SAPIENZA - UNIVERSITÀ DI ROMA

ANNALI DEL DIPARTIMENTO DI METODI E MODELLI PER L'ECONOMIA, IL TERRITORIO E LA FINANZA

2015

Perspectives on Behavioural Sciences

ISBN: 978-88-555-3333-1 ISSN: 2385-0825

PÀTRON EDITORE
Bologna 2015

PÀTRON Editore - Via Badini, 12
Quarto Inferiore, 40057 Granarolo dell'Emilia (BO)
Tel. 051.767003
Fax 051.768252
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Stampa: Rabbi s.r.l., Bologna per conto di Pàtron editore, dicembre 2015.

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A PROPOSAL FOR MEASURING GLOBAL SUBJECTIVE SATISFACTION

Abstract: In many fields of social studies it would be useful to measure the degree of global satisfaction (or agreement) of a population based on individual opinions about several aspects. Some examples regard, for instance, the measurement of customer satisfaction, of students' or graduates' opinions about university teaching, of subjective well-being and so on. This problem concerns the more general framework of measuring complex phenomena that are not directly observable and hence approximated by a set of elementary (one-dimensional) indicators. The problem of synthesizing a set of variables thus arises. A common solution is to construct a composite indicator, which synthesizes all the elementary indicators by means of a weighted average. There is a widely developed statistical literature on this subject. Here we propose an alternative approach. We consider a set of elementary variables that are observed in a population. Each variable represents the individual degree of satisfaction about one of several aspects related to the complex phenomenon. In aiming to synthesize them, starting from their joint distribution, we define a new variable that measures global individual satisfaction. To evaluate the degree of global satisfaction for the entire population, we propose a normalized index, which takes values between 0 (maximum dissatisfaction) and 1 (maximum satisfaction). Specifically, this index measures the distance between the observed distribution of the new variable and the theoretical one, which refers to the situation of maximum dissatisfaction (all individuals are not at all satisfied for every aspect).

Keywords: degree of satisfaction, composite indicators, distance measures.

1. Introduction

The problem of measuring the degree of satisfaction of a population based on individual opinions has been widely discussed in literature; see for example, the measurement of customer satisfac-

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tion, of students' or graduates' opinions about university teaching, of subjective well-being and so on (Bini et al., 2009; Cassel and Eklöf, 2001; Hayes, 1998; Krueger, 2009; Piccolo, 2008).

This problem concerns the more general framework of measuring complex phenomena that are not directly observable and hence approximated by a set of elementary (one-dimensional) indicators. Moreover, such indicators are often measured by ordinal categorical data, since they express attitudes or opinions that are surveyed by means of a questionnaire and are encoded in a Likert scale (Likert, 1932) going from 'strongly disagree' (or 'strongly dissatisfied') to 'strongly agree' (or 'strongly satisfied'), or vice-versa. In other words, their attributes can be ordered (by ranks) but cannot generally be interpreted on a numeric scale, except in some specific cases. This occurs, for instance, when it can be assumed that the distances between any two adjacent categories are equal. In such cases, the original ordinal variables can be quantified by assigning numerical scores to their categories. In literature, there are several quantification methods, such as transformation by expert ratings, estimation from item text and optimal scaling (Bălătescu, 2002; Hensler and Stipak, 1979; Herzel, 1974; Casacci and Pareto, 2014).

Referring to the general problem of measuring a multidimensional phenomenon, there are different approaches on how to use information coming from elementary indicators. On the one hand, it can be useful to aggregate them in order to construct a synthetic measure of the phenomenon intensity. A common solution for quantitative elementary indicators is to construct a composite indicator by means of a weighted average. This approach requires a subjective choice about the aggregation formula and the weighting structure. There is a widely developed statistical literature on this subject (for instance, see OECD, 2008; Saltelli, 2007). On the other hand, it seems appropriate to use original data, without any aggregation, since elementary indicators are often heterogeneous and the aggregation could imply an information loss.

We propose an index that measures the degree of global satisfaction of a population synthetically, based on n individual evaluations of several aspects. In particular, we consider the case where such evaluations are expressed on an ordinal scale. Here we avoid introducing any arbitrary hypotheses about the categories, and then we consider the ordinal nature of elementary

indicators. Concerning the construction of a synthetic measure, we adopt a different approach from the classical methodology of composite indicators, as we illustrate in the next section.

2. The proposed method

Let us consider a set of elementary indicators that represent the degree of individual satisfaction (or the level of agreement) about a set of aspects of life, or a product, or a service. Let us suppose that each variable takes a finite number of ordered levels (for instance going from 'strongly satisfied' to 'strongly dissatisfied') and that it is observable on a population. As a particular case, we can consider the four levels used in the Istat *Multipurpose survey on households - aspects of daily life*: 1) 'very very satisfied'; 2) 'quite satisfied'; 3) 'not much satisfied'; 4) 'not at all satisfied' (Istat, 2006).

In the literature, many methods were suggested for constructing indices of satisfaction on a specific aspect. For example, referring to previous four levels scale, a simple index can be obtained by dividing the number of satisfied people (very or quite) by the number of respondents. This index, however, underestimates the effective degree of satisfaction, since it does not consider 'not satisfied' people. Recently Casacci and Pareto proposed an interesting method (Casacci and Pareto, 2015). They considered data from the aforementioned Istat survey regarding the citizen's opinions about the level of satisfaction for some aspects of daily life (financial position, health, family relations, friend relations, free time). For each variable, the authors compared the percentage distribution of the Italian population in year t with the analogous distribution referring to a hypothetical population where all people were 'not at all satisfied'. Their idea was that, as the observed distribution was further from the one of maximum dissatisfaction, so the level of satisfaction would be higher for the aspect corresponding to that variable. Consequently, the authors proposed measuring the degree of satisfaction by calculating the distance between the two distributions. Specifically, they applied a quadratic index of dissimilarity for ordinal data (Leti, 1983). Moreover, for evaluating how high the degree of satisfaction was, they applied a normalization procedure to this index, thus obtain-

ing the corresponding normalized one with values between 0 and 1 (meaning, respectively, maximum dissatisfaction and maximum satisfaction). They applied this index separately to each variable. This method seems preferable compared to other ones already known in the literature. In particular, it does not need to quantify the values of ordinal variables and it makes it possible to take into account the different levels of satisfaction that have been declared by the satisfied people, without aggregating them into one category as other methods do.

However, currently the authors have only focused on a one-dimensional perspective of satisfaction, calculating the values of the normalized index for one variable at a time. In fact, individual satisfaction about life, or a product, or a service, concerns *simultaneously* all the different aspects that affect it. Consequently, it could be interesting to measure the degree of global satisfaction from a multidimensional perspective, taking into account all the aspects simultaneously.

In trying to generalize the proposal of Casacci and Pareto into a multidimensional perspective, we could pursue this aim by defining a new variable, whose values depend on the specific degree of satisfaction expressed for each of the elementary indicators simultaneously, i.e. a variable able to express the general degree of individual satisfaction. We could then compare its observed distribution with the theoretical one corresponding to maximum dissatisfaction, i.e. to the hypothetical situation where all people were 'not at all satisfied' for all the aspects simultaneously.

Let K, M and n be the number of elementary indicators, of their categories and of individuals, respectively. Without losing generality, we assume that the first and the M-th category correspond, respectively, to the best and the worst judgements, as in the case of the ISTAT *Multipurpose survey*.

Starting from the K indicators let us define a new variable X that takes the same M categories, as follows:

$X = 1$ if all the indicators are equal to 1;
$X = 2$ if all the indicators are equal to 2;
…;
$X = M$ if all the indicators are equal to M.

Let ${}_Xf_m$ and ${}_XF_m$ be the relative frequency and the corresponding cumulative one, respectively, for the m-th level of X, $1 \leq m \leq M$.

Furthermore, let ${}_Xf_m^*$ and ${}_XF_m^*$ be the analogous quantities for the aforesaid theoretical distribution, where: ${}_Xf_m^* = {}_XF_m^* = 0$ for $m \neq M$, while ${}_Xf_M^* = {}_XF_M^* = {}_XF_M = 1$.

A first index of global satisfaction for the entire population could be:

$$IS_1(h) = \sqrt[h]{\sum_{j=1}^{M-1} \left| {}_X F_j - {}_X F_j^* \right|^h} = \sqrt[h]{\sum_{j=1}^{M-1} {}_X F_j{}^h} \tag{1}$$

where h is the distance order and takes positive values (in the distance methods literature, the most commonly used values are 1 and 2). This distance will be maximum and equal to $\sqrt[h]{M-1}$ when all people were 'very satisfied' for all the aspects simultaneously (hypothetical situation of maximum satisfaction), that is when ${}_XF_1$ and obviously all the other ${}_XF_m$'s will be equal to 1.

The normalized index can be obtained simply by using the following ratio:

$$is_1(h) = \frac{IS_1(h)}{\sqrt[h]{M-1}} \tag{2}$$

A limitation shown by this index is that in measuring the degree of global satisfaction, it does not take into account those individuals that are partially satisfied for at least one aspect, though dissatisfied for other ones. Thus the index underestimates the global degree of satisfaction.

We can overcome this problem by defining another variable Y, that represents the global satisfaction and that takes $M+1$ categories as follows:

$Y = 1$ if all the indicators are equal to 1;
$Y = 2$ if all the indicators are at most equal to 2 and at least one of them is equal to 2;
…
$Y = m$ if all the indicators are at most equal to m and at least one of them is equal to m;
…
$Y = M$ if all the indicators are at most equal to M and at least one of them is equal to M but not altogether;
…
$Y = M+1$ if all the indicators are equal to M.

An example with $K = 2$ indicators and $M = 4$ is illustrated in Table 1.

Tab. 1 - Example of the possible values for the variable Y 'global satisfaction', in the case of two indicators with four order levels.

Indicator1 Indicator 2	1	2	3	4
1	**Y = 1**	**Y = 2**	**Y = 3**	**Y = 4**
2				
3				
4	**Y = 4**			**Y = 5**

Now, analogously with the procedure for $IS_1(h)$, let ${}_Yf_m$ and ${}_YF_m$ be the relative frequency and the corresponding cumulative one, respectively, for the m-th level of Y, $1 \le m \le M+1$. Furthermore, let ${}_Yf_m^*$ and ${}_YF_m^*$ be the analogous quantities for the aforesaid theoretical distributions, where: ${}_Yf_m^* = {}_YF_m^* = 0$ for $m \neq M$, while ${}_Yf_m^* = {}_YF_m^* = 1$ for $m = M+1$.

Then, a second index of global satisfaction for the entire population could be defined as follows:

$$IS_2(h) = \sqrt[h]{\sum_{j=1}^{M+1} \left| {}_Yf_j - {}_Yf_j^* \right|^h} = \sqrt[h]{\sum_{j=1}^{M} {}_Yf_j{}^h + \left| {}_Yf_{M+1} - 1 \right|^h} \qquad (3)$$

where the distance is measured between the two relative frequency distributions, instead of between the two cumulative ones. In this way we avoid the multiple counting of a same fraction of individuals as occurs in the $IS_1(h)$ index. For example, referring to Table 1, the cumulative frequency F_2 also includes F_1 (the left upper area) and the cumulative frequency F_3 also includes F_2 (the two left upper areas). In reality the relative frequencies f_j correspond to the differences $F_j - F_{j-1}$.

The maximum value of $IS_2(h)$ index is equal to $\sqrt[h]{2}$ and therefore the normalized index has the following expression:

$$is_2(h) = \frac{IS_2(h)}{\sqrt[h]{2}} \qquad (4)$$

A further formulation of the global satisfaction index can be ob-

tained from $IS_2(h)$, by excluding the individuals completely dissatisfied for all the aspects (the right buttom in Table 1), coherently with the purpose of the index and with some classical satisfaction indices. Consequently, we define:

$$IS_3(h) = \sqrt[h]{\sum_{j=1}^{M} \left| {}_Y f_j - {}_Y f_j^* \right|^h} = \sqrt[h]{\sum_{j=1}^{M} {}_Y f_j{}^h} \tag{5}$$

whose maximum value is equal to 1. Therefore, $IS_3(h)$ is just a normalized index.

3. Conclusions

We have proposed a method for measuring the global degree of satisfaction (or agreement) for an entire population on the basis of several elementary indicators simultaneously. More precisely, we have defined three different indices, each focusing on specific subsets of individuals to be included/excluded from the analysis.

Future work will concern the distributional properties of the indices, so that they could be used for statistical inference.

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