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SOCIAL BENEFITS OF CRITICALITY-INDUCED PSYCHOLOGICAL REWARD

Abstract: This article deals with the issue of evolutionary game theory by adopting the key ingredient of criticality borrowed from the field of phase transitions to establish the social benefit of altruism. The theoretical perspective that we propose rests on the dynamics of a Nash system modulated by the behavioral dynamics of the units of this system. The players of the network under study are naturally led to make the selfishness choice, but due to the imitation principle they may also make the altruism choice if some of their neighbors are altruist. We prove that if the imitation strength K is assigned a special value K_c a phase transition occurs to the emergence of altruism. This form of phase transition is similar to the phase transition processes of physics but it is significantly extended, due to the fact that the sociological system does not fit the thermodynamic limit condition. As a consequence at criticality the concentration of altruists is characterized by large fluctuations that have the effects of significantly enhancing the financial benefits generated by the psychological reward for altruism recently introduced by Gintis for a modified version of Nash game theory. We argue that the feedback of the financial level on the behavioral level will lead to a plausible explanation of the emergence of altruism in human societies.

Keywords: Nash game theory, criticality, emergence of altruism, temporal complexity, psychological reward for altruism.

1. Introduction

A big revolution is taking place in Science (Laland, Wray, 2014) leading biologists, psychologists, behavioral scientists and anthropologists to rethink the Darwinian concept of evolution by natural selection. In this article we shall illustrate what the contribution of physicists to this ambitious interdisciplinary project

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may be. The interesting discussion of Laland, Wray (2014) rests on the remarkable results of research work illustrated in the exemplary books (Pigliucci, Muller eds., 2010; Jablonka, Lamb, 2014). The book of Pigliucci, Muller eds. (2010) addresses the ambitious purpose of extending the modern synthesis, the theoretical interpretation that has been used to define the evolutionary theory since the 1940s. This ambitious purpose is addressed also by Jablonka, Lamb (2014) using four dimensions to generate a new evolutionary theory: genetic, epigenetic, behavioral and symbolic. A key idea emerging from this discussion is that the survival of the fittest has not to be interpreted as a property of competing individuals but as referring to communities struggling against unfavorable environmental conditions, thereby making cooperation rather than competition the key ingredient for survival and for evolution. This basic concept emerges also from the remarkable essay by Wade (2009), who illustrates the possible evolutionary origin of religion as a powerful tool to strengthen societal cooperation.

These observations lead us to go beyond the evolutionary game theory of Maynard Smith (1958) and the non-cooperative game theory of Nash (1950). In this paper we focus on an important condition that, in spite of its relevance, has not been properly considered by the researchers. The game played by the units of the currently theories suggests the existence of an organized society. In fact, the players, either cooperators or defectors, are human beings and we cannot ignore the fact that they belong to an organized society. Physicists have studied for many years the transition from the state where the units, atoms and molecules in their cases, are virtually independent the ones from the others, to the state where they are closely correlated. This is the subject of phase transitions that generated a revolution in physics with the main achievements of renormalization group theory, eventually yielding the end of reductionism era, as the important direction to do science. It is reasonable to assume that a phase transition exists also for human society, with the plausible conjecture that the interaction between the players, which is apparently local, as in the traditional game theory, is actually influenced by the long-range correlation emerging with phase transition.

How to combine societal organization and Nash game theory? How to explain the origin of cooperation? The main idea that is leading our investigation is based on the widely accepted conviction that cooperation increases the efficiency of an organized society. We think that this has the effect of yielding an evolutionary process where the survival of the fittest has to be interpreted as the survival of the fittest community rather than the survival of a single individual.

This is a very ambitious goal going much beyond the results of this article. We limit ourselves to show the effect that criticality may have on the time evolution of Nash model, and we show that although altruism may emerge due to the imitation tendency of the network units, the financial benefit depends also on the psychological reward for the choice of altruism. We adopt a version of the prisoner's dilemma proposed by Gintis in his excellent Game Theory Evolving book (2000) to take into account the observation made by Kiyonary, Tamida and Yamagishi (2000) that people prefer to cooperate if their partners are cooperators. Gintis used this observation to introduce the concept of psychic gain λA for Alice making the choice of cooperation. The main result of our article is that the psychic gain may generate significant societal benefits at criticality, as a result of the extension of phase transition theory to the realistic condition of a society with a non excessively large number of units.

In Section 2 we introduce the readers to sociological criticality using as an example the imitation-induced transition to altruism. In Section 3, to make this paper self-contained, we first concisely review the effect of psychological reward for the choice of cooperation (Gintis, 2000) and then we evaluate the societal benefit on the basis of the model of the earlier Section in thermodynamic limit. In Section 4 we show the joint effect of criticality and psychological reward for the choice of cooperation. Finally, we devote Section 5 to summarize the main results of this article and to illustrate the research directions that should be followed to move from the limited purpose of this paper to the more ambitious issue of contributing to the emergence of cooperation as a result of an evolutionary process.

2. Two-state model in the all-to-all coupling case

In this section, we illustrate a very simple model generating imitation-induced phase transition. For simplicity we make the simplifying assumption that all the units interact with all the other units, All-To-All (ATA) coupling condition, but we do not make the traditional assumption that the number of units, N, is infinitely large.

2.1. Critical slowing down

Let us consider the case where the N units of a network have to make a decision on whether to select the state A, which contains the fraction q or the state B, which contains the fraction $p = 1 - q$ of the entire population. We assume that the choice of state B corresponds to the choice of altruism and that the state A represents shelfisness. The master equation for the time evolution of the two probabilities p and q is given by

$$\dot{p} = -\gamma p + \omega q \tag{1}$$

$$\dot{q} = -\omega p + \gamma q, \tag{2}$$

which are, of course, compatible with the condition $p + q = 1$ at all times. The master equation is equivalent to the single equation

$$\dot{p} = -(\gamma + \omega) p + \omega, \tag{3}$$

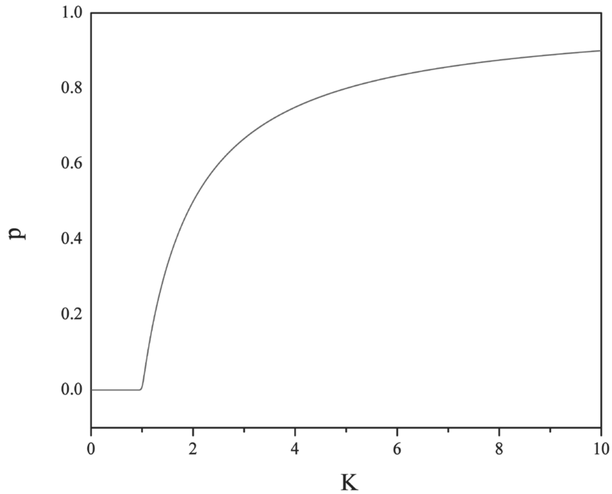
which is derived from Eq. (1) by replacing q with $1 - p$. In the absence of the natural tendency to imitation that we hypothesize to characterize human beings (West, Turalska, Grigolini, 2014), namely when $\omega = 0$, a unit in the state A will remain there forever, and a unit in the state B will jump to the state A with a finite transition rate γ. In this case all the units end in the state A and altruism gets extinct.

We hypothesize that imitation is a typical property of the individuals of a human society. Consequently we assume that the transition probability of a given unit from the state A to the state B does not vanish, but it is given by

$$\omega = K \frac{M_B}{M}, \tag{4}$$

where M is the number of its neighbors and M_B is the number of them in the state B.

Fig. 1 - A sketch representing the phase transition from the selfishness to the altruism condition, γ = 1.



In the All-To-All (ATA) case M coincides with N. In the ATA thermodynamic limit, $N = \infty$, we have

$$\omega = Kp \tag{5}$$

and the time evolution of p is given, according to Eq. (3) by

$$\dot{p} = (K - \gamma)p - Kp^2. \tag{6}$$

With some algebra it is possible to prove that the solution of this equation is:

$$p(t) = \frac{p_0\,(K-\gamma)}{(K-\gamma-Kp_0)\,e^{-(K-\gamma)t} + Kp_0}. \tag{7}$$

Although simple, this equation is a powerful description of the consequences of imitation. In fact, it shows that for $K < \gamma$, when imitation is weak, the equilibrium is given by $p = 0$. This indicates that a few individuals in the state B cannot attract a large part of population into this state and these individuals will jump to the state A before attracting any of their neighbors to the state B. For $K > \gamma$, on the contrary, the fraction of individuals in the state B is given by

$$p = 1 - \frac{\gamma}{K}. \tag{8}$$

The individuals who may select for fortuitous reasons the state B may attract many other individuals and for $K \to \infty$ the whole society ends into the state B.

Criticality emerges at

$$K = \gamma. \tag{9}$$

In this condition the regression to equilibrium is given by

$$p(t) = \frac{1}{Kt + \frac{1}{p_0}}, \tag{10}$$

which in the long-time limit leads to $p(t) \alpha 1/t$. As a consequence, it takes an infinite time for the system to regress to equilibrium and this phenomenon is well known as critical slowing down.

The surprising similarity with the second-order phase transitions in physics is clearly illustrated by Fig. (1) which shows p as a function of K.

As earlier mentioned, in this article we interpret the states A and B as corresponding to selfishness and altruism, respectively. We see that in the thermodynamical limit of $N = \infty$ this model generates the altruism extinction even if the supercritical condition $K > \gamma$ is adopted. In fact, $p = 0$ remains an equilibrium state of the model even when $K > \gamma$ and the stable equilibrium is given by Eq. (8). In practice, we need to set a special boundary condition at $p = 0$ that in the case $K > \gamma$ may prevent the altruism extinction and lead the system to the stable condition of Eq. (8).

2.2. Temporal complexity

In the case of cooperative systems with a finite number of units, critical slowing down is associated to the important property of temporal complexity (Turalska, West, Grigolini, 2011) that is of fundamental importance for the transfer of information from one to another complex network (Grigolini et al., 2015; Lukovic et al., 2014). To illustrate temporal complexity let us consider the model of Eq. (3) with ω given by Eq. (4) in the crucial case when the number of units is not infinitely large. The algorithm we use to generate the time evolution of the network works with the following prescription. Let us assume that a given unit at time t is in the state A. We have to establish whether at the next time $t+1$ it is still in the state A or it jumps to the state B. The probability of jumping to the state B is given by ω of Eq. (4). In the case of a number of units that is not infinitely large ω reads

$$\omega = K(p+f), \tag{11}$$

where f is a fluctuation of intensity proportional to $1/\sqrt{N}$. In the case of a finite number of units, the process is described by

$$\frac{d}{dt}p = -(\gamma - K)p - Kp^2 + Kf(1-p). \tag{12}$$

We note that the condition $p = 0$, in the absence of stochastic force f would lead to the extinction of altruism. Thus, the adoption of a condition far from the thermodynamic limit $N = \infty$ has the beneficial effect of preventing altruism extinction because the stochastic force f will make the system depart from the unstable condition $p = 0$ when $K > \gamma$. However, if we interpret p as being a stochastic variable, called x, the condition $x < 0$ is forbidden due to the fact that x is a probability or a concentration of altruist. For this reason the numerical calculations of this article rest on the non-linear Langevin equation

$$\dot{x} = -\frac{d}{dx}V(x) - (\gamma - K)x - Kx^2 + K\sigma\xi(t)(1-x). \tag{13}$$

The potential $V(x)$ is assigned the form

$$V(x) = \frac{2e^{-\left(\frac{x}{a}\right)^2}}{a\sqrt{\pi}} \tag{14}$$

with a so small as to make the minimum a_{min} of the potential

$$V_{eff}(x) = V(x) + (\gamma - K)\frac{x^2}{2} + K\frac{x^3}{3} \tag{15}$$

extremely small. For the numerical calculations of this article we selected $a = 0.001$ yielding $a_{min} = 0.0054$. The variable $\xi(t)$ is a dichotomous fluctuation with the values $\xi = 1$ and $\xi = -1$, equivalent to a fair coin tossing, and

$$\sigma = \left(\frac{1}{\sqrt{N}}\right) \tag{16}$$

so as to correspond to the fluctuation $f(t)$, the intensity of which is indeed expected to be proportional to $1/\sqrt{N}$.

As a result of the reflective barrier at $x = 0$ the variable x is kicked out of the extinction condition $x = 0$ and gets positive values significantly larger than the minimum value a for extended times. Using the same theoretical approach as that adopted in Being et al. (2015) we show that the time duration of these states with a significantly large number of altruists is given by a waiting time distribution density $\psi(\tau)$, with the inverse power law $\mu = 1.5$. More precisely, the numerical calculations bases on the non-linear Langevin equation, as shown in Fig. 2, yield

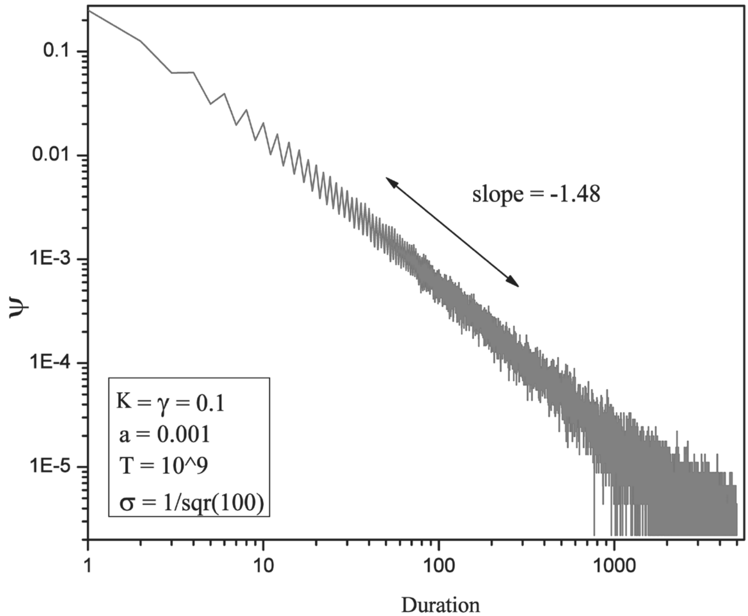
$$\psi(\tau) = 0.5\frac{1}{\tau^{1.5}}. \tag{17}$$

In fact, in either the subcritical and supercritical condition the effective potential of Eq. (15) yields equilibrium through a restoring force proportional to x that is as intense as the stochastic force when x gets the same order as σ. At criticality, on the contrary, the restoring force is proportional to x^2 thereby generating a restoring force of the order of σ^2, which is negligible compared to σ when $\sigma \ll 1$, as it happens in the case where $N \gg 1$. Thus the process is a merely diffusional one, thereby leading, see Being et al. (2015), to the inverse power law of Eq. (17). The numerical result, according to the fitting of Fig. (2), yields $\mu = 1.48$, which is satisfactorily close to the theoretical prediction $\mu = 1.5$. This is the criticality-induced temporal complexity that the research work of our group finds to be of big importance for the transfer of information from one to another network (Grigolini et al., 2015).

3. Nash theory and psychological reward for the choice of cooperation

In this Section we review the Nash game theory with psychological reward for the choice of cooperation (Gintis, 2000) and we evaluate the societal financial benefit by assuming that the concentration of altruists p is established by the model of Section 2

Fig. 2 - The waiting time distribution density $\psi(\tau)$ as a function of τ. τ is the time distance between two consecutive regressions to the hard repulsive wall of the potential V_{eff} of Eq. (15).



in the thermodynamic limit $N = \infty$. It is known (Gintis, 2000) that the financial benefit for player 1, called Alice, interacting with player 2, called Bob, depends on whether the two players adopt the cooperator or the defector choice. If α and β denote the probabilities of making the cooperator choice for Alice and Bob respectively, the financial benefit for Alice is given by

$$\pi_A = \beta(1+t) - \alpha\left[s(1-\beta) + \beta t\right], \tag{18}$$

where $t>0$ denotes the reward for defection and $-s$ the penalty for a cooperator playing with a defector. Since $\beta<1$, the quantity between square brackets is positive, and the defector choice, $\alpha = 0$, is the most convenient for Alice. If the Nash game is played assigning the psychological reward $\lambda_A>0$ to each player, the prediction of Eq. (18) turns into

$$\pi_A = \beta(1+t) - \alpha\left[s - \beta(s+\lambda_A)\right], \tag{19}$$

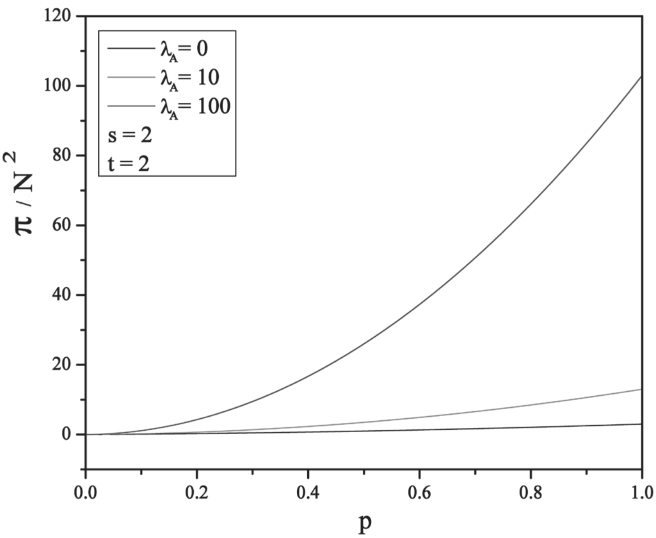
showing that if $\beta > s/(s+\lambda_A)$, for Alice cooperation is more convenient than defection, thereby stressing the importance of the psychological reward for the choice of altruism.

Let us proceed now to the evaluation of the societal benefit with the help of the model of Section 2 with $N = \infty$. We make the assumption that an individual in the state A is a defector and an individual in the state B is a cooperator. This leads us to establish that the fraction of cooperators is given by $p = 0$ for $K<\gamma$ and by of Eq. (8) for $K>\gamma$. We notice that the maximal benefit for society is reached when all the players are cooperators. In that case using Eq. (19) we obtain

$$\pi_{max} = 1 + t + \lambda_A. \tag{20}$$

In the ideal case where $\lambda_A \gg 1$ we see that the financial benefit for society is linearly proportional to the psychological reward for altruism. However, this ideal condition implies that the system is in the supercritical regime where all the units are cooperators. The statistical weight of the condition $\alpha = 1$ and $\beta = 1$ is p^2. When $p<1$ we have to consider also the condition $\alpha = 1$, $\beta = 0$, yielding economical disadvantage, $\pi = -s$, with the statistical weight pq, the condition $\alpha = 0$, $\beta = 1$ with $\pi = 1+t$, with the statistical weight

Fig. 3 - Financial benefit vs. p for three values of λ_A.



qp, identical to the earlier condition and the defectors benefitting from the cooperators, $\alpha = 0$, $\beta = 0$, yielding no society gain, with the statistical weight *qq*.

In conclusion the societal benefit is given by

$$\pi = N^2(p^2(1+t+\lambda) + p(1-p)(1+t-s)). \tag{21}$$

It is interesting to notice that this formula shows that the beneficial effects of psychological reward are negligible at criticality, in the limiting case of an infinitely large number of units. In this condition the beneficial role of psychological reward for altruism is perceived when $p > 0.5$. In fact, in this case the first term on the right hand side of Eq. (21) becomes more important than the second, which is affected by the economical disadvantage generated by the suckers.

To facilitate the reader's understanding in Fig. 3 we illustrate the financial benefit as a function p for different values of λ_A.

4. Joint action of criticality and psychological reward for altruism

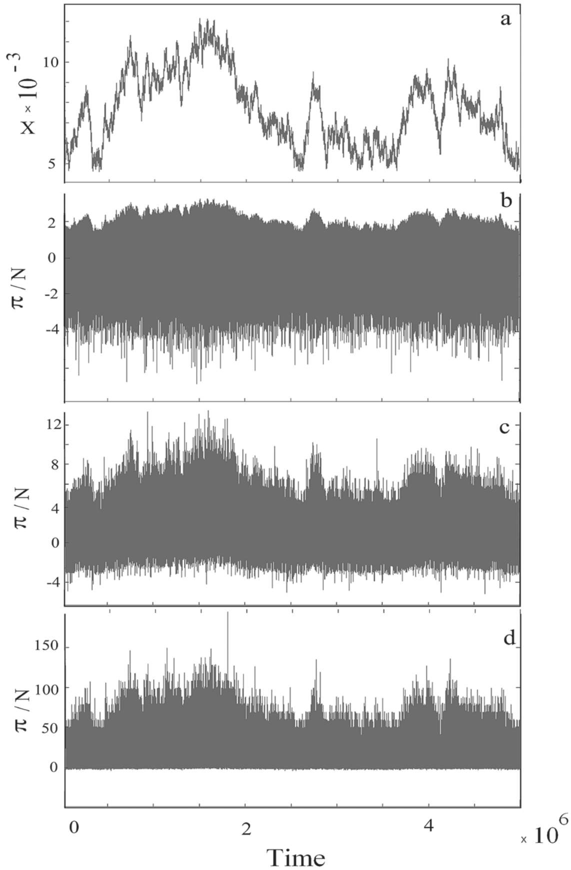
In this Section we illustrate the central result of this article, namely, the important role of criticality and of the criticality-induced temporal complexity, when $N < \infty$. To prove this important property, as done in the earlier Section, we assume that the individuals in the state *A* are defectors and the individuals in the state *B* are cooperator. However, since $N < \infty$ generates fluctuations, rather than adopting the prescription of Fig. 1, we evaluate the concentration *p* that, in the case of strong fluctuations is replaced by the stochastic variable $x(t)$. In other words, this means that we run the non-linear Langevin equation of Eq. (13).

To make it easier for the readers to understand the joint role of criticality and psychological reward for the choice of altruism-cooperation, we notice that to make the altruistic choice of Alice financially convenient, the concentration of altruist *p* must be larger than the critical threshold value

$$\Theta = \frac{s}{s + \lambda_A}. \tag{22}$$

If $\lambda_A = 0$, the threshold value become equal to 1, which is the max-

Fig.4 - K = γ = 0.5, a = 0.001, σ = 0.1, s = 2, t = 1, (a) : x, namely the number of altruists vs. time; (b): λ_A = 0; (c): λ_A = 100; (d): λ_A = 1000.



imal possible value of x, which, as the readers should remember, is the stochastic probability p. It is then evident that the optimal value of λ_A is given by

$$\lambda_A = s\left(\frac{1}{a} - 1\right) \approx \frac{s}{a}. \qquad (23)$$

We run Eq. (13) and at any time t we evaluate the financial benefit $\pi(t)$ of Eq. (19) by considering all possible pairs of units. This is equivalent to evaluating Eq. (21) with p replaced by $x(t)$. The results of this numerical work are illustrated in Fig. 4. Fig. 4 is worth of an extended comment. It refers to $K = \gamma$, which is the criticality condition. In the thermodynamic limit $N = \infty$, $p = x = 0$. In the condition $N < \infty$ criticality generates big fluctuations, and consequently a significantly large number of altruists that in the Nash game theory play the role of cooperators. Yet, the benefit for society is reduced by the play between defectors and suckers. The benefit π may also be negative, namely, the society may suffer financial loss as well as enjoying financial benefits. Fig. 4 shows that increasing λ_A has the effect of reducing loss till to its total extinction when λ_A is so large as to make the threshold Θ of Eq. (22) very close to the minimal possible value of altruists so as to get the maximal possible benefit from the criticality-induced emergence of altruists. In conclusion, the crucial property of Fig. 4 is that at criticality the adoption of λ_A of the order of Eq. (23) has the beneficial effect of canceling the losses produced by the action of the suckers playing with defectors.

5. Concluding remarks

It is important to stress that results of this article do not fully justify the ambitious title that we have adopted. We make the conjecture that further research work along these directions will allow us to fulfill the important goal of evolutionary game theory suggested by this title. In fact, in this article the Nash game theory of Eq. (19) is modulated by the imitation model that, as we have seen in Section 2.2., yields Eq. (13). Although in the limiting case of $N = \infty$, the imitation-induced concentration of altruists vanishes, thereby annihilating the societal financial benefit, temporal complexity generates so big fluctuations of p as to make it possible to make the psychic gain λ_A of Gintis (2000) annihilate the sucker-induced losses, as clearly illustrated by Fig. 4. In this article the dynamics of the behavioral level, namely the complex networks of individuals who have to choose either the state A or the state B drives the evolution of Nash game theory with no feedback. We expect that the addition of a feedback will favor the spontaneous evolution towards altruism, thereby making it possible for our theoretical approach to afford important contributions to the new field of evolutionary game theory.

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