

APPLICATION OF A TWO LAYER MODEL WITH THE AID OF A SLOPE COLLAPSE MODEL TO THE NATURAL LANDSLIDE DAM OUTBURST PROCESS

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ABSTRACT

Earthquakes and heavy rainfall result in the formation of landslide dams following a massive collapse or landslide. Landslide dams can result in catastrophic outburst floods or debris flows when the dam breaches with overtopping erosion. Therefore, it is important to determine the discharge rate and area of flooding due to overtopping erosion to mitigate disasters triggered by landslide dams. This study applied a two-layer model that incorporates a slope-collapse model to the erosion process at landslide dams. The two-layer model was proposed by Takahama and deals with the process of deposition-erosion with fully dispersed debris flow and sediment sheet flow. A two-layer model is derived to unify these two kinds of flow. Moreover, Takahama applied this model to a two-dimensional numerical simulation of debris flow, and found that the model can analyze phenomena in which the velocity direction of the upper-layer water flow differs from that of the lower layer sediment-water mixture flow. Furthermore, we introduced the slope-collapse model proposed by Sekine to express gradual collapse of the slope due to erosion in the riverbed. The model shows the process that maintains a constant angle at the side bank. We examined the erosion and deposit process of a landslide dam with a numerical simulation.

KEY WORDS: landslide dam, outburst flood, numerical simulation, and slope-collapse

INTRODUCTION

Outburst of a landslide dam that is formed after an earthquake may cause catastrophic disasters on the downstream area (e.g., COSTA *et alii.*, 1988; KORUP, 2004). Estimation of the damage caused to the downstream area requires accurate prediction of the overtopping erosion of the landslide dam and the downstream flood runoff.

By taking the outburst of the landslide dam and downstream flow into consideration, one can predict rapid changes in the sediment movement pattern, including the transition from debris flow to sediment sheet flow and bed load. Previously, a method based on one-dimensional simulation was proposed for the prediction and assessment of degree of erosion; this model was used to estimate the peak discharge rate and changes in the riverbed after the overtopping erosion of the landslide dam (SATOFUKA *et alii.*, 2007). This method was successfully used for reproducing the discharge rate and changes in a riverbed in the event of a landslide dam outburst (e.g., MORI *et alii.*, 2010; SATOFUKA *et alii.*, 2010). However, since this method is based on one-dimensional analysis, it cannot be used for obtaining data on the extended width of the water channel during the overtopping erosion of a landslide dam. Hence, a twodimensional simulation model must be used for a detailed assessment of the flow around a landslide dam and for predicting the erosion process.

TAKAHASHI *et alii.* (1993) proposed a two-dimen-

sional method for assessing a landslide dam; this method involves the classification of sediment transportation patterns and the application of the friction law to each pattern. This model takes into account the side bank erosion caused by the shear stress. With this method, the shear stress is estimated to be one-half of that in the direction of the riverbed, and the erosion velocity is assumed to decrease uniformly across the calculated area because of the relative elevation of the collapsed part of the dam on the side bank. On the other hand, TAKAHAMA *et alii* (2003) assumed that the behavior of the sediment moving layer and water flow layer under unsteady conditions is reflective of the constitutive law for each layer and proposed an analysis method based on a two-layer model. In this model, a governing equation is considered for each layer, on the basis of the volume conservation law and momentum conservation law; further, it is assumed that the sediment sheet flow comprises a low-concentration layer (water flow layer) and a high-concentration layer (sediment moving layer) separated by an interface. TAKAHAMA *et alii* (2004) extended this model to a two-dimensional simulation. In this method, classification of the sediment transportation patterns is not necessary and direct calculation of the unsteady flow in the sediment moving layer and water flow layer can be carried out. SEKINE (2003) took into account the process of slope collapse and proposed for predicting the degree of erosion, resulting in the same gradient of slope; this method could be used for the accurate prediction of the collapse of the side bank.

In this study, we incorporated the slope-collapse

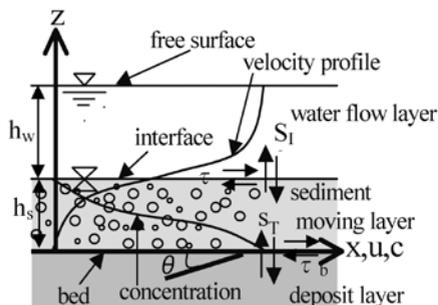


Fig. 1 - Pattern diagram for the two layer model

model into the two-dimensional model proposed by TAKAHAMA *et alii* (2004) and performed a two-dimensional simulation to study the overtopping erosion of landslide dams.

CALCULATION MODEL

EQUATION FOR TWO-LAYER MODEL

In this analysis, the two-dimensional two-layer model proposed by TAKAHAMA *et alii* (2004) is used. The model has been designed by taking into account the essential differences in the constitutive laws for the sediment moving layer and the water flow layer in the sediment sheet flow, as shown in Figure 1. Further, a governing equation based on the volume conservation law and momentum conservation law is used in this model. The conservation law is proposed by taking into account the water flow flux (s_I) and momentum flux at the interface between two layers; these two fluxes are dependent on the velocity vector at the interface. When this model is extended to two dimensions in the horizontal plane by using the X-Y coordinate system, the dominant equations can be given as follows.

Equation of continuity for the water flow layer:

$$\frac{\partial h_w}{\partial t} + \frac{\partial M_w}{\partial x} + \frac{\partial N_w}{\partial y} = s_I \tag{1}$$

Equation of continuity for the sediment moving layer:

$$\frac{\partial h_s}{\partial t} + \frac{\partial M_s}{\partial x} + \frac{\partial N_s}{\partial y} = s_T - s_I \tag{5}$$

Equation of continuity for the sediment:

$$\frac{\partial(c_s h_s)}{\partial t} + \frac{\partial(\gamma_x c_s M_s)}{\partial x} + \frac{\partial(\gamma_y c_s N_s)}{\partial y} = c_s s_T \tag{3}$$

Equation for the riverbed level:

$$\frac{\partial z_b}{\partial t} = -s_T \tag{4}$$

Equation of motion for the water flow layer (x-direction):

$$\frac{\partial M_w}{\partial t} + \frac{\partial(\beta_{wx}u_w M_w)}{\partial x} + \frac{\partial(\beta_{wy}v_w M_w)}{\partial y} - s_I u_I = \frac{1}{\rho_w} \left\{ -p_I \frac{\partial(h_s + z_b)}{\partial x} - \frac{\partial P_w}{\partial x} - \tau_{wx} \right\} \quad (5)$$

Equation of motion for the water flow layer (y-direction):

$$\frac{\partial N_w}{\partial t} + \frac{\partial(\beta_{wy}u_w N_w)}{\partial x} + \frac{\partial(\beta_{wy}v_w N_w)}{\partial y} - s_I v_I = \frac{1}{\rho_w} \left\{ -p_I \frac{\partial(h_s + z_b)}{\partial y} - \frac{\partial P_w}{\partial y} - \tau_{wy} \right\} \quad (6)$$

Equation of motion for the sediment moving layer (x-direction):

$$\frac{\partial(\gamma' \rho_s M_s)}{\partial t} + \frac{\partial(\beta_{sx} \rho_s u_s M_s)}{\partial x} + \frac{\partial(\beta_{sy} \rho_s v_s M_s)}{\partial y} + \rho_w s_I u_I = -p_b \frac{\partial z_b}{\partial x} + p_I \frac{\partial(h_s + z_b)}{\partial x} - \frac{\partial P_s}{\partial x} + \tau_{wx} - \tau_{bx} \quad (7)$$

Equation of motion of the sediment moving layer (y-direction):

$$\frac{\partial(\gamma' \rho_s N_s)}{\partial t} + \frac{\partial(\beta_{sx} \rho_s u_s N_s)}{\partial x} + \frac{\partial(\beta_{sy} \rho_s v_s N_s)}{\partial y} + \rho_w s_I v_I = -p_b \frac{\partial z_b}{\partial y} + p_I \frac{\partial(h_s + z_b)}{\partial y} - \frac{\partial P_s}{\partial y} + \tau_{wy} - \tau_{by} \quad (8)$$

where the indexes s, w denote the physical value of the sediment moving layer and the water flow layer, respectively. $M = uh$; $N = vh$; where u , and v denote the average layer flow velocity in the x- and y-directions, respectively. h denotes the flow layer thickness; ρ , the average layer density; and P , the integrated pressure. p_I and p_b are the pressures at the interface and in the riverbed, respectively. c_s and c_d denote the average sediment concentration of the sediment moving layer and sediment concentration of the deposit layer. z_b and s_{\pm} represent the riverbed height and erosion velocity,

respectively. In this study, we use the equation (TAKAHAMA *et alii.*, 2004) obtained by extending the erosion velocity equation proposed by EGASHIRA *et alii.* (1988) to the two-layer model. Further, we use τ_{wx} and τ_{wy} as the x- and y-direction elements of the shear stress at the interface. τ_{bx} and τ_{by} are the x- and y-direction elements of the shear stress at the riverbed surface, respectively, both of which are obtained from the constitutive law proposed by EGASHIRA *et alii* (1997) under conditions of uniform concentration. γ , γ' , and β are correction coefficients based on the flow velocity and concentration distribution and are set to 1 for the all analyses in this study.

Details of the shear stress at the riverbed and the interface and the formula for erosion-deposition velocity used in this study can be found in TAKAHAMA *et alii* (2003). Note that in this study, we introduce the following approximation and assumptions (TAKAHAMA *et alii*, 2003):

1 The concentration of the sediment layer is uniform (c_s / c_s^*).

2 The pressure at the riverbed is p_d , which is attributed to the pressure exerted by the particles; the value of this parameter is zero. Thus, at equilibrium, the stress in the riverbed is the same for the approximation solution and exact solution. The ratio of p_d to the skeleton pressure (p_s) in the flow layer is not constant and is given as a function of concentration (EGASHIRA *et alii*, 1997): $p_s / (p_s + p_d)^{1/5} = (c_s / c_s^*)^{1/5}$

3 When the sediment moving layer thickness calculated by (1) is greater than the total flow thickness, the sediment moving layer is considered as total sediment moving layer, with $c_s \geq c^* / 2$.

4 When the yield stress in the flow layer exceeds the external force when calculating the flow velocity distribution and friction coefficient of the sediment moving layer, the friction coefficient is calculated by using the equilibrium gradient corresponding to the average concentration in the total layer (c). This friction coefficient is the same as the equivalent friction coefficient obtained by MIYAMOTO *et alii* (2002), who carried out a numerical calculation of debris flow under the condition that the yield stress is greater than the external force.

In this study, we investigate the stopping and remigration of the sediment layer by using the method proposed by MIYAMOTO (2003) and carry out a two-dimensional simulation of the soil mass movement.

The stopping conditions used in this study correspond to the scenario where the momentum value calculated by excluding the yield stress lies in a circle whose radius is a product of the yield stress and pitch time and whose center is the point of origin (MIYAMOTO, 2003). Further, when the momentum calculated in the time step adjacent to the stopping step exceeds the maximum stiction, remigration occurs in this model (TAKAHAMA, 2004).

SLOPE COLLAPSE MODEL

In this study, s_t represents the changes occurring in the riverbed in the vertical direction; in the two-dimensional simulation, such changes do not occur in the region where there is no flow at the horizontal surface. For example, if a water channel, as in a natural landslide dam, causes riverbed decrease (flat head of landslide dam erosion) to such an extent that the relative height difference becomes very large and there is no flow into the nearby mesh, no further decrease occurs in the mesh, and a very steep gutter is generated in the water channel. For a simple representation of the side-bank collapse caused by erosion in the flat head of the landslide dam, we use the slope-collapse model proposed by SEKINE (2003) and generate a mesh flow channel.

SEKINE (2003) determined the value and direction of the steepest slope among the four slopes around the point of origin (O), as shown in Figure 2. When the steepest slope is directed from point O to point S, the vector for the point of appearance of this slope is as shown below.

$$\overline{OS} = (s_t \Delta x, (1 - s_t) \Delta y, s_t z_e + (1 - s_t) z_n) \tag{9}$$

where $s_t = \frac{(\Delta y)^2 z_e}{(\Delta x)^2 z_n + (\Delta y)^2 z_e}$, $z_n = \eta_{i,j+1} - \eta_{i,j}$,
 $z_e = \eta_{i+1,j} - \eta_{i,j}$.

The steepest slope denoted by $\tan \psi$ is calculated from the slope angle in the horizontal and vertical directions (denoted by $\tan \alpha$ and $\tan \omega$, respectively):

$$\tan \psi = \sqrt{\tan^2 \alpha + \tan^2 \omega} \tag{10}$$

According to SEKINE (2003), side-bank collapse occurs in the direction of the steepest gradient when the gradient exceeds $\tan \phi$, which is determined from the repose angle. In this study, we obtain the average gradient by estimating the bank slope of the water channel generated by the overtopping erosion of an actual landslide dam.

In this study, we assume that slope collapse proceeds until the position indicated by point O is decreased by a factor of ϵ . The collapsing surface is shown by the shaded ΔTAN in Figure 2 (a); ϵ denotes the extent of decrease in the vertical direction at point O and is given by

$$\epsilon = \sqrt{(s_t \Delta x)^2 + (1 - s_t)^2 (\Delta y)^2} \times (\tan \psi - \tan \phi) \tag{11}$$

Ground level at the point O deducts that for ϵ . Next, the collapse induced by changes in the ground form during one calculation time step is similar to the collapse that goes to completion within a time step of the same duration. Here, the amount of sediment

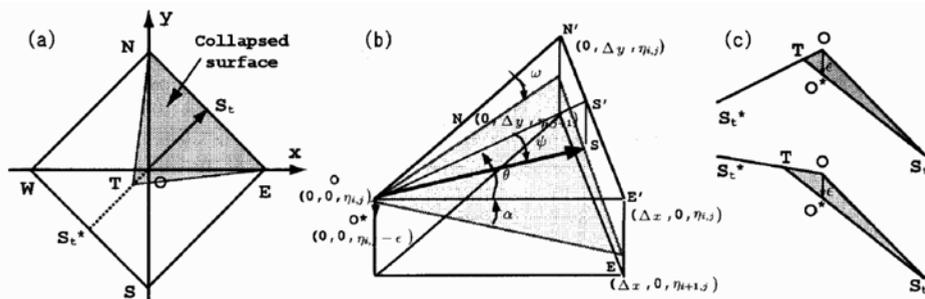


Fig. 2 - Conceptual diagram for slope-collapse model (SEKINE, 2003)

supplied by the collapse per unit time is calculated as follows:

$$Q_{collapse} = \frac{(1 - \lambda) \cdot (\varepsilon \Delta x \Delta y)}{6 \Delta t} \quad (12)$$

The sediment discharge rate is different in the x- and y-axis directions as shown in formula (9), and the discharge is toward the steepest gradient; the vector corresponding to the discharge rate is as shown below.:

$$\vec{q}_{collapse} = Q_{collapse} \times \left(\frac{\cos \theta}{\Delta y}, \frac{\sin \theta}{\Delta x} \right) \quad (13)$$

To solve the equation of continuity for the sediment, Sekine (2003) took into account the sediment discharge rate (shown in formula (13) determined from the sediment discharge function. We use this concept to calculate the equilibrium of sediment caused by the side-bank collapse from the summation ($Q_{collapse IN}$) of $q_{collapse}$, which flows into the rectangular Mesh A from four directions. As the sediment is lost from Mesh A ($Q_{collapse OUT}$) $Q_{collapse}$, the equilibrium ($Q_{collapse}$) is calculated as follows:

$$Q_{collapse IN} = \sum \vec{q}_{collapse} \quad (14)$$

$$Q_{collapse} = Q_{collapse IN} - Q_{collapse OUT} \quad (15)$$

Next, we consider the amount of sediment on the collapsed side bank in each mesh and re-solve the following equations of continuity. Note that the sediment amount on the side bank is considered to be completely included in the nearby mesh in the collapse direction.

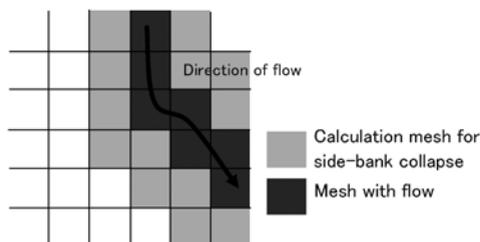


Fig. 3 - Placement of the calculation mesh

The equation of continuity for the riverbed level is

$$\frac{\partial z_b}{\partial t} = -\varepsilon \quad (16)$$

The equation of continuity for the total layer is

$$\frac{\partial h_t}{\partial t} = Q_{collapse} + S_T \quad (17)$$

The equation of continuity for the sediment is

$$\frac{\partial c_s h_s}{\partial t} = c_s \cdot (Q_{collapse} + S_T) \quad (18)$$

$$c_i = \frac{c_s \cdot h_t}{h_i} \quad (19)$$

The degree of side-bank collapse is estimated for the mesh in the vicinity of the mesh in which the flow occurs, as shown in Figure 3. In other words, if a mesh in which no flow occurs is lowered in height owing to the side-bank collapse and a new flow is generated in the process of calculation, the adjacent mesh is taken into account for investigating the side-bank collapse

CONDITIONS FOR NUMERICAL CALCULATION

DETERMINATION OF ANGLE OF THE SIDE BANK

The slope at the beginning of the side bank collapse is determined by examining the cross-sectional profile of the actual landslide dam formed during the Iwate-Miyagi inland earthquake (YOSHINO *et alii*, 2010). Figure 4 shows the relation between the relative height of the water channel of the landslide dam and the slope of the side shore

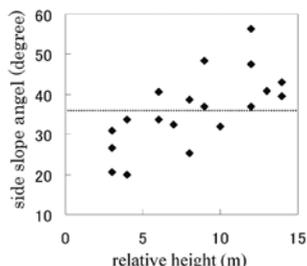


Fig. 4 - Relation between the relative height of the water channel and the angle of inclination of the side shore

bank. The examination results indicate that the slope varies by around 20 degree even for the same relative height and that the slope increases with relative height of the water channel. Therefore, it is necessary to consider this effect when calculating the slope of the side bank; however, for a simple representation of the model in this study, we perform calculations by setting the minimum angle required for the occurrence of the side bank collapse to $\phi_c = 36$ degree, which is the average angle of the water channel after overtopping erosion.

CALCULATION CONDITIONS

The calculation conditions we used are as shown in Table 1. Calculations were performed for the following three cases. The diagrams at the top in Figures 5 and 6 show the cross-sectional profile of each side bank, and the diagrams at the bottom show the longitudinal profile. In addition, 1 shows the longitudinal profile of the side bank, and 2 shows the longitudinal profile of the water channel in each Figure.

(1) Case 1: Calculations are performed for the water channel with a gradient changing point at 3° on the upstream and 14° on the downstream. 2-cm-high, 20-cm-wide water channel is formed at the gradual slope. In this case, the side bank collapse is taken into account.

(2) Case 2: Calculations are performed under conditions similar to those considered in Case I, except that the side bank collapse in the water channel is not taken into account.

(3) Case 3: Calculations are performed for the

water channel with a gradient changing point at 14° on the upstream and 3° on the downstream. 2-cm-high, 20-cm-wide water channel is formed at the steep slope. In this case, the side bank collapse is taken into account.

Note that the in the above-mentioned three cases, the flow depth does not reach 2 cm, which is the height of the water channel, at the given supply discharge rate; further, there is no flow over the water channel. Therefore, flow-induced riverbed erosion does not result in any increase in the width of the water channel.

| Parameter | Value | Explanation |
|-----------|------------------------|------------------------------|
| dx, dy | 5 cm | Mesh size |
| dt | 0.001 s | Calculation pitch time |
| ρ | 1.0 g/cm ³ | Density of spacing fluid |
| σ | 2.65 g/cm ³ | Sediment density |
| d_m | 0.2 cm | Average particle diameter |
| c_s | 0.6 | Sediment layer concentration |
| ϕ_s | 38° | Internal friction angle |
| ϕ_c | 36° | Side bank angle |
| q_m | 100 cm ³ /s | Inflow rate |
| T | 120 s | Total calculation time |

Tab. 1 - Calculation conditions

| | Upstream gradient (degree) | Downstream gradient (degree) | Occurrence of side bank collapse |
|--------|----------------------------|------------------------------|----------------------------------|
| Case 1 | 3 | 14 | yes |
| Case 2 | 3 | 14 | no |
| Case 3 | 14 | 3 | yes |

Tab. 2 - Cases for calculation

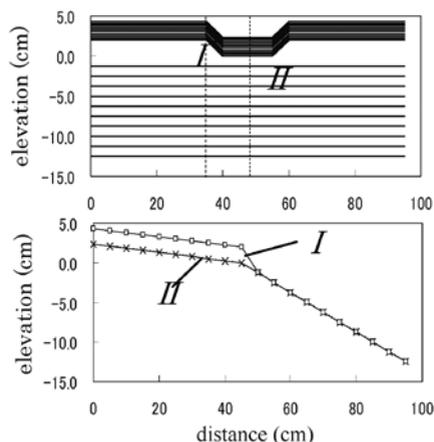


Fig. 5. Case 1,3 (top: cross-sectional view bottom: longitudinal section view)

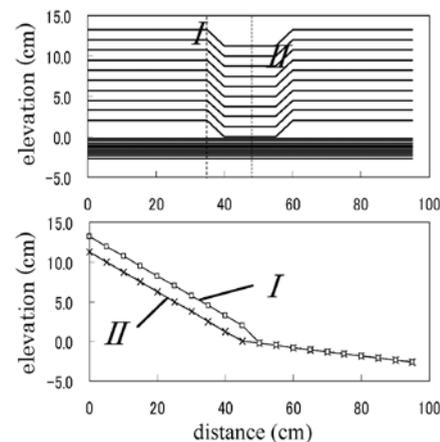


Fig. 6 - Case 2 (top: cross-sectional view; bottom: longitudinal section view)

RESULTS OF NUMERICAL SIMULATION

Figures 7, 8, and 9 show the riverbed level for each case after T = 10, 30, and 60 s. Figure 10 shows the riverbed level and water surface level determined by our calculation.

In Case 1, side bank collapse occurs as riverbed erosion advances from the gradient changing point. Then, the riverbed erosion proceeds gradually to the upstream to cause decrease at the upper end (flat head

of landslide dam). The width of the upper part of the water channel is maximum at the gradient changing point. The water channel first becomes narrow in the downstream direction and then becomes wide as we proceed further downstream. As shown in Figure 10, this is because riverbed erosion proceeds in the downstream direction, where the flow spreads over a wider region in the transverse direction. On the other hand, because side bank collapse is not taken into account in

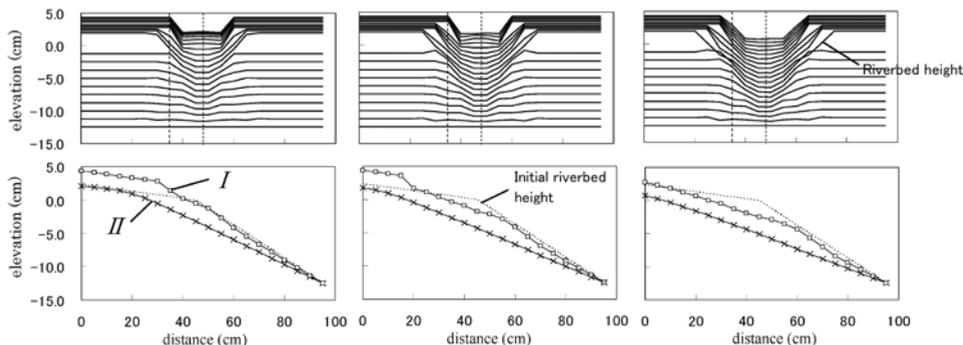


Fig. 7 - Variation of riverbed level with time (Case 1)

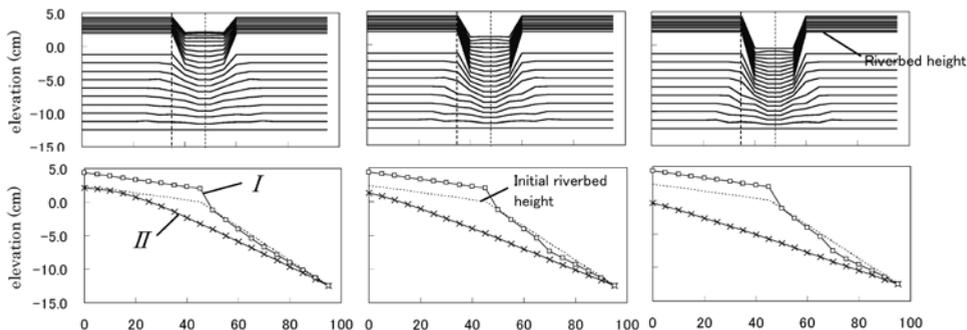


Fig. 8 - Variation of riverbed level with time (Case 2)

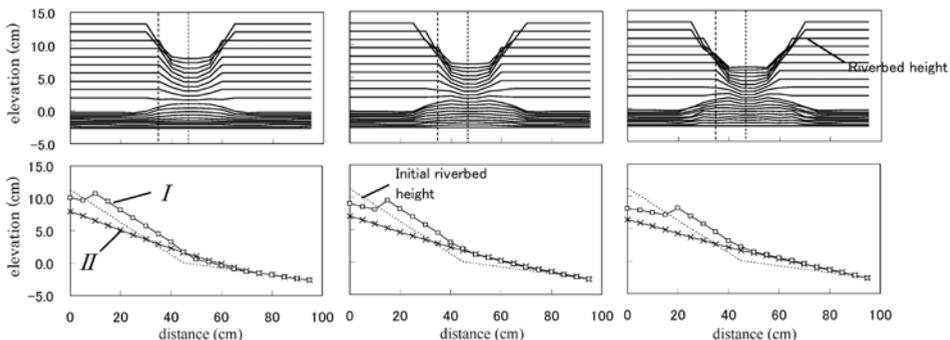


Fig. 9 - Variation of riverbed level with time (Case 3)

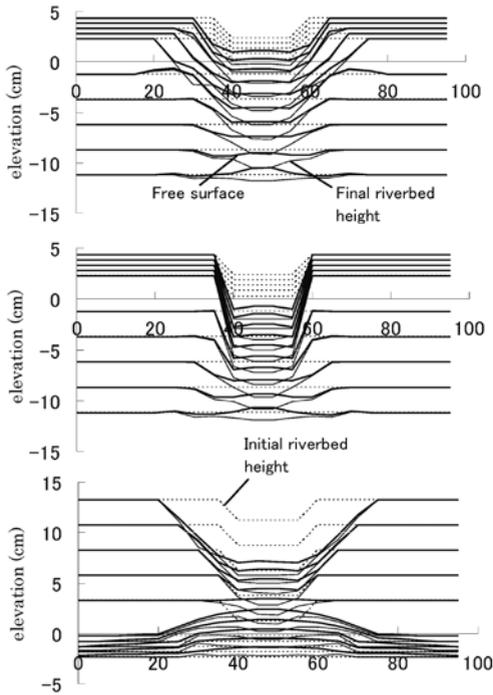


Fig. 10 - The riverbed level and water surface level (top: Case 1; middle: Case 2; bottom: Case 3)

Case 2, it is natural that the width of the water channel does not increase.

Moreover, as there is no sediment intake from the side bank, the extent of riverbed decrease at a given time is greater than in Case 1. In Case 3, as riverbed erosion advances from the upper end (flat head of the landslide dam), the side bank collapses. Then, the erosion advances gradually to the downstream. In addition to this, erosion occurs again in the sediment deposited area, and the water channel is gradually formed.

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CONSIDERATIONS AND FUTURE CHALLENGES

In this study, we developed a two-dimensional two-layer model by taking into account side bank collapse and used the model to carry out calculations by assuming overtopping erosion of a landslide dam. We simulated a side bank collapse process in which erosion advances in the vertical direction. We also found that the water flow layer is separated according to the sedimentation pattern, resulting in a low-concentration flow on the ground form after sedimentation; this eventually leads to recurrence of erosion. Thus, we could simulate the conditions under which a water channel is formed. It has been reported that once landslide dam breach occurs, a high concentration flow during which eroded sediment erosion occurs is usually induced; because of this type of flow, sediments are often strong unsteady phenomenon property (TAKAHAMA *et alii*, 2004). In addition, we considered the side bank collapse for a rational evaluation of the increase in the water channel width in the erosion area.

In this study, we also determined the collapsing slope of the side bank on the basis of the slope of the water channel formed by overtopping erosion in the case of the landslide dam, which in turn was formed during the 2008 Iwate-Miyagi Nairiku Earthquake. We also provided a simple method for calculating the average gradient. In reality, this angle may vary with the relative height of the water channel, materials that make up the landslide dam, apparent cohesion, etc. In the future, it is necessary to study multiple cases of landslide dam breach by carrying out measurements with the help of a LiDAR data set; in this case, emphasis should be laid on various factors such as the relative height of the water channel, angle of the side bank, and materials that make up the landslide dam.

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