,(1+17n)(-2-5n+16n^2)$
$\tilde{\varepsilon}_1^{NA,AUM} = n(1+8n)(-1+3n+16n^2)\big(4+n(-39+2n(243+656n))\big)$
$\hat{\varepsilon}_{1,r}^{NA,AUM} = n(1+8n)(-1+3n+16n^2)(6-3n(-26+n(139+528n)))$
$\hat{\varepsilon}_{1,t}^{NA,AUM} = n(1+8n)(-1+3n+16n^2)\,(6-78n-3n^2(139+528n))$
$\eta_0^{NA,AUM} = (1+17n)(-2-5n+16n^2)(2+n\big(26-n(139+528n)\big))$
$\tilde{\eta}_0^{NA,AUM} = 3n(1+17n)(-2-5n+16n^2)(-1+3n+16n^2)$
$\hat{\eta}_0^{NA,AUM} = (1+17n)(-2-5n+16n^2)(-2-23n+130n^2+480n^2)$
$\eta_1^{NA,AUM} = n(-1+3n+16n^2)\,(1+17n)(-2-5n+16n^2)$
$\tilde{\eta}_1^{NA,AUM} = n(-1+3n+16n^2)\big(4+n(-39+2n(243+656n))\big)$
$\hat{\eta}_1^{NA,AUM} = n(-1+3n+16n^2)(6-78n-3n^2(139+528n))$

A.2.7 “No Agreement” – “Price Discrimination”

$\alpha_0^{NA,PD} = (11+n\,(786+n\,(-3267+64\,n\,(-349+n\,(609+16\,n\,(207+128\,n))))))$
$\beta_0^{NA,PD} = (1+2\,n)\,(-1+16\,n)\,(-7+n\,(-227+16\,n\,(21+4\,n\,(55+32\,n))))$
$\alpha_1^{NA,PD} = (-2+n\,(366+n\,(-1149+16\,n\,(-623+4\,n\,(201+16\,n\,(87+64\,n))))))$
$\beta_1^{NA,PD} = (1+2\,n)\,(1+8\,n)\,(-1+16\,n)\,(-8+n\,(13+112\,n)$
$\gamma^{NA,PD} = (6+n\,(305+n\,(-2335+64\,n\,(-148+n\,(505+16\,n\,(115+64\,n))))))$
$\theta^{NA,PD} = (-1+4\,n)\,(5+16\,n)\,(6+n\,(305+n\,(-2335+64\,n\,(-148+n\,(505+16\,n\,(115+64\,n))))))$
$\delta_0^{NA,PD} = -4+n\big(-119+64n(3+34n+32n^2)\big)(-7+n\Big(-227+16n\big(21+4n(55+32n)\big)\Big)$
$\rho_0^{NA,PD} = 3\,(1+8\,n)\,(-8+n\,(13+112\,n))\,(1+n\,(-55+8\,n\,(3+16\,n\,(7+8\,n))))$
$\delta_1^{NA,PD} = 2(-3+2\,n\,(-667+n\,(1198+n\,(28917+16\,n\,(-859+4\,n\,(-6071+32\,n\,(-105+8\,n\,(77$
$+64\,n))))))))$
$\rho_1^{NA,PD} = 3\,(1+8\,n)\,(-8+n\,(13+112\,n))\,(1+n\,(-55+8\,n\,(3+16\,n\,(7+8\,n))))$
$\sigma_0^{NA,PD} = 3(1+8\,n)\,(-1+n\,(3+16\,n))\,(-7+n\,(-227+16\,n\,(21+4\,n\,(55+32\,n))))$
$\sigma_1^{NA,PD} = 3(1+8\,n)\,(-8+n\,(13+112\,n))\,(-2+n\,(-91+16\,n\,(15+8\,n\,(13+8\,n))))$
$\varphi_0^{NA,PD} = (-1+4n)(1+2\,n)\,(-1+n\,(3+16\,n))\,(-7+n\,(-227+16\,n\,(21+4\,n\,(55+32\,n))))$
$\varphi_1^{NA,PD} = (-1+4n)(1+2\,n)\,(-8+n\,(13+112\,n))\,(1+n\,(-55+8\,n\,(3+16\,n\,(7+8\,n))))$
$\varepsilon_0^{NA,PD} = 3(1+2n)(1+8\,n)(-1$
$+4n)(-1+n\,(3+16\,n))^2\,(-7+n\,(-227+16\,n\,(21+4\,n\,(55+32\,n))))^2$
$\varepsilon_1^{NA,PD} = 3(1+2n)(1+8\,n)(-1+4n)(-8+n\,(13+112\,n))^2\,(1+n\,(-55+8\,n\,(3+16\,n\,(7+8\,n))))^2$
$\eta_0^{NA,PD} = 3(-1+4\,n)\,(5+16\,n)(1+2n)(-1+16n)(-2-5n+16n^2)(-1+n\,(3+16\,n))\,(-7$
$+n\,(-227+16\,n\,(21+4\,n\,(55+32\,n))))$
$\eta_1^{NA,PD} = \frac{1}{3}(-1+4\,n)\,(5+16\,n)(-1+n)(1+2n)(1+8n)^2(-1+16n)\,(-8+n\,(13+112\,n))\,^2(-2$
$+n\,(-91+16\,n\,(15+8\,n\,(13+8\,n))))$

$$\xi^{NA,PD} = (-2-5n+16n^2)(-1+3n+16n^2)\Big(-7+n\ \big(-227+16\,n\,(21+4\,n\,(55+32\,n))\big)\Big)(9(-2$$
$$-5n+16n^2)(-1+3n+16n^2)\left(-7+n\ \big(-227+16\,n\,(21+4\,n\,(55+32\,n))\big)\right)$$
$$+2\big(-8+n\,(13+112\,n)\big)\Big(-1$$
$$+n\ \Big(276+n\ \Big(3+64\,n\ \big(-124+n\,(-123+16\,n\,(33+32\,n))\big)\Big)\Big)\Big)$$
$$+\big(-8+n\,(13+112\,n)\big)^2\Big(-1$$
$$+n\ \Big(276+n\ \Big(3+64\,n\ \big(-124+n\,(-123+16\,n\,(33+32\,n))\big)\Big)\Big)\Big)^2$$

$$\phi^{NA,PD} = (n-1)\sigma_0^{NA,PD} + (n-1)\sigma_1^{NA,PD} + \varepsilon_0^{NA,PD} + \varepsilon_1^{NA,PD} + \eta_0^{NA,PD} + \eta_1^{NA,PD} + (1/2)\xi^{NA,PD}$$

A.2.8 "Vertical Collusion" – "Airlines in the upstream market"

$\alpha^{C,AUM} = 9n^2 + 18n(1+16n)(1+17n)$

$\beta^{C,AUM} = (1+8n)(2(1+17n)(1+14n)+3n)$

$\tilde{\alpha}^{C,AUM} = 3n(1+17n)(1+8n)$

$\theta^{C,AUM} = 2(1+17n)(1+14n)(1+20n)$

$\tilde{\alpha}_r^{C,AUM} = 3n$

$\tilde{\beta}_t^{NA,AUM} = (1+14n)$

$\hat{\alpha}_r^{C,AUM} = (1+17n)$

$\hat{\beta}_t^{NA,AUM} = -(1+17n)$

$\gamma^{C,AUM} = (1+17n)$

$\tilde{\alpha}_t^{C,AUM} = (3n(1+8n))/(1+17n)$

$\tilde{\beta}_t^{C,AUM} = (1+22n+112n^2)/(1+17n)$

$\hat{\alpha}_t^{C,AUM} = (-1-25n-136n^2)/(1+17n)$

$\hat{\beta}_t^{C,AUM} = (1+43n+424n^2)/(1+17n)$

$\delta_0^{C,AUM} = 9n(2+67n+552n^2)$

$\tilde{\delta}_0^{C,AUM} = 3n(1+8n)(1+17n)$

$\rho_0^{C,AUM} = (1+8n)(2+65n+518n^2)$

$\delta_1^{C,AUM} = 3n(2+59n+416n^2)$

$\tilde{\delta}_1^{C,AUM} = 9n(1+33n)$

$\rho_1^{C,AUM} = 2+87n+1242n^2$

$\varphi_0^{C,AUM} = -\frac{3}{2}n$

$\tilde{\varphi}_0^{C,AUM} = 3n(2(1+17n)(1+14n)+3n)/2(1+17n)$

$\hat{\varphi}_0^{C,AUM} = 9n^2(1+8n)/2(1+17n)$

$\varphi_1^{C,AUM} = 9n^2/(1+17n)$

$\tilde{\varphi}_1^{C,AUM} = 3n$

$\hat{\varphi}_1^{C,AUM} = 42n^2/(1+17n)$

$\varepsilon_0^{C,AUM} = 9n(2+67n+552n^2)$

$\tilde{\varepsilon}_0^{C,AUM} = 3n(1+8n)(2+65n+518n^2)$

$\hat{\varepsilon}_0^{C,AUM} = 9n^2(1+8n)(1+17n)$

$\varepsilon_1^{C,AUM} = -3n(3n(1+8n)(1+17n))^{1/2}$

$\tilde{\varepsilon}_1^{C,AUM} = (1+17n)\ (3n(1+8n)(1+17n))^{1/2} =$

$\hat{\varepsilon}_1^{C,AUM} = -(1+14n)(3n(1+8n)(1+17n))^{1/2}$

$\eta_1^{NA,AUM} = -3n(3n)^{1/2}$

$\tilde{\eta}_1^{C,AUM} = (1+17n)\ (3n)^{1/2}$

$\hat{\eta}_1^{C,AUM} = -(1+14n)(3n)^{1/2}$

A.2.9 "Vertical Collusion" – "Price Discrimination"

$\alpha_0^{C,PD} = (1+n\,(-7+2\,n\,(-569+16\,n\,(-345+4\,n\,(233+8\,n\,(187+128\,n))))))$

$\beta^{C,PD} = (1+8\,n)\,(-1+n\,(-51+2\,n\,(-151+16\,n\,(149+4\,n\,(123+64\,n)))))$

$\alpha_1^{C,PD} = (-1+16n^2(15+16n))$

$\beta_1^{C,PD} = (1+8\,n)\,(1+14\,n)$

$\theta^{C,PD} = 6\,n\,(11+16\,n\,(11+8\,n))\,(-1+4\,n\,(-3+2\,n\,(9+32\,n)))$

$\gamma^{C,PD} = 2\,n\,(11+16\,n\,(11+8\,n))$

$\delta^{C,PD} = 3(1+n\,(1+2\,n\,(-365+8\,n\,(-539+8\,n\,(-65+512\,n\,(1+n))))))$

$\rho^{C,PD} = 3(1+8\,n)\,(1+14\,n)\,(-1+16\,n)\,(1+n\,(17+24\,n))$

$\sigma^{C,PD} = (1+8\,n)(1+14\,n)n\,(11+16\,n\,(11+8\,n))$

$\lambda^{C,PD} = 3n(-1+n\,(-51+2\,n\,(-151+16\,n\,(149+4\,n\,(123+64\,n)))))$

$\varphi^{C,PD} = (1+2\,n)\,(-1+4\,n)\,(1+14\,n)\,(-1+16\,n)\,(1+n\,(17+24\,n))$

$\varepsilon^{C,PD} = 3(1-16n)^2(1+2n)(-1+4n)(1+8n)(1+14n)^2(1+n(17+24n))^2$

$\eta^{C,PD} = \frac{1}{2}(-1+n)(1+8n)^2(1+14n)^2$

$\mu^{C,PD} = 3n(1+8n)(-1+n\,(-51+2\,n\,(-151+16\,n\,(149+4\,n\,(123+64\,n)))))^2$

$$\xi^{C,PD} = 3\left(-1+n\,\left(-51+2\,n\,\left(-151+16\,n\,\left(149+4\,n\,(123+64\,n)\right)\right)\right)\right)^2(9n^2+(1+14n)^2)+2n(1+14n)\left(-1+n\,\left(-51+2\,n\,\left(-151+16\,n\,\left(149+4\,n\,(123+64\,n)\right)\right)\right)\right)$$

$\phi^{C,PD} = 3(n-1){\sigma^{C,PD}}^2 + \varepsilon^{C,PD} + \eta^{C,PD} + \mu^{C,PD} + (1/2)\xi^{C,PD}$

A.2.10 "Airlines in the upstream market" – "Price Discrimination"

$$\alpha^{AUM,PD} = \left(2 + n\left(-1 + 8n\left(-61 + n\left(89 + 8n(133 + 128n)\right)\right)\right)\right)$$

$$\beta^{AUM,PD} = (1 + 8n)(-2 - 23n + 130n^2 + 480n^3)$$

$$\tilde{\alpha}^{AUM,PD} = (1 + 8n)(-3n + 9n^2 + 48n^3)$$

$$\gamma^{AUM,PD} = n(-43 + n(-557 + 16n(147 + 796n + 512n^2)))$$

$$\tilde{\alpha}_r^{AUM,PD} = (1 + 17n)(-2 + n(-5 + 16n))$$

$$\tilde{\beta}_r^{AUM,PD} = 4 + n(-39 + 2n(243 + 656n))$$

$$\hat{\alpha}_r^{AUM,PD} = 6(-1 + n(-14 + n(67 + 272n)))$$

$$\hat{\beta}_r^{AUM,PD} = 6 - 78n - 3n^2(139 + 528n)$$

$$\gamma_r^{AUM,PD} = -4 + n(-54 + n(273 + 1072n))$$

$$\tilde{\alpha}_t^{AUM,PD} = (1 + 8\,n)\,(-1 + n\,(3 + 16\,n))\,(1 + n\,(-12 + n\,(-511 + 16\,n\,(-73 + 4\,n\,(51 + 64\,n)))))$$

$$\tilde{\beta}_t^{AUM,PD} = 1 + n(56 + n\,(517 + 2\,n\,(-2713 + 8\,n\,(-3637 + 8\,n\,(509 + 8\,n\,(1171 + 16\,n\,(143 + 64\,n)))))))$$

$$\hat{\alpha}_t^{AUM,PD} = n\,(-3\,(1 + 8\,n)\,(-1 + n\,(3 + 16\,n))\,(-21 + n(-271 + 16\,n\,(75 + 4\,n\,(99 + 64\,n)))))$$

$$\hat{\beta}_t^{AUM,PD} = n(3\,(22 + n\,(697 + n\,(1921 + 32\,n\,(-2069 + 2\,n\,(-1747 + 32\,n\,(505 + 12\,n\,(117 + 64\,n))))))))$$

$$\gamma_t^{AUM,PD} = (-1 + 4n(-3 + 2n(9 + 32n)))(-4 + n(-54 + n(273 + 1072n)))$$

$$\delta_0^{AUM,PD} = n\left(-2 + n(-17 + 16n(7 + 24n))\right)\left(1 + n\left(-12 + n\left(-511 + 16n(-73 + 4n(51 + 64n))\right)\right)\right)$$

$$\tilde{\delta}_0^{AUM,PD} = 3n(-1 + n(3 + 16n))(-1 + n\left(-177 + 8n(-241 + 8n(11 + 8n)(17 + 64n))\right))$$

$$\rho_0^{AUM,PD} = n\left(2 + n(119 + n(1359 + 2n(-5249 + 32n(-2593 + 2n\left(1047 + 16n(1687 + 8n(425 + 192n))\right))))\right)\right)$$

$$\theta^{AUM,PD} = 3n\left(-1 + 4n(-3 + 2n(9 + 32n))\right)(-43 + n(-557 + 16n(147 + 796n + 512n^2)))$$

$$\delta_1^{AUM,PD} = 3(-2 + n(-1 + 8n)(25 + 64n))\left(1 + n\left(-12 + n\left(-511 + 16n(-73 + 4n(51 + 64n))\right)\right)\right)$$

$$\tilde{\delta}_1^{AUM,PD} = 3(1 + 8n)(-1 + 16n)(1 + n(17 + 24n))(-2 - 23n + 130n^2 + 480n^3)$$

$$\rho_1^{AUM,PD} = 3(1 + 8n)(-1 + 16n)(1 + n(17 + 24n))(-3n + 9n^2 + 48n^3)$$

$$\sigma_1^{AUM,PD} = n(1 + 8n)(11 + 16n(11 + 8n))\left(2 + n(26 - n(139 + 528n))\right)$$

$$\tilde{\sigma}_1^{AUM,PD} = n(1 + 8n)(11 + 16n(11 + 8n))(-2 - 23n + 130n^2 + 480n^3)$$

$$\hat{\sigma}_1^{AUM,PD} = n(1 + 8n)(11 + 16n(11 + 8n))(-3n + 9n^2 + 48n^3)$$

$$\varphi_0^{AUM,PD} = 3n(-1 + n(3 + 16n))\left(1 + n\left(-12 + n\left(-511 + 16n(-73 + 4n(51 + 64n))\right)\right)\right)$$

$$\tilde{\varphi}_0^{AUM,PD} = 3n(-1 + n(3 + 16n))(-1 - 51n - 302n^2 + 4768n^3 + 15744n^4 + 8192n^5)$$

$$\hat{\varphi}_0^{AUM,PD} = 3n(-1 + n(3 + 16n))n(63n + 813n - 3600n^2 - 19008n^3 - 12288n^4)$$

$$\varphi_1^{AUM,PD} = n(-1+16n)(-1+2n+8n^2)\big(1+n(17+24n)\big)\Big(2+n\big(26-n(139+528n)\big)\Big)$$
$$\tilde{\varphi}_1^{AUM,PD} = n(-1+16n)(-1+2n+8n^2)\big(1+n(17+24n)\big)(-3n+9n^2+48n^3)$$
$$\hat{\varphi}_1^{AUM,PD} = n(-1+16n)(-1+2n+8n^2)\big(1+n(17+24n)\big)(-2-23n+130n^2+480n^2)$$
$$\zeta_1^{AUM,PD} = 3n(1+8n)^2\big(11+16n(11+8n)\big)^2\Big(2+n\big(26-n(139+528n)\big)\Big)$$
$$\tilde{\zeta}_1^{AUM,PD} = 3n(1+8n)^2\big(11+16n(11+8n)\big)^2\big(3n(-1+3n+16n^2)\big)$$
$$\hat{\zeta}_1^{AUM,PD} = 3n(1+8n)^2\big(11+16n(11+8n)\big)^2(-2-23n+130n^2+480n^3)$$
$$\varepsilon_0^{AUM,PD} = (1+8n)\big(-1+n(3+16n)\big)\bigg(-1+n\bigg(12+n\Big(511-16n\big(-73+4n(51+64n)\big)\Big)\bigg)\bigg)$$
$$\tilde{\varepsilon}_0^{AUM,PD} = (1+8n)\big(-1+n(3+16n)\big)(-1-51n-302n^2+4768n^3+15744n^4+8192n^5)$$
$$\hat{\varepsilon}_0^{AUM,PD} = (1+8n)\big(-1+n(3+16n)\big)n(63n+813n-3600n^2-19008n^3-12288n^4)$$
$$\varepsilon_1^{AUM,PD} = n^{1/2}(1+8n)(-1+16n)^2(-1+2n+8n^2)\big(1+n(17+24n)\big)\Big(2+n\big(26-n(139+528n)\big)\Big)$$
$$\tilde{\varepsilon}_1^{AUM,PD} = n^{1/2}(1+8n)(-1+16n)^2(-1+2n+8n^2)\big(1+n(17+24n)\big)(-2-23n+130n^2+480n^2)$$
$$\hat{\varepsilon}_1^{AUM,PD} = n^{1/2}(1+8n)(-1+16n)^2(-1+2n+8n^2)\big(1+n(17+24n)\big)\big(3n(-1+3n+16n^2)\big)$$
$$\eta_0^{AUM,PD} = \big(-1+n(3+16n)\big)\bigg(-1+n\bigg(12+n\Big(511-16n\big(-73+4n(51+64n)\big)\Big)\bigg)\bigg)$$
$$\tilde{\eta}_0^{AUM,PD} = \big(-1+n(3+16n)\big)(-1-51n-302n^2+4768n^3+15744n^4+8192n^5)$$
$$\hat{\eta}_0^{AUM,PD} = \big(-1+n(3+16n)\big)n(63n+813n-3600n^2-19008n^3-12288n^4)$$
$$\eta_1^{AUM,PD} = (1+8n)^2\big(11+16n(11+8n)\big)^2\Big(2+n\big(26-n(139+528n)\big)\Big)$$
$$\tilde{\eta}_1^{AUM,PD} = (1+8n)^2\big(11+16n(11+8n)\big)^2\big(3n(-1+3n+16n^2)\big)$$
$$\hat{\eta}_1^{AUM,PD} = (1+8n)^2\big(11+16n(11+8n)\big)^2(-2-23n+130n^2+480n^3)$$