# DAM-BREAK FLOWS OF DRY GRANULAR MATERIAL ON GENTLE SLOPES

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### ABSTRACT

This work examines dam-break flows of dry granular material and investigates the suitability of the depth-averaged models with particular attention being given to the description of the shear stresses and pressure terms. The experimental results of dam-break flows down a gently sloped channel have been reported. Tests were carried out on both a smooth Plexiglas bed as well as a rough one. Measurements of the flow depth profiles and the front wave position were obtained using two digital cameras. In order to compare the prediction of the depth-averaged approach with granular avalanche tests, a specific mathematical and numerical model was implemented. The momentum equation was modified in order to take into account the resistances due to the side walls. The numerical integration of the shallow water equations was carried out through a TVD finite volume method. In order to address the importance of a good estimate of the stress distribution inside the pile, several numerical simulations were performed, calculating with different formulas the pressure coefficient that relates longitudinal and vertical normal stresses in the momentum equation. The simulations present, in general, agree with experimental data. The differences have been outlined between the smooth and rough bed cases.

Key words: dry granular, dam-break, pressure coefficient, avalanche, depth averaged model

#### **INTRODUCTION**

Debris flows as well as snow and rock avalanches are fast-moving flows that occur in many areas of the world. They are particularly dangerous to life and property because they move with high velocities, destroy infrastructures in their paths, and often strike without warning.

In some real world cases, geophysical flows can be triggered by phenomena that are very similar to a dam break. Water flows generated by a dam break have been widely studied and mathematical models for water dam-break waves are available in many textbooks and research papers. Compared to water dambreak waves, debris flow waves display a wider variability and, for their mathematical description, require models with a much greater complexity. As in the case of clear water, particular attention must be given to their numerical integration because of the frequent developing of steep gradients and shock waves.

Owing to this complexity, a number of simplifications are put in place and tested under laboratory conditions: the results of the tests are then used to improve the rheological models that underlie the numerical simulations.

In the specific case of debris flows, flows arising from a dam-break-like event of dry granular material can help in the process of model validation. Moreover, a good understanding of the mechanics of dry granular flows is also essential in order to set up twophase debris-flow models because two separate modelling of the solid and fluid phases are needed. The propagation of a dam break of dry granular material can be modelled by a depth averaged SAINT-VENANT (1871) approach. The material is assumed to be incompressible, with the mass and momentum equations being written in a depth-averaged form. This analysis is valid under the assumption that the flowing layer is thin compared to its lateral extension, as often occurs in the case of geophysical flows. A depth averaged approach makes it possible to avoid a complete three-dimensional description of the flow field. In fact, it is only necessary to specify a single term describing the frictional stress between the flowing material and the boundary surface.

Depth-averaged equations were introduced in the context of dry granular flows (S-H model) by SAVAGE & HUTTER (1989). In their model, the interaction between the granular flow and the boundary surface is described by a simple Mohr-Coulomb vield criterion: the shear stress at the bottom is proportional to the normal stress by a constant friction coefficient. The longitudinal normal stress  $\sigma_{\rm x}$  is considered proportional to the normal stress  $\sigma_{y}$  exerted on an element normal to the bed, through a coefficient K suitably calculated. Besides, because the aspect ratio  $\varepsilon = H/L$ of the flow is most likely small (H and L are respectively, the typical thickness and the typical length of the avalanche), the terms of order  $\varepsilon$  are omitted in the y-momentum equation (i.e. the momentum equation projected along the normal direction to the flow). As a consequence the the normal stress has an hydrostatic distribution over the flow depth, analogously to the De Saint Venant equations.

It is worth to state that the S-H model has been widely used in some two-phase debris flow models (i.e. IVERSON, 1997; IVERSON & DENLINGER, 2001) in order to describe the mechanics of the solid phase.

As observed by HUTTER *et alii* (2005), the model works well when the surface of the plane is sufficiently smooth. The model is able to predict the motion and spreading of a granular mass on steep slopes in one and two dimensions (SAVAGE & HUTTER, 1989).

Several experimental observations (POULIQUEN & FORTERRE, 2002; HUTTER *et alii*, 2005) show that the Savage-Hutter approach is no longer suitable for the flow of granular material on rough surfaces (where the roughness of the bed is of the order of the particle size) as well as moderate inclinations (i.e.



Fig. 1 - The experimental apparatus

less than 30°). The aim of this paper is to present an experimental investigation into dry granular flows down a gently inclined channel, with specific attention being given to the suitability of depth-averaged models. Furthermore, an additional aim has been to examine the commonly assumed hypotheses about stress distributions and, in particular, some different approaches to estimate the coefficient K were implemented and compared.

## **EXPERIMENTAL APPARATUS**

An 8-m-long chute (Fig. 1), designed for the study of both mud and dry granular flows, was set up at the LIDAM (Laboratory of Hydraulic, Environmental and Maritime Engineering of the University of Salerno). The channel inclination, which is constant for its whole length, can be varied between 0° and 23°, by rotating the structure around its lower end through an hydraulic ram controlled by a pumping system. The channel width can be adjusted between 0 and 80 cm, as the right side wall position can be moved by a screw system. The 90-cm-high side walls and the bottom are both made of Plexiglas and are suitably supported by structural steels. At the upper end of the chute, there is a wide tank, integral with the chute structure, with a capacity of 3 m<sup>3</sup>. In addition, at the lower end the channel ends in a collector tank, with about the same capacity as the upper one.

After setting up the apparatus, dam-break tests were carried out on dry granular material. The channel width was set to 24 cm, which seemed to be large enough to somehow reduce the influence of the side walls on the flow mechanics.

For each run, the slope was trigonometrically calculated through geometric measurements. Its accuracy is good, due to it being valued less than 0.1°

Density	1410 kg/m <sup>3</sup>
Mean diameter	3.35 mm
Maximum diameter	3.9 mm
Minimum diameter	2.8 mm
Modulus of elasticity	2700 MPa
Water absorption	0.65%
Deposit concentration	0.63
Internal friction angle	26.5°
Sliding Friction coefficient	0.32
(with steel surface)	

#### Tab. 1 - Features of the granular material

and results sufficient beyond parameter sensitivity.

For these tests, which involved a mass of material of only 100 kg, the upper tank was not used. The 2-m-long upper part of the channel was used to store the material.

At the beginning of each run, a granular pile was restrained in the upper part of the chute by a small wooden gate, which was placed at exactly 2 m from the beginning of the chute. When closed, the gate is perpendicular to the channel bed and able to release the material when rapidly rotated counter-clockwise due to a spring mechanism. This opening apparatus was designed to open quickly in order to avoid any significant influence on the forming dam-break wave. The total opening time results less than 2/12 s and, only after 1/12 s, the captured frames show that there is no contact between the gate and the upstream material. Therefore, the influence on the flow seems to be negligible.

## CHARACTERSTCS OF THE GRANULAR MATE-RIAL

The tests reported in this paper were carried out using lens-like shaped acetalic resin beads (HERAFORM R900), with a maximum diameter and minimum diameter respectively equal to 3.9 mm and 2.8 mm. In Table 1, the main features of the material are reported. At each run, a mass of 100 kg of granular material was suddenly released by opening the wooden gate.

Particular attention was given to set the initial position of the pile upstream the gate in order to impose the same initial condition in each test. A trapezoidalshaped initial deposit was used for all the runs and is reported in Fig. 2.

Two kinds of test were carried out. The first type of flows were carried out on a smooth Plexiglas bed. In the second, on a bed with a lining of coarse sandpaper (grit P40 FEPA/ISO 6344).

The following procedure was carried out to meas-



Fig. 2 - Initial deposit of the granular material. x-axis is parallel to the sloping bottom of the chute

ure the bed friction angles  $\delta$  between the two surfaces and the granular material and the internal friction angle  $\varphi$  of the granular material. A single layer of plastic beads was glued onto a thin plywood sheet. Moreover, three wooden plates were lined with Plexiglas, sandpaper and granular material by gluing a single layer. Then, the plywood sheet was gently placed with the granular layer downward onto the inclined surface with a fixed slope. The friction angle, which depends on the granular material and the surface, is considered equal to the angle at which the static equilibrium of the plywood sheet on the inclined surface is no longer possible.

The measured internal angle is  $\varphi$ =26.5° and can be considered a good estimate of the constant-volume friction angle (i.e. the dynamical friction angle). In fact, just prior to failure, the two overlapping layers of material are weakly packed and thus the effect of interlocking is negligible.

For the smooth bed of Plexiglas, $\delta$ = 19.5° was measured. An angle much greater than the internal friction angle of the granular material was observed for the sandpaper bed ( $\delta$ = 36.5°). Therefore, in this latter case, failure occurs inside the pile and only the friction angle of material instead of the bed friction angle should be considered in order to calculate the basal shear stress.

These last two angles are good estimates of the static friction angles. Nevertheless, POULIQUEN & FOR-TERRE (2002) observed that the dynamic friction angle, called "stop angle", is roughly one degree below the static friction value. Owing to this, it was decided to lower the estimates by 1°. Thus, the friction angle of the Plexiglas bed was set equal to 18.5°.

#### MEASURING INSTRUMENTS

The motion was recorded by two Sony Super HAD CCD video cameras at 12 frames per second, connected to a digital video recorder. The first camera was installed at the side of the channel and, thanks to the side wall transparency, allowed for the view of about 80 cm downstream the gate. The effective resolution of the cameras was about 450 lines, with a precision of 1 cm being assured in the chosen field of view. In order to rectify the images from the first camera, a 2 cm grid was put on the opposite side wall.

The second one was located over the chute and was able to capture the front wave position in the first 2 m downstream the gate. The same rectification of the images was implemented using fixed reference lines on the channel bed.

Afterwards, the recorded images were digitally processed. At first, the Barrel deformations were minimized by using a photo editing software. Then, the frames were subjected to a perspective rectification. Image rectification from the side camera was accomplished by exploiting the fixed spots of the grid behind it. Strictly speaking, it was sufficient to choose 4 point in order to rectify an image. Nevertheless, to minimize any errors due to uncertainty, a set of 8 points was taken and a residual evaluation was carried out. The same procedure was carried out for the frames recorded by the front camera. However, any errors due to rectification were less than 3 mm, with it being possible that global accuracy was within 1 cm.

### MATHEMATICAL MODEL

The flow under study develops predominantly in the longitudinal direction. It is therefore natural to use depth-averaged type models (SAVAGE & HUTTER, 1989; IVERSON, 1997; POULIQUEN, 1999; POULIQUEN & FORTERRE, 2002). These models can be obtained by integrating the mass and momentum conservation equations over the flow depth.

In this work, the Savage-Hutter 1D model was used, which is described by the two following hyperbolic partial differential equations, written in the conservative form:

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0$$

$$\frac{\partial (hu)}{\partial t} + \frac{\partial (\cos\alpha \cdot K \cdot g/2 \cdot h^2 + \beta \cdot hu^2)}{\partial x} = gh \cdot (S_0 - S_f)$$
(1)

where  $S_0 = \sin \alpha$  represents the gravity force component in the flow direction per unit mass and  $S_f = \cos \alpha$  tan  $\delta |u|/u$  is the bed friction, *h* is the flow depth, u the mean velocity, *x* and *y* are respectively the direction parallel and normal to the bed,  $\alpha$  the channel slope

angle,  $\delta$  the basal friction angle,  $\beta$  the Boussinesq momentum coefficient. *K* is the active/passive pressure coefficient, i.e. the ratio of the normal stress  $\sigma_v$  exerted on an element normal to the bed and the normal stress  $\sigma_v$  exerted on an element parallel to it.

$$K = 2\sec^2\phi \left( l \,\mu \left( l - \cos^2\phi / \cos^2\delta \right)^{l/2} \right) - 1 \tag{2}$$

where  $\phi$  is the internal friction angle and the bed friction angle. Eq. 2 should be taken with the minus sign if  $\partial u$  $/\partial x > 0$  (i.e. the pile is elongating), the plus sign if  $\partial u / \partial x < 0$  (i.e. the pile is compressing).

This expression is obtained under the hypotheses that failure simultaneously occurs at the bed and inside the material, with the shear stress linearly varying with depth as the normal stress  $\sigma_y$  does. Moreover, it is worth noting that the expression (2) is only valid when  $.\delta \le \phi$ .

If  $\delta > \phi$ , IVERSON (1997) proposed using Rankine formula, instead of (2):

$$K = \frac{1 \mu \sin \phi}{1 \pm \sin \phi}$$
(3)

which is also used in the Voellmy-fluid model proposed by BARTELT *et alii*. (1999). It could also be obtained from (2) by putting  $\delta \rightarrow 0$ . In this case, failure only occurs inside the granular material and the  $\tau_{xv}$  distribution along the y-direction is unlikely to be linear. The rationale of (3) is that it is assumed that  $\sigma_x$  and  $\sigma_v$  are principal stresses, as if  $\tau_{xv} = 0$ . Moreover, many studies have also proposed a completely isotropic stress tensor (Bartelt, 1999; POULIQUEN & FORTERRE, 2002), that is K=1.

In this work, the above-stated three different approaches were implemented and compared in order to estimate the coefficient *K*.

For simplicity, the Boussinesq coefficient  $\beta$  was set equal to 1. This is exact only if the velocity is constant over the depth. It is therefore expected to be valid when the flow is sheared in a thin basal layer, as when the bed is sufficiently smooth (i.e  $\delta < \varphi$ ). However, it was found that eqs. (1) are quite insensitive to  $\beta$  changes (SAVAGE & HUTTER, 1991), with further studies being required when a large bed roughness is considered and a complete sheared flow expected.

#### RESISTENCE DUE TO SIDE WALLS

The equations described above are for a 1D flow. Therefore, the only resistance accounted for is due to the bed friction. Nevertheless, in order to simulate a dam-break flow down a chute, it is necessary to consider the resistances due to side walls. In order to take into account the side wall resistance, an approach similar to the one reported in (SAVAGE & HUTTER, 1991) was adopted. Along the side walls, it is assumed that there is slip, therefore the Coulomb failure law holds  $\tau = \sigma_z$ ,  $tan \delta_{lat}$  with z being normal to the side walls and  $\delta_{lat}$  the friction angle between the granular material and side walls. The side walls are made of Plexiglass like the channel bottom, so  $\delta_{lat} = 18.5^{\circ}$ . Assuming that  $\sigma_z$  is linearly distributed over the depth, as it is  $\sigma_y$ , there should be a pressure coefficient  $K_z$  so that:

$$\sigma_z = K_z \cdot \sigma_y = K_z \cdot \rho \cdot g \cdot \cos \alpha \cdot h \tag{4}$$

Then, integrating  $\tau_{xz}$  over the depth, the resistance per unit length of the channel due to a single side wall is the following:

$$T_{lat} = \frac{1}{2} \cdot K_z \cdot \rho \cdot g \cdot \cos \alpha \cdot h^2 \cdot \tan \delta_{lat}$$
<sup>(5)</sup>

Since no pile elongation occurs along the z-direction during the flow,  $K_z$  should be close to the at-rest pressure coefficient. In the simulations presented, the Jaky formula (JAKY, 1944) was used to estimate it:

$$K_z \approx K_0 = 1 - \sin \phi = 0.554$$
 (6)

Taking into account the effect of (5) for both the walls in eq. (1), the following modified right-hand member of the momentum equation results:

$$gh(\sin \alpha - \cos \alpha \cdot \tan \delta_{bed} \cdot |u| / u - K_z \cdot \cos \alpha \cdot \tan \delta_{bat} \cdot h / W \cdot |u| / u)$$
  
(7)

where  $\delta_{bed}$  is the bed friction and *W* the width of the chute. In order to keep the same expression of eq. (2), when the bed is made of the same material as the side walls (i.e.  $\delta_{bed} = \delta_{lat}$ ), an equivalent bed friction angle could be defined as follows:

$$\tan \delta_{eff} = \tan \delta \left( 1 + K_* \cdot h / W \right) \tag{8}$$

This expression agrees with the one used by SAV-AGE & HUTTER (1991), where for glass side walls and rough bed, the coefficient K was set equal to 0.453.

## NUMERICAL METHOD

Savage-Hutter equations were numerically solved using a shock-capturing Finite Volume method. The spatial domain was divided by an equally spaced mesh  $\Delta x$ . In addition, for each step,  $\Delta t$  was calculated keeping the maximum value of the CFL number constant. The generic volume element is represented by the pair (xj,ti) and the solution at (xj,ti) is the integral mean value over the volume element.

An explicit Total Variation Diminishing scheme was used, which is generally second order accurate in space and time and can capture the shocks due to a local decrease of the order of accuracy near the discontinuities.

The explicit formula to update the solution vector  $U=(h, hU)^{T}$  is the following:

$$U_{j}^{i+1} = U_{j}^{i} - \frac{\Delta t}{\Delta x} \left( F_{j+1/2}^{TVD} - F_{j-1/2}^{TVD} \right) + \Delta t \cdot R_{j}^{i}$$
(9)

where  $R_j^i$  is the source term and  $F^{TVD}$  are the numerical fluxes. These fluxes are obtained as a convex combination of the first order Godunov flux $F^i$  and the second order Lax-Wendroff flux  $F^{ij}$ :

$$F_{j+1/2}^{TVD} = F_{j+1/2}^{I} + \Phi_{j+1/2} \cdot \left(F_{j+1/2}^{II} - F_{j+1/2}^{I}\right)$$
(10)

$$F_{j+1/2}^{I} = \frac{1}{2} \Big[ A_{j+1/2} \cdot (U_{j} + U_{j+1}) - |A_{j+1/2}| \cdot (U_{j+1} - U_{j}) \Big]$$
(11 a)

$$F_{j+1/2}^{II} = \frac{1}{2} \left[ A_{j+1/2} \cdot \left( U_j + U_{j+1} \right) - \frac{\Delta t}{\Delta \mathbf{x}} \cdot A_{j+1/2}^2 \cdot \left( U_{j+1} - U_j \right) \right]$$
(11 b)

where  $A=\partial F/\partial U$  is the Jacobian matrix of the flux F,  $|A|=R|\Gamma| R^{-1}$  is obtained through eigen-decomposition, R is the right-eigenvector matrix associated to A and  $\Gamma$  the diagonal eigenvalues matrix. Both the approximated Jacobian matrix A and |A| at interfaces are calculated through a local linearization of the system of equations using the following Roe's approximations (ROE, 1981):

$$h_{j+1/2} = \frac{h_j + h_{j+1}}{2}, \qquad \qquad u_{j+1/2} = \frac{\sqrt{h_j} \cdot u_j + \sqrt{h_{j+1}} \cdot u_{j+1}}{\sqrt{h_j} + \sqrt{h_{j+1}}}$$
(12)

The limiter  $\Phi$  in eq. (11) is a 2x2 diagonal matrix, whose elements can range between 0 and 1. Its purpose is to lower the order of accuracy in high gradient zones. The *Minmod* function was chosen as a limiter function for the calculation of each element of the matrix  $\Phi$ , since it seems to be the best among the available TVD limiters for this particular problem. Its expression is reported as following:

$$\phi_{j+1/2}^{kk}\left(\vartheta^{k}\right) = \max\left[0, \min\left(1, \vartheta^{k}\right)\right]$$
(13)

where superscript k specifies the k-th component and  $\theta^{k}$  at the interface j+1/2 is the ratio:

$$\mathcal{P}_{j+1/2}^{k} = \frac{\left[R_{j+1/2-s}^{-1} \cdot \left(U_{j+1-s} - U_{j-s}\right)\right]^{k}}{\left[R_{j+1/2}^{-1} \cdot \left(U_{j+1} - U_{j}\right)\right]^{k}}, \qquad s = sign\left(\widetilde{a}_{j+1/2}^{k}\right) \tag{14}$$

being R<sup>-1</sup><sub>i+1/2</sub> the left eigenvector matrix and

 $\widetilde{a}_{j+1/2}^{k}$  the k-th eigenvalue at interface j+1/2, obtained using the Roe's formulas

To avoid any convergence to a non physical weak solution violating the entropy principle (Leveque, 2002), a correction to absolute values of the eigenvalues was applied (HARTEN & HYMAN, 1981). Firstly, let:  $\varepsilon_{j+1/2}^{k} = \max \left[0, \left(\widetilde{a}_{j+1/2}^{k} - \widetilde{a}_{j}^{k}\right), \left(\widetilde{a}_{j+1}^{k} - \widetilde{a}_{j+1/2}^{k}\right)\right] k = 1,2$  (15)

The entropy correction for k-th eigenvalue, at the interface between j and j+1 is the following:

$$\Psi_{j+1/2}^{k} = \max\left(\widetilde{a}_{j+1/2}^{k}, \varepsilon_{j+1/2}^{k}\right) \tag{16}$$

Special attention was given to managing the transition of the avalanche to the zones without material (wet/dry zone transition). In particular, a very thin layer of material ( $10^{-5}$  m) was considered over the dry zones as an initial condition. Furthermore, during the calculation, in cells where h< $10^{-4}$  m, velocity was kept at zero without solving the momentum equation. In order to test the suitability of the chosen threshold values, calculations were compared with the ones obtained using lower threshold values of h, without noting any significant differences.

Regarding the lower boundary condition, in order to avoid any wave reflection, a non-reflective condition was applied by imposing the two following ghost values at the right end of the domain:

$$U_{n+1}^{i+1} = U_n^{i+1}, \quad U_{n+2}^{i+1} = U_n^{i+1}$$
 (17)

where the subscript n denotes the last cell at the right end of the domain.

Regarding the upper boundary condition, it is not expected the flow to go upstream and so a management of wave reflections should not be necessary. Anyway, in order to preserve the mass balance, a solid wall condition was imposed at the top boundary all the same, because it is needed to correctly allow the reflection of the small spurious waves, due to the thin layer located in dry zones. To do so, the following ghost values were imposed at the left end of the domain:

$$h_{0}^{i+1} = h_{1}^{i+1}, \quad h_{-1}^{i+1} = h_{2}^{i+1}$$
(18)

$$(hu)_{0}^{i+1} = -(hu)_{1}^{i+1}, \quad (hu)_{-1}^{i+1} = -(hu)_{2}^{i+1}$$
<sup>(19)</sup>

Due to the channel geometry (constant slope), it results  $\partial u / \partial x > 0$  in the whole domain and for any given time (i.e. the pile elongates). Therefore, in eqs. (1) only the active pressure coefficient  $K_{act}$  appears and numerical issues due to the jump discontinuity between  $K_{act}$  and  $K_{bass}$  do not arise.

Volume balance is always respected within an error below 0.05%. Simulations were carried out, imposing  $\Delta x$ = 0.02 m and a  $\Delta t$  for each step such as CFL=0.2.

# EXPERIMENTAL DATA AND COMPARI-SONS

The aim of this work is to investigate how the pressure coefficient *K* should be calculated in order to have the best fitting of the experimental data. An additional aim is to explain the results obtained considering the influence of the bed roughness on the flow.

At first, a set of tests with a smooth bed was carried out at the following slopes of the chute: 19°, 20°, 22.7°. Then, a set of runs on a sandpaper bed was carried out with the same slopes.

In order to compare the numerical solutions with the experimental data, the zero time (t=0) was taken as corresponding to the first frame where the gate is no longer in contact with the upstream pile. This moment is 1/12 s after the start of the movement of the gate. Considering that at this point, the pile is practically still motionless, the initial condition used in the numerical simulations was the initial deposit depicted in Fig. 2.

For each run, three numerical simulations were carried out: the first one with calculated according to the Savage & Hutter formula (2), the second one with K = I assuming the stress tensor is spherical, the last one with  $K_{act}$  calculated through the Rankine formula (3). The other model parameters for the smooth bed runs were the following:  $\delta_{bed} = 18.5^{\circ}$ ,  $K_z = 1$ -sin( $\phi$ ) =0.554. For the rough bed simulations, instead, the bed friction angle is  $\delta_{bed} = \phi = 18.5^{\circ}$ , due to failure occurring inside the granular material. For these runs, the same value of  $K_z = 0.554$  was used.

#### SMOOTH BED TESTS

Comparisons between the numerical simulations



Fig. 3 - Flow depth profiles, comparison among experimental data and numerical simulations (slope 19°, smooth bed).



Fig. 4 - Flow depth profiles, comparison among experimental data and numerical simulations (slope 20°, smooth bed)



Fig. 5 - Flow depth profiles, comparison among experimental data and numerical simulations (slope 22.7°, smooth bed)

and the smooth bed tests are reported in Figs. 3, 4, 5. The following notation applies to all the diagrams: Ksh=0.67 is the pressure coefficient calculated through the Savage-Hutter formula; K=1 is calculated under the hypothesis of spherical stress tensor, Kr=0.38 calculated through the Rankine formula. The simulations using  $K_{act}$  calculated according to the Savage & Hutter formula gave the best fitting. It is important to state that at the very first times (below 1.5s), the simulated flow is faster than the observed one. In this window time, the best fitting seems to be achieved using  $K_{act}$ calculated through the Rankine formula. However, it is possible that the different behaviour of the simulations at the first stages is linked more to the failure of many of the hypotheses made (linear distribution of stresses, rate independence of basal friction) rather than to a real



Fig. 6 - Flow depth profiles, comparison among experimental data and numerical simulations (slope 19°, rough bed).



Fig. 7 - Flow depth profiles, comparison among experimental data and numerical simulations (slope 20°, rough bed



Fig. 8 - Flow depth profiles, comparison among experimental data and numerical simulations (slope 22.7°, rough bed)

fitting to the Rankine formula.

The numerical simulation cannot accurately reproduce the thickness profile at the end of the avalanche (8 s and 10 s) in the 20°-run. This could be due to an error in taking into account the side wall stress.

## ROUGH BED TESTS

Comparisons between the numerical simulations and the rough bed tests are reported in Figs. 6, 7, 8. For these runs *Ksh*=1.49, because it is imposed  $\delta = \phi = 26.5^{\circ}$ in expr. (7); *Kr*=0.38, which depends only on  $\phi$  and not on the bed friction. In this case, the thickness profiles are very similar for different slopes. This happens because the source terms in the right-hand member (7) of the momentum equation, representing the resistances are much greater than the term representing the gravity force. Therefore, varying the slope produces only small differences on the dam-break waves. For these simulations, it is worth noting that the numerical simulations using K = I are in greater accordance with the empirical observation as was found also by POULIQUEN & FOR-TERRE (2002), even though there is a systematic light underestimation of the thickness. On the other hand, using calculated according to the Savage & Hutter produces a faster dam-break wave and a thinner deposit profile. This could be due to the fact that flows on a rough bed are more sheared than on a smooth bed and therefore the hypothesis of linear distribution of stress-



es appears to be less realistic. At the first stages of the motion (0-1.5 s), as similarly observed in the smooth runs, the best fitting is obtained using Kr:

#### POSITION OF THE WAVE FRONT

A comparison between the observed positions of the wave front and the simulated ones is depicted in Fig. 9. Only the comparison for the run at 22.7° for both the smooth and rough beds is reported, since the other profiles are very similar at these first stages. The numerical simulation chosen for the comparison is calculated using Kr=0.38, which allows for the best fitting. The experimental position of the wave front is determined by analyzing the frames from the camera located over the chute. Unfortunately, the front camera was only able to capture a limited field of view, i.e. the first 2 m downwards the position of the gate. For the numerical simulations, determining the wave front position using a criterion based on the front depth could be ambiguous because of the thin layer of material used to describe the dry/wet transitions. Therefore, the front has been defined as the point corresponding to the first cell in the mesh, upstream from the right boundary, where the velocity is zero. As shown in the graph (Fig. 9), the numerical model is able to predict fairy well the velocity of the wave front, as the two diagrams have approximately the same slope. Nevertheless, it is worth noting that the numerical simulations are faster than the observed avalanches. However, this disagreement could be partially due to either a time shift error in determining the zero-time or to the particular technique used to determine the position of the wave front in the numerical simulations



## CONCLUDING REMARKS

In this work the experimental data of a dam-break on a gently sloped chute was compared to numerical solutions obtained using the S-H model with various expressions of the pressure coefficient K. Runs with smooth and rough beds were carried out. Side wall resistance was considered, assuming that the lateral pressure coefficient is close to the at-rest pressure coefficient. Comparisons of smooth bed runs have confirmed, as many works have already stated (SAVAGE & HUTTER, 1989; SAVAGE & HUTTER, 1989; HUTTER & *alii*, 2005), the fitness of the S-H model in describing such a type of dry granular avalanche.

On the other hand, comparisons of the rough bed runs have shown that an isotropic stress tensor assumption gives a better fitting of the experimental data and suggests further investigations of the role of the stresses and velocity distributions over the depth. In fact, it is not clear if the best fitting obtained using K=1is simply due to either an empirical compensation of some other effects ignored by the model or to a truly substantiated hypothesis about the stress distribution.

However, under these conditions, the flowing pile is likely to have a wide fluidized layer above a layer of material already deposited. At first, this suggests giving a more accurate estimate of the Boussinesq momentum coefficient.

The stress tensor distribution over the depth is likely to be too complex to be described by a simple pressure coefficient K. In order to overcome this difficulty, it could be interesting to develop a multi-layer depth-averaged model which joins the simplicity of the depth-averaged approach and a better capability to describe this variation.

In this perspective, interesting experimental data may be obtained by measuring the velocity profiles as well as a wider window of flow depth profiles. In order to achieve these results the test equipment needs to be modified and completed.

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