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MONITORING NEAR-RIVERBED SEDIMENT BEHAVIOR OF DEBRIS FLOWS USING HYDROPHONES

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ABSTRACT

Hydrophones are steel pipes containing a microphone, and they can be used to measure bedload transport intensity. Bedload discharge and average grain diameter can be calculated analytically using sound pressure data. In this study, hydrophones were used to identify debris flows. The proportional relationship between the output voltage corresponding to a grain collision and its momentum was used to analyze electric pressure distribution, which was then used to calculate the mean diameter of colliding grains. Flume experiments were conducted to verify the effectiveness of this method in recognizing the time change of the near-riverbed sediment discharge from debris flows and low concentrated flows, including their transition ranges. Total sediment discharge can also be calculated if the collision rate upon the hydrophones is evaluated by setting the interface. In addition, the time change of the average grain diameter can be calculated. Large grains were detected in the debris flow surge, and the analytic values were in rough agreement with the experimental values.

Key words: *debris flow monitoring, hydrophone, sound pressure*

INTRODUCTION

The flow characteristics of debris flows are determined by the balance between external forces and shear stresses. Research on the constitutive equations of debris flows has been carried out to construct models of interactions among particles. Thus, many constitutive equations have been proposed (e.g., Takahashi, 1980; Shen *et alii*, 1982; Tsubaki *et alii*, 1982; Drew, 1983; Egashira *et alii*, 1989). However, in these equations, the influence of riverbed roughness has not been taken into consideration. Suzuki *et alii* (2003) and Suzuki & Hotta (2006) confirmed that the coefficient of resistance increases as riverbed roughness increases with larger riverbed particles. Thus, the coefficient of resistance is large when the sediment concentration is large or the relative flow depth is small. Therefore, it is necessary to construct a flow model that can evaluate the influence of riverbed roughness. To do so, the near-riverbed sediment behavior of debris flows must be clarified. Unfortunately, there is no established method to measure this behavior because it is generally very difficult to measure sediment concentration and sediment load. There is no sensor that can measure sediment concentration or sediment load directly. Moreover, a direct sampling measurement method requires a huge amount of effort and is tremendously costly, and long-term continuous measurement is impossible. Therefore, sediment concentration and load are estimated indirectly by measuring other physical quantities. One class of methods for the indirect estimation involve the use of hydrophones (microphones within steel pipes), and their effectiveness in the measurement of bedload transport intensity has been verified (Barzinger & Burch, 1990; Hegg & Rickenmann, 1998; Rickenmann, 1998; Mizuyama *et alii*, 2002). An example of such a method is the "pulse" method, in which the sound of sand colliding with the steel pipe containing the microphone is analyzed and transformed into a pulse; this pulse represents the number of times the sound level crosses a certain threshold. Thus, the sediment transport intensity is estimated on the basis of the premise that it is positively correlated with the pulse. However, this method has some problems. For example, when the sediment rate is high, the sound level is continuously high. Thus, the number of pulses decreases or becomes zero (MIZUYAMA *et alii*, 2008).

On the basis of the above information, SUZUKI *et alii* (2010) attempted to use an analytical method involving the use of sound pressure data. According to these researchers, bedload discharge and average diameter can be obtained by considering the reduction in sound pressure caused by the interference of sound waves. While hydrophones can be used to evaluate debris flow, the method of SUZUKI *et alii* (2010) has measurement limitations with regard to the number of grain collisions. Thus, this method needs to be modified before it can be used for debris flow measurements. The aim of this paper is to develop a method for calculating the mean grain diameter by analyzing the electric pressure distribution and to apply it to debris flows. The method was verified by performing flume experiments.

EXPERIMENTS

EXPERIMENTAL DEVICES

The variable slope channel of the Civil Engineering Research Laboratory was used for the experiment. The channel is 9 m long and 10 cm wide, with glazed sides (Fig. 1). The sides in the downstream part of the channel (4.5 m) are as high as 10 cm. A sand roughness was positioned at the downstream part (4.5 m); the upstream part (1.8 m) was filled with sediment. Water was regularly supplied from the upper end and a debris flow was generated.

MEASUREMENT METHOD

An ultrasonic sensor was used to measure the time change of the flow surface level. This sensor was installed 1 m above the lower end. A hydrophone was installed at the lower end of the channel, as shown in Fig. 2. The output voltage of the microphone was amplified to the range of ±10 V.

Around the surge front, a sample of the debris flow was taken at the downstream end and the sampling time was measured. The debris flow discharge, Q, was calculated by dividing the sample volume, Vlm, by the sampling time. The sediment concentration, c, was calculated as follows:

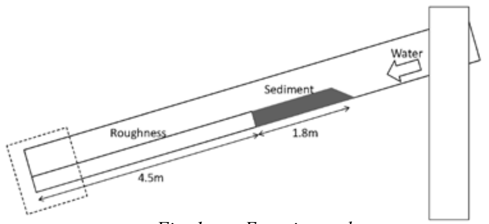

Fig. 1 - Experimental setup

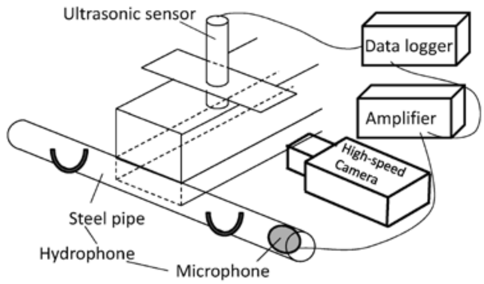

Fig. 2 - Schematic design of the hydrophone setup

$$c = \frac{V_s}{V_s + V_w} \tag{1}$$

In Eq. (1), V_s is the sediment volume in the sample and V_w is the water content. The passing time of a large grain was measured with high-speed video (500 frames per second) installed 1 m above the lower end.

CONDITIONS

Four tests were performed. In all cases, the channel slope θ was set to 13°. The supplied sediments in the upstream part of the channel had different grain distributions. In two cases, a uniform grain size was used, with d = 0.294 cm and σ = 2.65 (σ is the specific gravity of the sediment) (Case-Uni.). In the other two cases, together with a sediment of d = 0.294 cm, larger grains were used with d = 1.76 cm and located at 15 cm intervals on the surface (Case-Large). The total number of larger sediment grains was 10, one grain per 15 cm interval. The supplied water discharges were 1 and 3 L/sec under each grain distribution condition.

NUMERICAL SIMULATION

A continuous sequence of sediment discharge cannot be identified through experimental measurements alone. Therefore, a one-dimensional numerical simulation was conducted to replicate the flume experiments.

The basic debris flow equations are shown below (MIYAMOTO & ITO, 2002). The momentum equation is:

$$\frac{\partial M}{\partial t}+\frac{\partial (uM)}{\partial x}=-gh\frac{\partial H}{\partial x}-\frac{\tau_0}{\rho_m} \quad (2)$$

The continuity equation for the total volume of the debris flow is:

$$\frac{\partial h}{\partial t}+\frac{\partial M}{\partial x}=E \quad (3)$$

and the continuity equation for the particles is:

$$\frac{\partial (\bar{c}h)}{\partial t}+\frac{\partial (c_t M)}{\partial x}=Ec_* \quad (4)$$

Changes in the bed surface elevation can be determined using the following equation:

$$\frac{\partial z_b}{\partial t}=-E \quad (5)$$

For Eqs. (2)-(5), h is the flow depth, u is the flow velocity, $M = uh$, g is the acceleration due to gravity, ρ_m is the density of the debris flow, $H = h + z_b$, z_b is the bed elevation, E is the erosion velocity, τ_0 is the riverbed shear stress, $\bar{c}$ is the average sediment concentration, c_t is the flux sediment concentration, and c_* is the sediment concentration of the movable bed layer.

The equations of MIYAMOTO & ITO (2002) were applied for τ_0 and c_t, and the following erosion rate equation of SUZUKI *et alii* (2009) was applied for E:

$$E=\frac{1}{T\cdot c_*}\left(h_e\bar{c}_e-h\bar{c}\right) \quad (6)$$

In Eq. (6), T is the relaxation time of erosion or deposition, and e is the equilibrium sediment concentration, expressed as follows:

$$\bar{c}_e=\frac{\rho\tan\theta}{(\sigma-\rho)(\tan\phi_s-\tan\theta)} \quad (7)$$

In Eq. (6), h_e is the equilibrium flow depth when τ_0 is equal to the external force. In the equations of miyamoto & ito (2002), τ_0 was expressed as the following equation with the sum of the Coulomb friction shear stress, τ_{0y}, and dynamic shear stress, τ_{0D}:

$$\tau_0=\tau_{0y}+\tau_{0D}=\tau_{0y}+\rho f_b u^2 \quad (8)$$

$$\tau_{0y}=\alpha\rho(\sigma/\rho-1)\bar{c}gh\cos\theta\tan\phi_s \quad (9)$$

$$\alpha=\left(\frac{\bar{c}}{c_*}\right)^{1/5} \quad (10)$$

In Eq. (9), f_b is the coefficient of resistance.

$$f_b=\frac{25}{4}\left(k_g\left(1-e^2\right)\frac{\sigma}{\rho}c^{\frac{1}{3}}+k_f\frac{(1-c)^{\frac{5}{3}}}{c^{\frac{2}{3}}}\right)\left(\frac{h}{d}\right)^{-2} \quad (11)$$

In Eq. (11), k_g is the empirical constant that is specified as 0.0828, e is the coefficient of restitution, and k_f is an experiment constant of 0.16.

Substituting Eq. (8) with $\bar{c} = c_e$, $h = h_e$, and $u = M/h_e$ into the following Eq. (12), h_e is obtained,

$$\rho_m g h_e\sin\theta=\tau_{0y}+\rho f_b\left(\frac{M}{h_e}\right)^2 \quad (12)$$

Numerical simulations were conducted using the above- mentioned method.

Model parameters were calibrated to obtain the best match between simulation and experimental results in terms of time-series variation of h and sediment concentration around the surge front.

In Figs. 3 and 4, the experimental and simulation results are compared; c-Exp and h-Sensor refer to the measured c and h, and c-Cal. and h-Cal. refer to the simulation results of c and h using calibrated parameters. From 72 to 85 sec of the 1 l-case and from 31 to 34 sec of the 3 l-case, the h-Sensor decreased drastically. These changes were due to the transition from debris flow to low concentrated flow after most of the grains were eroded. Thus, the simulation results agree well with the experimental results. Therefore, these simulation results are useful as comparison values for hydrophone measurements.

ANALYTICAL METHOD

Raw data obtained with a hydrophone are shown in Fig. 5. To reduce the electrical noise, circumferential frequency components were extracted with a band-pass filter (Fig. 6). Sound pressure data correspond to the line connecting the local maximum points of the extracted data (Fig. 7), and Sp is the average value.

SUZUKI *et alii* (2010) confirmed the relationship between Sp and bedload discharge, Qs, as follows:

$$Sp=\alpha\cdot Qs\cdot R \quad (13)$$

$$R=f(N) \quad (14)$$

where α is the proportionality coefficient, R is the detection rate, and N is the number of collisions per second. Equation (14) indicates that R is a function of N. The relationship between R and N can be obtained from experimental results under a wide range of conditions. However, it is unrealistic because a tremendous amount of data is necessary for experimental accuracy. Therefore, SUZUKI *et alii* (2010) proposed a method for estimating the relationship between R and N using superposition simulations. Their method is described in the following.

First, a uniform random number, $rd(t)$, is given every one-millionth of a second, where t is the elapsed time (sec). The threshold value, Th, is set at $Th = N/100000$ for arbitrary N. When $rd(t)$ is lower than Th, an individual collision wave datum, which is obtained by preliminary experiment, is added to the wave data being produced. R is calculated using Eq. (13) from the data computed in this way. The relationship between R and N is obtained when N is changed over a wide range. Thus, R decreases as N increases due to the effects of sound wave interference (Fig. 8).

Qs is expressed as

$$Qs = \frac{\pi d^3}{6} N \tag{15}$$

Substituting Eqs. (14) and (15) into Eq. (13), Eq. (16) is obtained,

$$Sp = \alpha \cdot \frac{\pi d^3}{6} N \cdot f(N) \tag{16}$$

This paper aimed to develop a method for calculating Qs and d using just sound data obtained with a hydrophone; this was attempted because it is difficult to measure the time-series variation of grain size distribution. Thus, Eq. (16) has two unknown values, d and N, while Sp is measured variable. Therefore, it is impossible to obtain Qs from Sp only through Eq. (16). Suzuki *et alii* (2010) proposed a method for calculating the transformed data that satisfies Eq. (17) by dividing the original data equally and summing them linearly as follows:

$$Qs_{\text{tfd}} = mQs \tag{17}$$

in which Qs_{tfd} is the bedload discharge value of the transformed data and m is an arbitrary counting number.

T_2, second transformed data, is calculated using T_1, second original data. The relationship between T_1 and T_2 is expressed as follows:

$$T_1 = mT_2 \tag{18}$$

The original data are divided evenly into m parts, and the pth part is designated as $V_1(p,t)$, where p is the counting number from 1 to m and t is the elapsed time from the starting point of every part. Transformed data are designated as $V_2(t)$ and calculated as Eq. (19),

$$V_2(t) = \sum_{p=1}^{m} V_1(p,t) \tag{19}$$

In Fig. 9, an example with $T_1 = 5.0$, $T_2 = 0.5$, and m = 10 is shown.

With regard to transformed data, Eq. (20) is developed using a method similar to Eq. (16),

$$Sp_{\text{tfd}} = \alpha \cdot \frac{\pi d^3}{6} mN \cdot f(mN) \tag{20}$$

where Sp_{tfd} is the sound pressure value of the transformed data. Equations (16) and (20) are solved as a set of simultaneous equations to calculate Qs and d. However, this method has measurement limitations with respect to the number of grain collisions. Thus, the method needs to be modified before it can be used for debris flows. In the present study, a new method was developed to calculate the grain diameter distribution by analyzing the electric pressure distribution. This new method is detailed in the following.

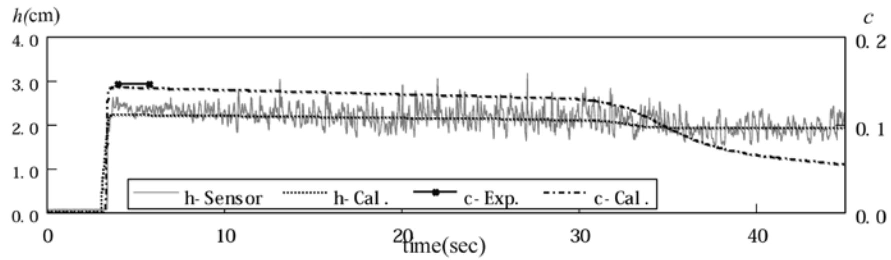

Fig. 3 - Experimental and simulation results of flow height, h, and sediment concentration, c (Case-Uni.-1l)

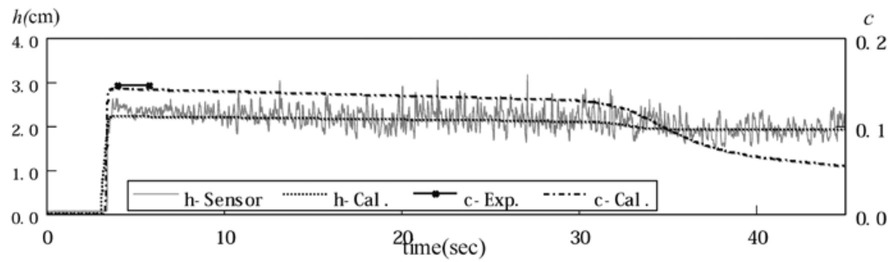

Fig. 4 - Experimental and simulation results of flow height, h, and sediment concentration, c (Case-Uni.-3l)

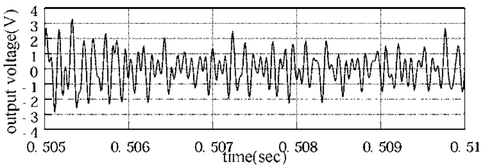

Fig. 5 - Raw wave data

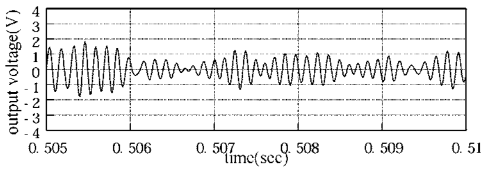

Fig. 6 - Wave data extracted with a band-pass filter

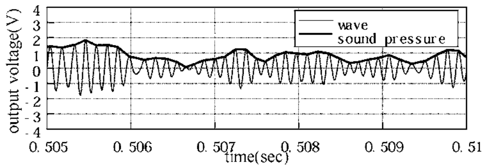

Fig. 7 - Sound pressure data

First, output voltage data were divided by 0.0001 V. The voltage of the *i*th section, V_i, is

$$V_i = 0.0001i \tag{21}$$

Suzuki *et alii* (2010) confirmed that the maximum voltage of the individual collision sound, V_{max}, was proportional to the momentum of the grain. Thus, the relationship between V_{max} and d is as follows:

$$V_{max} = \beta \cdot d^3 v \tag{22}$$

where β is the proportionality coefficient and v is the velocity of a grain. Assuming that v is constant in certain durations, the diameter corresponding to the *i*th section, d_i, is given as follows:

$$d_i = \gamma \cdot v^{\frac{1}{3}} \cdot V_i^{\frac{1}{3}} \tag{23}$$

where γ is the proportionality coefficient and is found to have a value of 0.6β from the results of the present preliminary investigations.

Assuming the number of local maximum voltage values in the ith section, Nm_i, is proportional to the number of grains in the *i*th section, the volume of grains in the ith section, Vol_i, is expressed as follows:

$$\mathrm{Vol}_i = k \cdot Nm_i \frac{\pi d_i^3}{6} \tag{24}$$

where k is the proportionality coefficient. Therefore, the ratio of Vol_i to the total volume, r_i, is given as follows:

$$r_i = \frac{\mathrm{Vol}_i}{\sum \mathrm{Vol}_i} = \frac{Nm_i V_i}{\sum Nm_i V_i} \tag{25}$$

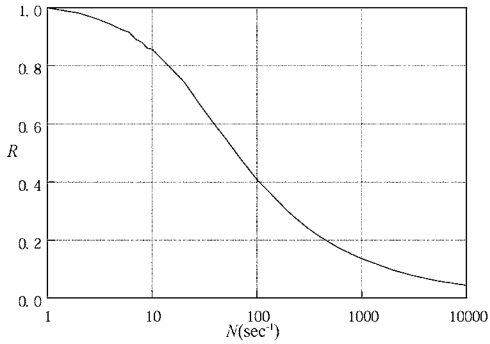

Fig. 8 - Relationship between the number of collisions, N, and the detection rate, R

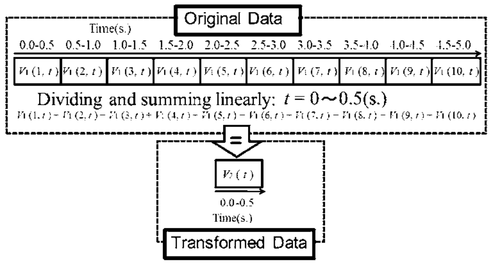

Fig. 9 - Schematic diagram of the data transformation

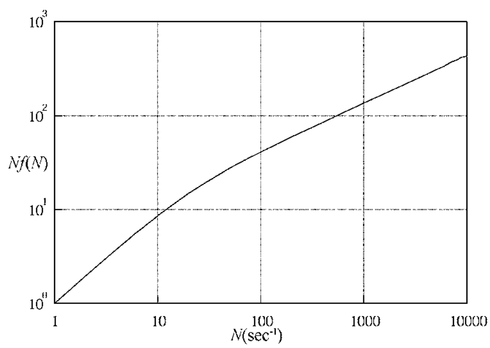

Fig. 10 - Relationship between the number of collisions, N, and Nf(N)

The grain size accumulation curve is obtained by accumulating r_i, and d is calculated as the volume mean diameter using Eq. (26),

$$d = \sum r_i d_i \tag{26}$$

Substitutions allow for the identification of d in Eq. (16), enabling the value of $Nf(N)$ to be calculated. The relationship between N and $Nf(N)$ (Fig. 10) was computed easily from the $Nf(N)$ relationship (Fig. 9). $Nf(N)$ was substituted into the $Nf(N)$ relationship, and N was calculated. Finally, substituting d and N into Eq. (15) allowed Qs to be obtained.

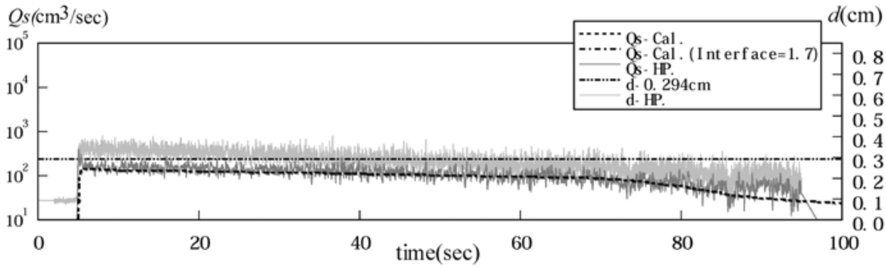

Fig. 11 - Analytic results of sediment discharge, Qs, and average grain diameter, d (Case-Uni.-1l)

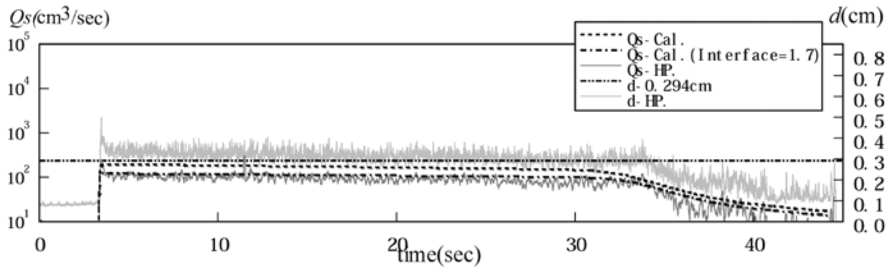

Fig. 12 - Analytic results of sediment discharge, Qs, and average grain diameter, d (Case-Uni.-3l)

RESULTS

CASE-UNI

In Figs. 11 and 12, the analyses and simulation results are shown and compared, with *Qs*-Hp and *d*-HP referring to the analytic results of *Qs* and *d* (see the previous paragraph), and *Qs*-Cal. and *d*-Cal. referring to the simulation results of *Qs* and *d* (see the section titled "Numerical Simulation"). In the 1*l*-case, *Qs*-HP and *Qs*-Cal. agreed quantitatively. However, *Qs*-HP of the 3*l*-case was lower than *Qs*-Cal. The underestimation of *Qs* for the 3*l*-case can be explained fairly well if one considers that, during the experimental tests, the grains in the upper layer likely did not collide with the hydrophone. Assuming that only grains under a certain interface height, IF, collide with the hydrophone, the collision rate, r_c, was evaluated as follows:

$$r_c = \frac{\int_0^{\mathrm{IF}} u(z)dz}{\int_0^h u(z)dz} \tag{27}$$

where $u(z)$ is the velocity at z. Substituting the typical velocity distribution of a debris flow [Eq. (28)] into Eq. (27), r_c is expressed as Eq. (29),

$$u(z) = \frac{5}{3}U\left(1-\left(1-\frac{z}{h}\right)^{3/2}\right) \tag{28}$$

$$r_c = \frac{\mathrm{IF} + 2h\left\{(1-\mathrm{IF}/h)^{5/2} - 1\right\}}{3h} \tag{29}$$

In this study, *Qs* and *d* were calculated every 0.01 sec because the duration of one collision sound was about 0.02 sec. For *Qs*, a 0.1 sec moving average was computed.

In Figs. 11 and 12, simulation results when IF was 1.7 cm [*Qs*-Cal(Interface = 1.7)] are shown, and they are in agreement with the analyzed results.

When *c* was high, the analytic results of *d* were close to 0.294 cm. However, when *c* was low (low concentrated), the analytic results of *d* were lower. This was due to the noise of the water, and the problem should be rectified in future studies.

CASE-LARGE

Qs-Hp, *d*-HP, *Qs*-Cal., and *d*-Cal. around the surge are shown in Figs. 13 and 14, in which pass timings of large grains determined from high-speed video also are shown. *Qs* and *d* increased instantaneously when large grains were considered to have collided with the hydrophone. About five grains in the 1*l*-case and two grains in the 3*l*-case were discriminated. It is presumed that other grains did not collide with or graze the hydrophone. The maximum analysis value of *d* was about 0.8 cm, which is lower than the actual value of 1.76 cm. This is because the analytic value of *d* was the volume mean diameter. The volume mean diameter, d_{av}, when large grains collide is obtained as follows:

$$d_{av} = \frac{Qs_S \cdot d + Qs_L \cdot d_L}{Qs_S + Qs_L} \tag{30}$$

where Qs_S is the sediment discharge of 0.294 cm grains, d_L is the diameter of a large grain, and Qs_L is the sediment discharge of large grains. Qs_L was obtained with Eq. (31),

$$Qs_L = \frac{\pi d_L^3 / 6}{t_{d_L}} \quad (31)$$

$$t_{d_L} = \frac{d_L}{v} \quad (32)$$

Substituting Eqs. (31) and (32) with d = 0.294, d_L = 1.76, and the simulation results of Qs_S into Eq. (30), d_{av} was calculated from 0.82 to 1.14 cm. Therefore, the analytic values of d_{av} were slightly lower than the actual values, but they are reasonably close.

CONCLUSIONS

In this study, hydrophones, which have been developed for bedload sediment analysis, were used to analyze debris flows. An analytical method for calculating the grain diameter distribution from measurements of the electric pressure distribution was developed by considering the fact that the output voltage corresponding to a grain collision is proportional to its momentum. Also introduced herein was the existing method of Suzuki *et alii* (2010). Flume experimental and analytical results confirmed that sediment discharge and the average diameter of near-riverbed sediment can be measured quantitatively by this method. Total sediment discharge can also be calculated if the collision rate on the hydrophones is evaluated by setting the interface.

The hydrophone itself has some limitations. For example, the measurable sound level is limited and its range depends on the performance of the microphones. Because of this, hydrophones are applicable within a specific range. The method described in this study can be used to perform highly accurate measurements of the time change of sediment discharge within experimental results of this study . However, this method also has some limitations. For example, when sediment concentration is low, the values of the grain diameter obtained are lower than the actual values. Moreover, it is presumed that the interface varies with the scale of the debris flows, the conditions under which the hydrophone is installed, and the terrain conditions at the installation site. Therefore, further improvement of the method is necessary to resolve these problems.

In addition, the method described in this study was developed for flume experiments performed as part of basic studies of debris flows. However, it may also be applied to field monitoring. In such cases, the hydrophone's steel pipe must be large enough to endure the impact of realistically scaled debris flows. Therefore, verification of the applicability of large-size hydrophones is a necessary next step in the development of the method.

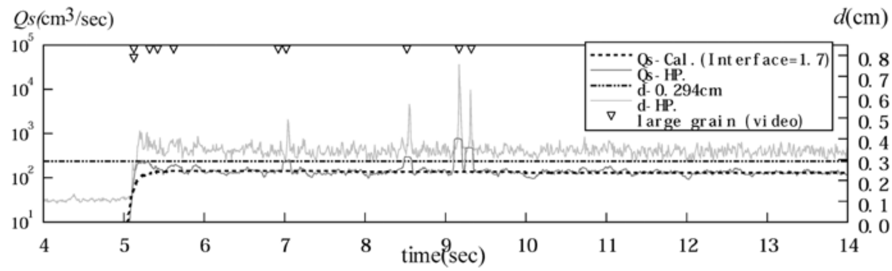

Fig. 13 - Analytic results of sediment discharge, Qs, and average grain diameter, d (Case-Large.-1l)

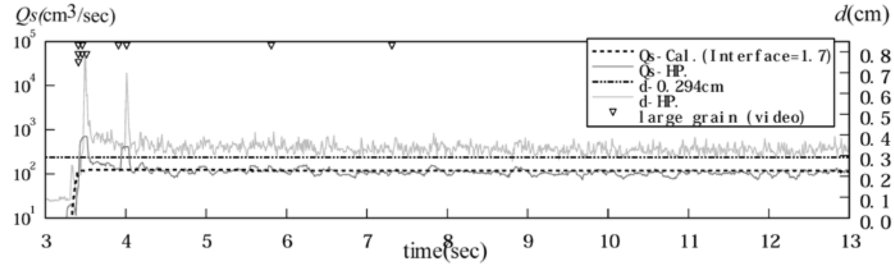

Fig. 14 - Analytic results of sediment discharge, Qs, and average grain diameter, d (Case-Large.-3l)

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