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MODELING OF RUNOUT LENGTH OF HIGH-SPEED GRANULAR MASSES

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ABSTRACT

The power balance of a high speed granular mass sliding along planar surfaces is written by taking into account its volume, the slopes of the surfaces (runout and runup), an assigned basal fluid pressure and different possibilities for the energy dissipation. In particular, collisions acting within a thin layer ("shear zone") at the base of the mass and shear resistance due to friction along the basal surface induce the dissipation of energy. The solution of the ODE describing the mass displacements vs time is numerically obtained. The runout length and the speed evolution of the sliding mass depend on the involved geometrical, physical and mechanical parameters as well as on the rheological laws assumed to express the energy dissipation effects. The well known solutions referred to the Mohr-Coulomb or Voellmy resistance laws are recovered as particular cases. The runout length of a case is finally back analysed, as well as a review of some relationships expressing the runout length as a function of the volume V of the sliding mass.

Key words: *sliding granular mass, granular temperature, shear layer, collisions*

INTRODUCTION

The analysis of the complex mechanisms of the chaotic movement of high speed granular masses sloping along mountain streams till their arrest is necessary to identify hazardous areas. The runout length of high speed granular mass often depends on the interstitial pressures at the base of the mass, that can vary between null and higher than hydrostatic values (Iverson, 1997), due to possible water pressure excess, related to very rapid changes of pore volumes, often localized along a thin layer in proximity of the sliding surface. Experimental observations showed in fact the growth of a basal "shear zone" where initially great deformations and then dilation and collisions occur, differently from the top (Hungr, 1995).

Field observations denote the dependence of the runout length on the debris flow volume, but the usual Mohr-Coulomb (M-C) shear resistance criterium doesn't allow to obtain this result.

Thus, more complex resistance laws must be developed to describe the rapid sliding of a granular mass because high speed relative motion and collisions between solid grains take place within the basal shear layer, causing a fluidification effect (Hungr & Evans, 1996) coupled with energy dissipations.

To this purpose, Bagnold (1954) defined "dispersive pressure" the stress component normal to the boundary that encloses a set of particles in grain-inertial regime:

$$p_{dis} = a_i \rho_s \lambda^2 d_p^2 \left(\frac{du}{dy}\right)^2 \cos\phi \qquad (1)$$

being:

a_i, the "*Bagnold coefficient*"; Bagnold (1954) and Takahashi (1981) suggest the value *0.042*;

ρ_s, the solid fraction mass density;

λ, the "*linear concentration*", that is a function of solid fraction v_s;
d_p, the characteristic diameter of the grain;
$(du/dy)^2$, the square of the velocity gradient;
Φ, the internal dynamic friction angle of granular bulk.

If a linear change of velocity, along the orthogonal direction of the motion, is assumed, the following dependence is obtained between the Bagnold's definition of dispersive pressure and the rate $\dot{x}$ of the sliding mass: $p_{dis} \sim x^2$.

Ogawa (1978) defined "*granular temperature*" as the mean square deviance of the relative velocities of sliding and colliding particles with respect to the mean value

$$T_g = \langle \delta v^2 \rangle \quad (2)$$

Several Authors apply the Voellmy law (V-M): a turbulent resistance is added to the M-C resistance, to estimate the runout length of debris flows.

To overcome these limitations, the rapid sliding of a granular mass along planar surfaces is analytically modelled in the paper, by taking into account the effects of *granular temperature* and *dispersive pressure*, acting within the basal '*shear zone*'. An original model is firstly proposed, based on some simplifying hypotheses. The governing equations are formulated by introducing the parameters describing the *granular temperature* and the *dispersive pressure*. After an evaluation of the model parameters, some parametric results are developed. The comparisons among solutions obtained according to the General (G-M), Coulomb (C-M) and Voellmy Models (V-M) are then shown. The schematic back analysis of a well described avalanche is carried out through the G-M and a comparison among solutions of G-M, Rickenmann's empirical formula (1999) and Corominas's results (1994) of analyses are finally developed.

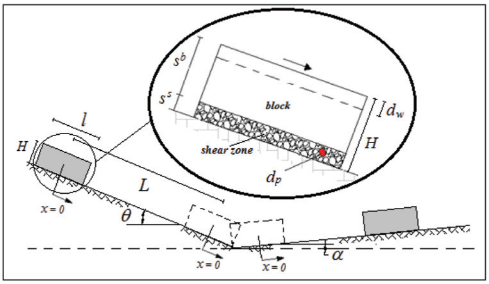


Fig. 1 - Problem's setting and reference systems. The origin (x = 0) of each reference system coincides with the projection of the position of the gravity centre of the sliding mass along the sliding surfaces

BASIC ASSUMPTIONS

• The granular sliding body is composed by two masses of equal basal area Ω and length l; they represent respectively the "*shear zone*" (thickness s^s) and the superimposed mass (thickness s^b,"*block*"). The total height of the sliding mass is $H = s^b(t) + s^s(t)$. The global geometry (Ω, l and H) does not change; erosion or deposition processes are neglected.

• The "*shear zone*" is composed by particles that, moving at high velocity and colliding each with others, induce appreciable fluctuations of their velocities (granular temperature); the "*block*" is dominated by inertial forces and quasi-static stress.

• The sum of the masses of the shear zone (m^s) and the overlying block (m^b) equals the total sliding mass m_0. Both masses vary during the sliding and may change their volume:

$$m^s(x(t)) = \rho^s s^s(x(t))\Omega \qquad m^b(x(t)) = \rho^b s^b(x(t))\Omega \qquad m_0 = m^s(x(t)) + m^b(x(t)) \quad (3)$$

ρ^s, ρ^b simply assume constant values although it is possible to define their dependence upon the sliding rate.

• The thicknesses $s^b(x(t))$, $s^s(x(t))$ are not a priori known along the travelled distance x, at time t. Their values may be obtained by imposing the equilibrium in the direction orthogonal to the sliding planes: the resulting N_{tot} (= $W\cos\zeta$, $\zeta=\theta$ or α, Fig. 1) must be balanced by the lithostatic stresses, σ_{lit}, as well as by the dispersive pressures, p_{dis}, introduced by Bagnold (1954). The equilibrium equation is therefore written as follows:

$$W\cos\zeta = \bar{r}(\dot{x})\sigma_{lit}\Omega + r(\dot{x})p_{dis}\Omega \quad (4)$$

being:

$$\sigma_{lit} = \rho^b g H \cos\zeta \quad (5)$$

$H=s_0^b$, *block* initial thickness;
the functions $\boldsymbol{r}$ and $\bar{\boldsymbol{r}}$ are defined to allow a rational splitting of the force N_{tot} between the lithostatic force and the resultant of the colliding forces. They are written as a function of the rate of the sliding masses($\dot{x}$):

$$r(\dot{x}) = \frac{1}{\pi}\{\operatorname{atan}[\eta(\dot{x} - v_{crit})] - \operatorname{atan}[-\eta(\dot{x} + v_{crit})]\} \quad (6)$$

$$\bar{r}(\dot{x}) = 1 - r(\dot{x}) \quad (7)$$

being:

η, parameterϵ[0.005,0.5];
$v_{crit,}$ the critical value of the speed for which the regime dominated by the inertial forces turns towards a regime governed by the collisions.
By recalling the expression [4], it is obtained:

$$s^b(\dot{x}) = \bar{r}(\dot{x})s_0^b + r(\dot{x})Q_1\lambda^2\dot{x}^2 \tag{8}$$

$$s^s(\dot{x}) = \frac{m_0 - \rho^b s^b(\dot{x})\Omega}{\rho^s\Omega} \tag{9}$$

being:

$$Q_1 = \frac{a_i\rho_s d_p^2 cos\phi}{\rho_b g\ cos\zeta} \tag{10}$$

• The angle ζ of the slope may assume only two values: $\zeta = \theta$, runout; $\zeta = \alpha$, runup. Therefore, the total length traveled by a high speed sliding granular mass is obtained through the analysis of three sliding phases. *(I):* the granular mass runs along the first slope (θ and length L are assigned) and progressively accelerates; *(II)* intermediate section: the granular mass runs at the same time along both slopes (θ and α, see Fig. 1); *(III):* the granular mass runs only along the counterslope (α); its speed decreases up to stop.

ENERGY AND POWER BALANCES

The energy balance of the sliding mass is expressed by the equation:

$$E(t) = E_p(t) + E_k(t) + E_{coll}(t) + E_{gt}(t) + E_{fr}(t) = E_{p,0} \tag{11}$$

$E_{p,0,}$ initial potential energy; E_p, potential energy; E_k, kinetic energy of the sliding mass; E_{fr}, energy lost due to the (Coulomb's) friction along the sliding surface; E_{coll} and $E_{gt,}$ energies transferred from the "*block*" to the basal "*shear zone*" to support the grain inertial regime. Deriving the eq. [11], the Power Balance is obtained:

$$\dot{E}_p(t) + \dot{E}_k(t) + \dot{E}_{fr}(t) + \dot{E}_{coll}(t) + \dot{E}_{gt}(t) = 0 \tag{12}$$

POTENTIAL AND KINETIC ENERGY: E_P AND E_K

The potential energy is expressed as follows (*b = block; s = shear zone):*

$$E_p = E_p^b + E_p^s \tag{13}$$

The corresponding power is obtained deriving [13]:

$$\dot{E}_p = \dot{E}_p^b + \dot{E}_p^s \tag{14}$$

First slope:

$$E_p^b = m^b(t)g\left[(L - x(t))sin\theta + \left(s^s(t) + \frac{s^b(t)}{2}\right)cos\theta\right] \tag{15}$$

$$\dot{E}_p^b = \rho^b\Omega g\left\{\dot{s}^b(t)\left[(L - x(t))sin\theta + \left(s^s(t) + \frac{s^b(t)}{2}\right)cos\theta\right] + s^b(t)\left[-\dot{x}(t)sin\theta + \left(\dot{s}^s(t) + \frac{\dot{s}^b(t)}{2}\right)cos\theta\right]\right\} \tag{16}$$

$$E_p^s = m^s(t)g\left[(L - x(t))sin\theta + \left(\frac{s^s(t)}{2}\right)cos\theta\right] \tag{17}$$

$$\dot{E}_p^s = \rho^s\Omega g\left\{\dot{s}^s(t)\left[(L - x(t))sin\theta + \left(\frac{s^s(t)}{2}\right)cos\theta\right] + s^s(t)\left[-\dot{x}(t)sin\theta + \left(\frac{\dot{s}^s(t)}{2}\right)cos\theta\right]\right\} \tag{18}$$

Counterslope:

$$\dot{E}_p^b = \rho^b\Omega g\left\{\dot{s}^b(t)\left[\left(x(t) + \frac{l}{2}\right)sin\alpha + \left(s^s(t) + \frac{s^b(t)}{2}\right)cos\alpha\right] + s^b(t)\left[\dot{x}(t)sin\alpha + \left(\dot{s}^s(t) + \frac{\dot{s}^b(t)}{2}\right)cos\alpha\right]\right\} \tag{19}$$

$$\dot{E}_p^s = \rho^s\Omega g\left\{\dot{s}^s(t)\left[\left(x(t) + \frac{l}{2}\right)sin\alpha + \left(\frac{s^s(t)}{2}\right)cos\alpha\right] + s^s(t)\left[\dot{x}(t)sin\alpha + \left(\frac{\dot{s}^s(t)}{2}\right)cos\alpha\right]\right\} \tag{20}$$

Transition zone:

$$E_{p,\theta}^b = \rho^b\Omega(t)g\left[\frac{l_1(t)}{2}sin\theta + \left(s^s(t) + \frac{s^b(t)}{2}\right)cos\theta\right] \tag{21}$$

$$E_{p,\theta}^s = \rho^s\Omega(t)g\left[\frac{l_1(t)}{2}sin\theta + \left(\frac{s^s(t)}{2}\right)cos\theta\right] \tag{22}$$

$$E_1 = E_{p,\theta}^b + E_{p,\theta}^s \tag{23}$$

$$E_{p,\alpha}^b = \rho^b\Omega(t)g\left[\frac{l_1(t)}{2}sin\alpha + \left(s^s(t) + \frac{s^b(t)}{2}\right)cos\alpha\right] \tag{24}$$

$$E_{p,\alpha}^s = \rho^s\Omega(t)g\left[\frac{l_1(t)}{2}sin\alpha + \left(\frac{s^s(t)}{2}\right)cos\alpha\right] \tag{25}$$

$$E_2 = E_{p,\alpha}^b + E_{p,\alpha}^s \tag{26}$$

In the proposed model, to simplify the numerical solution, it is directly assumed a linear combination on powers of energies:

$$\dot{E}_{12} = \dot{E}_1\frac{l_1(t)}{l} + \dot{E}_2\frac{l_2(t)}{l} \tag{27}$$

being

$$l_1(t) = l - x(t) \tag{28}$$

$$l_2(t) = x(t) \tag{29}$$

The kinetic energy and the power of the kinetic energy are expressed as follows:

$$E_k = E_k^b + E_k^s \tag{30}$$

$$\dot{E}_k = \dot{E}_k^b + \dot{E}_k^s \tag{31}$$

$$\dot{E}_k^b = \frac{1}{2}\rho^b\Omega(\dot{s}^b\dot{x}^2 + 2s^b\dot{x}\ddot{x}) \tag{32}$$

$$\dot{E}_k^s = \frac{1}{2}\rho^s\Omega(\dot{s}^s\dot{x}^2 + 2s^s\dot{x}\ddot{x}) \tag{33}$$

EFFECTS OF COLLISIONS

E_{gt} , $E_{coll.}$ The energy E_{gm} transferred to the "*shear* *zone*" is partly lost due to repeated grain inelastic collisions *(E_{coll})* and partly stored as granular temperature (E_{gt}). ZHANG & FODA (1997) have shown that the power of the energy lost in granular collisions is related to the *granular temperature* (T_g) according to the relation: $E\ coll \sim T_g^{(3/2)}$. OGAWA (1978) observed that the energy stored in the grain-inertial regime is proportional to the *granular temperature*: $E_{gt} \sim T_g$.

Granular temperature, in turn, is proportional to the mean velocity of the grains composing the shear zone (SAVAGE & JEFFREY, 1981), according to the relation: $T_g \sim x^2$. These relations are respectively applied to formulate the powers of energies lost due to grain inelastic collisions (E_{coll}) as well as stored in granular temperature (E_{gt}):

$$\dot{E}_{coll} = M\dot{x}^3 \qquad (34)$$

$$\dot{E}_{gt} = K\dot{x}\ddot{x} \qquad (35)$$

being, according to previous analyses (FEDERICO & FAVATA, 2011):

$$K = \frac{k}{k-1} m_0 \qquad (36)$$

$$k = \frac{\dot{E}_{coll}}{\dot{E}_{gt}} = \frac{1}{4}(1-e^2)\beta^2; \quad k \in [0,1] \qquad (37)$$

$$M = 8(1-e^2)\left(\frac{\rho_s \upsilon_s d_p^2}{\left(\varpi^3 (s^s(t))\right)^2}\right) G(\upsilon_s)\Omega \qquad (38)$$

e being the restitution coefficient (ϵ[0,1]); ω, ZHANG-FODA coefficient; *β*, coefficient *ϵ[0,2]* and υ_s being the solid franction; d_p, the characteristic diameter of the grains, ρ_s, the solid phase density

$$G(\upsilon_s) = \frac{1-0.5\upsilon_s}{(1-\upsilon_s)^3} \qquad (39)$$

INTERSTITIAL PRESSURES

The interstitial pressure p_w(x) at the base of the mass affects the friction dissipated energy. Isopiezic lines are assumed orthogonal to the motion direction and p_w (x) is assumed constant along the planar sliding surface (IVERSON, 1997):

$$p_w = \gamma_w (H - d_w)\cos\theta \qquad (40)$$

γ_w is the specific weight of the water; d_w= 0 if the mass is saturated; d_w=*H* if the mass is dry; p_w may exceed the hydrostatic value due to the mechanical effects associated with the rapid change of pore volumes, corresponding growth of interstitial water pressures excess (MUSSO *et alii*, 2004). To simulate this effect, d_w< 0 values must be assigned. The length d_w varies in the range: $d_{(w,min)} \le d_w \le d_{(w,max)}$. If the sliding granular mass always transfers positive normal stresses to the basal surface, the minimum value $d_{w,\min}$ can be deduced by imposing the equilibrium along the direction perpendicular to the sliding surface:

$$d_{w,min} = H\left(1 - \frac{\gamma_s}{\gamma_w}\right) \qquad (41)$$

γ_s is the unit weight of the sliding mass. The interstitial pressure's resultant is expressed as follows:

$$U = \bar{r}(\dot{x})\, p_w\, \Omega \qquad (42)$$

THE ROLE OF BASAL FRICTION

The role of basal friction**:** *Efr***.** *Efr* is a function of the weight W of the sliding mass, the dynamic friction angle Φ_b at the base of the block. Dispersive and interstitial pressures reduce the friction energy dissipation.

By including the effect of interstitial pressures, the basal friction resistance is expressed as follows:

$$T_{fr,w} = (W\cos\zeta - U)\tan\varphi_b \qquad (43)$$

φ_b assumes a constant value along the slopes. The power related to the energy dissipated due to the friction along the sliding basal surface is:

$$\dot{E}_{fr} = \frac{d}{dt}\int_0^x T_{fr}dx = (W\cos\zeta - U)\tan\varphi_b\,\dot{x} = (\rho_b g s^b(\mathrm{t})\Omega\cos\zeta - \bar{r}(\dot{x})p_w\Omega)\tan\varphi_b\,\dot{x} \qquad (44)$$

RESULTS OF PARAMETRICAL ANALYSES

The proposed model depends on few parameters pertaining to the micromechanical behaviour: *β (ϵ[0,2])*, e (restitution coefficient *ϵ[0,1]*), *k* (in *Ėgt*'s expression [36]) and d_p (grain diameter). First of all, by changing arbitrarily the parameters *e* and *β*, we investigate the effect of the parameter k, for the assigned values of remaining parameters: $L=1000\ m;\ \theta=38°;\ \alpha=0°,\ \Phi_b=18°,\ l=300\ m;\ \Omega=15000\ m^2;\ H=35\ m;\ d_p = 0.1\ m;$ interstitial pressure resultant $U \neq 0\ (d_w = 0)$. In Table 1, the set of selected values is shown

-	e	k	β
β = 1.75	0.3	***0.7***	-
	0.5	***0.57***	-
	0.7	***0.39***	-
e = 0.3	-	***0.51***	1.5
	-	***0.23***	1
	-	***0.06***	0.5

Tab. 1 - Assigned values to the parameters e, β, k

Results in terms of the (runout and runup) total length (*x*), *block* and *shear zone* thicknesses (s^b, s^s), collisional energy (E_{coll}) and energy related to granular temperature (E_{gt}), for different values of the parameter k are shown in the figures 2, 3, 4 and 5.

The thicknesses of the *block* and the '*shear zone*' slightly change if the parameter k increases (Fig. 2). Collisional energy E_{coll}, set the coefficient of restitution *(e = 0.3)*, to decrease of the parameter k (and therefore of β), increases, while holding almost unchanged, to the decrease of k, fixed the coefficient β (Fig. 3).

The energy associated with granular temperature E_{gt} decreases if k decreases (Fig. 4). It is worth observing that if the parameters e, β provide small values of k, the sliding mass reaches unrealistic high speeds (more than 40 m/s), usually obtained by Coulomb Model. Fixed parameter β, a decrease of k (and thereby the increase of e) gets an increase of both maximum speed and runout length, while fixed e, if k decreases, the runout length decreases too (Fig. 5).

Effect of the parameter *dp*. (the parameters e, β and k are: $e = 0.3$; $\beta = 1.75$; $k = 0.7$).

The following values of the parameter d_p are assigned: d_p=0.05 m; d_p=0.10 m; d_p = 0.15 m). The results are reported in terms of runout length, block and shear zone thicknesses, collisional energy and energy related to granular temperature, according to d_p.

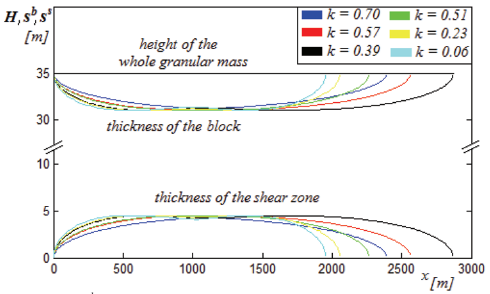


Fig. 2 - Curves [sb(x(t)), x(t)];[ss(x(t)), x(t)];)];[H, x(t)] for different values of parameter k

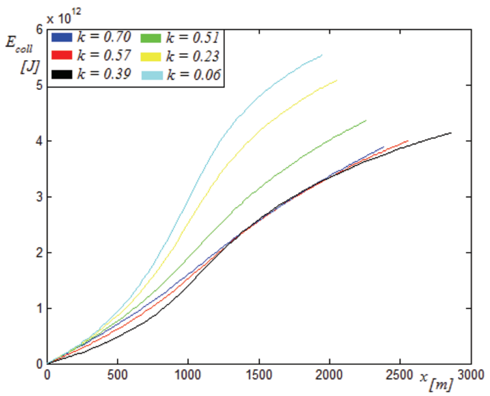


Fig. 3 - Collisional energy for different values of k

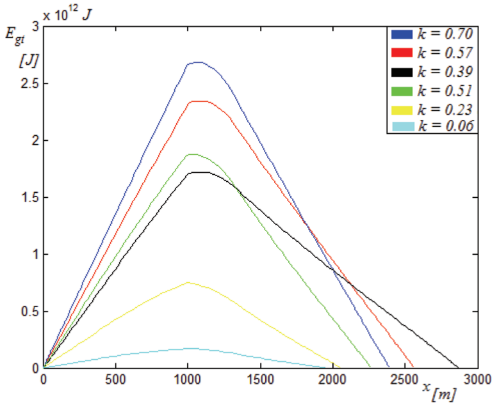


Fig. 4 - Energy E_{gt} related to the granular temperature T_g, for different values of parameter k

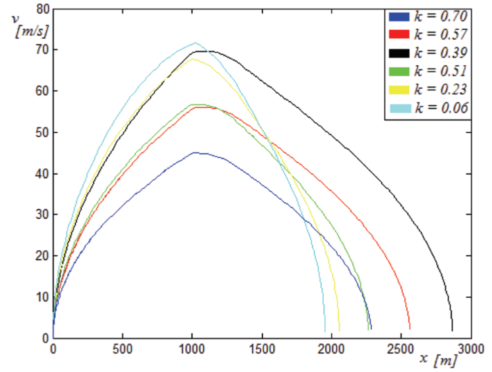


Fig. 5 - Rate v of the sliding granular mass, for different values of parameter k

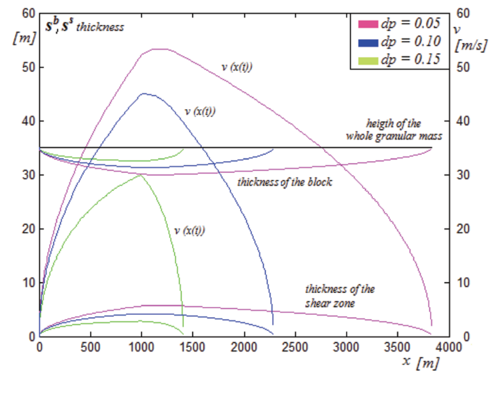


Fig. 6 - Curves [s^b(x(t)), x(t)]; [s^s(x(t)), x(t)]; [v, x(t)] for different values of parameter d_p

If the diameter d_p decreases, the thickness of the '*shear zone*' becomes smaller; as a result, being the sum of s^b and s^s equal to the height H of the debris flow (constant value), the thickness of the sustained block increases (Fig. 6). Instead, the collisional energy, to an increase of d_p, increases (Fig. 7). If d_p increases, the distance traveled, the maximum speed (Fig. 6) and the energy associated with granular temperature E_{gt} (Fig. 7) decrease.

COMPARISON WITH RESULTS OBTAINED ACCORDING TO CONVENTIONAL RHEOLOGICAL MODELS

Results obtained through the General-Model (G-M) are compared with solutions of Coulomb-Model (C-M) and Voellmy-Model (V-M). The motion equations for the V-M and C-M models are given by the ODE:

$$W \sin\zeta - W\cos\zeta \tan\varphi_{b,w} - \bar{M}\dot{x}^2 - m\ddot{x} = 0 \quad (45)$$

$\bar{M}=0$ describes the Coulomb Model (C-M); $\bar{M}\neq 0$, the Voellmy Model (V-M). The $\bar{M}$ parameter can be related to the ξ turbulence coefficient of Voellmy through the equation (FEDERICO & FAVATA, 2011):

$$\bar{M} = \frac{\gamma_s}{\xi} \quad (46)$$

γ_s being the specific weight of the bulk mass. The Voellmy's turbulent component of resistance describes, in a schematic manner, the energy dissipation due to granular collisions. Therefore, in the Voellmy Model, the term $\bar{M}x^2$, describes the energy dissipation in granular collisions (E_{coll}). For a better comparison between the results obtained through the models (G-M, C-M, V-M), the coefficient $\bar{M}$ has been determined on the base of the coefficient M of the General Model, replacing $s^s(t)$ with $(s^s_{max}/2)$:

$$\bar{M} = 8(1-e^2)\left(\frac{\rho_s \upsilon_s d_p^2}{(\varpi^3 (s^s_{max}/2)^2}\right) G(\upsilon_s)\Omega \quad (47)$$

If the [47] applies, the energy dissipated due to collisions is almost equal for both G-M and V-M models.

The following values of the parameters are assigned: *θ = 30°, α = 0°; L = 1000 m; γ_w=10 kN/m³; ρ^b=2105 Kg/m³; d_w = 0; H = 25 m; V = 18750 m³; m = 4•10⁷ kg; k = 0,7; φ_b = 18°; d_p = 0.05 m; Ω = 750 m²; l = 100 m; e = 0.3; β = 1.75; in V-M, $\bar{M}$=8 x 10⁴ Kg/m; in C-M, $\bar{M}$ = 0 kg/m* is assumed. The figures 8, 9, 10 and 11 show the G-M, V-M and C-M results.

C-M model gets *run out* and velocity values greater than G-M and V-M models (Fig. 8); the reduction of the sliding rate is caused by the additional shear resistance due to grains dissipation. In Fig. 9 a comparison among the models is shown in terms of traveled distance. In G-M model, the duration of the motion is greater than in the others cases. In Fig. 10, the energies concerned with the G-M model are shown; the initial potential energy partly becomes kinetic energy, partly is stored as granular temperature, partly is dissipated owing to grains collisions and friction sliding. In Fig. 8, the block and '*shear zone*' thicknesses concerned with the General Model are shown. It is further investigated the influence of the volume *V* of the sliding mass on the total runout length, for some assigned values of parameters: *L=1630 m; θ=30°; α=-15°, φ_b=18°, d_p = 0.05 m; d_w = 0, γ_w=10 kN/m³; ρ^b=2105 Kg/m³*. Results of computations are shown in Fig. 11. C-M run out doesn't depend on the volume of the sliding mass. In G-M and V-M, the solutions depend on the granular volume; for small value of *V*, the run out

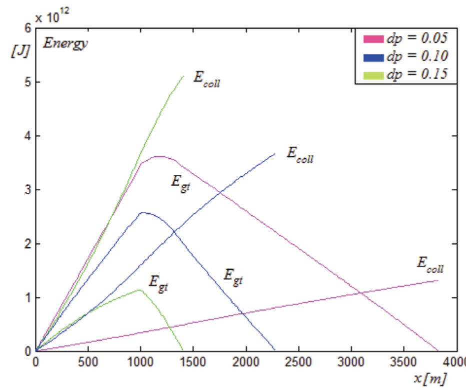


Fig. 7 - Collisional energy (E_{coll}) and energy related to the granular temperature (E_{gt}) for different values of dp

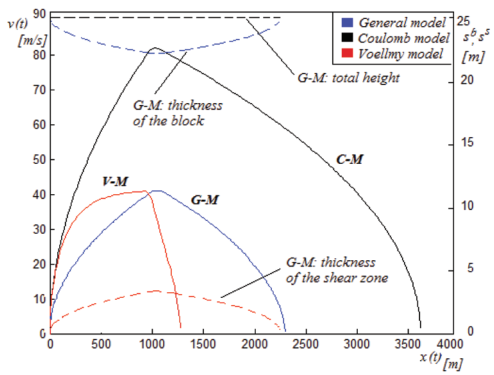


Fig. 8 - Curves [v(t), x(t)]; [s^b(x(t)), x(t)]; [s^s(x(t)), x(t)]: comparison between General Model, Coulomb Model and Voellmy Model

length increases significantly with the volume *V*; for great values of *V*, G-M and V-M runout length tend to the C-M runout and the travelled distance progressively becomes almost independent on the sliding mass volume.

BACK ANALYSES

Through the proposed model, the Frank slide is first back analysed; the comparison among solutions of G-M, Rickenmann's empirical formula and Corominas's analyses are then carried out.

Frank slide. The Frank slide occurred on the morning April 29, 1903, in the south western Alberta (British Columbia, Canada). The original unstable rock mass volume was estimated as 30x10^6 m^3. The debris moved down from the east face of Turtle Mountain across the entrance of the Frank mine, the Crowsnest River, the southern end of the town of Frank, the main road from the east, and the Canadian Pacific mainline through the Crowsnest Pass.

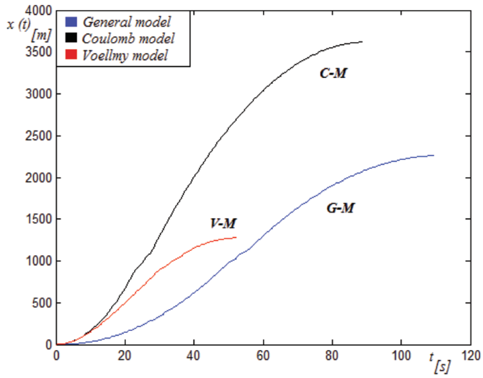


Fig . 9 Curves [x(t), t]; comparison between General Model, Coulomb Model and Voellmy Model

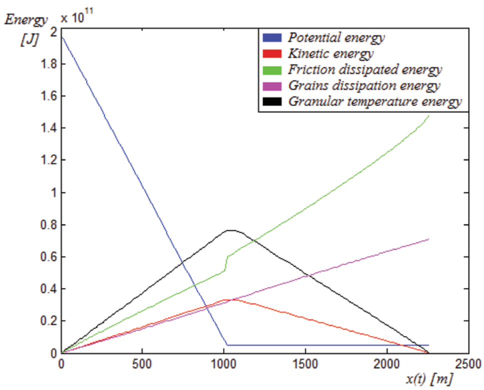
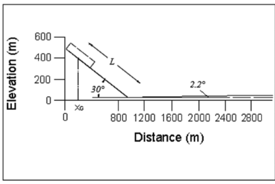


Fig. 10 - Energies concerned with G-M solution

The separated rock mass has been shattered by impacts against the side of the mountain during its sliding, and probably long before it reached the bottom, into myriads of fragments, some of which were flung far out into the valley. Immediately after the slide, an inspection was made by the Geological Survey of Canada: the slide occurred across rather than along bedding planes and the primary cause for the slide was found in the structure of the mountain. Water action in summit cracks and severe weather conditions also contributed to the disaster. In Table 2, the input parameters obtained from conventional back analysis of the event (Cruden & Hungr, 1986) are shown.

The runout length (x) and the rate of sliding mass (v), for the assigned parameters $d_p = 0.05$ *m;* $d_w = 0$*;* $\gamma_w = 10$ *kN/m*^{3}*;* $\rho^b = 2105$ *Kg/m*^{3}*;* $e = 0.3$*;* $\beta = 1.75$ are shown in figures 12, 13; in V-M, $\bar{M} = 1.5 \times 10^7$ *Kg/m; in C-M,* $\bar{M} = 0$ *kg/m* is assumed. The maximum speed and the runout length computed according to the Coulomb Model appear remarkably greater than the corresponding values obtained through the G-M and V-M models (Fig. 12). The Voellmy Model gets values of runout less than the values obtained by the General Model (Fig. 13) although the corresponding rate appears too high. The



GEOMETRIC PARAMETERS	VALUES
first slope angle	30°
second slope angle	2.2°
length of first slide	920 m
debris flow length	520 m
debris flow height	75 m
debris flow base area	400000 m^2
MECHANICAL PARAMETERS	-
dynamic friction angle	18°
parameter k	0.7
RUN OUT LENGTH	2800 m

Tab. 2 - Frank slide. Input parameters for back analysis

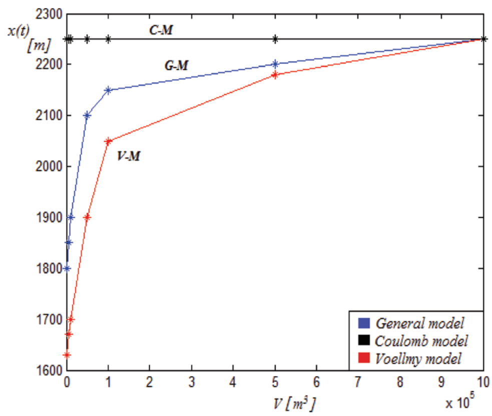


Fig. 11 - Curves: [x(t), V]; comparison between General Model, Coulomb Model and Voellmy Model

General Model gets the runout length greater than that one computed through the Voellmy Model and close to that one in situ observed. The time interval of the motion is about 120 s for the General Model and about 70 s for the Voellmy Model (Fig. 13). The larger time interval for the G-M derives from the stored energy as 'granular temperature' that sustained the runup phase, allowing a more gradual reduction of the sliding rate, if compared to the reduction pertaining to the V-M or C-M models.

EMPIRICAL RELATIONSHIPS.

A comparison between the General-Model results and runout lengths obtained through some empirical relationships proposed in literature is carried out. After back analyses on "*160 debris flows*", Rickenmann (1999) proposed an empirical relation to estimate the length D (horizontal distance traveled by sliding

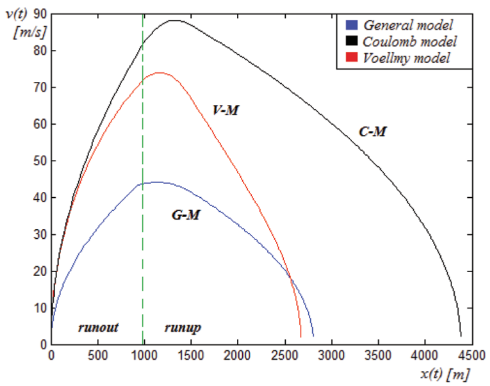


Fig. 12 - Frank slide back analysis. Curves [v(t), x(t)] obtained through the General, the Coulomb and the Voellmy Models

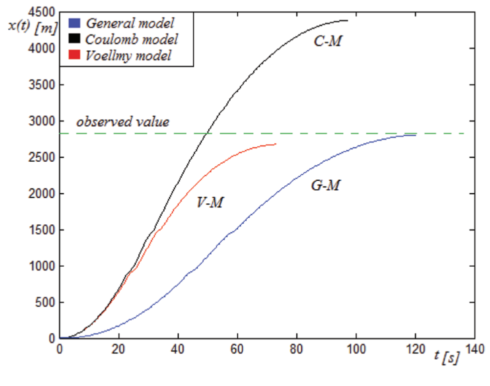


Fig. 13 - Frank slide back analysis. Curves [x(t), t] obtained through the General, the Coulomb and the Voellmy Models

mass):

$$D = 1{,}9\, V^{0{,}16} H^{0{,}83} \tag{48}$$

being: V, volume of the mass; H, difference in elevation (see Fig. 14).

The geometry of the problem is fixed and the following values of the parameters are assigned: $\theta = 38°$, $\alpha = 0°$; $L = 1000\ m$; $\gamma_w = 10\ kN/m^3$; $\rho^b = 2105\ Kg/m^3$; $d_w = 0$; $\varphi_b = 18°$; $e = 0.3$; $\beta = 1.75$. By applying the (G-M) model, different values of parameters H, d_p, e, l, and Ω are assigned. The Fig. 14 shows the (runout and runup) total length computed as a function of the volume V. Following a suitable choice of the assigned parameters, the travelled length estimated through the General-Model is quite close to that one estimated by the empical formula [48]. While the empirical formula for runout length depends explicitly on the volume V, in the G-M model, the runout length depends on the volume through the parameters H, d_p, e, l, and Ω.

Scheidegger (1973) correlated the volume V of a sliding mass to the dimensionless variable f, this one defined as the tangent of the angle Ψ (Fig. 15) formed by the horizontal line and the straight line joining the point of greatest potential energy of the system under static conditions and the lower end of the debris flow, after its arrest. The Scheidegger s relationship is expressed as follows

$$f = 10^{C_1 + C_2 \log V} \tag{49}$$

The constants C_1 and C_2 are determined through interpolation of data on real landslides. After a careful back analysis on several debris flows, Corominas (1994) obtained the following values: $C_1 = -0.034$; $C_2 = -0.101$.

By interpolating values of f obtained from the val-

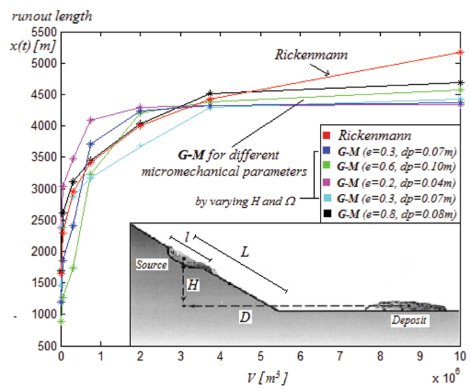


Fig. 14 - Curves [x,V]; comparison between General Model, and RICKENMANN's relationship

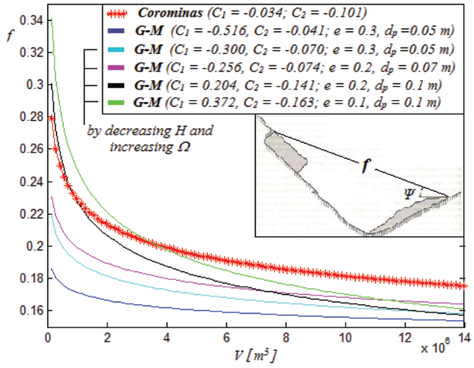


Fig. 15 - Curves [f, V]; comparison between General Model and COROMINAS's results

ues of the runout length related to cases analyzed with the General-Model, for different values of the micromechanical parameters e and d_p, the results shown in Fig. 15 are obtained.

In Fig. 15, the values of the parameter f as a function of the volume V related to the cases analyzed with the G-M and the results obtained through the [49] are shown (Cesali, 2013). An appreciable agreement between these results is observed. At the same time, the role of micromechanical parameters is highlighted; their influence on the definition of empirical relationship cannot be neglected, although it is not easily knowable, since the values of the runout length, (and of the parameter f), were obtained by varying other parameters such as H and Ω (height and basal area of the debris flow). In particular, by decreasing e and increasing d_p, thanks to a suitable choice of H and Ω, the trends obtained by the G-M approximates the trend obtained by Corominas (1994).

CONCLUDING REMARKS

An original analytical model (*General Model*) based on energy-balance equations and some simplified assumptions, to estimate the runout length of granular debris flow or avalanches, is proposed. Hypotheses concern the geometry of the sliding mass (parallelepipedal shape), sliding surface (planar surfaces) and energy dissipation (friction, collisions). The model takes into account several experimental results reported in technical literature. Specifically, in a granular material sliding at high rate along a basal surface, a thin (*'shear'*) zone, with variable thickness, whose behaviour is characterized by a regime dominated by the presence of collisions, hosting the "granular temperature" phenomenon, generates and develops in proximity of the basal surface. The material composing the *shear zone* exchanges energy and mass with the remaining upper material (*block*), characterized by a regime dominated by inertial forces. Through the balance of the involved mechanical powers, the travelling of the granular mass along the planar surfaces is describes by a system of ODE, that have been numerically integrated. Parametric analyses allowed to identify the role of geometrical and mechanical parameters, such as the diameter of grains (d_p), the coefficient of restitution (e) and the parameter k. Finally, comparisons between the results obtained through the General Model, the Coulomb and Voellmy Models, as well as the back analysis of a case and a critical examination of well known empirical relationships are shown. The main limits of the proposed model lie in the oversimplified geometry of the debris body, in the assumption of constant total mass, in definition of the micro-mechanical parameters.

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