DOI: 10.4408/IJEGE.2011-03.B-055

DERIVATION OF CRITICAL RAINFALL THRESHOLDS FOR DEBRIS FLOW WARNINGS THROUGH MATHEMATICAL AND NUMERICAL MODELING

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ABSTRACT

The aim of the work is to develop a system capable of providing debris flow warnings in areas where historical events data are not available as well as in the case of changing environments and climate. For these reasons, critical rainfall threshold curves are derived from mathematical and numerical simulations rather than the classical derivation from empirical rainfall data. The possible formation of debris flow is simulated through infinite-slope stability analysis. Land instability is governed by the increases of groundwater pressures due to rainfall. The simulations are performed in a virtual basin, representative of the one studied, taking into account the uncertainties linked with the definition of the characteristics of the soil. A large number of calculations are performed which take into account the entire range of the governing input dynamic variables (rainfall characteristics) and different combinations between them. The dynamic variables considered are the antecedent rainfall, the intensity of the triggering rainfall and its duration. The multiple combinations of the input dynamic variables giving failure is therefore obtained. For each failure, the corresponding debris flow volume is estimated. The resulting database is elaborated in order to obtain rainfall threshold curves. These curves may be used for the real time evaluation of possible debris flow events on the basis of observed and forecasted rainfalls.

Key words: *debris flows, warning, critical rainfall thresholds*

INTRODUCTION

Debris and mud flow events may affect vulnerable areas causing a major natural risk. In some areas, the morphological, geological and climatic conditions leading to debris flow formation are quite widespread and extensive downstream areas result being prone to debris flow risk. In these cases, the risk reductions through the building of structural countermeasures may not only be too expensive but also create environmental concern. Moreover, in some cases, the rigid topography of the interested areas, or the lack of space, makes it difficult to find engineering design and construction countermeasure solutions.

For these reasons in many cases, non structural countermeasures, such as warnings through real time hazard assessment and civil protection measures are more suitable in reducing the risks.

The most common approach adopted in literature for real time debris flow hazard assessment (Wieczorek & Glade, 2005) is based on empirically derived rainfall thresholds. These approaches are based on the observations of past events, with the derived rainfall thresholds therefore depending on the particular characteristics of the basin from which they have been derived, and their application on different basins possibly giving incorrect results. This means that, theoretically, these approaches may be adopted only for those basins where a certain amount of observed debris flow events is available for the derivation of the threshold line.

Another drawback of empirically derived thresholds is that they cannot anticipate how debris flow hazards may change in response to changing climate, land use or large forest fires.

In order to overcome these limitations, it is necessary to obtain a debris flow warning system through a model that reflects the physics of the phenomenon. Such a system should provide the link between rainfall and possible debris flow hazards. The system should be based on simple rules in order to be fast enough to make possible real time applications.

Theoretical models of rainfall triggered debris flows are based on the infinite-slope stability analysis in which land instability is governed by the increase in groundwater pressure due to rainfall.

These models are usually implemented in discrete landscape cells and give the security factor for each one. The implementation of these models requires an accurate characterization of the spatial distribution of the soil properties, which, in many practical applications, is not available.

Many of the approaches proposed in literature are based on the hypothesis of steady groundwater flow conditions (Montgomery & Dietrich, 1994).

A simplified model that accounts for transient groundwater flow conditions has been proposed by Iverson (2000).

Comparisons with observed scares of debris flow formation areas (Godt *et alii*, 2008) have shown that accounting for the transient effects of rainfall infiltration on pore water response and consequent effects on slope stability improves the effectiveness of regional shallow landslide hazard maps.

However, the number of false positives and false negatives in the predicted unstable cells is still far higher than it should be for a wide and safe application of the method.

The implementation of a distributed model, based on the stability analysis for each grid cell of the basin, is not feasible in the case of warnings due to the long running time required for this kind of model as well as the lack of detailed information on the spatial distribution of the properties of the material in many practical cases.

Moreover, with the aim of giving debris flow warnings, it is not necessary to know the distribution of instable elements along the basin but only if a debris flow may affect the vulnerable areas in the valley. The capability of a debris flow of reaching the downstream areas depends on many factors linked with the topography, the solid concentration, the rheological properties of the debris mixture and the flow discharge as well as the occurrence of liquefaction of the sliding mass. In relation to a specific basin, many of these factors may be considered as not time dependent. The most rainfall dependent factors are flow discharge and correlated total debris volume. In the present study, the total volume that is instable, and therefore available for the flow, is considered as the governing factor from which it is possible to assess whether a debris flow will affect the downstream areas or not.

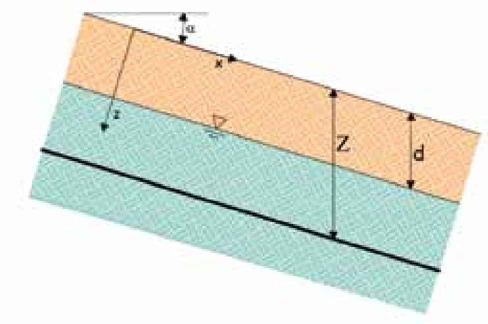


Fig. 1 - Schematic of the slope stability model

The approach presented is based on the simulation of a large number of cases covering the entire range of the governing input dynamic variables (rainfall characteristics), considering the different possible combinations between them. For any possible combination of rainfall intensity, duration and antecedent rain, the total debris volume, available for the flow, is estimated. The resulting database is elaborated in order to obtain rainfall threshold curves. When operating in real time, if the observed and forecasted rainfall exceeds a given threshold, the corresponding probability of debris flow occurrence may be estimated. Warning for possible debris flow occurrence may be given congruently with such result.

SIMULATION METHODOLOGY

MATHEMATICAL MODEL

The proposed approach is based on the simulation of a large number of possible debris flow occurrences, depending on different values of the driving input (rainfall characteristics). Consequently the model to assess possible land instability must be extremely fast. Another point is that, in practical cases, the availability of data about the characteristics of the soil is quite scarce.

For these reasons, the selected simulation methodology is extremely simplified, thus the requested data is limited and the computational time is quite short.

The possible triggering debris flow is simulated, in a generic element of the basin, by an infinite slope stability analysis (Iverson, 2000; Taylor, 1948). At any depth from the surface (Z), and at any time (*t*), the factor of safety (*FS*) is computed by the ratio between the resisting Coulomb friction and the driving stresses induced by gravity:

$$FS(Z,t) = \frac{tan\varphi}{tan\alpha} + \frac{c - \psi(Z,t)\gamma_w tan\varphi}{\gamma_s Z sin\alpha cos\alpha} \tag{1}$$

where *α* is the slope degree, Z is the vertical coordinate, positive downward, *c* is the soil cohesion,*φ*is the angle of internal friction, γ_s is the depth averaged soil unit weight, γ_w is the unit weight of ground water and *ψ*(Z,t) is the ground water pressure head that depends on depth and time (t).

When a critical value of *FS* is reached (e.g. Fs=1) the soil over the *Z* depth is considered instable.

Many observed debris flow events have been triggered by a long term and low intensity rainfall followed by a short term-heavy rainfall (Crozier, 1989; Wieczorek & Glade, 2005). As a consequence, the triggering groundwater pressure is calculated by the superposition of the effect of an "antecedent" rainfall and an "event" rainfall. The groundwater pressure response to antecedent rainfall is used as the initial condition for the time-dependent computation of the groundwater pressure response to the event rainfall.

If the antecedent rainfall has a sufficiently low intensity and long duration, the steady state conditions are reached and the direction of the groundwater flux may be assumed to be slope parallel. Under this condition, the ground water pressure may be calculated by:

$$\psi(Z,0) = (Z - d)cos^2\alpha \tag{2}$$

where *d* is the water table depth, measured in the Z direction, in steady state conditions. Following Montgomery & Dietrich (1994), the mass conservation equation of groundwater gives the following:

$$Z_T - d = \frac{(I_z)_{steady}}{K_x} \frac{A}{b sin\alpha cos\alpha} \tag{3}$$

where Z_T is the depth of the impermeable bed, (*Iz*) steady is the infiltration rate at ground surface, in the slope normal direction, in steady conditions, K_x is the hydraulic conductivity in the slope parallel direction, *A* is the drained catchment, *b* is the width of the slope element along the direction tangent to the local topographic contour.

Following the approach of Iverson (2000), the short term response to rainfall may be assessed in the hypothesis of vertical infiltration. Moving from this assumption, under the hypothesis of wet initial conditions, and with the boundary conditions of transient groundwater vertical flux equal to zero at great depths below the water table and water entry at ground surface governed by Darcy's law, Iverson (2000) proposed an analytical solution of the Richards equation

$$\psi(Z,T) = \psi(Z,0) + Z\frac{I_z}{K_z}[R(T^*)] \tag{4}$$

where *ψ(Z,0)* is the ground water pressure head at the beginning of the event rainfall, I_z is the infiltration rate at ground surface, in the slope normal direction, K_z is the hydraulic conductivity in the slope normal direction and *R(T*)* is defined as follows:

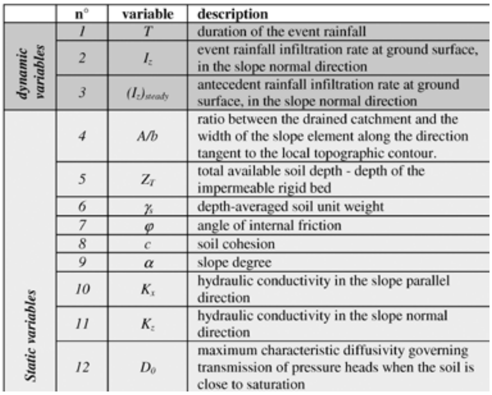
$$R(T^*) = \sqrt{T^*/\pi}\exp(-1/T^*) - erfc\left(1/\sqrt{T^*}\right) \tag{5}$$

in which:

$$T^* = \frac{T}{Z^2/(4D_0 cos^2\alpha)} \tag{6}$$

where D_0 is the maximum characteristic diffusivity, governing the transmission of pressure heads when the soil is near to saturation.

The input variables (Tab. 1) that feed the model presented above are divided into two main families, static and dynamic.



	n°	variable	description
dynamic variables	*1*	T	duration of the event rainfall
	2	I_z	event rainfall infiltration rate at ground surface, in the slope normal direction
	3	$(I_z)_{steady}$	antecedent rainfall infiltration rate at ground surface, in the slope normal direction
Static variables	*4*	A/b	ratio between the drained catchment and the width of the slope element along the direction tangent to the local topographic contour.
	5	Z_T	total available soil depth - depth of the impermeable rigid bed
	6	γ_s	depth-averaged soil unit weight
	7	φ	angle of internal friction
	8	c	soil cohesion
	9	α	slope degree
	10	K_x	hydraulic conductivity in the slope parallel direction
	11	K_z	hydraulic conductivity in the slope normal direction
	12	D_0	maximum characteristic diffusivity governing transmission of pressure heads when the soil is close to saturation

Tab. 1 - List of static and dynamic input variables

MODEL IMPLEMENTATION

The mathematical model described in the previous paragraph can assess possible instability of a basin element with given characteristics (static input variables) subjected to a given rainfall (dynamic input variables).

In order to asses if a specific basin may give place to the formation of a debris flow, the instability simulation, previously described, is performed for a certain amount (n) of computational elements that may represent the behaviour of the entire basin. The computational elements do not correspond to the real basin "pixel" but are virtual elements defined by a string of the static input variables.

The input of the model is therefore a matrix of static variables, representing the studied basin, compiled with the following procedure. The basin is divided into districts, each one with homogeneous geomorphological characteristics and consequently the same value of the soil variables: Z_T $\gamma s, \varphi$, c, K_x and K_z. The maximum characteristic diffusivity D_0 governing the transmission of pressure heads, when the soil is close to saturation, may assume any positive value depending on the closeness to complete saturation. On the basis of this consideration, D_0/Kz has been used as a calibration parameter.

The uncertainties in the evaluation of the soil variables are taken into account assigning to each variable an average value along with a confidence interval. The entity of the confidence interval is decided for each variable depending on the methodologies used for evaluating the variable and the consequent uncertainties in the evaluation results. The assignment of a specific soil variable value to a certain number of input strings follows the normal distribution function of that variable having the assigned average and confidence interval

The topographic variables α and A/b are computed for each basin pixel, with simple GIS instruments elaborating the Digital Elevation Model of the basin. Subsequently, the value of α and A/b are assigned to the strings coherently to the frequency distribution of the values in the real basin.

The number of matrix strings (n) may be set as an input parameter, thus regulating on one hand the computational time and on the other, the representativeness of the input matrix.

The dynamic input variables T, I_z, (I_z)steady are assigned by the definition of the lower and upper values of the range of possible values along with the total number of values for that specific variable, m_i $i \in (1,3)$. The simulation is then performed for each row of the static input matrix as well as for each combination of the dynamic input variables. The total number of simulations (N) is therefore given by:

$$N = n \prod_{i=1}^{3} m_i \qquad (7)$$

For each simulation, the number of instable elements and the corresponding volume of available debris are provided as outputs.

The obtained data are then elaborated, by simple interpolation, in order to build, for each value of antecedent rain $(I_z)_{\text{steady},}$ a graph representing the intensity-duration rainfall curves producing a fixed value of the percentage of instable cells or a fixed value of the total debris volume.

APPLICATION OF THE MODEL TO A STUDY CASE

STUDY AREA

The area under study is a basin on the Amalfi Coast (Sambuco Basin), in the south of Italy. The basin is about 6.3 km^2, delimited by mountains of about 800-1000 m a.s.l, heading prevalently north-south. The conoid of the basin hosts the ancient village of Minori. The outlet of the basin flows into the Mediterranean Sea.

Historical debris flow events are documented in 1910, 1924 and 1954 (Papa & Trentini, 2010).

Morphological and geo-pedological studies have been conducted on the triggering areas of ancient debris flow on the Amalfi Coast, particularly in 1954 and 2005 (Carbone, Iamarino & Gallo, paper in preparation). From these studies, matches were found between morphotypes, depositional processes and soil characteristics, with a detailed map of the soil deposits being subsequently drawn up. The map was further elaborated, and matched with information from literature (Basile *et alii*. 2003; Iamarino & Terribile, 2008; Bilotta *et alii*., 2005) in order to obtain the geographical distribution of the soil parameters (depth of soil layers, geotechnical properties, hydraulic conductivity, etc.) relevant to the stability analysis.

The basin has been divided into 21 homogeneous

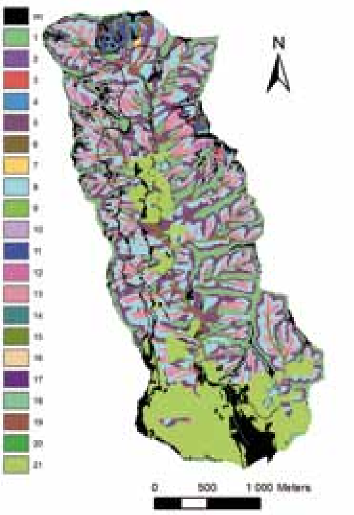


Fig. 2 - Map of the homogenous geo-morphological districts (nn stands for absence of soil layer)

district	Z_T	γ_s	φ	c	K_x	K_z
	m	kg/m³	°	Pa	mm/s	mm/s
1	0.5	1500	32	10000	0.96	0.96
2	0.5	1400	35	5000	0.36	0.06
3	0.5	1400	35	5000	0.36	0.06
4	1	1400	35	5000	0.36	0.06
5	1.5	1400	35	5000	0.36	0.06
6	1	1400	35	5000	0.36	0.06
7	1.5	1400	35	5000	0.36	0.06
8	2	1400	35	5000	0.36	0.06
9	5	1400	35	5000	0.36	0.06
10	1	1500	32	10000	0.22	0.11
11	1.5	1500	32	10000	0.22	0.11
12	2	1500	32	10000	0.22	0.11
13	3.5	1500	32	10000	0.22	0.11
14	5	1500	32	10000	0.22	0.11
15	1	1800	35	0	0.68	0.68
16	1.5	1800	35	0	0.68	0.68
17	3.5	1800	35	0	0.68	0.68
18	5	1800	35	0	0.68	0.68
19	4.5	1500	32	10000	0.18	0.10
20	5	1800	35	0	0.68	0.68
21	4	1500	32	10000	0.09	0.09

Tab. 2 - Average values of the soil input variables

geomorphological districts (Fig. 2), for each district the average values and the confidence intervals of the soil variables: Z_T γ_s, φ, c, K_x and K_z. have been estimated.

The average values of the soil variables are reported in Tab. 2. The confidence intervals were set, for γ_s and j equal to the 10% of the average while for the Z_T, c, K_x and $K_z$equal to the 20% of the average.

The topographic variables a (Fig. 3) and *A/b* are calculated by elaborating a Digital Elevation Model with a resolution of 5 m.

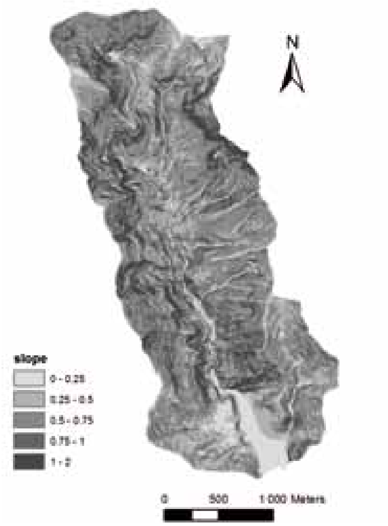


Fig. 3 - Map of the slope degree

From the reconstruction of the areas that were mobilized in 1954 (Carbone, Iamarino & Gallo, paper in preparation), it resulted that 2.8% of the total basin area was affected. The 1954 event caused a large amount of damage to the downstream village of Minori. Observation of the 2005 event showed that about 0.3% of the basin area was mobilized but the generated debris flow did not reach the downstream village of Minori. It may be concluded that the failure percentage of the order of magnitude of 0.1% does not constitute a hazard for the village. The threshold value of the failure percentage should be between 0.3% and 2.8 %.

RAINFALL THRESHOLDS

The static input variables matrix has been compiled using the data presented in the previous paragraph. The number of computational elements (number of strings of the static input variables) has been set equal to 1000

The calibration parameter D_0/K_z has been adjusted to 0.1 by comparison of simulated results with the historical event observations (see below).

The dynamic input variables have been set with reference to the local climatology in order to cover the entire range of possible values (Tab. 3).

The total number of simulations (N) resulted to be 25×10^6.

The obtained data set has been elaborated in order to draw intensity-durations curves for any fixed value of the antecedent rain.

Any intensity duration rainfall curve corresponds

	minimum value	maximum value	number of values (m_i)
T	0 hours	20 hours	10
I_z	0 mm/hour	100 mm/hour	50
$(I_z)_{steady}$	0 mm/month	600 mm/month	50

Tab. 3 - Range of values of the dynamic input variables

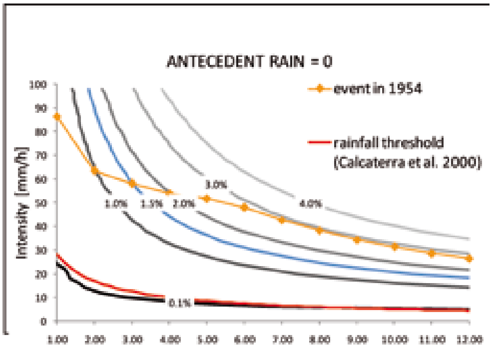


Fig. 4 - Intensity duration curves for different percentages of instable cells, with antecedent rain equal to zero

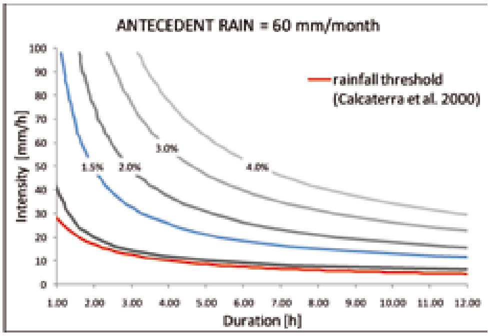


Fig. 5 - Intensity duration curves for different percentages of instable cells, with antecedent rain equal to 60 mm/month

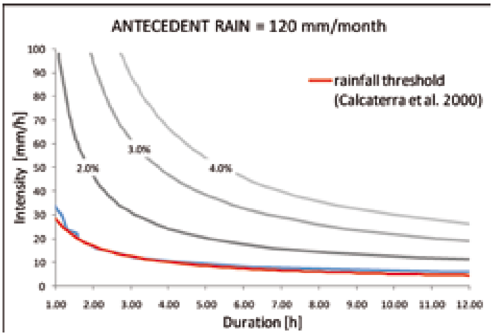


Fig. 6 - Intensity duration curves for different percentages of instable cells, with antecedent rain equal to 120 mm/month

to a fixed value of the ratio between the amount of computational elements that result to be instable over the total amount of elements (failure percentage).

The simulation results are compared with the rainfall curve relative to one observed debris flow event that occurred in October 1954 (Fig. 4). The months before that event were dry and therefore the comparison was carried out with the results obtained for the antecedent rain equal to zero. The comparison shows a very good agreement, with it resulting through the simulation that for an event rainfall with a duration in the range of 8-11 hours, the failure percentage is about 3%.

In the same graph of Fig. 4, a rainfall threshold line, derived through the elaboration of empirical data (Calcaterra *et alii*, 2000) is also reported. In this case, the antecedent rainfall does not explicitly appear as a parameter. From the comparison with the simulation results, it may be concluded that, in the case of antecedent rainfall equal to zero, the threshold line proposed by Calcaterra *et alii* (2000) corresponds to a failure percentage of about 0.1%. Following the consideration made in the previous paragraph, this failure percentage should not represent a significant hazard for the downstream village

If the same comparison is made with the simulation results obtained with antecedent rainfall equal to 60 mm/month (Fig. 5), it results that the threshold line proposed by Calcaterra *et alii* (2000) corresponds to a percentage of instable area equal to 1% of the totalbasin area. For antecedent rainfall equal to 120 mm/month (Fig. 6) the same percentage increases up to 1.5%.

Once the volume threshold is fixed, a graph similar to the one shown in (Fig. 7) may be used as a rule for DF warnings. In this kind of graph, the simulation results are elaborated in order to show, for any antecedent rain, the intensity duration rainfall curve giving place to a fixed value of the total amount of available debris volume.

In the example of the DF in the Sambuco basin in 1954, the total debris flow volume that reached the downstream village was 300˙000 m^3. The threshold line proposed by Calcaterra *et alii* corresponds to about 5˙000 m^3.

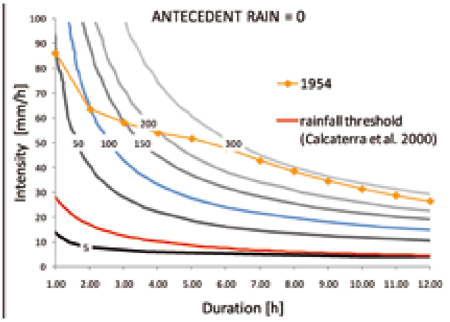


Fig.7 - Intensity duration curves for different total debris volume (1000 m^3), with antecedent rain equal to 0

CONCLUDING REMARKS AND FURTHER DEVELOPPEMENTS

In the present study, a simple system was developed, to be used as a debris flow warning tool. The system is based on critical rainfall thresholds, obtained from a mathematical model through numerical simulations.

In contrast to widespread critical rainfall thresholds derived from past debris flow events measurements, this system may be adopted not only in areas where historical events data are not available but also may take into account changing environments and climate.

The derivation of the critical rainfall thresholds curves has to be performed off line. The mathematical and numerical models, to be used for this derivation, are quite simple and fast, with it therefore being possible to apply the method to wide areas.

A set of rainfall threshold curves is provided, one for each considered value of antecedent rainfall.

In real time, it is possible to provide replies in very short time intervals through the simple comparison of observed and forecasted rainfalls with the graphs derived off line.

The model used for describing the mechanism of debris flow formation is very simple and many improvements could be made by removing some of the hypotheses made for the sake of simplicity. A complete solution of the transient underground water flow may be implemented instead of the considered simple cases of slope parallel or vertical flow. In order to assess if a local instability may develop into a debris flow, a further condition should be added to the model, taking into account the liquefaction process.

In many cases, a great increment of debris flow volume occurs as a consequence of channel erosion during the run-out process. It means that a huge volume of sediment could be produced even if the debris flow is of a small size in the occurrence area. In these cases the volume of debris flow calculated by the method proposed here would be underestimated. On the other hand an over estimation of the total debris flow volume may occur because the single computational element that results to be instable is not really going to move if it is surrounded by stable elements. A criterion should be found to fix correctly the critical percentage of instable cells taking into account both these problems.

The system has been tested through comparison with only one test basin and one debris flow event. Further validation and calibration of the system is required

A more detailed study should be carried out, dealing with the antecedent rainfall. The reference length of the time interval to be taken into account should be defined depending on the basin characteristics, while the loss for evapo-transpiration should be taken into account in the comparison with past or future events.

An assessment of the “false alarm” that may be given by the system can be carried out through the comparison with observed intense rainfall events that did not gave place to instability.

ACKNOWLEDGEMENTS

The work described in this study was supported by the European Community Seventh Framework Programme through the grant of the Collaborative Project IMPRINTS (IMproving Preparedness and RIsk maNagemenT for flash floods and debriS flow events), Contract FP7-ENV-2008-1-226555.

The authors would like to thank the dr Antonio Carbone, Antonio Gallo and Michela Iamarino for having kindly provided some partial results of their ongoing studies on the identification and characterization of the landslides after the intense rainstorm of October 1954 in the province of Salerno.

REFERENCES

Basile A., Mele G. & Terribile F. (2003) -*Soil hydraulic behaviour of selected benchmark soil involved in the landslide of Sarno 1998*. Geoderma, **117:** 331-346.

Beven K. (2004) - *Rainfall-runoff modelling: the primer.* John Wiley & Sons Inc.

Bilotta E., Cascini L., Foresta V. & Sorbino G. (2005) - *Geotechnical characterisation of pyroclastic soils involved in huge flowslides.* Geotechnical and Geological Engineering, **23:** 365–402

Calcaterra D, Parise M, Palma B & Pelella L. (2000) -*The influence of meteoric events in triggering shallow landslides in pyroclastic deposits of Campania, Italy.* In: Proceedings of the 8th International Symposium on Landslides, (Bromhead E, Dixon N, Ibsen ML, eds). Cardiff, Wales: A.A. Balkema, **1:** 209-214

Crozier M.J. (1989) – *Landslides: Causes, Consequences and Environment.* Routledge, London.

Godt J.W., Baum R.L., Savage W.Z., Salciarini D., Schulz W.H., Harp E.L. & Richard U. (2008) - *Transient deterministic shallow landslide modeling: Requirements for susceptibility and hazard assessments in a GIS frame work.* Engineering Geology, **102:** 214-226.

Iamarino M. & Terribile F. (2008) - *The relevance of andic soils in mountain ecosystems in Italy: a pedological investigation.* European Journal of Soil Science, **59:** 1284-1292.

Iverson M. (2000) - *Landslide triggering by rain infiltration.* Water Resources Research, **36**(7): 1897-1910.

Lambe T.W. & Whitman R.V. (1979) - *Soil mechanics.* Wiley, New York.

Montgomery D.R. & Dietrich W.E. (1994) - *A physically-based model for topographic control on shallow landsliding.* Water Resources Research **30**(4)**:** 1153-1171.

Medina V., Bateman A. & Hürlimann M. (2008) - *FLATModel: a 2D finite volume code for debris-flow modelling. Application to events occurred in the Eastern Pyrenees.* Int. J. Sediment Research, **23** (4)**:** 348–360.

O'Brien J.S., Julien P.Y. & Fullerton W.T. (1993) - *Two-dimensional water flood and mudflow simulation.* Journal of Hydraulic Engineering, **119**(2)**:** 244-261.

Papa M.N, & Trentini G. (2010) - *Valutazione della pericolosità connessa a fenomeni di correnti detritiche.* Proceedings of the XXXII Convegno Nazionale di Idraulica e Costruzioni Idrauliche, Palermo, 14-17 settembre (in press).

Taylor D.W. (1948) - F*undamentals of soil mechanics,* Wiley, New York.

Wieczorek G.F. & Glade T. (2005) - *Climatic Factors Influencing Occurrence of Debris Flows.* In: Jakob M. & Hungr O., eds., Debris-flow Hazards and Related Phenomena, Praxis, Springer, Chichester, UK, Ch. 14: 325-362.