Italian Journal of Engineering Geology and Environment
www.ijege.uniroma1.it

MULTIVARIATE REGRESSION MODEL FOR INDIRECT ESTIMATION OF UNIAXIAL COMPRESSIVE STRENGTH OF LIMESTONE

Asma KEMHIS(*), Allaeddine ATHMANI(**) & Hatem SEBOUI(**)

(*)National Higher School of Technology and Engineering - Annaba (Algeria)
(**)Badji Mokhtar-Annaba University - Department and Laboratory of Civil Engineering, Faculty of Technology - P. box, 12 - 23000 Annaba (Algeria)
Corresponding author: allaeddine.athmani@univ-annaba.dz

EXTENDED ABSTRACT

La resistenza a compressione uniassiale (UCS) della roccia, è un parametro fondamentale in varie applicazioni geomeccaniche. Comprendere l'UCS è fondamentale per valutare la stabilità e l'idoneità delle rocce per scopi ingegneristici, poiché influenza direttamente il comportamento delle rocce sotto carico. Tuttavia, determinare con precisione l'UCS può essere difficile, soprattutto nei tipi di roccia in cui il campionamento e la preparazione sono difficili o poco pratici. Per superare queste sfide, sono stati sviluppati metodi indiretti per la stima dell'UCS che si basano su correlazioni empiriche tra UCS e proprietà misurabili delle rocce, utili in situazioni in cui i test diretti sono irrealizzabili o proibitivi in termini di costi.Tra i vari metodi di stima indiretta, i modelli di regressione multivariata hanno guadagnato importanza per la loro capacità di incorporare molteplici fattori predittivi e generare previsioni UCS accurate. Considerando diversi parametri influenti, come la densità della roccia, la porosità e le proprietà meccaniche, questi modelli offrono un approccio completo alla stima dell'UCS. I progressi negli strumenti computazionali e nelle tecniche statistiche hanno facilitato lo sviluppo di robusti modelli di regressione in grado di gestire set di dati complessi e fornire previsioni affidabili. Una stima accurata dell'UCS è fondamentale per le rocce calcaree, che sono ampiamente utilizzate nelle costruzioni per la loro durabilità e il loro aspetto estetico. Tuttavia, la natura eterogenea delle formazioni calcaree e le loro proprietà meccaniche variabili pongono sfide significative per la determinazione dell'UCS. Lo sviluppo di modelli di stima robusti su misura per le rocce calcaree è essenziale per garantire la sicurezza e la stabilità delle strutture ingegneristiche costruite su o all'interno di queste formazioni. In questo contesto, il presente studio propone un modello di regressione multivariata implementato in Matlab per stimare l'UCS del calcare attraverso due fattori predittivi chiave: la velocità dell'impulso ultrasonico (UPV) e il numero di rimbalzo da martello di Schmidt (SRN), riconosciuti per la loro correlazione con le proprietà di resistenza della roccia. I campioni di calcare provenienti dal centro storico di Annaba, in Algeria, rivestono un significato particolare a causa del loro contesto storico, culturale e geologico. Annaba, precedentemente nota come Hippo Regius, vanta, infatti, una ricca storia che risale ai tempi antichi, con resti delle civiltà romana, bizantina e ottomana incorporati nel suo tessuto urbano. Le formazioni calcaree prevalenti in questa regione sono servite da materiali da costruzione per secoli, contribuendo al patrimonio architettonico e al carattere della città. Pertanto, comprendere il comportamento meccanico e le caratteristiche di resistenza della pietra calcarea del centro di Annaba non solo contribuisce alla conoscenza geotecnica, ma aiuta anche a preservare e gestire il patrimonio culturale della regione. Sfruttando la relazione tra questi fattori e l'UCS, il modello sviluppato mira a fornire previsioni accurate e affidabili sulla resistenza delle rocce calcaree. Per valutare le prestazioni del modello, vengono utilizzate tre metriche di regressione: coefficiente di determinazione (R^2), errore quadratico medio (RMSE) ed errore assoluto medio (MAE), che offrono informazioni sulla sua capacità predittiva e sul potenziale di applicazione pratica. Inoltre, è fondamentale esaminare i residui per garantire che le ipotesi del modello non vengano violate e per identificare eventuali motiviche potrebbero indicare problemi con il modello.

Per collocare questi risultati nel quadro dello stato dell'arte internazionale, il nostro modello è stato confrontato con altri modelli consolidati da studi precedenti. Nonostante l'utilizzo di meno parametri, le prestazioni del nostro modello sono comparabili, con un RMSE inferiore di 3.562 MPa, evidenziandone l'applicabilità pratica e l'efficienza. Questo confronto sottolinea la robustezza e la rilevanza del nostro modello nel prevedere l'UCS utilizzando metodi di test non distruttivi. Un vantaggio significativo è la capacità di utilizzare metodi di prova non distruttivi, come i test UPV e il martello di Schmidt, che sono più convenienti e richiedono meno tempo rispetto ai test UCS diretti. Questo approccio non distruttivo preserva l'integrità dei campioni di roccia e consente test più estesi, coprendo una gamma più ampia di condizioni della roccia. Inoltre, l'integrazione dei dati UPV e il martello di Schmidt fornisce una comprensione olistica delle proprietà delle rocce, acquisendo caratteristiche sia elastiche che di durezza superficiale fondamentali per una stima UCS accurata.

Concentrandosi su campioni di calcare provenienti dal centro storico di Annaba, in Algeria, questa ricerca mira a far luce sulle proprietà geomeccaniche di un materiale storicamente e geologicamente significativo. L'applicazione di tecniche di modellazione di regressione multivariata per stimare la resistenza a compressione uniassiale (UCS) di queste rocce calcaree è promettente per scopi di ingegneria, per gli sforzi di conservazione e le iniziative di sviluppo urbano ad Annaba e non solo.

IJEGE, 1 (2024): 67-77, DOI: 10.4408/IJEGE.2024-01.O-05
E-ISSN 2035-5688 | ISSN 1825-6635 / ©Author(s)

ABSTRACT

The strength of rock under uniaxial compression, commonly known as Uniaxial Compressive Strength (UCS), plays a crucial role in various Geomechanical applications such as designing foundations, mining projects, slopes in rocks, tunnel construction, and rock characterization. However, sampling and preparation can become challenging in some rocks, making it difficult to determine the UCS of the rocks directly. Therefore, indirect approaches are widely used for estimating UCS. This study presents a multivariate Regression Model implemented in Matlab to calculate the UCS of Limestone rocks. To validate the proposed model's effectiveness, a comprehensive dataset of 22 limestone rock samples is collected from the old city center of Annaba, Algeria. The dataset included measurements of Ultrasonic Pulse Velocity (UPV), Schmidt rebound number (SRN), and actual UCS values from laboratory tests. Three regression metrics, including Coefficient of Regression (R^2), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE), were used to evaluate and compare the performance of the models. The results indicate a high predictive capability of the developed model, with an R^2 value of 0.848, an RMSE of 3.562 MPa, and an MAE of 2.962 MPa. To place these findings in the frame of the international state of the art, our model was compared with other established models from previous studies. Our model demonstrates significant practical applicability by effectively predicting UCS with a minimal number of parameters, showcasing a lower RMSE compared to many existing models.

Keywords: *Uniaxial Compressive Strength (UCS), geomechanical properties, limestone rock strength, Schmidt Hammer*

INTRODUCTION

A plethora of studies have explored different approaches for UCS estimation, ranging from direct laboratory testing to indirect methods based on empirical correlations and predictive models. Direct methods for UCS determination involve conducting laboratory tests on intact rock samples under uniaxial compression to measure their strength properties (ASTM 2007). While direct testing provides accurate and reliable results, it can be time-consuming, expensive, and impractical, particularly in cases where large quantities of samples are required or where rock accessibility is limited (Cai *et alii*, 2004).

In contrast, indirect methods offer alternative means of estimating UCS by correlating measurable rock properties with strength parameters. These methods leverage empirical relationships derived from statistical analyses of experimental data, field observations, and theoretical frameworks (Seboui *et alii*, 2023). Indirect estimation techniques encompass a wide range of approaches, including empirical equations, artificial intelligence algorithms, and multivariate regression models (Dehghan *et alii*, 2010; Barham *et alii*, 2020; Mahmoodzadeh *et alii*, 2021; Ali & Hin Lai, 2023; Taghavi *et alii,* 2023; Zhao *et alii*, 2024).

Among the indirect methods, the Point Load Test (PLT) is particularly significant for its rapid and economical application in situ. The PLT involves applying a concentrated load to a rock specimen until failure occurs, providing a Point Load Strength Index (I_s50) that can be empirically correlated with UCS (Çobanoğlu & Çelik, 2008; Tandon & Gupta, 2015). This test is advantageous because it requires minimal sample preparation and can be performed on irregular rock specimens, making it highly practical for field applications. Numerous studies have demonstrated the effectiveness of the PLT in providing quick and reliable estimates of UCS, which is crucial for the rapid characterization of rock strength in geotechnical and mining projects (Ore 2020; Garrido *et alii*, 2022) .

Artificial intelligence (AI) algorithms, including neural networks, fuzzy logic, and genetic algorithms, have emerged as powerful tools for UCS prediction in recent years (Momeni *et alii*, 2015; Barham *et alii*, 2020). These algorithms utilize computational models to learn complex relationships between input parameters and output variables from training data, enabling them to make accurate predictions without relying on explicit mathematical equations (Mahmoodzadeh *et alii*, 2021; Gowida *et alii*, 2021). AI-based approaches offer flexibility, scalability, and adaptability to diverse datasets, making them suitable for nonlinear and complex modeling tasks (Yilmazkaya , 2023).

Multivariate regression models represent another category of indirect estimation methods that have gained popularity in geomechanics and rock engineering (Zhang & Goh, 2014; Rahimi Shahid *et alii,* 2022). These models utilize statistical techniques to establish relationships between multiple independent variables (predictors) and a dependent variable (UCS), allowing for the simultaneous consideration of various factors influencing rock strength (Aladejare *et alii*, 2021).

The selection of appropriate predictive factors plays a crucial role in the development of multivariate regression models for UCS estimation. Several rock properties and testing parameters have been investigated as potential predictors, including Schmidt rebound number (SRN), and Ultrasonic Pulse Velocity (UPV) (Tandon & Gupta, 2015). These indirect methods offer a non-destructive alternative, making them highly valuable in situations where sample collection is constrained by environmental or logistical factors. The Schmidt Hammer test, which measures the rebound hardness of the rock, is a simple, quick, and widely used technique. The P-wave velocity test, which involves measuring the speed at which compressional waves travel through rock, provides insights into the material's elasticity and density. By integrating these two measurements, it is possible to develop a robust predictive model for UCS. In this regard, this study presents a multivariate regression analysis to predict UCS from SRN and UVP measurements using a dataset of limestone samples. The performance of the regression model is evaluated using statistical metrics, including the Coefficient of Determination (R^2), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE). This research aims to validate the reliability of using Schmidt Rebound and P-wave Velocity as indirect indicators of UCS, contributing to more efficient and practical Geomechanical assessments.

In order to validate the robustness of our proposed model, it is compared with other well-established models from previous studies. Specifically, the model incorporates Ultrasonic Pulse Velocity (UPV) and Schmidt Rebound Number (SRN) as input parameters to predict the Uniaxial Compressive Strength (UCS). This comparison underscores the effectiveness of our model in the context of the current international state of the art, ensuring its relevance and applicability in the field of Geomechanical engineering

RECENT STUDIES

The integration of Ultrasonic Pulse Velocity (UPV) and Schmidt rebound number (SRN) for predicting the Uniaxial Compressive Strength (UCS) of limestone has been extensively studied in recent years. Research on UPV has demonstrated its effectiveness in estimating UCS for limestone. For instance, a study on dolomitic limestone from Khyber, North Pakistan (Abbas *et alii*, 2021), found that UPV correlated well with UCS, with most samples showing pulse velocities in the range of 1800-3800 m/s and a mean value of 2751 m/s. Another recent investigation by Kamran *et alii* (2022) examined the use of UPV and other non-destructive techniques to estimate the compressive strength of various stones, including limestone. This study reinforced the utility of UPV as a reliable predictor of UCS in limestone samples.

Similarly, studies on SRN have shown strong correlations with UCS in limestone samples. A comprehensive study by Mohammed *et alii* (2020) specifically focused on the reliability of empirical equations to predict UCS of limestone rocks using Schmidt hammer rebound number. The researchers analyzed 112 core samples of limestone from four different areas in Iraq. They found that among various existing empirical equations, the one developed by Arslan *et alii* (2015) was the most reliable for predicting UCS from SRN for limestone rocks. This study also demonstrated the potential for enhancing prediction accuracy by incorporating local measurement data.

While recent studies have shown the individual effectiveness of UPV and SRN in predicting UCS for limestone, it's important to highlight that the combined use of these methods often provides more accurate estimates compared to using either method alone. This conclusion is supported by earlier research, such as the study by Selçuk & Nar (2015), which demonstrated that the combined method offers better UCS predictions across various rock types. In this context, linear multivariate regression models have been extensively employed to predict the uniaxial compressive strength (UCS) of rocks, leveraging various combinations of rock properties to enhance predictive accuracy and applicability in geotechnical engineering (Aladejare *et alii*, 2021). Among the numerous models developed, those incorporating Ultrasonic Pulse Velocity (UPV) and Schmidt Rebound Number (SRN) have demonstrated significant promise due to the strong correlation of these parameters with UCS. For instance, Çobanoğlu & Çelik (2008) developed a model integrating UPV, SRN, and point load strength, achieving an impressive R^2 value of 0.99. This high correlation indicates the model's strong predictive capability and reliability in estimating UCS from easily measurable rock properties.

Heidari *et alii* (2018) further explored the potential of these parameters by creating a regression model that combines UPV, SRN, block punch index (BPI), and point load strength, resulting in an R^2 value of 0.91. This model highlights the benefits of integrating multiple mechanical properties to capture the complex behavior of rocks more accurately. Similarly, Jalali *et alii* (2017) achieved an R^2 value of 0.91 using a combination of UPV, SRN, block punch index, and point load strength, demonstrating consistent accuracy and reinforcing the importance of these predictors in UCS estimation.

Dehghan *et alii* (2010) developed a more complex model incorporating UPV, SRN, porosity, and point load strength, achieving an R^2 value of 0.64. While this model has a lower R^2 value compared to the others, it provides a detailed understanding of the interactions between various rock properties and their collective impact on UCS. This complexity can offer deeper insights into the factors influencing rock strength, particularly in cases where rock properties exhibit non-linear relationships.

Additionally, a significant reference by Selçuk & Nar (2015) developed a model that exclusively uses UPV and SRN, making it unique among the discussed models. This model, with a correlation coefficient R^2 of 0.87 and a root mean square error (RMSE) of 11.01, indicates good predictive capability using just these two parameters.

These studies collectively underscore the efficacy of using UPV and SRN in multivariate regression models for accurately predicting UCS across various rock types and geological contexts. The high R^2 values achieved by these models indicate their robustness and practical applicability in real-world scenarios, providing valuable tools for geotechnical engineers to assess rock strength efficiently. By leveraging the strong correlations between UCS, UPV, and SRN, these models facilitate more accurate and reliable predictions, enhancing the safety and effectiveness of engineering projects involving rock materials.

The use of linear multivariate regression models in predicting UCS is advantageous not only for its predictive accuracy but also for its cost-effectiveness and time efficiency. Laboratory tests to determine UCS can be time-consuming and expensive, particularly for large-scale or remote projects. In contrast, UPV and SRN measurements can be conducted quickly and with minimal equipment, making these models an attractive alternative for preliminary assessments and ongoing monitoring.

LOCATION AND GEOLOGICAL CONTEXT

The Hippo Regius, the ancient Roman city now known as Annaba, is located in northeastern Algeria. Its geographical coordinates are approximately 36°52’57”N 07°45’00”E. The city is situated near the mouth of the Seybouse River and lies on the coast of the Mediterranean Sea, close to the border with Tunisia (Fig. 1).

The geological context of Annaba is characterized by its proximity to the Edough Mountains, which have significantly influenced the area’s geology. The region’s geological composition is diverse, including metamorphic formations in the Edough Mountains (such as migmatite, garnet mica schists, marble benches, and various schists), sedimentary deposits (recent and ancient alluvium, dunes, and lakes or swamps), and volcanic intrusions from the Miocene age (Gheris, 2023). Limestone is a prominent feature, with Jurassic-period black shale and limestone dolomite formations present. This geological diversity has played a crucial role in the city’s development, particularly in construction. The limestone from the Edough Mountains was extensively used in Roman buildings, especially for lime mortars (Gheris, 2023). Archaeological evidence shows that these mortars contain minerals like quartz, feldspar, biotite, mica, plagioclase, and muscovite, likely derived from the Edough Mountains (Gheris, 2023). The Romans also created hydraulic mortars by combining local limestone with natural pozzolanic materials for water-resistant structures (Djerad *et alii*, 2022). Local quarries in the Edough Mountains provided a steady supply of high-quality limestone, which was integral to Hippo Regius’s architectural development. This geological context not only influenced the city’s ancient construction techniques but also contributed to the fertile plains surrounding the area, shaping its historical and economic significance.

.

METHODOLOGY

Samples collection and preparation

Firstly, a total of seven limestone stones (Fig. 2) were carefully extracted from various locations within the historic city center of Annaba, considering factors such as rock type, lithological variation, and accessibility. Annaba’s geological context includes sedimentary rock formations from the Miocene epoch. Due to limited tools and resources, Petrographic and Mineralogical analysis could not be conducted. The research laboratory utilized for

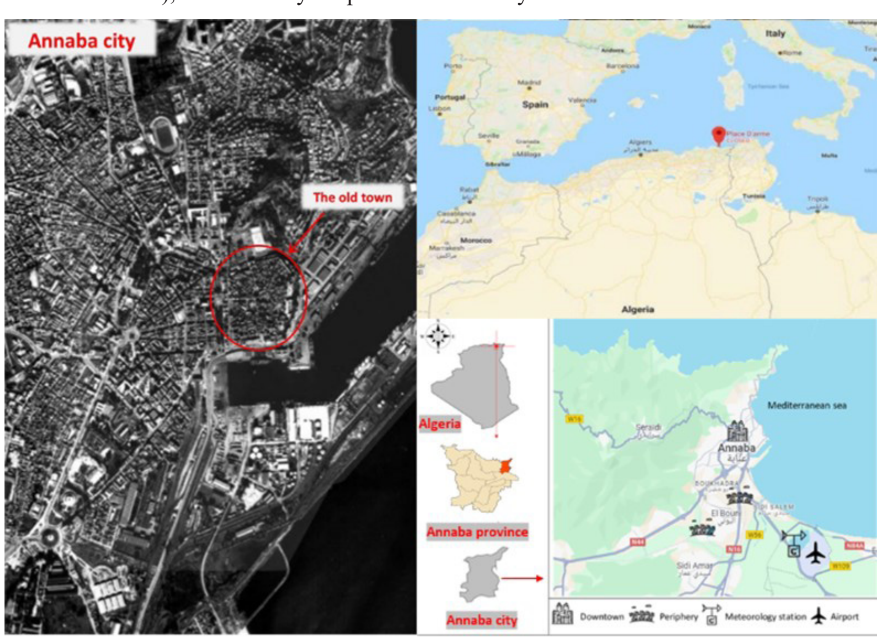

Fig. 1 - Geographic location of the study area

this study is primarily equipped for mechanical and non-destructive testing, which is essential for the objectives of this study. Despite this limitation, we employed comprehensive non-destructive testing methods (UPV and SRN) to capture the mechanical properties of the limestone samples, which are crucial for the development of our UCS estimation model. The collected stones were cut, numbered, and prepared for testing following the TS EN 1926 standard. Each stone was cut into cubes with a side length of 70 mm (Fig. 3) to

Fig. 2 - Collecting the limestone rocks from filed

Fig. 3 - Cutting and numbering the cubes of the limestone samples

ensure uniformity and consistency in sample size and geometry.

Laboratory tests

The experimental program consisted of a combination of non-destructive and destructive tests to determine the physical and mechanical properties of the limestone samples. Initially, Ultrasonic Pulse Velocity (UPV) tests (Fig. 4) were performed to measure the velocity of ultrasonic waves through the samples, providing insights into their internal structure and integrity. Additionally, Schmidt hammer tests (Fig. 5) were conducted to measure the rebound hardness of the limestone surfaces, which served as a supplementary indicator of their mechanical properties.

Following these non-destructive tests, uniaxial compression tests (Fig. 6) were carried out to directly measure the Uniaxial Compressive Strength (UCS) of the limestone specimens under controlled loading conditions (Fig. 5). These tests were performed in the laboratory of the Civil Engineering Department at Badji Mokhtar – Annaba University (Algeria) using a MATEST S.p.A. testing machine with a loading capacity of 2000 kN. This equipment ensured precise and controlled loading conditions during the

Fig. 4 - Ultrasonic Pulse Velocity test on the prepared limestone samples

Fig. 5 - Schmidt Hammer Rebound test on the prepared limestone samples

compression tests, allowing for accurate measurement of the UCS. The uniaxial compressive strength can be computed using the ultimate compressive load (P) resisted by the sample and the sample's cross-sectional area ($A= a^2$) using Eq. 1:

$$UCS=P/a \tag{1}$$

Fig. 6 - Uniaxial Compressive Strength test on the prepared limestone samples

Data analysis

Throughout the testing process, strict adherence to standardized procedures and protocols was maintained to ensure the reliability and validity of the experimental results. The dataset consists of UCS values, Schmidt Hammer Rebound Numbers, and P-wave Velocities for 22 limestone samples, identified as A1 to I2. Detailed measurements are provided in Table 1.

Sample ID	Rebound (N)	Vp (Km/Sec)	UCS (MPa)
A1	40.33	4.67	38.28
A2	34.00	4.00	24.94
B1	38.00	4.70	34.35
B2	34.00	3.83	18.54
B3	34.00	3.89	21.34
B4	41.67	4.56	36.79
C1	37.00	4.58	26.32
C2	38.00	4.64	40.28
C3	38.00	4.83	29.80
D1	35.00	4.02	21.45
D2	42.67	4.96	43.94
D3	41.00	4.96	34.54
D4	30.67	3.80	16.51
D5	40.67	4.76	36.67
D6	40.00	4.76	32.54
D7	37.00	4.76	39.23
D8	33.33	3.87	27.47
F1	38.00	4.22	29.00
F2	30.00	3.70	16.09
F3	51.00	5.00	49.00
I1	32.67	3.30	19.28
I2	32.27	3.37	21.28

Tab. 1 - Measurement values of the UCS, SRN, and UVP for the 22 limestone samples

Descriptive statistics for Schmidt Rebound, P-wave Velocity, and UCS measurements are necessary to enable a comprehensive understanding of the data's central tendency, variability, and distribution. Mean, median, range and standard deviation provide a concise data summary, allowing the identification of patterns, trends, and outliers. Descriptive statistics are summarized in Table 2.

Figure 7 displays the correlation matrix, highlighting the relationships between Schmidt Rebound, P-wave Velocity, and UCS. The high correlation coefficients between these variables indicate strong linear relationships, justifying their use in the regression model. Specifically, UCS shows a high correlation with both Schmidt Rebound (0.8968) and P-wave Velocity (0.8612), reinforcing their predictive power for UCS.

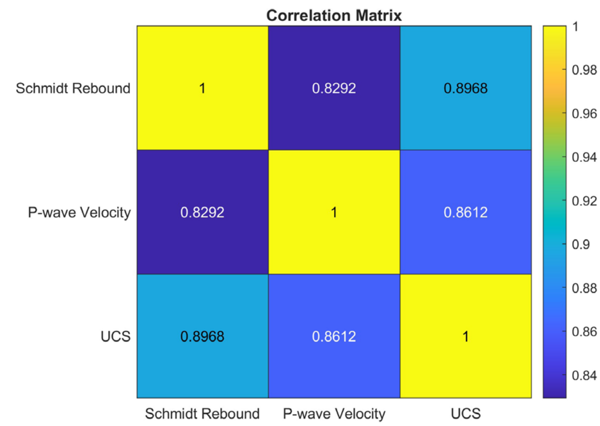

Fig. 7 - Correlation Matrix between the three parameters (UCS, SRN, UVP)

Regression models and validation

The multivariate regression approach in this study focuses on predicting UCS using Schmidt Rebound (SRN) and P-wave Velocity (UVP) as independent variables. It is crucial to ensure the data meets the regression assumptions of linearity, independence, homoscedasticity, normality, and absence of multicollinearity. The regression model was developed using the least squares method (Eq. 2), aiming to minimize the sum of squared differences between observed and predicted UCS values.

$$UCS = \beta_0 + \beta_1 \text{ x } SRN + \beta_2 \text{ x } UVP + \varepsilon \quad (2)$$

Where:

- β_0 is the intercept;
- β_1 is the coefficient for the measured Schmidt Rebound;
- β_2 is the coefficient for the measured P-wave Velocity in (km/s);
- ε is the random error.

To evaluate the regression model's performance, we employed a range of established metrics, including the coefficient of determination (R^2), root mean square error (RMSE), and mean absolute error (MAE). These metrics provided a comprehensive

Variable	Mean	Median	Standard Deviation	Variance	Minimum	Maximum
Schmidt Rebound	37.24	37.50	4.76	22.71	30.00	51.00
P-wave Velocity	4.32	4.57	0.53	0.2878	3.30	5.00
UCS	29.89	29.40	9.36	87.787	16.09	49.00

Tab. 2 - Descriptive statistics of the performed analyses assessment of the model's accuracy and predictive capability.

To ensure the model's assumptions held and to identify any potential discrepancies, we conducted a thorough residual analysis. This involved generating plots such as Residuals vs. Fitted Values, QQ Plot, Histogram of Residuals, and Box Plot of Residuals. The MATLAB Software was utilized for data analysis, regression modeling, and visualization.

RESULTS AND ANALYSIS

The scatter plot in Fig. 8 displays the relationship between the actual Unconfined Compressive Strength (UCS) values and the predicted UCS values obtained from the multivariate regression model. The scatter plot shows that most data points cluster around both the regression line and the line of perfect prediction, indicating a strong predictive capability of the model. However, some deviations from these lines suggest instances of overestimation or underestimation by the model, highlighting the residuals or differences between the actual and predicted values, which are crucial for assessing the model's precision and accuracy.

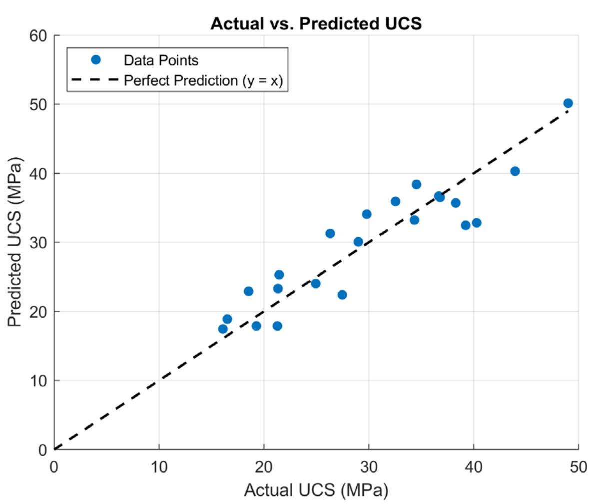

Fig. 8 - Actual vs. Predicted UCS

The regression coefficients are as follows:
- intercept: -41.36;
- Schmidt Rebound: 1.15;
- P-wave Velocity: 6.57.

These coefficients indicate the contribution of each predictor to the UCS prediction. The positive coefficients suggest that higher Schmidt Rebound and P-wave Velocity values correspond to higher predicted UCS values. The regression equation derived from the analysis is as follows (Eq. 3):

$$UCS = 1.15 \times SRN + 6.57 \times UVP - 41.36 \qquad (3)$$

The high R^2 value of 0.848 indicates that the model explains a significant portion of the variability in UCS, demonstrating its robustness and predictive capability. The low RMSE (3.562) and MAE (2.962) values further confirm the model's accuracy in estimating UCS based on UPV and SRN. This high level of agreement between actual and predicted values highlights the effectiveness of using UPV and SRN as predictors for UCS in limestone rocks.

Residual Analysis

Residual analysis was conducted to check the model's assumptions and accuracy. The residuals were plotted against Schmidt Rebound and P-wave Velocity (Fig. 9), and a QQ plot of the residuals was created (Fig. 10). Additionally, a box plot and a histogram (Fig. 11) of the residuals were produced to assess the distribution and variance.

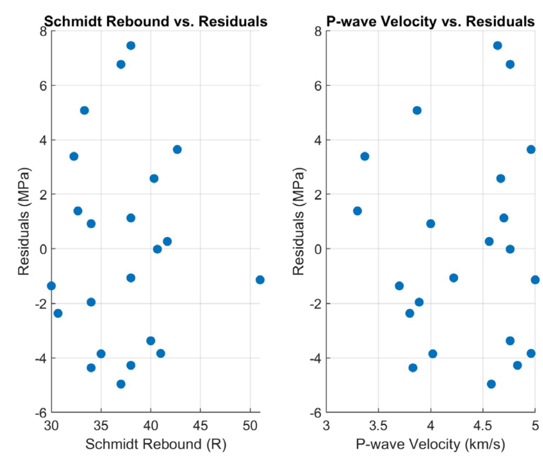

Fig. 9 - Schmidt Rebound and P-wave Velocity vs. Residuals

The residual plots (Fig. 9) indicate that the residuals are randomly distributed with respect to both Schmidt Rebound and P-wave Velocity, which implies that the model does not exhibit systematic bias. This randomness is crucial for validating the model's accuracy and ensuring that it captures the essential relationships between the predictors and the response variable without overfitting.

The QQ plot compares the quantiles of the residuals with the standard normal quantiles. The alignment of points along the red dashed line, which represents the theoretical normal distribution, indicates that the residuals approximately follow a normal distribution. This supports the assumption of normality for the residuals in our regression model.

As shown in Fig. 11, the box plot of residuals indicates a median close to zero and a narrow interquartile range, suggesting that most residuals are confined to a small range. The

Fig. 10 - QQ plot of Residuals

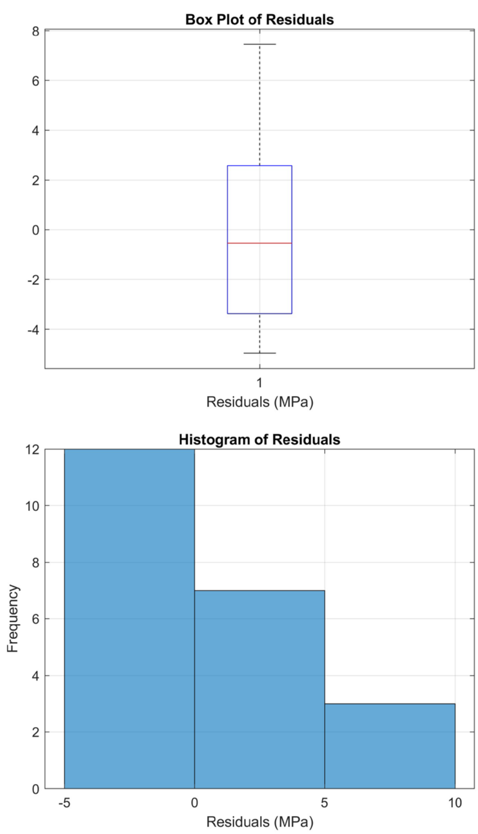

Fig. 11 - Box Plot of Residuals (top) and Histogram of Residuals (bottom)

presence of minimal outliers implies that the model handles the data well without significant deviations. Furthermore, the histogram of residuals in the same figure illustrates a frequency distribution concentrated around zero, with a slight skewness towards negative values, indicating a tendency for the model to underestimate UCS in some cases slightly. Notably, the majority of residuals fall within the -5 to 5 MPa range, further confirming the model's accuracy.

The unstandardized residuals in Fig. 12 are distributed around the zero line, indicating unbiased model predictions on average, with no evident pattern or trend suggesting that the model captures the systematic relationship between the variables.

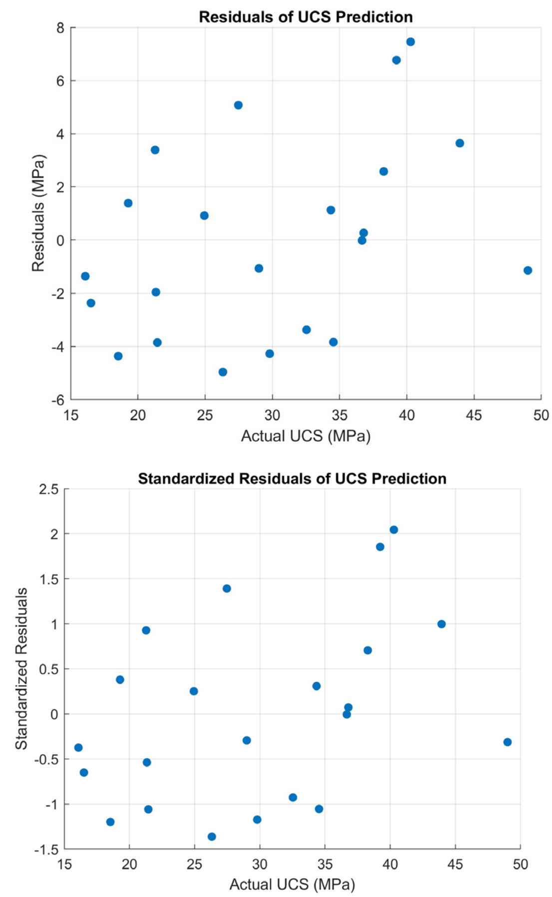

Fig. 12 - Residuals (top) and Standardized Residuals of UCS Prediction (bottom)

The spread of residuals appears consistent across different levels of UCS, meeting the assumption of homoscedasticity, although a few larger prediction errors are present. The standardized residuals (Fig. 12), which help identify points that deviate significantly from the predicted values, appear randomly scattered around zero, further supporting the model's unbiased predictions. The lack of a clear pattern in the standardized residuals suggests that the model appropriately captures the relationship between UCS and the predictors, and the relatively uniform spread of residuals indicates homoscedasticity. With no significant outliers visible, both plots collectively suggest that the multivariate regression model performs well in predicting UCS, with no major violations of regression assumptions evident in the residuals.

COMPARISON WITH PREVIOUS STUDIES

In this section, we compare the results of our model with those reported in various studies, as summarized in Table 3. Our model's regression coefficients, R-squared value, and RMSE are evaluated against these studies to understand its performance and reliability in predicting uniaxial compressive strength (UCS).

Multivariate regression model	Relationship	No of data	Input parameters	R^2	RMSE
(Cobanoğlu and Çelik 2008)	$UCS = 4.14\ Is\ (50) + 29.8V_p + 0.54N - 116$	150	Point load strength Is (50), P-wave velocity Vp, Schmidt hardness rebound (N)	0.98	1.96
(Heidari et al. 2018)	$UCS = 1.277N + 2.186BPI + 16.41Is\ (50) + 0.011Vp - 82.436$	108	Schmidt hardness number, Block punch index, point load strength, P-wave velocity	0.91	10.80
(Dehghan et al. 2010)	$UCS = -595.303 - 442.363V_p + 45.338V_p^2 - 6.1n + 0.52n^2 + 28.314Is\ (50) - 4.061Is^2\ (50) + 115.822N - 2.007N^2$	30	P-wave velocity, porosity, point load strength, Schmidt hardness number	0.64	/
(Jalali et al. 2017)	$UCS = 1.277N + 2.86BPI + 16.41Is\ (50) + 0.011Vp - 82.436$	53	Schmidt hardness number, Block punch index, point load strength, P-wave velocity	0.91	11.71
(Selçuk and Nar 2015)	$UCS = 12.92Vp + 11.29RN - 42.29$	42	P-wave velocity, Schmidt hammer rebound	0.87	11.01
Our model	$UCS = 6.57UVP + 1.15SRN - 41.36$	22	P-wave velocity, Schmidt hammer rebound	0.84	3.56

Tab. 3 - Comparison of multivariate regression models

The comparison of our proposed multivariate regression model with existing literature demonstrates the efficacy and relevance of our approach in predicting the Uniaxial Compressive Strength (UCS) of rocks. Our model utilizes P-wave velocity (UVP) and Schmidt Hammer Rebound (SRN) as input parameters. With an R^2 value of 0.84 and an RMSE of 3.56, our model shows strong predictive capability, particularly considering the limited dataset of 22 samples.

To ensure a meaningful comparison, we selected existing models that also use P-wave velocity and Schmidt hammer rebound among their input parameters, aligning with our approach. Çobanoğlu & Çelik (2008) developed a model achieving a very high R^2 of 0.98 with an RMSE of 1.96 using a larger dataset of 150 samples and three parameters: point load strength ($I_s(50)$), P-wave velocity (Vp), and Schmidt hardness rebound (N). This indicates a robust model but highlights the challenge of acquiring such comprehensive data.

Heidari *et alii* (2018) presented a model with an R^2 of 0.91 and an RMSE of 10.80, employing Schmidt hardness number, Block punch index (BPI), point load strength, and P-wave velocity across 108 samples. While their model performs well, it incorporates more complex parameters.

Similarly, the model by Dehghan *et alii* (2010), uses P-wave velocity, porosity (n), point load strength, and Schmidt hardness number, achieving an R^2 of 0.64 with a smaller dataset of 30 samples. This model includes non-linear terms, indicating a more complex relationship which may not be easily generalizable.

Jalali *et alii* (2017) proposed a similar model to Heidari *et alii* (2018), with a slight modification in the coefficient values, achieving an R^2 of 0.91 and an RMSE of 11.71 across 53 samples.

Lastly, Selçuk & Nar (2015) developed a model with an R^2 of 0.87 and an RMSE of 11.01 using 42 samples, focusing on P-wave velocity and Schmidt Hammer Rebound, similar to our approach but yielding a higher RMSE.

Our model's lower RMSE of 3.56, despite a smaller dataset, suggests a strong predictive power with fewer parameters, which simplifies data collection and processing. This makes our approach practical and efficient for predicting UCS in rock mechanics, providing a valuable contribution to the existing body of literature. Further research with larger datasets could help validate and potentially enhance the model's accuracy and generalizability.

While our study focused on limestone samples from Annaba, Algeria, it is important to acknowledge the limitations of applying this model to other rock types. The correlation between Uniaxial Compressive Strength (UCS) and Ultrasonic Pulse Velocity (UPV) can vary significantly depending on the rock's composition and structure. Future studies should explore the applicability of this multivariate regression model to different rock types, particularly polymineralic and clastic rocks. Previous research has shown that the relationship between UCS and UPV is often less robust for these more complex rock types (Bolla & Paronuzzi, 2021). For instance, sedimentary rocks with varying grain sizes and compositions may exhibit different wave propagation characteristics, potentially affecting the accuracy of UCS predictions based on UPV measurements. Additionally, metamorphic rocks with strong foliation or banding might display anisotropic behavior, further complicating the UCS-UPV relationship. Igneous rocks with varying crystal sizes and mineral compositions could also present challenges in establishing consistent correlations (Yasir *et alii,* 2021). To advance this field of study, future research should:

1. Investigate the UCS-UPV relationship in a wide range

of rock types, including sedimentary, metamorphic, and igneous rocks;

2. Explore the influence of mineralogical composition, texture, and structural features on the UCS-UPV correl ation (Yasir *et alii*, 2021);
3. Develop rock-specific models that account for the uni que characteristics of different lithologies.

By addressing these aspects, future studies can contribute to a more comprehensive understanding of the limitations and potential applications of UCS estimation using non-destructive testing methods across diverse geological settings.

CONCLUSION

This study successfully developed a multivariate regression model for estimating the Uniaxial Compressive Strength (UCS) of limestone rocks using Ultrasonic Pulse Velocity (UPV) and Schmidt Hammer Rebound Number (SRN) as predictive factors. The limestone samples, sourced from the old city center of Annaba, Algeria, underwent comprehensive laboratory testing to provide the necessary data for model development. The resulting model demonstrated strong predictive accuracy with a high coefficient of determination (R^2) of 0.848, indicating that 84.8% of the variability in UCS could be explained by UPV and Schmidt Hammer Rebound Number.

Our model's simplicity and strong performance, with a low root mean square error (RMSE) of 3.562 MPa, highlight its practical applicability. This lower RMSE compared to other models suggests more precise predictions, which is crucial for practical applications in the field. Visual and statistical analyses of the model's residuals indicated unbiased and normally distributed predictions, further validating the robustness and reliability of the regression model. The strong correlation between actual and predicted UCS values underscores the effectiveness of using non-destructive testing methods for UCS estimation in limestone rocks.

The findings of this study have significant implications for Geomechanical and engineering applications. The ability to accurately estimate UCS using non-destructive methods offers a practical and efficient alternative to direct UCS testing, particularly in situations where sample collection and preparation are challenging. This approach facilitates better-informed decision-making in the design and implementation of engineering projects involving limestone formations.

In conclusion, the multivariate regression model developed in this study provides a valuable tool for predicting the UCS of limestone rocks. While other models may include additional predictors and exhibit higher R-squared values, our model's simplicity and comparable performance underscore its practical utility.

Future research should explore the UCS-UPV relationship across various rock types, including sedimentary, metamorphic, and igneous rocks, examine how mineralogical composition, texture, and structural features impact the UCS-UPV correlation, and develop rock-specific models that account for the unique properties of different lithologies. This study contributes to the advancement of UCS estimation techniques, promoting the use of non-destructive testing methods in rock mechanics and engineering geology.

REFERENCES

Abbas N., Akhter Qureshi J., Mir Z. & Khan A (2021) - *18 c Correlation of Schmidt Hammer Rebound Numbers with Ultrasonic Pulse Velocity and Slake Durability Index of Dolomitic Limestone of Khyber, North Pakistan.* International Journal of Economic and Environmental Geology, **13**: 18-22. https://doi.org/10.46660/ijeeg.v13i1.13.

Aladejare A.E., Alofe E.D., Onifade M., et alii (2021) - *Empirical Estimation of Uniaxial Compressive Strength of Rock: Database of Simple, Multiple, and Artificial Intelligence-Based Regressions.* Geotechnical and Geological Engineering, **39**: 4427-4455. https://doi.org/10.1007/s10706-021-01772-5.

Ali M. & Hin Lai S. (2023) - *Artificial intelligent techniques for prediction of rock strength and deformation properties – A review.* Structures, **55**:1542-1555. https://doi.org/10.1016/j.istruc.2023.06.131.

Arslan M., Khan M.S., Yaqub M. (2015) - *Prediction of durability and strength from Schmidt rebound hammer number for limestone rocks from Salt Range, Pakistan.* Journal of Himalayan Earth Sciences, **48**: 9-13.

ASTM (2007) - *Designation: D 2938 – 95.* Standard Test Method for Unconfined Compressive Strength of Intact Rock Core Specimens.

Barham W.S., Rabab'ah S.R., Aldeeky H.H. & Al Hattamleh OH (2020) - *Mechanical and Physical Based Artificial Neural Network Models for the Prediction of the Unconfined Compressive Strength of Rock.* Geotechnical and Geological Engineering, **38**: 4779-4792. https://doi.org/10.1007/S10706-020-01327-0.

Bolla A. & Paronuzzi P. (2021) - *UCS field estimation of intact rock using the Schmidt hammer: A new empirical approach.* In: IOP Conference Series: Earth and Environmental Science, 1-8. IOP Conference Series.

Cai M., Kaiser P.K., Uno H., et alii (2004) - *Estimation of rock mass deformation modulus and strength of jointed hard rock masses using the GSI system.* International Journal of Rock Mechanics and Mining Sciences, **41**: 3-19. https://doi.org/10.1016/S1365-1609(03)00025-X.

Çobanoğlu I. & Çelik S.B. (2008) - *Estimation of uniaxial compressive strength from point load strength, Schmidt hardness and P-wave velocity.* Bulletin of Engineering Geology and the Environment, **67**: 491-498. https://doi.org/10.1007/S10064-008-0158-X.

Dehghan S., Sattari G., Chehreh Chelgani S. & Aliabadi M.A. (2010) - Prediction of uniaxial compressive strength and modulus of elasticity for Travertine samples using regression and artificial neural networks. Mining Science and Technology 20:41–46. https://doi.org/10.1016/S1674-5264(09)60158-7.

Djerad M.S., Boufenara K., Courtils J. DES, et alii (2022) - *Multianalytical Characterisation And Provenance Investigation Of Natural Pozzolana In Roman Lime Mortars From The Archaeological Site Of Hippo Regius (Algeria).* Mediterranean Archaeology and Archaeometry, **22**:231-231. https://doi.org/10.5281/zenodo.7412076.

Garrido M.E., Petnga F.B., Martínez-Ibáñez V., et alii (2022) - *Predicting the Uniaxial Compressive Strength of a Limestone Exposed to High Temperatures by Point Load and Leeb Rebound Hardness Testing.* Rock Mech. Rock Eng., **55**:1-17. https://doi.org/10.1007/s00603-021-02647-0.

Gheris A. (2023) - *New dating approach based on the petrographical, mineralogical and chemical characterization of ancient lime mortar: case study of the archaeological site of Hippo, Annaba city, Algeria.* Herit, Sci., **11**:103. https://doi.org/10.1186/s40494-023-00942-3.

Gowida A., Elkatatny S., Gamal H. (2021) - *Unconfined compressive strength (UCS) prediction in real-time while drilling using artificial intelligence tools.* Neural Comput. Appl., **33**: 8043-8054. https://doi.org/10.1007/S00521-020-05546-7.

Heidari M., Mohseni H. & Jalali S.H. (2018) - *Prediction of Uniaxial Compressive Strength of Some Sedimentary Rocks by Fuzzy and Regression Models.* Geotechnical and Geological Engineering, **36**: 401-412. https://doi.org/10.1007/S10706-017-0334-5.

Jalali S.H., Heidari M. & Mohseni H. (2017) - *Comparison of models for estimating uniaxial compressive strength of some sedimentary rocks from Qom Formation.* Environ. Earth. Sci., **76**:753. https://doi.org/10.1007/s12665-017-7090-y.

Kamran A., Ali L., Ahmed W., et alii (2022) - *Aggregate Evaluation and Geochemical Investigation of Limestone for Construction Industries in Pakistan: an approach for sustainable economic development.* Sustainability, **14**. https://doi.org/10.3390/su141710812.

Mahmoodzadeh A., Mohammadi M., Hashim Ibrahim H., et alii (2021) - *Artificial intelligence forecasting models of uniaxial compressive strength.* Transportation Geotechnics, **27**. https://doi.org/10.1016/j.trgeo.2020.100499.

Mohammed D.A., Alshkane Y.M. & Hamaamin Y.A. (2020) - *Reliability of empirical equations to predict uniaxial compressive strength of rocks using Schmidt hammer.* Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, **14**: 308-319. https://doi.org/10.1080/17499518.2019.1658881.

Momeni E., Jahed Armaghani D., Hajihassani M. & Mohd Amin M.F. (2015) - *Prediction of uniaxial compressive strength of rock samples using hybrid particle swarm optimization-based artificial neural networks.* Measurement (Lond), **60**: 50-63. https://doi.org/10.1016/j.measurement.2014.09.075.

Ore R.T.I. (2020) - *The effect of anisotropy orientation on the sedimentary rock strength estimated by point load testing strength*. Pilbara, Australia.

Rahimi Shahid M., Karami M. & Lashkaripour G.R. (2022) - *Use of multivariate regression for assessing rock mass permeability in Khersan 2 dam site using discontinuity system parameters.* New Findings in Applied Geology, **16**: 32-51. https://doi.org/10.22084/nfag.2021.23832.1456.

Seboui H., Athmani A. & Formisano A, (2023) - *Literature review on mechanical properties estimation of historical masonry buildings: application of an evaluation method for the Algerian case.* International Journal of Masonry Research and Innovation, **8**: 293-332. https://doi.org/10.1504/ijmri.2023.129558.

Selçuk L. & Nar A. (2015) - *Prediction of uniaxial compressive strength of intact rocks using ultrasonic pulse velocity and rebound-hammer number.* Quarterly Journal of Engineering Geology and Hydrogeology, **49**: 67-75. https://doi.org/10.1144/qjegh2014-094.

Taghavi B., Hajizadeh F. & Moomivand H. (2023) - *Comparison of artificial intelligence and multivariate regression methods in predicting the uniaxial compressive strength of rock during the specific resistivity monitoring.* Bulletin of Engineering Geology and the Environment, **82**. https://doi.org/10.1007/S10064-023-03415-W.

Tandon R.S. & Gupta V. (2015) - *Estimation of strength characteristics of different Himalayan rocks from Schmidt hammer rebound, point load index, and compressional wave velocity.* Bulletin of Engineering Geology and the Environment, **74**: 521-533. https://doi.org/10.1007/S10064-014-0629-1.

Yasir M., Ahmed W. & Sajid M. (2021) - *The control of composition, texture and weathering on the physical and strength properties of selected intrusive igneous rocks from North Pakistan.*

Yilmazkaya E. (2023) - *Amperage prediction in mono-wire cutting operation using multiple regression and artificial neural network models.* Neural Comput. Appl., **35**:13343-13358. https://doi.org/10.1007/S00521-023-08443-X.

Zhang W. & Goh A.T.C. (2014) - *Multivariate adaptive regression splines model for reliability assessment of serviceability limit state of twin caverns.* Geomechanics and Engineering, **7**: 431- 458. https://doi.org/10.12989/GAE.2014.7.4.431.

Zhao J., Li D., Jiang J. & Luo P. (2024) - *Uniaxial Compressive Strength Prediction for Rock Material in Deep Mine Using Boosting-Based Machine Learning Methods and Optimization Algorithms.* CMES - Computer Modeling in Engineering and Sciences, **140**: 275-304. https://doi.org/10.32604/CMES.2024.046960.

Received March 2024 - Accepted June 2024