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WATER HAMMER IN WATER DISTRIBUTION SYSTEMS

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EXTENDED ABSTRACT

L'interesse scientifico verso il fenomeno del colpo d'ariete nasce in Europa agli inizi del 1900 allorquando la società si trova nel pieno di una rivoluzione industriale con una notevole crescita demografica e con la necessità di maggiore potenza elettrica e macchinari industriali. In quel periodo l'energia idroelettrica rappresentava la maggiore risorsa energetica per cui le principali compagnie di produzione di energia contribuirono allo sviluppo di studi e ricerche riguardanti i fenomeni transitori di moto vario che interessavano soprattutto le turbine. Gli stessi primi studi di Allievi sull'argomento nascono dagli incidenti causati dalle sovrappressioni dovute a manovre di valvole nelle fabbriche del Nord Italia. Il problema del colpo d'ariete fu per primo studiato da Menabrea (1858). Michaud (1878) esaminò l'uso delle casse d'aria e di valvole di sicurezza per contrastare il fenomeno. Successivamente ricercatori come Weston (1885), Carpenter (1893) e Frizell (1898) tentarono di sviluppare delle espressioni per descrivere la variazione di pressione e velocità nelle condotte. Un notevole contributo sull'argomento venne da Joukowsky (1908) e da Allievi (1902, 1913, 1934). Joukowsky (1908) sviluppò l'equazione considerata fondamentale negli studi del colpo d'ariete: $\Delta P = \pm \rho a\Delta V; \Delta H = \pm a \Delta V/g$, con a = celerità della perturbazione; P =pressione piezometrica; g = accelerazione di gravità; ρ = densità del fluido e V =velocità media della corrente. Allievi (1902, 1913, 1934) dimostrò che il termine convettivo dell'equazione di conservazione della quantità di moto si può considerare trascurabile, introdusse due importanti parametri adimensionali che caratterizzano le tubazioni e le valvole e propose un sistema di equazioni concatenate per la risoluzione delle equazioni che descrivono il problema.

Successivamente altri studi hanno riguardato la formulazione delle note equazioni del moto vario per correnti in pressione e l'integrazione numerica delle stesse mediante il metodo numerico delle caratteristiche, metodi numerici alle differenze finite e ai volumi finiti. Un metodo semplificato per lo studio del moto vario delle correnti in pressione è il metodo delle Altezze Piezometriche Virtuali teorizzato da Orabona (1950, 1956) e applicato in Castorani *et alii* (1994) e in Balacco *et alii* (2007). Un esaustivo stato dell'arte sugli studi proposti in letteratura riguardanti il colpo di ariete è presente in Ghidaoui *et alii* (2005).

Come già detto, gli studi sul colpo di ariete nascono per risolvere i problemi legati agli impianti idroelettrici e solo successivamente sono stati anche analizzati i fenomeni di moto vario che interessano condotte quali quelle di un acquedotto. In queste condotte le sovrappressioni dipendono da una molteplicità di fattori tra cui il materiale costituente la condotta ed il tempo e la legge di chiusura degli organi di regolazione/intercettazione.

Questo contributo tende a spostare l'attenzione sul fenomeno del colpo d'ariete verso le condotte di un acquedotto considerando che in letteratura gli studi del colpo di ariete hanno riguardato perlopiù le condotte degli impianti idroelettrici. Con semplici formulazioni si analizzano alcuni fenomeni di moto vario che si verificano nelle condotte di distribuzione di un acquedotto. In particolare, per una condotta esterna di avvicinamento all'origine della distribuzione urbana, si verifica che la chiusura lineare di Allievi per minimizzare il fenomeno del colpo di ariete risulta inaccettabile alla luce di rilevanze teoriche e sperimentali che in questo articolo sono illustrate arrivando a risultati corretti. L'utilizzo della teoria di Allievi comporta una valutazione della sovrappressione ben diversa da quella che si otterrebbe considerando la formula, qui riportata, che lega istante per istante la velocità con la sovrappressione, così come già evidenziato in Orabona (1967) con un esempio numerico. A valle dello stesso caso, si determinano, inoltre, le sovrappressioni che si generano al variare della legge e dei tempi di apertura di una valvola. Analizzando le esperienze di Damiani (1957-58) in quattro reti dell'Acquedotto Pugliese, si dimostra che sono solo gli ultimi giri della saracinesca quelli temibili ai fini delle sollecitazioni della condotta a causa del moto perturbato. Inoltre, per gli ultimi giri del volantino prima della chiusura totale, emerge chiara l'influenza della legge di chiusura.

Infine si esamina il fenomeno del colpo di ariete in una condotta elevatoria con sollevamento meccanico. In particolare, si analizzano alcuni dispositivi di protezione comunemente utilizzati come i volani, le casse d'aria e le condotte di *by-pass* con valvole di ritegno. L'impiego del volano come organo regolatore fa sì che a causa dell'inerzia delle masse rotanti l'arresto del flusso avviene in un intervallo più o meno lungo. La durata di detto arresto può, perciò, protrarsi a volontà, entro determinati limiti pratici, aumentando le masse rotanti con una massa rotante aggiuntiva o volano. Nell'ipotesi che il massimo valore negativo della sovrappressione Δh cada nella fase di colpo diretto, si riporta la formulazione presente in Frega (1967).

ABSTRACT

The hypothesis of Allievi about the uniform valve closure in the first part of an urban water distribution network in order to minimize water hammer, has proved to be unacceptable in view of the theoretical and experimental results that are shown in this paper. Further considerations are made about the unsteady flow in pump rising pipeline systems according to the most recent developments, to mitigate water hammer damages. Furthermore, it is noted that water hammer studies mainly concerned pipeline systems in hydroelectric power plants while less attention was paid to water supply pipeline systems considered in this work.

Keywords*: water hammer, water distribution systems, valve, pump rising pipeline system*

INTRODUCTION

The scientific interest in water hammer arose in Europe at the beginning of the 1900s when Europe was in the middle of the industrial revolution with growing urban populations and industries requiring electrical power for the new industrial machineries. In that period hydroelectric energy represented the major energy resource; for this reason, the main hydrogeneration companies contributed heavily to the development of studies and researches about unsteady flow phenomena that mainly affected the turbines.

Some of Allievi's early experiments were undertaken as a direct result of incidents and failures caused by overpressure due to rapid valve closure in northern Italian power plants. The problem of water hammer was first studied by Menabrea (1858). Michaud (1878) examined the use of air chambers and safety valves to control water hammer. Weston (1885), Carpenter (1893) and Frizell (1898) subsequently attempted to develop expressions relating pressure and velocity changes in a pipe. A significant contribution on the subject came from Joukowsky (1908) and from Allievi (1902,1913,1934). Joukowsky (1908) proposed the best known equation in transient flow theory called the "fundamental equation of water hammer":

$$\Delta P = \pm \rho a \Delta V; \; \Delta H = \pm a \Delta V/g$$

in which a is the water hammer wave speed, P is the piezometric pressure; H is the piezometric head, g is the gravitational acceleration, ρ is the fluid density and V is the cross-sectional average velocity. Allievi (1902, 1913, 1934) developed a general water hammer theory showing that the convective term in the momentum equation was negligible. He introduced two important dimensionless parameters that are widely used to characterize pipelines and valve behaviour and proposed a system of chained equations for the resolution of the equations describing the problem. Other studies subsequently concerned the formulation of the unsteady flow equations in pressure pipes whose description is well detailed in Chaudrhy (1987).

Numerical modelling of unsteady flow propagation in pressure pipes has rapidly developed in the last years shifting from 1-D to quasi 2-D and 2D models (Vardy & Hwang, 1991; Ohmi *et alii*, 1985; Wood & Funk ,1970; Bratland, 1986). The most applied model is the Method of Characteristics as reported in Evangelisti (1965), Goldberg & Wylie (1983) and Lai (1989), together with the difference finite method (Chaudhry & Hussaini 1985) and the Finite volume method (Zhao & Ghidaoui, 2004).

A simplified method for the study of unsteady flow in pressure pipes is the Virtual Piezometric Head proposed by Orabona (1950; 1956) and applied in Castorani *et alii* (1994) and Balacco *et alii* (2007). An exhaustive water hammer review is presented in Ghidaoui *et alii* (2005). As already mentioned, water hammer studies have generally regarded transient flow resolution in hydroelectric power plants, while the problems related to unsteady flow in water supply pressure pipes have been analysed only later. In these pipes the piezometric head rise depends on many factors among which pipe material, closure time and law regarding the valves or any other device. Yao *et alii* (2014) too analysed the effects of the valve closing time. Water hammer in viscoelastic pipes with different sections was showed in Meniconi *et alii* (2012). Recently, Evangelista *et alii* (2015) studied the transient transmission and reflection phenomena in high density polyethylene (HDPE) pipes.

In this paper water hammer in water distribution systems has been analysed using simplified methods. In particular, in the first part of the distribution networks, the hypothesis of Allievi about the uniform closure of a valve in order to minimize water hammer, has proved to be unacceptable in view of the theoretical and experimental results that are shown. The piezometric head rise has been calculated downstream, considering different valve closure times. Moreover, water hammer has been analysed in a pump rising pipeline system.

WATER HAMMER UPSTREAM A SLUICE VALVE IN WATER DISTRIBUTION NETWORKS

In the first part of a water distribution system perturbation is caused by the closure of a sluice valve. The wave speed of that perturbation is often called celerity c, that is different from water flow velocity:

$$c = \pm \sqrt{\varepsilon'/\rho} \qquad (1)$$

in which: $\varepsilon'=\varepsilon/[1+(\varepsilon D)/(E\delta)]$; is the equivalent bulk modulus of elasticity of the system fluid-pipe; ε is the bulk modulus; E is the Young's modulus of the pipe wall material; D the inner diameter of the pipe; δ is the wall thickness and ρ is water density.

The Figure 1 presents a simple pipe characterized by uniform flow regime with velocity V_0.

In pipes, it is usually possible to calculate the friction head loss as:

$$y_0 = k\,V_0^2 \qquad (2)$$

Moreover the presence of a sluice valve, with an opening degree equal to Ψ_0 , leads to a further head loss ξ_0. According to Borda's theorem and considering an unchanged diameter downstream of the gate valve, ξ_0 is:

$$\xi_0 = [(1/\Psi_0)-1]^2 \, V_0^2/2g \quad (3)$$

In the hypothesis that y_0 is concentrated at the gate cross section, and that ξ_0 occurs half upstream and half downstream of the gate, the total head line becomes the piezometric head line minus $y_0+\xi_0/2$, neglecting the kinetic energy per unit weight related to V_0.

For the sudden closing of the sluice gate with a time less than $2L/c$, the opening degree becomes Ψ_1 lower than Ψ_0. Consequently, considering the new values of the variables V_1, y_1, and ξ_1 it is possible to write:

$$y_1 = k \, V_1^2 \quad (4)$$

$$\xi_1 = [(1/\Psi_1)-1]^2 \, (V_1^2/2g) \quad (5)$$

Figure 2 shows that:

$$y_1 + (\xi_1/2) > y_0 + (\xi_0/2) \quad (6)$$

then the piezometric head rise is:

$$\Delta h_1 = [y_1+(\xi_1/2)] - [y_0+(\xi_0/2)] \quad (7)$$

Assuming the same sudden closing of the sluice valve, the following equation can be considered:

$$\Delta h_1 = (c/g) \, (V_0-V_1) \quad (8)$$

where c is perturbation celerity. Equation 8 is related to the phase that immediately follows the sudden closure of the valve. Another method considers the velocity V_0 of the uniform flow before the valve closure according to Torricelli's formula:

$$V_0 = \sqrt{2 \, g \, h_0} \quad (9)$$

Therefore, for an opening degree equal to Ψ_0, calculated as the ratio between the initial ω cross-section before the variation and the Ω cross- section upstream the gate:

$$\Psi_0 = \omega/\Omega \quad (10)$$

it is possible to write:

$$V_0 = \Psi_0 \sqrt{2 \, g \, h_0} \quad (11)$$

After the sudden closing, reducing the section to an opening degree Ψ smaller than the previous one, the head in the same cross-section varies going from h_0 up to $h_0+\Delta h$ due to the perturbation, so that:

$$V = \sqrt{2 \, g \, h} = \sqrt{2 \, g \, (h_0+\Delta h)} \quad (12)$$

$$V = \Psi \, (\omega/\Omega) \sqrt{2 \, g \, (h_0+\Delta h)} \quad (13)$$

It results:

$$V = \Psi \, V_0 \sqrt{1+(\Delta h/h_0)} \quad (14)$$

Equation 14 shows the relationship between velocity and overpressure. A simplified expression can be obtained from the analysis of the parable law representing the Equation 14 (Orabona, s.d.):

$$V \cong \Psi \, V_0 \, [1+ 0.5 \, (\Delta h/ h_0)] \quad (15)$$

Through this procedure (Eq. 14), the value of the overpressure Δh_1 appears very different from the one calculated by the Equation 8. In a numerical example reported by Orabona (1967), the value of Δh_1 calculated using the Equation 14, was about 300 times greater than that calculated by the Equation 8.

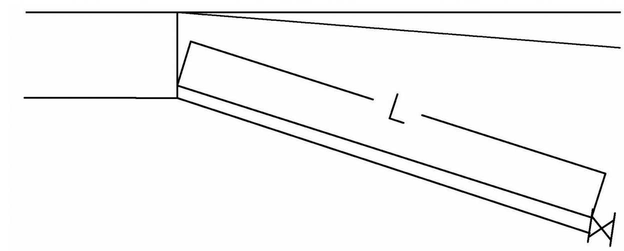

Fig. 1 - Sketch of the pipe

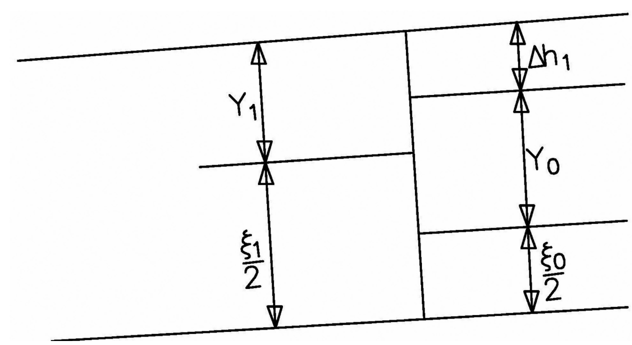

Fig. 2 - Piezometric head line downstream and upstream of the sluice valve

WATER HAMMER IN PIPES DOWNSTREAM OF THE GATE AT DIFFERENT PHASES

If the closing time is slower $t > 2L/c$, for each phase with a duration of $\tau=2L/c$ the following expression can be used:

$$\Delta h_n = -\Delta h_{n-1} + c/g \, (V_{n-1}-V_n) \quad (16)$$

The following equations have been applied instead of Equations 5 and 7:

$$\xi_n = [(1/\Psi_n)-1]^2 \, (V_n^2/2g) \quad (17)$$

$$\Delta h_n = [y_n+(\xi_n/2)] - [y_{n-1}+(\xi_{n-1}/2)] \quad (18)$$

The Equations 17 and 18 show that the perturbations are not affected by a very remarkable area reduction at the gate. As will be clarified below, the last turns of the handwheel of the sluice valve are the most dangerous due to the transient motion.

In this contest, the pipe downstream of the sluice valve has been analysed with a phase duration equal to $\tau=2L/c$, the same as the upstream one.

In the experiments carried out by Damiani (1957-58) in four networks of the Apulian Aqueduct, all the tests showed that the piezometric head variations were reduced when the valve started to operated. Only the perturbations resulting from the last sluice valve operations, before the gate valve was completely closed, were compared by Damiani (1957-58).

The influence of the closing law is clear. With regard to the test concerning the IV center in the aforementioned paper, the following results are obtained.

The gate valve had a strong, gradually increasing, resistance and therefore the handwheel did not rotate uniformly. For this

V_0	*Turns*	Ψ_0	t_c/τ	Δh	
				calculated	*observed*
0.7369	*1.1/2*	*0.08204*	*3.3889*	*60.56*	*26*
0.7847	*1*	*0.05476*	*4.2222*	*39.55*	*23.8*
0.9129	*2.1/2*	*0.13694*	*2.8889*	*92.94*	*30.0*
0.9834	*3.1/2*	*0.19091*	*3.6222*	*101.34*	*36.5*

Tab. 1 - Comparison between observed and calculated Δh from D*AMIANI* *(1957-58)*

reason, the calculated values were different from the observed ones as shown in the Table 1.

Pipeline characteristics are:

$$\tau = 2L/c = 3.6\ s;\ c/g = 114\ s;\ K = 11.4\ m^{-1}s^{2}$$

Figure 3 shows the diagram related to the I case.

To confirm the above assumptions, the first case data were re-calculated by considering a parabolic closure law in accordance with the diagram in Figure 4.

Starting from the initial value $\Psi_0 = 0.08204$ it can be observed that the successive openness degrees are:

$$\Psi_1 = 0.064291 \qquad \Psi_2 = 0.028574 \qquad \Psi_3 = 0.0071435$$

The application of the Equations 3, 5, 7 and 8 provides, after the rather long but very simple calculations, which for the sake of brevity are omitted, for the instants:

$$t_1 = 0.389\ t_c \qquad t_2 = 1.389\ t_c \qquad t_3 = 2.389\ t_c$$

the following values:

$$V_1 = 0.7281 \quad V_2 = 0.6475 \quad V_3 = 0.3076 \quad V_4 = 0$$
$$\Delta h_1 = 0.9918 \quad \Delta h_2 = 8.2137 \quad \Delta h_3 = 30.524 \quad \Delta h_4 = 4.52$$

Figure 5 shows a comparison between the values obtained with the above assumptions and the observed ones. In fact, if closing operations are decreasingly carried out at, overpressure is reduced by over 50%.

The slight discrepancy that exists (Fig. 5) between the overpressures calculated with the closing parabolic law and the observed ones, depends on the fact that closure gave rise to a law of openness degrees presenting upwards concavity even more accentuated than in the parabolic law.

If we accept the observed overpressures from Figure 5:

$$\Delta h_1 = 2.5 \quad \Delta h_2 = 13.4 \quad \Delta h_3 = 26 \quad \Delta h_4 = 9$$

with $c/g=114$, the application of the previously reported Equation 16 provides:

$$V_0 - V_1 = 2.5/114 = 0.02192$$
$$V_1 - V_2 = 0.13947$$
$$V_2 - V_3 = 0.34561$$
$$V_3 = 0.31579$$

from which it is possible to obtain $V_0 = 0.8228$ and subsequently:

$$V_1 = 0.8009 \quad V_2 = 0.6614 \quad V_3 = 0.3158$$

the value thus obtained for V_0 differs by about 10.5% from that of 0.7369, assumed above. This difference is not excessive if we consider that the discharge value, from which the velocity of 0.7369 is derived, has been obtained by a venturimeter as an average value of the observed data in a few minutes duration.

Simple calculations were employed to examine the experimental results, because the effects of the numerous wave reflections that occur in real networks under different consumption laws could not be taken into account.

It is important to note, however, that the closing of the gate valves must also be carried out very quickly during the initial operations of the handwheel, but very slowly in the final ones.

In practice, however, it is well known that very often the turncock starts to close the valve at a regular pace, and when he gets bored towards the end, he tends to accelerate, obtaining the above-mentioned dangerous results.

WATER HAMMER IN IN A PUMP RISING PIPELINE SYSTEM

Let's examine the case in which the pump is not equipped with a regulating device. An instantaneous energy disruption gives rise to a depression equal to

$$\Delta h = -cV_0/g \qquad (19)$$

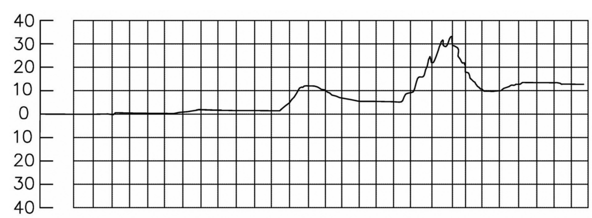

Fig. 3 - Overpressure diagram from D*AMIANI (1957-58) replotted*

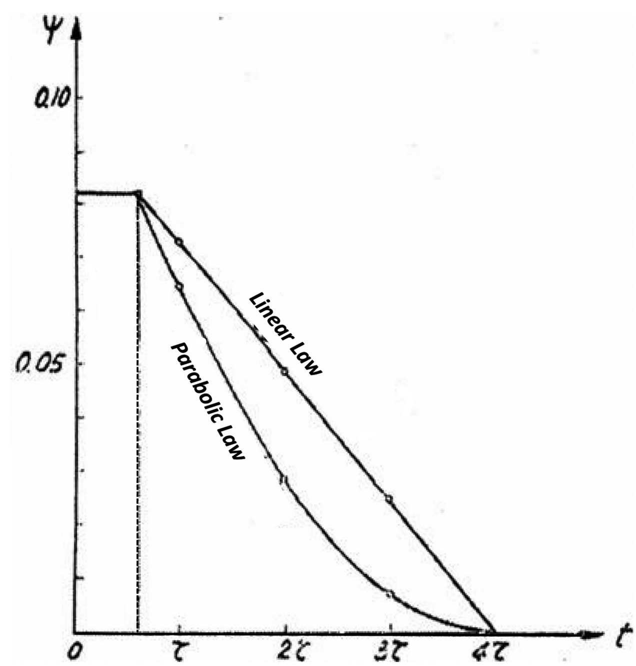

Fig. 4 - Closing laws from D*AMIANI (1957-58) replotted*

in the direct hit phase $\tau = 2\ (L/c)$.

Following the return of the pressure wave, an overpressure $+cV_0/g$ of equal absolute value occurs.

If there is a regulating device (flywheel, air chamber) its action is carried out in order to maintain the flow for a certain time after the engine stops, which reduces the amount of overpressure.

With reference to Figure 6, it is assumed that the ascending pipe flows into the reservoir at a lower level than the water surface, so that the pressure head line is constant at the inlet. In practice, the flow is usually above the water surface, but, for calculation's sake, the scheme adopted greatly simplifies the investigation, without giving resulting in errors that exceed the limits of tolerability. The most stressed cross section is the inlet section, where the perturbation is located and the velocities are to be calculated in the same versus as uniform velocities.

The use of a flywheel as a regulator implies that, due to inertial rotating masses, the flow is stopped at longer interval. The duration of the aforesaid block can, therefore, be prolonged at one's will, within certain practical limits, by increasing the rotating masses with an additional rotating mass or flywheel.

Assuming that the maximum negative value of the overpressure Δh occurs into the direct hit phase, Frega (1967) showed that the parameter proportional to the dynamic moment of the rotating masses GD^2K is equal to:

$$K=1/g\ GD_i^2\ (\pi/60)^2 \tag{20}$$

with G weight and D_i Inertia diameter of the flywheel, it results:

$$K=(N/D) \tag{21}$$

where:

$$N = 5.4\ \gamma\ Q_0\ h_0\ LR_0^2\ (1+R_0)$$

and

$$D=\eta_m cN_0^2(2R_0^3-0.25\ R_0^2)\ln[(h_0(1+2R_0)-h_{min})/(2h_{min}R_0)]+$$
$$+[(0.42(0.5+R_0)^2)/(2R_0^3-0.25R_0^2)]\ln(h_{min}/h_0)$$

with the following meaning of the symbols: γ= specific weight of water; c= celerity of the perturbation; Q_0= discharge; N_0= number of turns; h_0= geodetic prevalence; h_{min}= minimum pressure chosen value; L = length of the pipeline; V_0= velocity; η_m= yield; $R_0=cV_0/2gh_0$ (dimensionless).

Another important regulating device used to reduce water hammer is the so-called air chamber (Bianchi, 1985) or water chamber. There are also special valve systems that connect one end of the pipeline to atmospheric pressure, thus giving free flow to pressure waves coming from the outlet tank. With reference to Figure 7, we will consider the operation procedure of a simple check valve downstream of a by-pass in the initial section. In normal operation procedures, the check valve is kept closed by the higher downstream pressure. After instantaneous or abrupt closing, the inertial liquid flow in the supply line continues, a water demand originates that can only be satisfied by water recalled directly from the tank through the by-pass. Then the closing of the valve prevents this trend by modifying

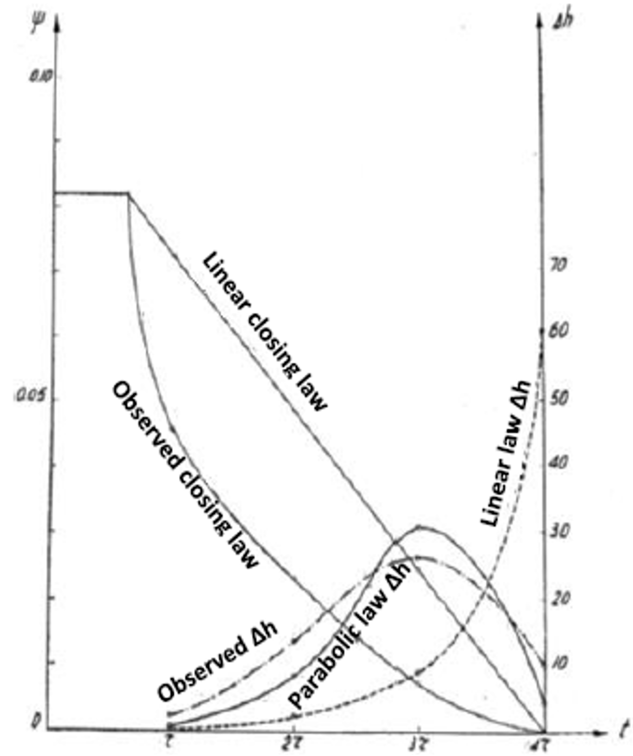

Fig. 5 - Diagram of the closure laws and comparison between calculated and observed overpressures from Damiani (1957-58) replotted

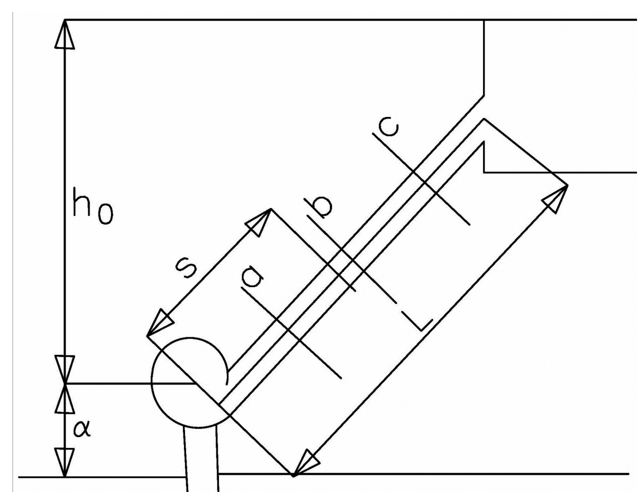

Fig. 6 - Sketch of the ascending pipe

the boundary conditions in the initial section to give rise to a transient with oscillations that can be evaluated.

According to Bianchi (1985), the maximum value of these oscillations is approximately equal to the height of the geodetic head H_g. In any case, as shown in the Figure 7, the pipe head should never go below the minimum value of 10.33 m, which could trigger cavitation.

CONCLUSIONS

Water hammer events that may occur in the pipeline system of an aqueduct have been examined using simplified formulas. In particular, in the first part of the distribution networks, the hypothesis of Allievi about the uniform closure of a valve in order to minimize water hammer, has been proved to be unacceptable in view of the theoretical and experimental results that have been shown. In a pipe downstream of the sluice valve, the piezometric head increase has been calculated considering different valve closure times. It has been underlined that the closing of the gate valves must be carried out very quickly during the initial operations, but very slowly in the final ones in order to minimize damages. Moreover, water hammer has been analysed in a pump rising pipeline system. Some commonly used protection devices such as flywheels, air chambers and by-pass check valve pipelines have been presented.

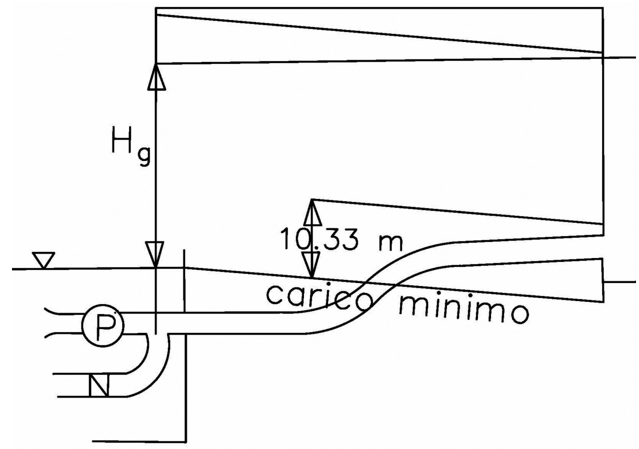

Fig. 7 - Check valve downstream of a by-pass

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