DOI: 10.4408/IJEGE.2011-03.B-096

NUMERICAL SIMULATION OF DEBRIS FLOW: A CASE STUDY OF THE DANIAO TRIBE DEBRIS FLOW IN EASTERN TAIWAN IN AUGUST, 2009

Ko-Fei LIU(*), Yu-Charn HSU(*), Hsin-Chi LI(**) & Hung-Ming SHU(***)

(*) Department of Civil Engineering, National Taiwan University, Taipei, Taiwan. E-Mail: d92521021@ntu.edu.tw
(**) Socio-Economic System Division, National Science & Technology Center for Disaster Reduction, Taipei, Taiwan.
(***) Graduate Institute of Disaster Prevention on Hillslopes and Water Resources Engineering, National Pingtung University of Science and Technology, Pingtung, Taiwan

ABSTRACT

In August 2009, Typhoon Morakot hit Taiwan and induced tremendous disasters, including large-scale landslides and debris flows. One of these debris flows was suffered by the Daniao tribe in Taitung, eastern Taiwan. The volume was in excesses of 500,000 m^3, much larger than the original design mitigation capacity. The DEBRIS-2D program developed by (Liu *et alii*, 2009) was applied in a hazard assessment at this particular site two years before the disaster. The model predicted a hazard zone that was close to the real disaster. This successful prediction seems to support the usefulness of DEBRIS-2D. However, there may be still factors that need to be discussed before identifying the success of the program. One of the important factors discussed was the total volume and its distribution. This paper showed that a 20 % variation in estimating the total volume in this particular site, would give rise to only a 2.75 % variation on the final front position. Therefore, volume is not very sensitive

Key words: *Typhoon Morakot, Numerical Simulation, Debris flow, DEBRIS-2D, Hazards assessment.*

INTRODUCTION

Debris flow is a mixture of water, gravel and soil. The motion of the mixture is induced by gravity and usually has high velocity, which causes catastrophic destruction in Taiwan. The common uncertainties during the planning of any countermeasures are the hazard zone area and the path of debris flow. There are numerous empirical formulas that can be used to obtain part of the information needed in the designing process. Nevertheless, empirical formulas can be inaccurate for complicated geographic regions. Numerical simulation is a better way to obtain the needed information.

In previous studies of debris flow, non-Newtonian fluid models have often been used. Early analytical or numerical studies of Bingham-like fluids have been limited mainly to one- or twodimensional spreading on an inclined plane. Liu & Mei (1989) presented a two-dimensional theory for the unidirectional slow flow of Bingham fluid on a slope. Huang & Garcia (1998) worked on the same problem for Herschel–Bulkley fluid. For three-dimensional flows, the slow and steady spreading of mud released from a point source on a plane was investigated by Hulme (1974) with a Bingham model, and by Coussot & Proust (1996) and Wilson & Burgess (1998) with a Herschel–Bulkley model. The static problem of the final deposit on an inclined plane has been studied experimentally by Coussot, Proust & Ancey (1996) and by Osmond & Griffiths (2001). For a horizontal plane bottom, Balmforth *et alii* (2000) derived analytical and numerical solutions for the radically symmetric evolution of isothermal lava domes. Reviews of these topics can be found in Coussot (1997), Griffiths (2000) and Mei *et alii* (2001). Balmforth *et alii* (2001) developed an analytical theory for the equilibrium shape of lava domes on an inclined plane.

For high-speed flows, LIU & MEI (1994) and NG & MEI (1994) examined the nonlinear formation of roll waves for a Bingham fluid and a power-law fluid, respectively. Similar problems regarding the avalanche of dry granules flowing down an inclined plane have been reported by Wieland, GRAY & HUTTER (1999) and POULIQUEN & FORTERRE (2002).

Most debris-flow models have focused on laboratory scale experiments or slow debris flow motion in a regular channel. However, debris flows occurring in the field are quite different from those in a controlled environment. It is difficult to simulate debris flow both numerically and experimentally. IVERSON *et alii* (2000) used a two-phase flow model to simulate debris flows moving from a huge flume (5 m by 100 m) to a wide deposition basin. O'BRIEN & JULIEN (1997) used a quadratic constitute to simulate high concentration flows. The DEBRIS-2D program was developed by LIU & HUANG (2006) for field debris flow simulation. This model had already been verified by a 1-D analysis solution, laboratory testing and a field case (LIU & HUANG, 2006). Additionally, this model has been used in many practical applications in Taiwan.

Debris flows at the Daniao tribe occurred during typhoon Morakot in August 2009 in eastern Taiwan. The typhoon dumped 740.5 mm of rainfall in 62 hours, and induced tremendous landslides and debris flow with volume exceeding 500,000 m^3. The authors

had performed a simulation for the Daniao tribe using DEBRIS-2D in 2007 under different design capacities (LIU *et alii*, 2009). However, the area of influence for the present event, with mitigation measures constructed, was almost the same as what was predicted before with no countermeasures. This proved the simulation ability of DEBRIS- 2D, but also induced questions on why it was the same. The challenges for finding the answers lie on the uncertainty of the input data. The geographical data was available but was not highly precise. The total amount of available soil that could be eroded or mobilized during heavy rainfall and the properties that could correctly represent the field material were also two major problems. Strictly speaking, if these parameters could not be precisely resolved, any modelling results would have errors. Therefore, this paper focused on only a few major factors.

RICKENMANN (1999) showed that in torrent hazard assessment, the debris flow volume is one of most important parameters. LIU & HSU (2008) studied sensitivity of three factors total volume, yield stress, and bottom slope for debris flow spread in lab scale. The paper concluded that total volume amount is more important than the others. It was found that a 20% change in total volume would induce 20% change for the maximum depth. Therefore, this paper uses different total volumes to simulate a real debris flow

DESCRIPTION OF DEBRIS-2D MODEL

The original DEBRIS-2D model was developed by LIU & HUANG (2006). The governing equations of the DEBRIS-2D model were adopted in the mass and momentum conservation. The constitutive relation proposed by JULIEN &LAN (1991) was used.

The original one-dimensional version was extended to a threedimensional version as.

$$\tau_{ij} = \left(\frac{\tau_0}{\left|\dot{\gamma}_{ij}\right|} + \mu_d + \mu_c \left|\dot{\gamma}_{ij}\right| \right) \dot{\gamma}_{ij} \,,\ \left|\tau_{ij}\right| > \tau_0 \tag{1}$$

$$\left|\dot{\gamma}_{ij}\right| = 0 \,,\ \left|\tau_{ij}\right| < \tau_0 \tag{2}$$

where τ_{ij} is the shear stress tensor and γ_{ij} is the strain rate tensor. $\tau\ 0$ is the yield stress, μ_d is the dynamic viscosity and μc is the turbulent-dispersive coefficient. τ_{ij} and γ_{ij} represent the second invariant of the shear stress and strain rate tensor, respectively. LIU & LAI (2000) defined the portion of debris flow with stress greater than the yield stress as the boundary layer. The depth ratio between the boundary layer and the main debris flow could be proved to be small. This implied that most of the flow region was in a weak stress condition, i.e. the plug region. The corresponding constitutive law is equation (2), which can be expressed as

$$\left|\dot{\gamma}_{ij}\right| = \sqrt{ 2\frac{\partial u_d}{\partial x}^2 + 2\frac{\partial v_d}{\partial y}^2 + 2\frac{\partial w_d}{\partial z}^2 + (\frac{\partial u_d}{\partial y} + \frac{\partial v_d}{\partial x})^2 + (\frac{\partial u_d}{\partial z} + \frac{\partial w_d}{\partial x})^2 + (\frac{\partial v_d}{\partial z} + \frac{\partial w_d}{\partial y})^2 } = 0 \,,\ \left|\tau_{ij}\right| < \tau_0 \tag{3}$$

where the x-axis coincides with the averaged bottom of the channel and is inclined at angle θ with respect to the horizon. The y-axis is in the transverse direction and the z-axis is perpendicular to both the xand y- axes. *u, v, w* are the velocity components in the *x, y, z* directions, respectively. Since debris flow in a lab or in the field can usually be considered as long waves, i.e. the depth scale is much smaller than the horizontal length scales; it can be obtained from equation (4) by neglecting the small terms

$$\frac{\partial u}{\partial z}=0\ ,\ \frac{\partial v}{\partial z}=0 \tag{4}$$

This implies that the portion of debris flow near the free surface where the stress free condition applies is a two-dimensional plug flow [i.e. $u \neq u(z)$ and $v \neq v(z)$]. Substituting (4) into the momentum equations obtains

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+g\sin\theta+\frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} \tag{5}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{zy}}{\partial z} \tag{6}$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g\cos\theta \tag{7}$$

The stress free condition is applied at the free surface $z = h(x, y,t)$. The upper boundary of the thin boundary layer near the bottom is defined as $z = B(x, y,t) +\delta (x, y,t)$ where the natural bottom of the debris flow is $z = B(x, y,t)$. As the thickness of the boundary layer is very small compared to the flow depth as discussed above, the natural bottom can be used as the boundary for the plug flow.

Equation (7) leads to static pressure in z

$$p=\rho g\cos\theta(h-z) \tag{1}$$

Integrating (5) and (6) in z from the bottom to the free surface obtains the results in conservative form as

$$\frac{\partial uH}{\partial t}+\frac{\partial u^2H}{\partial x}+\frac{\partial uvH}{\partial y}=-g\cos\theta H\frac{\partial(B+H)}{\partial x}+g\sin\theta H-\frac{1}{\rho}\frac{\tau_0 u}{\sqrt{u^2+v^2}} \tag{9}$$

$$\frac{\partial vH}{\partial t}+\frac{\partial uvH}{\partial x}+\frac{\partial v^2H}{\partial y}=-g\cos\theta H\frac{\partial(B+H)}{\partial y}-\frac{1}{\rho}\frac{\tau_0 v}{\sqrt{u^2+v^2}} \tag{10}$$

The depth integration of continuity equation gives

$$\frac{\partial H}{\partial t}+\frac{\partial \bar{u}H}{\partial x}+\frac{\partial \bar{v}H}{\partial y}=0 \tag{11}$$

where H=h(x,y,t)-B(x,y,t) is the flow depth. Equations (9), (10) and (11) could be used to solve the three unknowns H, u and v. This paper used the Adams-Bathforth 3rd order scheme for time and central differences and the upwind scheme in space. The upwind method is used for convective terms. The central difference method is used for all other terms. Mathematically, one condition each for *H, u* and *v* is needed in the physical boundary. For debris flow simulations in the field, it is usually necessary to find a computational domain, which contains the whole reach of the debris flows. In real applications, a large computation domain could be selected, so that debris flows would never reach the domain boundary. Thus the boundary conditions are

$$H(x,y,t)=0\ ,\ u=0\ ,\ v=0 \tag{12}$$

If debris flows are restrained in a fixed domain such as a flume, no normal flux condition will be used on all physical boundaries. However, (12) still applies to the front and tail of the debris flow. The tracking of the points with a velocity near zero is important. Corrections of overshooting the physical quantities are performed during every time step. The initial condition is the depth contour in the computation domain with all possible debris flow sources. The value of the rheological properties is also needed, which must be obtained from field sample measurements.

DESCRIOPTION OF DANIAO TRIBE DEBRIS FLOW

The Daniao tribe debris flow occurred in the debris flow potential stream DF097 in Taitung (eastern Taiwan). Stream DF097 has a high debris flow disaster potential according to the information from the Soil and Water Conservation Bureau in Taiwan. The watershed area of DF097 is roughly 0.86 km^2. A total of 71.7% of area has a slope greater than 15°, 18.6% of the area has a slope between 15°~6°, and only 28.3% has a slope less than 6° (see Fig. 1).

Field investigations in September 2006 discov-

Fig. 1 - Stream DF021 watershed where Daniao tribe debris flow occurred

Picture Series	Location (TWD67)		Solid Volume (m³)
	E	N	
1	239428	2477115	16,766
2	239536	2477089	498
3	239645	2477073	829
4	239661	2477026	358
5	239661	2477240	1,492
Total			19,943

Tab. 1 . Field investigation in 2007 before typhoon Morakot

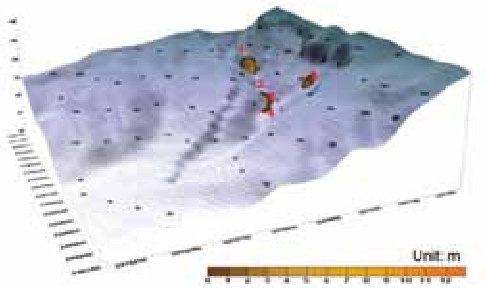
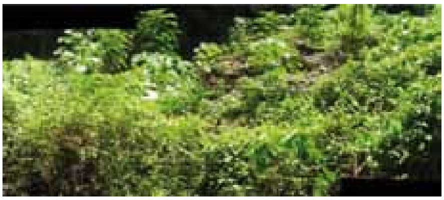


Fig. 2 - Debris flow source location (before typhoon Morakot)

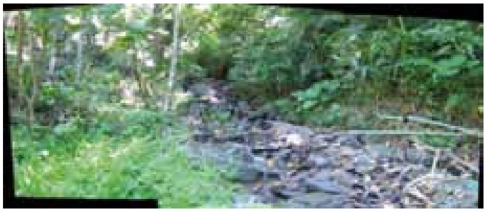
Pic. 1 - Landslide deposition on upstream hill



Pic. 2 - Ground with serious erosion on left bank hill

Pic. 3 - Mass source deposition on streambed

Pic. 4 - Landslide deposition on right bank hill



Pic. 5 - Debris deposition on branch streambed

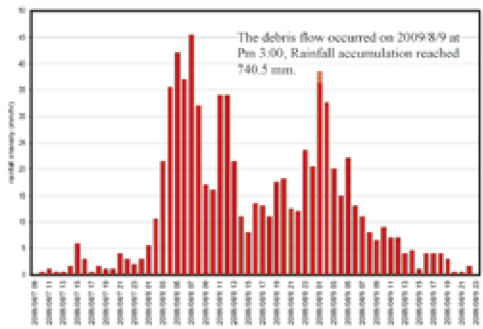


Fig. 3 - Rainfall intensity record for typhoon Morakot

ered several locations with large amounts of deposit. The total volume exceeded 19,943 m3 distributed on the slopes and streambed. (Tab. 1 and Fig. 2, pictures 1~5). A total of 63.1% of this material was located in regions with a slope greater than 15°, and only 7.8% of the material was located in regions with slope less than 6°. The formation of the mixture was composed of slate, mudstone, sandstone and weathered gravel, which are all easily movable under external forces.

In August 2009, typhoon Morakot hit Taiwan and dumped heavy rainfall. The maximum rainfall intensity of this event reached 45.5 mm/hour (see Fig. 3), and accumulated 759 mm of rainfall in 66 hours (from 2009/8/7 9:00 AM to 2009/8/09 11:00 PM). The

annual record for the maximum rainfall accumulated at 1, 2, 4, 6, 8, 12, 15, 24, 48 and 66 hours duration are shown in Fig. 4. On 2009/8/9 at 3:00 PM (with a rainfall accumulation of 740.5 mm in 62 hours), the rainfall induced tremendous landslides and debris flows. The aerial photo after the disaster is show in Fig. 5. Field investigations after the disaster revealed that almost 17.2 % (0.1485 km^2) of the watershed was buried, the total volume of debris flow exceeded 500,000 m^3 and almost 200,000 m^3 flowed out of the valley. The aerial photography before and after Typhoon Morakotare compared in Fig. 6 and Fig. 7.

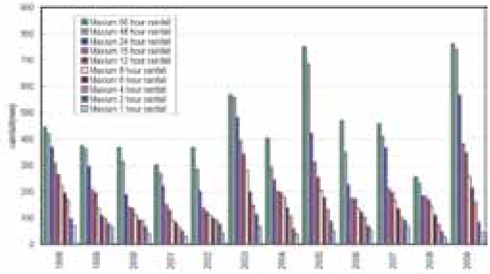


Fig. 4 - Annual record for maximum rainfall accumulated for 1, 2, 4, 6, 8, 12, 15, 24, 4 and, 66 hours duration in the DF021 stream watershed

Fig. 5 - Aerial photograph of Daniao tribe debris flow in Typhoon Morakot

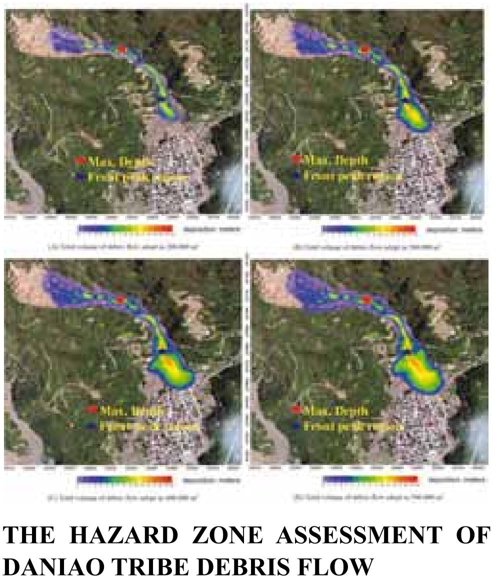


Fig. 6 - Aerial photograph before Typhoon Morakot hit

Fig. 7 - Aerial photograph after Typhoon Morakot

Fig. 8 - Final deposition contour maps adopt in different volume; (1)The maximum depth all almost equal to 15 m deposited on a watershed gap in medium stream; (2)The front peak all almost equal to 12~13 m deposited on the ran out of valley region of the watershed in down stream

THE HAZARD ZONE ASSESSMENT OF DANIAO TRIBE DEBRIS FLOW

Rickenmann (1999) showed that in torrent hazard assessment, the debris flow volume is one of the most important parameters. In reality, it is difficult to forecast real debris flow, as there are numerous uncertainties in a watershed. The DEBRIS-2D model was applied to assess a hazard zone with total amounts of 200,000 m^3, 300,000 m^3, 400,000 m^3 and 500,000 m^3 (see next page for volume estimation). A yield stress of the debris flow of 250 dyne/cm^2 was measured in the field. A time step of 0.01 seconds was set up, and the computational grid size adopted was 5m x 5m DTM of theDaniao tribe watershed. The initial debris sources were distributed atthe head of Taitung DF097 creek. Fig. 8 shows the final simulated deposit contour maps adopted in different volumes of debris flow sources. This paper compared hazard zones in differ-

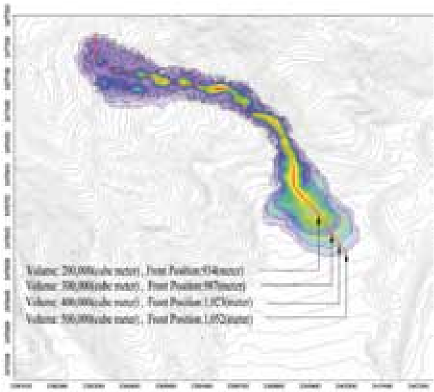


Fig. 9 - Compared hazard zones in difference volumes

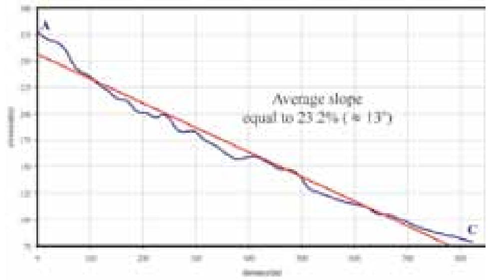


Fig. 10 - Average slope of debris flow path

ent volumes as shown in Fig. 9.

Form 2006 surveyed, we found there were at least 20,000 m^3 solid sources of debris flow deposited on the triggering areas of the Daniao tribe watershed. However, for heavy rainfall event, there should be more loose material can be created. Therefore, we adopt a different approach using accumulated rainfall to estimate the volume of debris flow in this site.

An equilibrium concentration conceptual of Takahashi (1980) was applied to estimate the debris flow volume amount. Takahashi (1980) derived the equilibrium concentration, which is

$$C_\infty = \frac{\rho_w \tan\theta}{(\rho_s - \rho_w)(\tan\phi - \tan\theta)} \tag{13}$$

where C_∞ is the equilibrium concentration, ρ_s is the solid density of debris, ρ_w is the liquid density of debris, φ is the rest angle of solids, and θ is the slope. In general, ρ_w is the water density, ρ_s and φ could be measured from field samples, and θ could bem calculated from the Digital Topographic Map (DTM). In the Daniao tribe debris flow watershed (Taitung DF097 watershed), the slope was calculated from the average creek bottom slope to be 23.2% (≈ 13°), as shown in Fig. 10. From the field samples, the solid density was ρ_s = 2.6 g/cm^3, and $\varphi \approx$ 30° . With the liquid density ρ_w = 1.0 g/cm^{3,} the equilibrium concentration was calculated as C_∞ = 41.6% from equation (13). With this equilibrium concentration, the amount of water needed to induce debris flow in this watershed could be estimated. If the amount of water was not enough to mobilize all the source material to form a debris flow, then the volume of the debris flows would be smaller.

This study assumes that water that can induce debris flow must come from rainfall in the area with loose deposition and in the correct flow direction. After slope and direction analysis, we found 71.7% of the Daniao tribe watershed (equal to 0.63 km^2 as shown in Fig.1) satisfies this requirement. For the particular event, a rainfall accumulated to 740.5 mm in 62 hours before the debris flow occurred. The last 12 years of records showed this value was satisfied in 22.3 return years from frequency analysis showed. Therefore, a water volume accumulation of about 296,916 m^3 occurring in 62 hours using rational formula can be found from the flow duration curve, and the debris flow volume of 508,417 m^3 could be found from the water volume divided by (*1*- C_∞). With all the uncertainties, 508,417 m^3 can be considered as the maximum possible amount. Therefore, 200,000 m^3, 300,000 m^3, 400,000 m^3 and 500,000 m^3 four cases are simulated. It happened that in 2006, we were using 50 year return frequency as design basis and these volumes are also cases we simulated. The following simulation results are for total volume 500,000 m^3.

The simulated debris flow depth contour and velocity vector at 1, 3, 6, 10, 50 and 100 minutes are shown in Fig. 11 and Fig. 12. The maximum depth of the deposits was in excess of 15 m. The sources of debris flows were distributed in the gap of the watershed (in the medium stream) and ran out of the valley region (in the down stream), as shown in Fig. 11 (a) and (b). The maximum velocity was in excess of 20 m/sec during the start of the debris flow, but began to slow rapidly when the debris flow passed the watershed gap (maximum velocity less than 3 m/sec), as shown in Fig. 12 (b), (c), (d) and (e).

The results showed that around 10 minutes, the

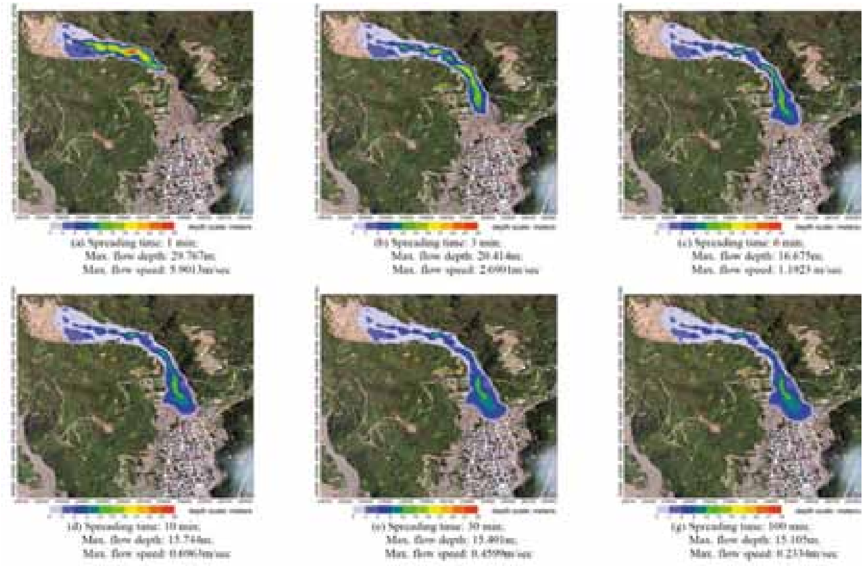


Fig. 11 - The debris flow depth contour maps at different time

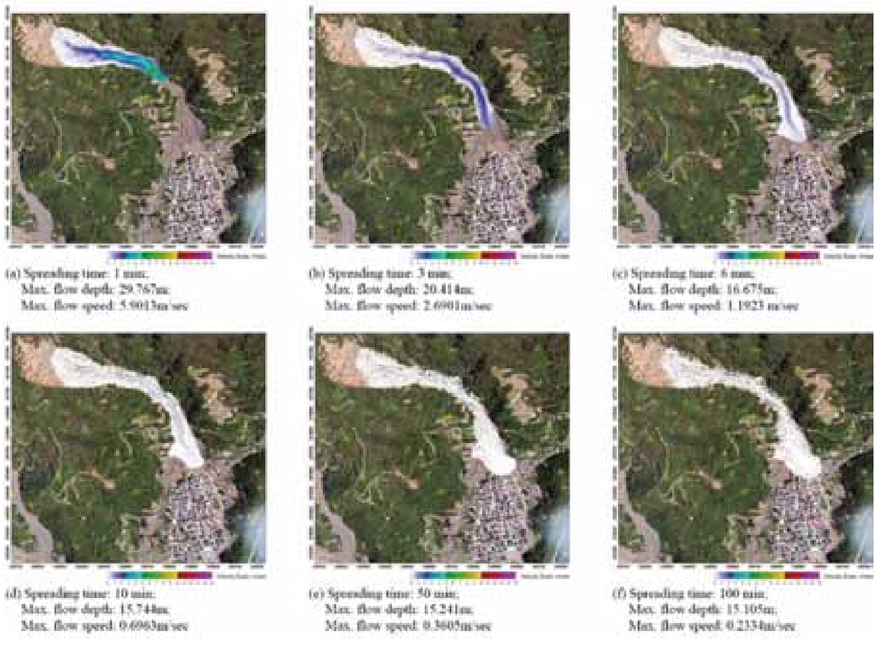


Fig. 12 - The debris flow velocity vector maps at different time

debris flow reached upstream of the Daniao tribe, as shown in Fig. 11 (d) and Fig. 12 (d). As the velocity of the debris flow slowed to less than 0.5 m/sec, as shown in Fig. 12 (f), and the debris flow front peak continued to maintain the same depth (≈ 15 m), as shown in Fig. 11 (d), (e) and (f). The final deposition fronts for all 4 cases are almost identical.

The final deposition area for both the simulation and the real event are shown in Fig. 13. The simulation results were very close to the field measurements. However, drainage ditches were constructed on both sides of the village after the simulation in 2006, so part of the debris flow spread along the ditches (see Fig. 13). As a result, the front travelled a shorter distance than

Fig. 13 - Region in red is area affected by Typhoon Morakot, and the blue region is the simulation result. The red star indicates where field depth estimation is available. The depth estimated by rescuer is between 12 m and 13 m. The simulated result for 50,000 m³ is 13 m

volume (m^3)	volume difference (m^3)	$\frac{\text{volume difference}}{\text{maximum volume}}$
200,000	300,000	60%
300,000	200,000	40%
400,000	100,000	20%
500,000	-	-

Front position (m)	difference for front position	$\frac{\text{position difference}}{\text{Total length}}$
934	118 m	11.22 %
987	65 m	6.18 %
1,023	29 m	2.76 %
1,052	-	-

Tab. 2 - Changes in front positions for different volumes

what was simulated. Since the only difference for different total volume is the front location. Therefore, this means the spread of the simulation from all 4 different total volumes are equally good. However, the maximum depth of debris flow for the final deposition was 15 m measured in the field and is 15.14 m for 500,000 m3 simulation and 15.02 m for 200,000 m^3 simulation. The location for the maximum predicted by numerical simulation is only 3 m away from the real location as shown in Fig. 8. One depth near the front in the field is available by the estimation from rescuers. The depth estimated is between 12 m and 13 m at the location marked by red star in Fig. 13. The simulation result is 13.06 m for 500,000 m^3 and 11.74 m for 200,000 m^3.

CONCLUSION

This study used a DEBRIS-2D model to simulate a real debris flow event before it occurs. The location of loose deposits was found through a field investigation. The total volume was obtained through hydrological methods and was verified with field estimation. The simulated result done in 2006 had a deposition area very close to the real event in 2009. Maximum depth and its location have practically no meaningful error between numerical result and real event. One reason for this successful prediction is due to the relatively insensitivity from volume estimation. This paper found that a 20 % variation in estimating the volume would only give rise to a 2.76 % variation on the final deposition front. This case study of Daniao tribe debris flow would give a support for the usability of numerical simulations in real engineering detailed designs.

REFERENCES

Bagnold R.A. (1954) - *Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear.* Proc. of the Royal Soc. of London, 49-63.

Balmforth N. J., Burbudge A. S., Craster R. V., Salzig J. & Shen A. (2001) - *Visco-plastic models of isothermal lava domes.* Journal of Fluid Mechanics, **403:** 37-65.

Coussot P., Proust S. & Ancey C. (1996) - *Rheological interpretation of deposits of yield stress fluids*. Journal of Non-Newtonian Fluid Mechanics, **66**(1)**:** 55-70.

Coussot Ph. (1997) - *Mudflow Rheology and Dynamics.* Publisher: Taylor & Francis Group.

Coussot Ph.& Proust S. (1996) - *Slow, unconfined spreading of a mud flow.* J. Geophys. Res., 101(B11): 25217-25229.

Griffiths R.W. (2000) - *The dynamics of lava flow.* Annual Review of Fluid Mechanics, **32:** 477-518.

Huang X. *&* Garcia M.H. (1998) - *A Herschel–Bulkley model for mud flow down a slope.* Journal of Fluid Mechanics, **374:** 305-333.

HULME G. (1974) - *The interpretation of lava flow morphology.* Geophysical Journal of the Royal Astronomical Society, **39**(2)**:** 361-383.

IVERSON R.M. (2000) - *Landslide triggering by rain infiltration.* Water Resources Research, **36**(7): 1897-1910.

JULIEN P.Y. & LAN Y. (1991) - *Rheology of Hyperconcentrations*. J. Hydr. Engrg., **117**(3)**:** 346-353.

LIU K.F, LAI K.W. (2000) - *Numerical simulation of two-dimensional debris flows.* Proceedings of the 2nd International Conference on Debris Flow Hazards Mitigation:531-53.

LIU K.F. & HSU Y.C (2008) - *Study on the sensitivity of parameters relating to debris flow spread.* Proceedings of the International Conference, Debris Flows: Disasters, Risk, Forecast, Protection, Pyatigorsk, Russia:19-22.

LIU K.F. & HUANG M.CH. (2006) - *Numerical simulation of debris flow with application on hazard area mapping*. Computational Geosciences, **10**(2)**:**221-240.

LIU K.F. & MEI C.C. (1989) - *Slow spreading of the sheet of Bingham fluid on an incline plane.* Journal of Fluid Mechanics, **207:** 505-529.

LIU K.F. & MEI C.C. (1994) - *Roll waves on a layer of a muddy fluid flowing down a gentle slope - A Bingham model.* Physics of Fluids, **6**(8)**:** 2577-2590.

LIU K.F., LI H.C. & HSU Y.C. (2009) - *Debris flow hazard assessment with numerical simulation.* Natural Hazards, **49**(1)**:** 137-161.

MEI C.C., LIU K.F.& YUHI M. (2001) - *Mud flow-slow and fast.* Geomorphological Fluid Mechanics, **582:** 548-577.

NG CH.O. & MEI C.C. (1994) - *Roll waves on a shallow layer of mud modelled as a power-law fluid.* Journal of Fluid Mechanics, **263:** 151-184.

O'BRIEN J.S. & JULIEN P.Y. (1997) - *On the importance of mudflow routing*. Proc. 1st Int. Conf. on Debris-Flow Hazards, Mitigation: Mechanics, Prediction and Assessment, ASCE, Reston, Va., 677-686.

OSMOND D.I. & GRIFFITHS R.W. (2001) - *The static shape of yield strength fluids slowly emplace on slope*. J. Geophys. Res., **106**(B8)**:** 16241-16250.

POULIQUEN O. & FORTERRE Y. (2002) - *Friction law for dense granular flows: application to the motion of a mass down a rough inclined plane.* Journal of Fluid Mechanics, **453:** 133-151.

RICKENMANN D. (1999) - *Empirical Relationships for Debris Flows*. Natural Hazards, **19**(1)**:** 47-77.

SAVAGE S.B. & MCKEOWN S.(1983), *Shear stress developed during rapid shear of concentrated suspension of large spherical particles between concentric cylinders.* Journal of Fluid Mechanics**, 127:** 453-472.

TAKAHASHI T. (1980)- *Debris flow on prismatic open channel.* J. Hydr. Div., **106**(3)**:** 381-396.

TSAI Y.F. (1999) - *Study on the configuration of debris-flow fan*. PhD Dissertation, Department of Hydraulic and Ocean Engineering, National Cheng-Kung University, Taiwan.

WIELAND M., GRAY J.M.N.T. & HUTTER K. (1999) - *Channelized free-surface flow of cohesionless granular avalanches in a chute with shallow lateral curvature.* Journal of Fluid Mechanics, **392:** 73-100.

WILSON S.D.R. & BURGESS S.L. (1998) - *The steady, spreading flow of a rivulet of mud*. Journal of Non-Newtonian Fluid Mechanics, **79**(1)**:** 77-85

