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APPLICATION OF TOPSIS MODEL BASED ON GAME THEORY TO RESERVOIR WATER QUALITY EVALUATION

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EXTENDED ABSTRACT

La valutazione della qualità dell'acqua per i bacini artificiali è un approccio efficace per affrontare i problemi dell'inquinamento idrico, della sicurezza idrica nazionale e dello sviluppo economico regionale. Pertanto, la scelta di metodi oggettivi e ragionevoli di valutazione della qualità dell'acqua diventa una base importante per garantire la sicurezza idrica urbana, nonché lo sviluppo e l'utilizzo razionale delle risorse idriche. In questo studio, gli indicatori di valutazione dell'inquinamento idrico del bacino di Mopanshan (Cina), hanno considerato il COD (domanda chimica di ossigeno), il pH, il DO (ossigeno disciolto), il NH_3-N (azoto ammoniacale) campionati da gennaio a dicembre 2020.

Un modello di valutazione TOPSIS per la qualità dell'acqua nel bacino idrico di Mopanshan è stato costruito combinando i pesi soggettivi del metodo AHP e i pesi oggettivi del metodo del peso dell'entropia, utilizzando la teoria dei giochi per superare gli inconvenienti del metodo di ponderazione unico. I risultati del modello TOPSIS sono stati confrontati con quelli del *grey correlation model*. I risultati hanno mostrato che i risultati della valutazione per luglio, agosto, settembre, ottobre e dicembre erano tutti identici a quelli della Classe II sulla base del modello TOPSIS di ponderazione delle combinazioni basato sulla teoria dei giochi, mentre solo pochi indicatori superavano gli standard della Classe II a novembre. I risultati di due valutazioni ponderate indipendenti generalmente erano allineati con quelli del modello ponderato combinato, basato sulla teoria dei giochi, mentre si sono riscontrate differenze nei risultati della valutazione a febbraio e novembre. I pesi sono stati assegnati solo utilizzando dati grezzi nella ponderazione dell'entropia. Quando si verificava una variazione nell'utilizzo dell'acqua, il significato di ciascuna indicazione cambiava. Poiché il bacino idrico di Mopanshan ha fornito una quantità significativa di acqua potabile alla città di Harbin, l'azoto ammoniacale è diventato un indicatore di valutazione essenziale. Tuttavia, la significatività relativa di questo indicatore non è stata rispecchiata dalla ponderazione entropica. Sebbene la competenza e l'esperienza degli esperti siano state prese in considerazione nell'assegnazione dei pesi attraverso l'approccio AHP, l'assegnazione dei pesi era soggetta ad arbitrarietà soggettiva e valutazioni di vari esperti potrebbero fornire risultati diversi. Quindi oltre a tenere conto della conoscenza e dell'esperienza degli esperti, il modello TOPSIS di ponderazione includeva anche informazioni intrinseche trovate nei dati oggettivi, mettendo la ponderazione più in linea con la teoria scientifica accettata. La qualità dell'acqua del bacino idrico di Mopanshan ha raggiunto il picco a marzo, suggerendo circostanze ideali per la qualità dell'acqua. Mentre a novembre il valore è sceso, suggerendo un leggero peggioramento della qualità dell'acqua. Tuttavia, lo stato generale della qualità dell'acqua è rimasto sostanzialmente invariato.

Nel bacino idrico di Mopanshan, la temperatura media invernale è rimasta sotto lo zero impedendo ai batteri di nitrificarsi, il che ha ridotto l'efficacia della rimozione dell'azoto ammoniacale che invece aumenta di livello da settembre a dicembre.

L'area di protezione delle fonti d'acqua di Mopanshan comprendeva anche aree coltivate, dove lo sviluppo agricolo era aiutato dall'applicazione di erbicidi, pesticidi e fertilizzanti sfusi. Il deflusso superficiale probabilmente ha trasportato gli inquinanti rimanenti nel serbatoio, il che potrebbe avere un impatto sulla qualità dell'acqua del serbatoio. Questa potrebbe essere un'ulteriore spiegazione per l'aumento della concentrazione di azoto ammoniacale tra settembre e dicembre. Alla luce delle circostanze sopra menzionate, sono state suggerite le seguenti raccomandazioni: (1) utilizzare la tecnologia informatica contemporanea per monitorare il bacino in tempo reale al fine di prevenire imprevisti con inquinamento della qualità dell'acqua; (2) stabilire adeguate misure di riscaldamento del giacimento al fine di aumentare la concentrazione di ossigeno disciolto e migliorare la reazione di nitrificazione; e (3) migliorare i metodi di semina agricola e utilizzare saggiamente gli agenti di crescita al fine di ridurre l'inquinamento da fonti non puntuali provenienti dall'agricoltura. Questo studio ha proposto un modello TOPSIS di ponderazione combinata basato sulla teoria dei giochi che ha tenuto pienamente conto delle informazioni effettive tra i diversi indicatori, ha ridotto l'arbitrarietà soggettiva nella ponderazione, ha aumentato la natura scientifica della ponderazione dell'indicatore e ha prodotto risultati più ragionevoli per la valutazione della qualità dell'acqua. Vale la pena applicare questo metodo a problemi simili di valutazione della qualità dell'acqua. Per creare una base per lo sviluppo delle risorse idriche e la gestione dell'inquinamento idrico, negli studi futuri dovrebbero essere utilizzati approcci più razionali e scientifici per responsabilizzare il processo decisionale.

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ABSTRACT

In view of the shortcomings of the single weighting method, the game theory method was adopted to optimize the combination of the subjective weight determined by the analytic hierarchy process (AHP) method and the objective weight determined by the entropy weight method. The technique for order preference by similarity to an ideal solution (TOPSIS) evaluation model based on the combination weighting of the AHP-entropy weight method-game theory was constructed for evaluating the water quality of Mopanshan Reservoir. The result obtained by the TOPSIS evaluation model was compared with the evaluation result of the grey relational model. The evaluation results showed that the water quality of Mopanshan Reservoir during the sampling period was Class I and II. The main factors affecting water quality were NH_3-N (ammonia nitrogen) and pH value. The game theory-based combination weighting TOPSIS model proposed in this paper fully considers the effective information among various indicators, reduces the subjective arbitrariness of weighting, makes indicator weighting more scientific, and obtains more reasonable water quality evaluation results. It is worthy of application in similar water quality evaluation problems.

Keywords: *entropy method, analytic hierarchy process, game theory, TOPSIS model, reservoir water quality evaluation, Mopanshan Reservoir*

INTRODUCTION

Water quality assessment is the foundation of water environment management and protection. With the development of society and economy, water pollution and scarcity pose serious threats to national water security and regional economic development (Medici *et alii*, 2023, Zuppi *et alii*, 2004). Constructing an artificial reservoir for collecting and storing water, especially for community water supply or irrigation, is one effective approach for addressing these issues. Reservoirs play a crucial role in flood control, water retention, storage, and utilization. Currently, China has constructed over 98,000 reservoirs of various types which have increasingly shown significant economic and social benefits. On one hand, reservoirs serve as water sources for many cities; on the other hand, they act as headwater reservoirs for major water diversion projects. For example, the Danjiangkou Reservoir serves as the source of China's South-to-North Water Diversion Project. Therefore, the selection of objective and reasonable water quality evaluation methods is an important basis for guaranteeing urban water safety. At present, there are many application achievements of evaluation methods. The current main methods for water quality evaluation include fuzzy comprehensive judgment method, technique for order preference by similarity to an ideal solution (TOPSIS) method, improved set pair analysis method, projection pursuit method, back-propagation neural network (BPNN), grey relational analysis method, and physical element model analysis method (Fu, 2006; Kong *et alii*, 2017; Luo *et alii*, 2020).

The TOPSIS method, which is a commonly used comprehensive evaluation method that fully utilizes the information of raw data. Its results accurately reflect the differences between various evaluation schemes. The determination of weights is the core issue in the TOPSIS model, and traditional TOPSIS models use a single weighting method. For example, Huang (2020) improved the TOPSIS model by objectively assigning weights and reasonably applying it to water quality assessment in Dongting Lake. Jiang *et alii* (2016) investigated the water quality of Puyang River using an improved TOPSIS model. Taking into account the limitations of single weighting, this study addresses both the difficulty of objective weighting methods in effectively reflecting indicator importance and the strong subjectivity of subjective weighting methods. In recent years, many scholars have summarized their experiences and proposed a combination weighting approach. For example, Hao (2016) determined weights by combining the AHP method with entropy weight method and introduced the concept of level characteristic value to determine water quality levels, thereby making the evaluation results more accurate and reasonable. Lian *et alii* (2021) evaluated the water quality of Tongli Ancient Town using an improved TOPSIS model with combined weights, resulting in a more discernible water quality assessment. However, there is still insufficient theoretical support for establishing comprehensive weights. The game theory combination weighting method can achieve a balance between subjectivity and objectivity, thereby enhancing the scientific rationality of weight-based decision-making (Zhang *et alii*, 2018; Wu *et alii*, 2021; Zhu *et alii*, 2019). Therefore, establishing an objective and reasonable water quality evaluation method is of great significance for ensuring urban water safety and the rational development and utilization of water resources.

This study applies the game theory combination weighting method to the TOPSIS model, forming a more scientific and reasonable weighting decision-making method to evaluate reservoir water quality, fully considering subjective and objective factors, improving the scientific rationality of weighting decision-making, and later verifying its effectiveness and accuracy in practical cases. The principal objectives of this study is: (i) to screen evaluation indicators and analyze the main factors affecting water quality grades based on the samples collected from Mopanshan Reservoir, (ii) to establish a TOPSIS model based on game theory combination weighting to comprehensively evaluate the water quality of Mopanshan Reservoir, providing an accurate and objective theoretical basis for the rational development and utilization of water resources, protection of the Mopan Mountain water environment, and control of water pollution.

OVERVIEW OF THE STUDY AREA

The Mopanshan Reservoir is a clay core wall earth rock dam, located near Shenjiaying, Shahezi Town, upstream of the Lalin River, a first-level tributary of the Songhua River in Wuchang City (44°24'N, 127°41'E). It is situated 180 kilometers away from Harbin City, the location and geometrical details of Mopanshan Reservoir is shown in Fig. 1.

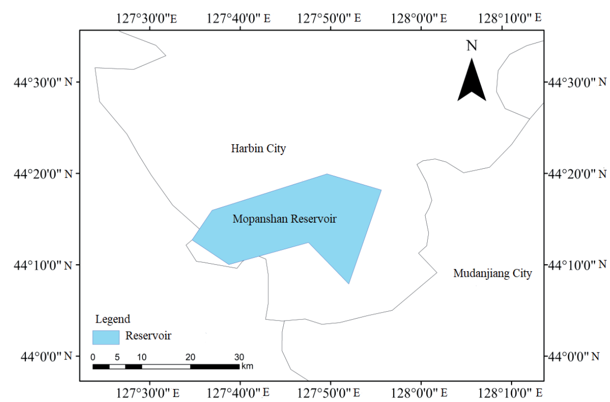

Fig. 1 - Location and geometrical details of Mopanshan Reservoir

The reservoir covers an area of 28 square kilometers and has a water catchment basin area of 1,151 square kilometers. The controlled watershed area above the Mopanshan Reservoir dam site is 1151 km^2. The maximum annual runoff of Mopanshan Reservoir is 902 million m^3, the minimum runoff is 281 million m^3, and the average annual runoff is 561 million m^3. The annual precipitation in the region is about 500~800 mm, with precipitation mainly concentrated from June to August, the winter lasts from November to March with little snowfall. The reservoir area belongs to a temperate continental monsoon-influenced region with cold weather and an average annual temperature around 3~4 °C. The reservoir is a large (Type II) reservoir primarily serving urban water supply, while also considering comprehensive utilization such as flood control and irrigation. The geological conditions of Mopanshan Reservoir is as follows: at the upper dam site, the fully weathered and strongly weathered rock zone is relatively thick, and some parts have high permeability. There is a fault with a width of about 20 meters at the axis.while, at the lower dam site, the permeability of the gravel mixed soil layer below the left bank gravel mixed soil layer is high, and it is buried deeper.

CONSTRUCTION OF RESERVOIR WATER QUALITY EVALUATION MODEL

TOPSIS model

The basic process involves first uniform the indicator type in the original data matrix and then normalizing indicators to eliminate the influence of different dimensions. The optimal and worst schemes are identified among limited options, and distances between each evaluation object and the optimal/worst schemes are calculated. This allows for determining the relative closeness of each evaluation object to the optimal scheme, serving as a basis for evaluating superiority or inferiority.

Specific steps of the TOPSIS comprehensive evaluation method are as follows.

Step 1: After conducting extensive data processing, the resulting matrix is as follows:

$$X=\begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}.$$

The purpose of standardizing matrix X is to eliminate the influence of different indicator dimensions. The standardized matrix is denoted as Z, and each element in Z:

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}} \quad (1)$$

The decision-maker provides a weight vector for each attribute: $w = [w_1, w_2, \ldots, w_n]^T$.

A weighting matrix is constructed:

$$Z = (z_{ij})_{m\times n}: z_{ij} = w_j\, z_{ij},$$

$$i = 1,2..,n; j = 1,2,..,m. \quad (2)$$

Step 2: Optimal scheme Z^+ and worst scheme Z^- are defined. The distance between each evaluation object and the optimal scheme (D_i^+) and the distance between each object and the worst scheme (D_i^-) are calculated. Finally, the score is normalized.

Define the optimal scheme:

$$Z^+ = (Z_1^+, Z_2^+, \ldots, Z_m^+)$$
$$= (max\,\{z_{11}, z_{21}, \ldots, z_{n1}\},\ max\,\{z_{12}, z_{22}, \ldots, z_{n2}\}, \ldots,\ max\,\{z_{1m}, z_{2m}, \ldots, z_{nm}\}) \quad (3)$$

Define the worst scheme:

$$Z^- = (Z_1^-, Z_2^-, \ldots, Z_m^-)$$
$$=(min\,\{z_{11}, z_{21}, \ldots, z_{n1}\},\ min\,\{z_{12}, z_{22}, \ldots, z_{n2}\}, \ldots,\ min\,\{z_{1m}, z_{2m}, \ldots, z_{nm}\}) \quad (4)$$

The distance between the *i-th* ($i=1,2,\ldots,n$) evaluation object and the maximum value is calculated:

$$D_i^+ = \sqrt{\sum_{j=1}^{m} (Z_j^+ - z_{ij})^2} \quad (5)$$

The distance between the *i-th* ($i=1,2,\ldots,n$) evaluation object and the maximum value is calculated:

$$D_i^- = \sqrt{\sum_{j=1}^{m} (Z_j^- - z_{ij})^2} \quad (6)$$

Step 3: The non-normalized score of the *i -th (i=1,2,...,n)* evaluation object is calculated:

$$S_i = \frac{D_i^-}{D_i^- + D_i^+} \quad (7)$$

Step 4: The score is normalized:

$$H_i = \frac{S_i}{\sum_{i=1}^{n} S_i} \quad (8)$$

The water samples are sorted in descending order based on the H_i value. The larger the H_i value, the better the water quality.

Calculation of comprehensive weight

Entropy weight method for determining weights

The entropy weight method is an objective weighting method, based on the principle that the smaller the variation degree of indicators, the less information they reflect, and the lower the corresponding weights (Yang *et alii*, 2018).

The calculation steps of the entropy weight method are as follows.
Step 1: After conducting extensive data processing, the resulting matrix is as follows:
Non-negative transformation is performed on the data:

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{bmatrix}$$

$$Z_{ij} = \frac{b_{ij} - min\{b_{1j}, b_{2j}, \ldots, b_{nj}\}}{max\{b_{1j}, b_{2j}, \ldots, b_{nj}\} - min\{b_{1j}, b_{2j}, \ldots, b_{nj}\}} \quad (9)$$

i=1,2,...,n; j=1,2,...,m.
Step 2: The data are normalized. The proportion of the *i-th* scheme to the *j-th* indicator is calculated to construct matrix p_{ij}:

$$p_{ij} = \frac{Z_{ij}}{\sum_{i=1}^{n} Z_{ij}} \quad (10)$$

Entropy E_j of different indicators is calculated:

$$E_j = \frac{1}{lnm} p_{ij} ln p_{ij} \quad (11)$$

Step 3: Information utility value d_j is determined and normalized, and then entropy weight W_j of every indicator is obtained:

$$d_j = 1 - E_j \quad (12)$$

$$W_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \quad (13)$$

AHP method for determining weights

The article employs a nine-level scale method to compare evaluation indicators and determine their relative importance (Tab.1).

Scale a_{ij}	Meaning
1	The two factors are equally important.
3	One factor is slightly more important than the other factor.
5	One factor is significantly more important than the other factor.
7	One factor is slightly more important than the other factor.
9	One factor is slightly more important than the other factor.
2, 4, 6, 8	The median of the above two neighboring judgments

Tab. 1 - Scale dividing and its corresponding meaning for AHP method

Taking into account the context of reservoirs, the weights of each indicator are calculated, and finally, consistency checks are conducted.
Step: The judgment matrix is constructed:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$$

In the matrix: a_{ij} *(i,j=1,2,...,n)* is the ratio of the importance of the *i-th* factor to the *j-th* factor.
Step 2: The weight is calculated by the geometric mean method, and the obtained weight vector is normalized.
The weight vector obtained by the geometric mean method is:

$$\omega_i = \frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}}{\sum_{k=1}^{n}\left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}} \quad (i, j, k = 1,2,\ldots, n) \quad (14)$$

Step 3: Consistency check is performed on the judgment matrix. Consistency index (*CI*) is calculated:

$$CI = \frac{\lambda_{\max} - n}{n-1} \quad (15)$$

The corresponding average random consistency index (*RI*) is found out to calculate consistency ratio (*CR*):

$$CR = \frac{CI}{RI} \quad (16)$$

If *CR* is smaller than 0.1, the consistency of the judgment matrix can be accepted; otherwise, the judgment matrix is corrected.

Game theory for determining combined weight

Based on game theory, the combination weighting method aims to achieve Nash equilibrium and reconcile conflicts between subjective and objective weights, seeking consistency and compromise among the weights. The process involves a combination of comparison and coordination. The weights are calculated separately using AHP and entropy weight methods. The game theory-based combination weighting method can comprehensively consider the intrinsic information among multiple indicators, reduce the one-sidedness of a single weighting method, and improve the rationality of indicator weighting. The steps for assigning weights are as follows (CHEN *et alii*, 2019).

Step 1: Subjective and objective weight vectors are calculated using AHP and entropy weight methods and denoted as W_1 and W_2. The indicator combined weight W expressed by the linear combination of W_1 and W_2 is:

$$\boldsymbol{W} = \begin{pmatrix} \lambda_1 w_{11} + \lambda_2 w_{21} \\ \lambda_1 w_{12} + \lambda_2 w_{22} \\ \vdots \\ \lambda_1 w_{1n} + \lambda_2 w_{2n} \end{pmatrix} = \begin{pmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \\ \vdots & \\ w_{1n} & w_{2n} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \lambda_1 \mathrm{w}_1 + \lambda_2 \mathrm{w}_2 \quad (17)$$

where λ_1 and λ_2 are linear combination coefficients

Step 2: Based on the principles of game theory, the objective function is established. The objective is to find the optimal linear combination coefficients λ_1* and λ_2*, with the minimum sum of deviations between the weight combinations W, W_1, and W_2. The resulting indicator combined weight will be optimal combined weight W*. The objective function and constraint condition are:

$$\min\left(\|\boldsymbol{W} - \boldsymbol{W}_1\|_2 + \|\boldsymbol{W} - \boldsymbol{W}_2\|_2 = \min\left(\|\lambda_1 \boldsymbol{W}_1 + \lambda_2 \boldsymbol{W}_2 - \boldsymbol{W}_1\|_2 + \|\lambda_1 \boldsymbol{W}_1 + \lambda_2 \boldsymbol{W}_2 - \boldsymbol{W}_2\|_2\right)\right.$$

making $\lambda_1 + \lambda_2 = 1,\ \lambda_1,\ \lambda_2 \gg 0.$ (18)

Step 3: According to the differential principle, the first-order derivative condition for minimizing equation (16) is:

$$\begin{cases} \lambda_1 \boldsymbol{W}_1 \boldsymbol{W}_1^{\mathrm{T}} + \lambda_2 \boldsymbol{W}_1 \boldsymbol{W}_2^{\mathrm{T}} = \boldsymbol{W}_1 \boldsymbol{W}_1^{\mathrm{T}} \\ \lambda_1 \boldsymbol{W}_2 \boldsymbol{W}_1^{\mathrm{T}} + \lambda_2 \boldsymbol{W}_2 \boldsymbol{W}_2^{\mathrm{T}} = \boldsymbol{W}_2 \boldsymbol{W}_2^{\mathrm{T}} \end{cases} \quad (19)$$

As W_1 and W_2 have been known, combination coefficients λ_1 and λ_2 are calculated.

Step 4: Combination coefficients λ_1 and λ_2 obtained from equation (17) are normalized:

$$\begin{cases} \lambda_1^* = \frac{|\lambda_1|}{|\lambda_1| + |\lambda_2|} \\ \lambda_2^* = \frac{|\lambda_2|}{|\lambda_1| + |\lambda_2|} \end{cases} \quad (20)$$

Then, the optimal combined weight of the evaluation indicators is obtained:

$$W^* = \lambda_1^* W_1 + \lambda_2^* W_2 \quad (21)$$

APPLICATION

Case analysis

The article is based on common surface water pollutants and the actual situation of pollution in Mopanshan Reservoir. There may be an impact from residents' daily life on the water body in the reservoir area. There are 16,100 people living in the Mopanshan source protection zone. Their domestic sewage and scattered livestock and poultry waste contain nutrients such as nitrogen and phosphorus. If this sewage is not treated, it will infiltrate into the ground and increase the nitrogen and phosphorus content in the water body, which is a potential influencing factor (14). Sampling was conducted at Mopanshan Reservoir. Based on the sampling results, chemical oxygen demand (COD), pH value, dissolved oxygen (DO), and ammonia nitrogen (NH_3-N) are chosen as evaluation indicators. The evaluation criteria are based on the 'Environmental Quality Standards for Surface Water' (GB3838-2002).

Determine combined weight

(1) *AHP method*

Based on experts' analysis of the local hydrological conditions and relevant data, a judgment matrix is obtained by ranking the evaluation indicators in terms of their importance:

$$A = \begin{bmatrix} 1 & 1/4 & 1/6 & 1/3 \\ 4 & 1 & 1/2 & 2 \\ 6 & 2 & 1 & 3 \\ 3 & 1/2 & 1/3 & 1 \end{bmatrix}$$

According to equation (12), subjective weight W_1 is calculated:

W_1 = [0.2802 0.0682 0.4854 0.1662].

According to equation (13), the *CI* is calculated as 0.0108. Using equation (14), the *CR* is calculated as 0.0122. Since *CR* is less than 0.10, matrix A's consistency can be accepted.

(2) *Entropy weight method*

The evaluation criteria and water quality monitoring results are combined to construct an initial matrix. The data are normalized using equation (8). Information entropy E is calculated by substituting the normalized data into equation (9). The information utility value is obtained from equation (10) with E as input, and subsequently the objective indicator weight is determined using equation (11):

W_2 = [0.3076 0.1325 0.3015 0.2584].

(3) *Determine the combined weight by game theory*

Optimal linear combination coefficients λ_1* = 0.7666, λ_2* = 0.2334 are obtained according to game theory combination weighting equations (15)-(18). Finally, optimal combined weight W* is obtained:

W* = [0.2866 0.0832 0.4425 0.1877].

TOPSIS evaluation

The water quality monitoring results from the Mopanshan Reservoir were combined with evaluation criteria to create an initial decision matrix. The type of evaluation indicators was converted into a maximization-type indicator, and the data was standardized using equation (1). The outcomes of this standardization process can be observed in Table 2.

According to equation (2), the standardized decision matrix and combined weights were utilized for constructing a weighted decision matrix. Distances D_i^+ and D_i^- between the optimal and worst schemes were calculated using equations (3)-(6) and then substituted into equation (9) to calculate the score and obtain S_i.

Time/grade	Standardized data			
	PH	DO	NH_3-N	COD
I	0.2921	0.3065	0.2645	0.2537
II	0.2191	0.2452	0.2144	0.2537
III	0.1460	0.2044	0.1430	0.2030
IV	0.0730	0.1226	0.0715	0.1015
V	0.0000	0.0817	0.0000	0.0000
January 30th	0.2775	0.2289	0.2530	0.2329
February 28th	0.2892	0.2779	0.2788	0.2644
March 20th	0.2921	0.3679	0.2788	0.2676
April 1st	0.2848	0.2763	0.2788	0.2740
May 31st	0.2819	0.2763	0.2802	0.2778
June 30th	0.2191	0.2771	0.2802	0.2884
July 12th	0.2629	0.2346	0.2716	0.3125
August 21st	0.2921	0.2338	0.2702	0.2504
September 25th	0.2468	0.2097	0.2573	0.2440
October 23rd	0.2483	0.2211	0.2502	0.2416
November 1st	0.2322	0.2023	0.2573	0.2314
December 1st	0.2337	0.2109	0.2502	0.2382

Tab. 2 - The outcomes of standardization process

Time/grade	Hi	Hi (from high to low)	Water quality grade
I	0.0708	0.0744	I
II	0.0597	0.0714	II
III	0.0422	0.0713	III
IV	0.0208	0.0709	IV
V	0.0000	0.0708	V
January 30th	0.0648	0.0688	II
February 28th	0.0709	0.0674	I
March 20th	0.0744	0.0670	I
April 1st	0.0713	0.0648	I
May 31st	0.0714	0.0635	I
June 30th	0.0670	0.0635	II
July 12th	0.0688	0.0619	II
August 21st	0.0674	0.0615	II
September 25th	0.0635	0.0597	II
October 23rd	0.0635	0.0422	II
November 1st	0.0615	0.0208	II
December 1st	0.0619	0.0000	II

Tab. 3 - Comprehensive evaluation results of water quality

S_i is normalized using equation (8). Normalization result H_i was ranked from high to low. The comprehensive evaluation index and the obtained water evaluation results are shown in Table 3.

RESULTS ANALYSIS

In this study, different weighting methods, namely the TOPSIS model and the grey relational model, were used to comprehensively evaluate water quality in the Mopanshan Reservoir. The evaluation result was compared with the result obtained from the game theory-based combination weighting TOPSIS model, as shown in Table 4. To visually depict the trend of water quality and model outcomes, a time series graph (Fig. 2) was plotted by comparing scores derived from the game theory-based combination weighting TOPSIS model with those from other models.

According to Table 4, when comparing the result of the game theory-based combination weighting TOPSIS model with the result of the grey relational model, it can be observed that the evaluation results for July, August, September, October, and

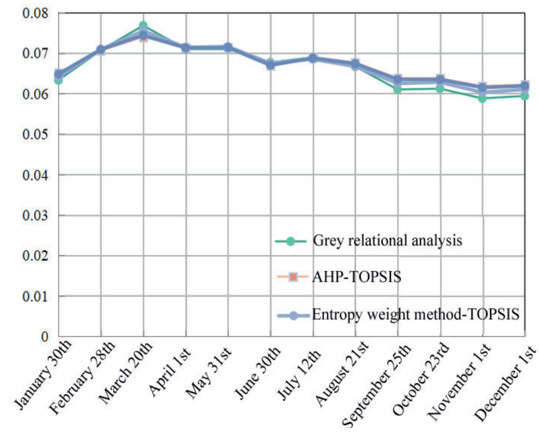

Fig. 2 - Time series graph in 2020

December were all identical - Class II water quality. Based on the actual measurement results of water quality in Table 2, it is evident that only a few indicators exceeded Class II standards in November. A study from Shu et al. Indicated that total nitrogen, and permanganate index were also the indicators that exceeded Class II standards in Mopanshan Reservoir. Under overall good water quality conditions, the influence of individual evaluation indicators were amplified. The reason for this is that the grey relational model evenly distributed weights during allocation process, which lacked objectivity and rationality. Meanwhile, more indicators should be focused on if possible.

Time	Water quality classification by different evaluation methods			
	Grey relational model	The entropy weight-based TOPSIS modeling	The AHP method-based TOPSIS model	The game theory-based combination weighting TOPSIS model
January 30th	II	II	II	II
February 28th	II	II	II	I
March 20th	I	I	I	I
April 1st	II	I	I	I
May 31st	II	I	I	I
June 30th	II	II	II	II
July 12th	II	II	II	II
August 21st	II	II	II	II
September 25th	II	II	II	II
October 23rd	II	II	II	II
November 1st	III	III	II	II
December 1st	II	II	II	II

Tab. 4 - Comparison of water quality classification by different evaluation methods

According to Table 4, it can be observed that the results of two separate weighted evaluations generally aligned with those of the game theory-based combined weighted model. However, in February and November, there were discrepancies in the evaluation results - the former was lower than the latter. The two separate weighting methods both have drawbacks. Entropy weighting only allocates weights based on raw data.

The importance of each indicator varies when there is a difference in water usage. As Mopanshan Reservoir serves as an important drinking water source for Harbin City, ammonia nitrogen becomes a crucial evaluation indicator. However, entropy weighting fails to reflect the relative importance of this indicator. Although the AHP method takes into account the knowledge and experience of experts in assigning weights, there is subjective arbitrariness in weight allocation, and different expert ratings may result in varying outcomes. Additionally, it is challenging to identify inherent patterns within the data. The proposed game theory-based combination weighting TOPSIS model not only takes into account the knowledge and experience of experts but also reflects intrinsic information inherent in objective data, making the weighting more scientifically reasonable.

Based on Figure 1, it is evident that the water quality of Mopanshan Reservoir reached its peak in March, indicating excellent conditions. However, there was a decrease in score during November, implying a slight decline in water quality. Nonetheless, the overall water quality remained relatively stable. According to the water quality results measured in Table 2, it can be observed that the ammonia nitrogen content increased from September to December. Possible reasons for this include:
(1) The Mopanshan Reservoir is located within Wuchang City, where the average winter temperature falls below 0°C. The low water temperature is unsuitable for nitrifying bacteria, resulting in a decrease in ammonia nitrogen removal efficiency and consequently an increase in ammonia nitrogen content.
(2) There are still some cultivated lands in the Mopanshan Water Source Protection Area, where bulk fertilizers, insecticides, and herbicides are used for crop growth. However, the residual chemicals can be carried into the reservoir through surface runoff, thereby potentially affecting the water quality of the reservoir.

In response to the situation mentioned above, the following suggestions are proposed: i) utilize modern information technology for real-time monitoring of the Mopanshan Reservoir in order to prevent sudden incidents of water quality pollution; ii) implement appropriate heating measures for the Mopanshan Reservoir to increase dissolved oxygen concentration and enhance nitrification reaction; iii) improve agricultural planting techniques and make rational use of growth agents to reduce non-point source pollution from agriculture.

CONCLUSIONS

The evaluation results from game theory-based combination weighting TOPSIS model indicated that the water quality was classified as Class I and II and ammonia nitrogen (NH_3-N) and pH value were the main factors affecting water condition. The water condition in Mopanshan Reservoir was able to ensure the safety and demand of local water use.

Based on game theory, the combination of subjective weights determined by the AHP method and objective weights determined by entropy weighting overcomes the limitations of using a single weighting method. This approach considers both subjective and objective factors, comprehensively evaluates relevant information among indicators, minimizes subjectivity in weight assignment, and enhances the scientific nature of indicator weighing process, resulting in a more rational assessment of water quality. The TOPSIS model based on game theory combination weighting objectively reflects the water condition and is worth applying in similar water quality evaluation problems. In future research, more scientific and reasonable methods to enhance empowerment decision-making should be applied to provide a basis for water resource development and water pollution control.

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