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POTENTIAL LINK BETWEEN THE GOLDEN RATIO AND THE AT-REST EARTH PRESSURE IN NORMALLY CONSOLIDATED SOILS

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EXTENDED ABSTRACT

Questo lavoro costituisce lo sviluppo di un precedente studio dell'autore relativo alla predizione del coefficiente di spinta a riposo nei terreni normalconsolidati usando il concetto di angolo di resistenza al taglio mobilizzato nella condizione di compressione vergine con deformazione laterale impedita.

Com'è noto, il coefficiente di spinta a riposo K_0 del terreno, dato dal rapporto $\sigma'_{h0}/\sigma'_{v0}$ tra le tensioni geostatiche efficaci orizzontale e verticale in un semispazio di terreno delimitato da un piano orizzontale, è un importante parametro geotecnico. Esso, infatti, consente di determinare lo stato tensionale iniziale del terreno medesimo, la cui conoscenza è necessaria nelle analisi geotecniche.

Per i terreni normalconsolidati, K_0 ($\equiv K_{0(NC)}$) rappresenta una misura della forza di gravità in essi trasmessa nella direzione orizzontale. Per il suo calcolo sono disponibili numerose equazioni nelle quali $K_{0(NC)}$ è – di norma e sorprendentemente – correlato con l'angolo di resistenza al taglio Φ' del terreno. Quest'ultimo è un parametro relativo a condizioni di stato limite in una massa di terreno, mentre $K_{0(NC)}$ di per sé rappresenta uno stato di sforzo non a rottura, ma intermedio tra due stati limite specificati dalle condizioni, attiva e passiva, entrambe relative a rottura, ovvero all'angolo Φ'. Di conseguenza, il cerchio di Mohr relativo allo stato tensionale $K_{0(NC)}$ è tangente ad una retta il cui angolo di inclinazione è minore dell'angolo Φ' e ne rappresenta la parte mobilizzata Φ'_{mob}.

L'angolo Φ'_{mob} non può essere misurato direttamente, ma può essere correlato con l'angolo Φ'. Varie sono le correlazioni tra Φ' e Φ'_{mob} – ottenute per via speculativa o empirica – e le corrispondenti equazioni per $K_{0(NC)}$ presenti in letteratura. Una di queste correlazioni era stata ottenuta dall'autore, senza assunzioni di sorta e in maniera diretta, attraverso "back-analysis" di una lunga serie di coppie di dati sperimentali ($K_{0(NC)}$, Φ') attinti dalla letteratura, unitamente a un'equazione per $K_{0(NC)}$ in funzione di $\Phi'_{mob} = m\Phi'$, m essendo il fattore di mobilizzazione dell'angolo di resistenza al taglio Φ'.

Il riesame dei valori numerici dei fattori di mobilizzazione m ($= \Phi'_{mob}/\Phi'$) di cui al precedente studio, ha mostrato che i medesimi sono in relazione inversa con le capacità predittive delle corrispondenti equazioni per $K_{0(NC)}$. Da semplici sviluppi analitici di tale tendenza è scaturita la possibile emergenza del Rapporto Aureo, noto come $\tau = 1.618$, quale rapporto tra l'angolo di resistenza al taglio Φ' e la sua porzione mobilizzata Φ'_{mob}, ovvero $\Phi'_{mob} = \Phi'/1.618 = 0.618\Phi' = m\Phi'$.

L'equazione risultante per $K_{0(NC)}$ ha la stessa base dell'omologa equazione in precedenza ottenuta, ma migliore capacità predittiva di quest'ultima e di alcune altre equazioni per $K_{0(NC)}$, inclusa la popolare equazione approssimata di Jaky, raccomandata nello EUROCODICE 7.

Un corollario dell'approccio seguito è relativo al coefficiente di Poisson ν, la cui valutazione sperimentale è assai difficoltosa. Peraltro, mentre per la maggior parte dei materiali il valore di ν può facilmente essere ottenuto da tabelle, ciò non è possibile per il terreno, per il quale i risultati sperimentali variano grandemente.

Ad ogni modo, la stessa connessione tra Φ' e Φ'_{mob}, combinata col legame tra il coefficiente K_0 e il rapporto di Poisson per i materiali elastici, ha consentito di ottenere una interessante relazione tra ν e Φ'. Non sono state reperite in letteratura genuine coppie di dati di laboratorio (ν, Φ'), indispensabili per la convalida sperimentale dell'equazione. La quale, pertanto, è da riguardare come caso limite per i terreni normalconsolidati.

ABSTRACT

The paper deals with the prediction of the at-rest earth pressure coefficient K_0 ($\equiv K_{0(NC)}$) for normally consolidated soils as a function of the mobilised proportion Φ'mob of the shearing resistance angle *Φ'* in the one-dimensional virgin compression process. By carrying out a back analysis of a wealth of published experimental ($K_{0(NC)}$, *Φ'*) pairs, the author previously derived, with no assumptions and in a straightforward manner, an empirical equation for $K_{0(NC)}$ in terms of the mobilised angle Φ'_{mob} of shearing resistance. Further focusing on the numerical value of the mobilising factor of *Φ'* in the one-dimensional virgin compression process showed the possible emergence of the Golden Ratio $\tau = 1.618$ as the ratio between the shearing resistance angle *Φ'* and its mobilised proportion in the above process, i.e. $\Phi'/\Phi'_{mob} = 1.618$ or $\Phi'_{mob} = 0.618\Phi'$.

The resulting empirical equation has the same basis, in terms of soil behaviour, as the previously obtained $K_{0(NC)}$ equation and a slightly better predictive capability for the full range of the *Φ'* values considered; it is also better than several other $K_{0(NC)}$ equations, including Jaky's approximate equation.

The link between K_0 and the Poisson's ratio *ν* for elastic materials, combined with the connection between *Φ'* and Φ'_{mob}, suggests a correlation between ν and *Φ'* for normally consolidated soils. However, this correlation is a borderline case for these soils, thus requiring experimental validation.

Keywords: *coefficient of at-rest earth pressure, normally consolidated soils, Golden Ratio, Poisson's ratio, mobilised angle of shearing resistance*

INTRODUCTION

The at-rest earth pressure coefficient K_0, i.e. the ratio of horizontal to vertical effective geostatic stresses, $\sigma'_{h0}/\sigma'_{v0}$ (note that, unlike σ'_{v0}, σ'_{h0} cannot be determined unequivocally from the equilibrium conditions), when the lateral strain is zero at a point in a semi-infinite homogeneous soil mass bounded by a horizontal plane, is an important parameter in Soil Mechanics, used to determine the initial stress state of a soil deposit. Its symbol (K_0) was introduced by Donath (1891).

For normally consolidated soils, K_0 ($\equiv K_{0(NC)}$), represents a measure of the force of gravity transmitted in the horizontal direction, and it can be estimated through a number of empirical expressions (see, e.g., Jaky, 1948; Brooker & Ireland, 1965; Alpan, 1967; Yamaguchi, 1972; Massarsch, 1979; Federico *et alii*, 2008). Moreover, theoretical expressions for $K_{0(NC)}$ have been developed over the years, the oldest being that proposed by Jaky (1944). A few other expressions have been developed by subsequent authors, including Rowe (1958), Hendron (1963), Burland & Roscoe (1969), Burland & Federico (1999) and Federico *et alii* (2009).

In the above expressions, both empirical and theoretical, the effective stress ratio $K_{0(NC)}$ is most often expressed as a function of the effective shearing resistance angle *Φ'*, i.e. of the failure stress conditions. However, since this ratio represents stress conditions well below the limit stress state given by *Φ'*, a more appropriate measure of such conditions is not *Φ'*, but, in principle, the mobilised proportion Φ'_{mob} of *Φ'* in this stress state, i.e. in the one-dimensional virgin compression process.

As for the Poisson's ratio *ν*, i.e. the ratio of the horizontal (ε_h) to the vertical strain (ε_v), although this elastic parameter may be readily obtained from tables for most materials, it is somewhat problematic for soils.

In its initial part, the paper lists the expressions of the mobilised shearing resistance angle found in the literature. Then, it will be shown that a minor change in the numerical value of the mobilisation factor – previously derived, with no assumptions, by Federico *et alii* (2008) – can elicit the possible presence of the Golden Ratio in the relationship between *Φ'* and its mobilized proportion Φ'_{mob} in the one-dimensional virgin compression process. The predictive capability of the corresponding equation of the at-rest coefficient of earth pressure $K_{0(NC)}$, is slightly better than that previously obtained by Federico *et alii* (2008) and better than several $K_{0(NC)}$ equations, including Jaky's popular approximate equation.

Moreover, given the link between K_0 and the Poisson's ratio ν in the theory of elasticity, the same connection between *Φ'* and Φ'_{mob} makes it possible to derive a correlation between ν and the shearing resistance angle *Φ'*, though as a borderline case for normally consolidated soils.

THE PARAMETERS K_0 AND *ν*

It is well known that, if a soil is assumed to be a linearly elastic, homogeneous and isotropic material, the stresses and strains inside it satisfy Hooke's law.

In the case of a triaxial test with no lateral deformation of this soil, the stresses must satisfy the condition:

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)] = 0 \tag{1}$$

$$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \nu(\sigma_1 + \sigma_2)] = 0 \tag{2}$$

where *E* and *ν* are the Young's modulus and Poisson's ratio, respectively, while σ_1 and $\sigma_2 = \sigma_3$ are the major and minor principal (effective) stresses.

Since the radial strains $\varepsilon_2 = \varepsilon_3 = 0$,

$$\sigma_2 = \sigma_3 = \frac{\nu}{1-\nu}\sigma_1$$

from which

and

$$\frac{\sigma_3}{\sigma_1} = \frac{\sigma_2}{\sigma_1} = K = K_0 = \frac{\nu}{1-\nu} \tag{3}$$

$$\nu = \frac{K_0}{1 + K_0} \qquad (4)$$

As indicated by Eq. (3), for an elastic material, K_0 depends solely on the Poisson's ratio ν.

Figure 1 shows K_0 as a function of Poisson's ratio ν. The relative curve is an arc of hyperbola with asymptotes at K_0 = -1 and ν = 1.

Poisson's ratio and Young's modulus are the more usual elastic parameters, and the elastic shear *G*, bulk *K* and constrained *D* moduli are related to them.

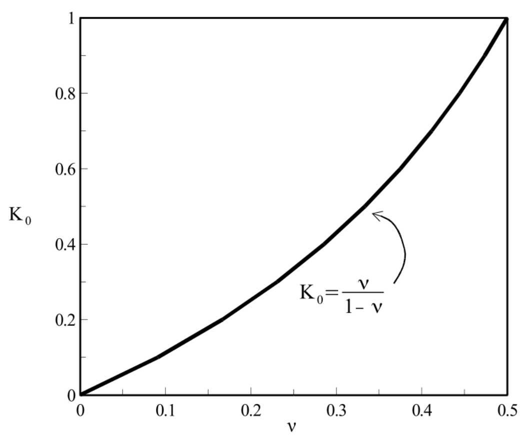

Fig. 1 - At-rest earth pressure coefficient (K_0) versus Poisson's ratio (ν) for elastic materials

For linearly elastic materials, Poisson's ratio ranges from 0 to 0.5 and thus the value of K_0 varies from 0 to 1. In fact, if a material has ν < 0, then a wire made out of it will get thicker upon stretching, while a compressed cylinder will get thinner. If ν > 0.5, an all-around increase in compression would result in an increase in volume (Bolton, 1979).

It is interesting to note that the product ($K_0 \cdot \nu$) between the parameters K_0 and ν (Eq. 4) is equal to their difference ($K_0 - \nu$). As a matter of fact, this ''property'' is peculiar to the pairs (x, y) of real numbers where y = x/(1+x).

Unlike $K_0 \equiv K_{0(NC)}$ – for which, just as for ν, drained conditions are considered here – it is very difficult to exactly determine a value of ν for soils that can be used in any situation (Muir Wood, 1990). Fortunately, it usually has a relatively small effect on engineering predictions (Lambe & Whitman, 1969). Moreover, the many tables and the few empirical relationships available in the literature are of little or no use (APPENDIX I).

THE MOBILISED SHEARING RESISTANCE ANGLE DURING ONE-DIMENSIONAL VIRGIN COMPRESSION

As is well known, two horizontal limit stresses are possible in a soil mass, as specified by active and passive conditions, both of which are related to failure conditions and thus to the shearing resistance angle *Φ'*.

In terms of magnitude of horizontal stress, the geostatic stress state K_0 represents an intermediate stress state that lies between the active and passive stress states. Its Mohr's circle of stress (Fig. 2) is tangent to a straight line whose inclination angle Φ'_{mob} is, of course, smaller than the soil angle *Φ'* at failure.

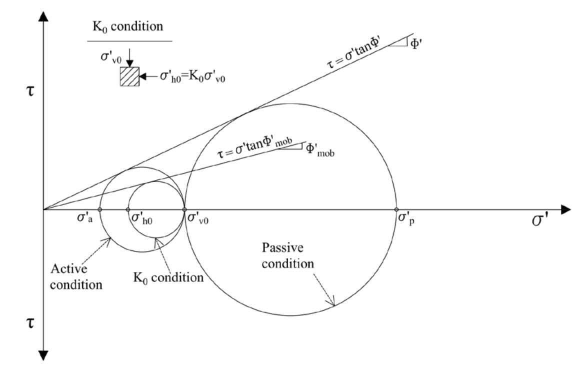

Fig. 2 - Mohr's circles of stress for soils at failure and under one-dimensional virgin compression

This angle represents the mobilised proportion Φ'_{mob} of the shearing resistance angle *Φ'* in the one-dimensional virgin compression process.

Using the obliquity relations (e.g. Taylor, 1948) for the geometry of the Mohr's circle, the at-rest coefficient of earth pressure $K_0(\equiv K_{0(NC)})$ can be geometrically related to the mobilised shearing resistance angle Φ'_{mob} via the expression:

$$K_0 \equiv K_{0(NC)} = \frac{\sigma'_{h0}}{\sigma'_{v0}} = \frac{1-\sin\Phi'_{mob}}{1+\sin\Phi'_{mob}} = \tan^2\left(45° - \frac{\Phi'_{mob}}{2}\right) \qquad (5)$$

or

$$K_{0(NC)} = \tan^2\left(45° - m\frac{\Phi'}{2}\right) \qquad (5')$$

m being the mobilisation factor of the shearing resistance angle *Φ'*. Note that the proper meaning of the ratio m between Φ'_{mob} and *Φ'* should be understood as a mobilisation factor rather than the inverse of a safety factor.

Equation (5) was originally formulated by Terzaghi (1923), and then interpreted by Rowe (1954 and 1958), and discussed by Mesri & Hayat (1993). Although the mobilized shearing resistance angle Φ'_{mob} in one-dimensional compression cannot be directly measured, it can be correlated with the effective shearing resistance angle *Φ'* of soils.

A few correlations between Φ'_{mob} and *Φ'* are found in the literature and are listed in Tab 1.

The corresponding expressions for $K_{0(NC)}$ can be obtained by substituting the equations of Tab. 1 in Eq. (5) (Tab. 2).

$\Phi'_{mob} = 1.15(\Phi' - 9°)$	Abdelhamid & Krizek (1976)	(6)
$\Phi'_{mob} = \Phi' - 11.5°$	Bolton (1991)	(7)
$\Phi'_{mob} = \sin^{-1}\frac{1}{\sqrt{2}}\sin\Phi'$	Simpson (1992)	(8)
$\Phi'_{mob} \approx 0.69\Phi'$	Eq. (8) rewritten for the 20°-35° range	(8bis)
$\Phi'_{mob} = 0.67\Phi'$	Hayat (1992)	(9)
$\Phi'_{mob} = 0.64\Phi'$	Federico *et alii* (2008); Federico & Elia (2009)	(10)
$\Phi'_{mob} = \beta\Phi'$	Lee *et alii* (2013)	(11)

Tab. 1 - Correlations between Φ'_{mob} and Φ'

$K_{0(NC)} = \tan^2\left[45° - \frac{1.15(\Phi' - 9°)}{2}\right]$	(6')
$K_{0(NC)} = \tan^2\left[45° - \frac{(\Phi' - 11.5)}{2}\right]$	(7')
$K_{0(NC)} = \tan^2\left[45° - \frac{1}{2}\left(\sin^{-1}\frac{1}{\sqrt{2}}\sin\Phi'\right)\right] \approx \tan^2\left(45° - 0.69\frac{\Phi'}{2}\right)$	(8')
$K_{0(NC)} = \tan^2\left(45° - 0.67\frac{\Phi'}{2}\right)$	(9')

Tab. 2 - $K_{0(NC)}$ versus Φ' equations

As regard Eq. (10), the Φ'_{mob} values were obtained through "back analysis", considering a significant number of experimental ($K_{0(NC)}$, Φ') pairs taken from the literature and relating to 59 out 66 soils, as subsequently described. The $K_{0(NC)}$ values were substituted in Eq. (5) and the resulting mobilised angles (Φ'_{mob} values) were compared with the corresponding experimental angles (Φ' values). This process resulted in Eq. (10), with a coefficient of determination $R^2 = 0.84$ between the two angles. This made it possible, in turn, to rewrite, <u>with no assumptions</u>, a new empirical $K_{0(NC)}$ equation (Federico *et alii*, 2008) for normally consolidated soils:

$$K_{0(NC)} = \frac{1 - \sin 0.64\Phi'}{1 + \sin 0.64\Phi'} = \tan^2\left(45° - 0.64\frac{\Phi'}{2}\right) \qquad (10')$$

This equation has a better predictive capability than Eqs. (6'), (7'), (8'), (9'), and Jaky's approximate equation (1948), i.e.

$$K_{0(NC)} = 1 - \sin\Phi' \qquad (12)$$

THE GOLDEN RATIO

The Golden Ratio (also known as Golden Section, Golden Mean, Divine Proportion), first described by Euclid, is a mathematical ratio well known for its unexpected occurrence in mathematics, science, biology, art, architecture, nature and beyond, to such an extent that it is regarded as "The World's Most Astonishing Number" (Livio, 2002 and 2003). Examples of the Golden Ratio in engineering are quoted by several authors (e.g., Lin & Lu, 2010; Puri & Jordan, 2006; Hueckel & Peano, 1987).

Incidentally, the Golden Ratio and the shearing resistance angle of soil share the same symbol Φ. To avoid confusion, the Golden Ratio will be identified hereafter with its old symbol τ.

The Golden Ratio is the irrational algebraic number τ = 1.6180339887... (Livio, 2002; Dunlap, 1997), and it can be derived from the division of a segment into two parts such that the ratio (τ) of the length of the whole segment (1) to its larger part (x) is equal to the ratio of its larger part (x) to its smaller part ($1 - x$):

$$\frac{1}{x} = \frac{x}{1 - x} \qquad (13)$$

Since $\tau = 1/x$ and hence $x = 1/\tau$, τ satisfies:

$$\tau^2 - \tau - 1 = 0 \qquad (14)$$

The Golden Ratio is the positive root of the above equation, i.e.,

$$\tau = \frac{(1 + \sqrt{5})}{2} = 1.6180339887... \qquad (15)$$

To 3 decimal places, the numerical values of τ and its reciprocal or conjugate, x, are 1.618 and 0.618. Curiously, the value of the reciprocal x is also the absolute value of the negative root. The Golden Ratio exhibits a number of other mathematical properties and a special relationship with Fibonacci numbers. For instance, the square of the golden ratio 1.6180339887... is 2.6180339887..., while the value of its reciprocal, as previously shown, is 0.618339887.... The numbers after the decimal point are exactly the same in all cases! The Golden Ratio is the only number whose square is equal to itself plus 1 and its reciprocal is equal to itself minus 1 (Livio, 2002).

Note that the value (0.618) of the reciprocal of the Golden Ratio is fairly close to the value (0.64) of the mobilising factor of the shearing resistance angle Φ' in Eq. (10').

THE GOLDEN RATIO AS THE POSSIBLE RATIO OF THE PEAK SHEARING RESISTANCE ANGLE Φ' TO ITS MOBILISED PROPORTION IN THE ONE-DIMENSIONAL VIRGIN COMPRESSION OF SOILS

As discussed above, the empirical Equation (10') was derived without any assumption. Comparing Eqs. (8'), (9'), and (10'), where the mobilising factor m of the angle of shearing resistance Φ' is 0.69, 0.67 and 0.64, respectively, showed (Federico *et alii*, 2008) that Eq. (10') has the highest predictive capability, which is inversely proportional to the numerical value of m. In order to better evaluate this trend, i.e. the effect of reducing the value of m, lower values − namely 0.63, 0.61, 0.60, 0.58, 0.56, and also the value (0.618) of the reciprocal of the Golden Ratio − were imposed on Eq. (5) by employing the same set of 59 soils as those used by Federico *et alii* (2008), enriched with a further 7 soils from the literature (listed in Tab. 4 later on). Results were compared in terms of:

i) Mean Discrepancy Ratio (or, Average of Model Uncertainty), $\bar{x}$:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}\frac{\text{Measured } K_{0(NC)}}{\text{Predicted } K_{0(NC)}} \qquad (16)$$

ii) Mean Absolute Percentage Error, $\bar{p}$:

$$\bar{p} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\text{Measured } K_{0(NC)} - \text{Predicted } K_{0(NC)}}{\text{Predicted } K_{0(NC)}}\right| \qquad (17)$$

iii) Coefficient of Determination R^2, Standard Deviation S_d and Coefficient of Variation *CV* in the linear regression of the measured $K_{0(NC)}$ on the predicted $K_{0(NC)}$.

Table 3 and Fig. 3 show the influence of the numerical value of the shearing resistance mobilisation factor *m*. Iterations of Eq. (5') using three different values of *m* (i.e. 0.64, 0.63 and 0.618), share the same high Coefficient of Determination ($R^2 = 0.83$).

Moreover, the same absolute error $\bar{p}$ (6.54%) is associated with the $m = 0.64$ and $m = 0.618$, whereas, if the Mean Discrepancy Ratio $\bar{x}$, the Standard Deviation S_d and the Coefficient of Variation *CV* are also considered, $m = 0.618$ yields a slightly better predictive accuracy. Moreover, the minor decrease (0.07%) of $\bar{p}$ (i.e. from 6.54 to 6.47 %) when $m = 0.63$ (i.e. where the inversion of the trend of the curve begins) is probably offset by the slightly higher Standard Deviation and Coefficient of Variation compared to $m = 0.618$.

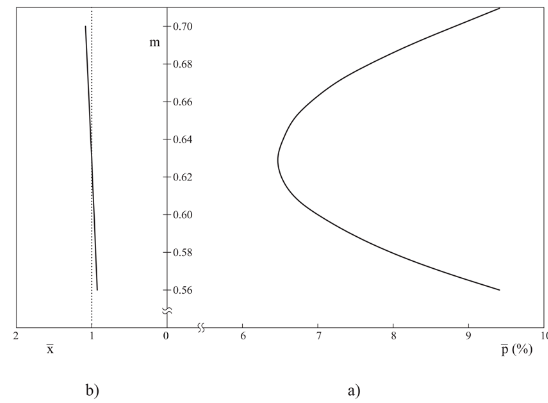

Fig. 3 - a) Mean Absolute Percentage Error, $\bar{p}$, and b) Mean Discrepancy Ratio, $\bar{x}$, versus the mobilisation factor, m, of the shearing resistance angle Φ'

Figure 4 displays the measured versus the predicted values of $K_{0(NC)}$ using Eq. (20), while Fig. 5 shows $K_{0(NC)}$ predictions as a function of Φ' (Eq. 20) compared with the measured $K_{0(NC)}$ data.

Eq. (5'): $K_{0(NC)} = \tan^2\left(45° - m\frac{\Phi'}{2}\right)$	Mean Predicted Value $K_{0(NC)}$	Mean Discrepancy Ratio (Eq. 16) $\bar{x}$	Mean Absolute Percentage Error (Eq. 17) $\bar{p}$(%)	Coefficient of Determination R^2	Standard Deviation S_d	Coefficient of Variation CV
Eq. (5') with m=0.69 i.e. Eq. (8')	0.494	1.071	8.22	0.73	0.093	0.188
Eq. (5') with m=0.67 i.e. Eq. (9')	0.504	1.047	7.24	0.79	0.087	0.173
Eq. (5') with m=0.64 i.e. Eq. (10')	0.520	1.013	6.54	0.83	0.080	0.154
Eq. (5') with m=0.63	0.526	1.001	6.47	0.83	0.078	0.149
Eq. (5') with m=0.618 i.e. Eq. (20)	0.532	0.989	6.54	0.83	0.076	0.143
Eq. (5') with m=0.61	0.537	0.980	6.68	0.82	0.075	0.139
Eq. (5') with m=0.60	0.542	0.969	7.00	0.80	0.073	0.136
Eq. (5') with m=0.58	0.553	0.948	7.97	0.75	0.071	0.129
Eq. (5') with m=0.56	0.565	0.928	9.40	0.67	0.070	0.124

Tab. 3 - Comparison of the predictive capability of Eq. (5') as a function of the mobilisation factor m of the shearing resistance angle Φ'

However, as the measured values of $K_{0(NC)}$ (or at least most of them, see Tab. 4) can be slightly lower than the true values, for reasons that are set out in what follows, on the basis of the Mean Discrepancy Ratio (Tab. 1, i.e. $\bar{x} = 1.013$ when $m = 0.64$, $\bar{x} = 1.001$ when $m = 0.63$ and $\bar{x} = 0.989$ when $m = 0.618$), $m = 0.618$ can provide a slightly higher accuracy of prediction of $K_{0(NC)}$.

Therefore, 0.618 being the reciprocal of the Golden Ratio τ,

$$0.618\Phi' = \frac{1}{\tau}\Phi' = \Phi'_{mob} \qquad (18)$$

or

$$\frac{\Phi'}{\Phi'_{mob}} = \frac{\Phi'_{mob}}{\Phi' - \Phi'_{mob}} = 1.618 = \tau \qquad (19)$$

Thus,

$$K_{0(NC)} = \frac{\left(1 - \sin\frac{\Phi'}{\tau}\right)}{\left(1 + \sin\frac{\Phi'}{\tau}\right)} = \tan^2\left(45° - \frac{1}{\tau}\frac{\Phi'}{2}\right) = \tan^2\left(45° - 0.618\frac{\Phi'}{2}\right) \qquad (20)$$

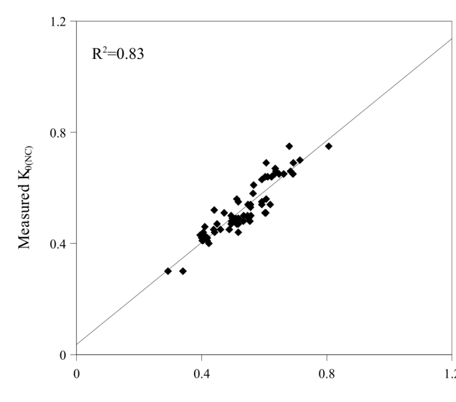

Fig. 4 - Correlation between the measured values of $K_{0(NC)}$ and the values obtained from Eq. (20)

Table 4 compares the measured and predicted $K_{0(NC)}$ values for a given value of the angle Φ' for Eqs. (20), (10'), (6'), (7')

N°	Soil	w_l (%)	PI (%)	Φ' (°)	$K_{0(NC)}$ Experimental data	Proposed Equation Eq. (20)	Federico et alii (2008) Eq. (10')	Abdelhamid & Krizek (1976) Eq. (6')	Bolton (1991) Eq. (7')	Simpson (1992) Eq. (8')	Hayat (1992) Eq. (9')	Jaky (1948) Eq. (12)	ν Eq. (21)	References
1	Remolded Boston Blue Clay	33	15	27.5	0.54	0.548	0.536	0.468	0.568	0.509	0.520	0.538	0.354	Ladd (1965)
2	Remolded Weald Clay	46	24	26	0.61	0.566	0.555	0.499	0.600	0.529	0.539	0.562	0.362	Skempton & Sowa (1963)
3	Remolded Vicksburg Buckshot Clay	63	39	24	0.54	0.592	0.581	0.543	0.644	0.556	0.566	0.593	0.372	Ladd (1965)
4	Undisturbed Kawasaki Clay I and II	80	38	37	0.52	0.440	0.427	0.305	0.398	0.398	0.409	0.398	0.305	Ladd (1965)
5	Undisturbed Brobekkvein Oslo Clay	39	18	30.5	0.47	0.512	0.499	0.410	0.509	0.472	0.482	0.492	0.338	Simons (1960)
6	Undisturbed Skabo Clay	52	29	30	0.47	0.517	0.505	0.419	0.518	0.478	0.488	0.500	0.341	Landva (1962)
7	Hokkaido silt 1 (slurry)	52	21	37.2	0.45	0.438	0.425	0.302	0.395	0.396	0.407	0.395	0.304	Mitachi & Kitago (1976)
8	Hokkaido silt 2 (slurry)	51	21	35.1	0.45	0.460	0.447	0.333	0.428	0.418	0.430	0.425	0.315	Mitachi & Kitago (1976)
9	Hokkaido Clay (slurry)	72	32	36.1	0.47	0.449	0.436	0.318	0.412	0.407	0.419	0.411	0.310	Mitachi & Kitago (1976)
10	Spestone Kaolinite (slurry)	72	32	22.6	0.64	0.611	0.600	0.575	0.677	0.576	0.586	0.616	0.379	Parry & Nadarajah (1974)
11	Kawasaki clay-mixture M-10 (slurry)	28	11	39.2	0.42	0.418	0.404	0.274	0.365	0.375	0.386	0.368	0.295	Nakase & Kamei (1988)
12	Kawasaki clay-mixture M-15 (slurry)	35	15	38.7	0.40	0.423	0.409	0.281	0.373	0.380	0.392	0.375	0.297	Nakase & Kamei (1988)
13	Kawasaki clay-mixture M-20 (slurry)	43	19	40.6	0.41	0.404	0.391	0.256	0.346	0.361	0.373	0.349	0.288	Nakase & Kamei (1988)
14	Kawasaki clay-mixture M-30 (slurry)	55	29	40.8	0.41	0.402	0.389	0.253	0.343	0.359	0.371	0.347	0.287	Nakase & Kamei (1988)
15	Kawasaki clay M-50 (slurry)	84	51	41.6	0.43	0.395	0.381	0.243	0.332	0.351	0.363	0.336	0.283	Nakase & Kamei (1988)
16	Ohita Clay (slurry)	42	13	41.2	0.43	0.398	0.385	0.248	0.337	0.355	0.367	0.341	0.285	Nakase & Kamei (1988)
17	Nagoya Clay (slurry)	45	24	39.3	0.41	0.417	0.403	0.273	0.364	0.374	0.385	0.368	0.294	Nakase & Kamei (1988)
18	Sakaiminato Clay (slurry)	54	28	40.5	0.44	0.405	0.392	0.257	0.347	0.362	0.374	0.351	0.288	Nakase & Kamei (1988)
19	Niigata Clay (slurry)	71	38	40	0.46	0.410	0.397	0.264	0.354	0.367	0.378	0.357	0.291	Nakase & Kamei (1988)
20	Toyama Clay (slurry)	78	43	41.1	0.42	0.400	0.386	0.249	0.338	0.356	0.368	0.343	0.285	Nakase & Kamei (1988)
21	Aomori Clay (slurry)	82	48	40.2	0.43	0.408	0.395	0.261	0.351	0.365	0.376	0.354	0.290	Nakase & Kamei (1988)
22	Kobe Clay (slurry)	86	56	31.9	0.48	0.495	0.483	0.385	0.483	0.455	0.466	0.472	0.331	Nakase & Kamei (1988)
23	Whitefish Falls	-	-	27	0.48	0.554	0.542	0.478	0.578	0.516	0.526	0.546	0.356	Delory & SalvaS (1969)
24	Wallaceburg	-	-	23	0.51	0.605	0.595	0.566	0.668	0.571	0.580	0.609	0.377	Delory & SalvaS (1969)
25	Marine Clay	-	-	34	0.51	0.472	0.459	0.350	0.446	0.431	0.442	0.441	0.320	Koutsoftas & Ladd (1985)
26	Vicksburg Buckshot Clay (slurry)	57	36	26.7	0.50	0.557	0.546	0.484	0.585	0.520	0.530	0.551	0.358	Donaghe & Townsend (1978)
27	Louisiana EABPL Clay	79	53	21.7	0.64	0.623	0.613	0.597	0.699	0.589	0.599	0.630	0.384	Donaghe & Townsend (1978)
28	Sidney Kaolin	50	16	30.7	0.48	0.509	0.497	0.407	0.505	0.469	0.480	0.489	0.337	Poulos (1978)
29	Hydrite 10 Kaolinite (flocculated sample)	62	28	17.8	0.75	0.680	0.670	0.701	0.802	0.649	0.657	0.694	0.405	Abdelhamid & Krizek (1976)
30	Hydrite 10 Kaolinite (dispersed sample)	62	28	16.9	0.69	0.693	0.684	0.727	0.828	0.664	0.672	0.709	0.409	Abdelhamid & Krizek (1976)
31	Hydrite PX Kaolinite	-	-	16.9	0.65	0.692	0.684	0.727	0.828	0.664	0.672	0.709	0.409	Edil & Dhowian (1981)
32	Australian Kaolin 1	75	40	23	0.56	0.606	0.595	0.566	0.668	0.571	0.580	0.609	0.377	Moore & Cole (1977)
33	Australian Kaolin 2	58	32	30	0.44	0.517	0.505	0.419	0.518	0.478	0.488	0.500	0.341	Moore & Cole (1977)
34	Kaolin	-	-	23.2	0.64	0.603	0.592	0.561	0.663	0.568	0.577	0.606	0.376	Parry & Wroth (1976)
35	Spestone Kaolin	76	37	20.7	0.66	0.637	0.627	0.622	0.724	0.604	0.613	0.647	0.389	Sketchley & Bransby (1973)
36	Kaolin	-	-	23	0.69	0.606	0.595	0.566	0.668	0.571	0.580	0.609	0.377	Burland (1967)
37	Kaolin	55	23	23.3	0.51	0.602	0.591	0.559	0.660	0.566	0.576	0.604	0.376	Singh (1971)
38	London Clay	95	65	20	0.65	0.647	0.637	0.641	0.742	0.615	0.624	0.658	0.393	Skempton & Sowa (1963)
39	London Clay	65	38	17.5	0.66	0.684	0.675	0.710	0.811	0.654	0.662	0.699	0.406	Brooker & Ireland (1965)
40	Weald Clay	41	21	22	0.54	0.619	0.609	0.590	0.692	0.585	0.594	0.625	0.382	Brooker & Ireland (1965)
41	Weald Clay	-	-	26.2	0.58	0.564	0.552	0.494	0.595	0.526	0.537	0.558	0.361	Skempton & Sowa (1963)
42	Bearpawe Shale	101	78	15.5	0.70	0.714	0.706	0.770	0.870	0.687	0.695	0.733	0.417	Brooker & Ireland (1965)
43	Bearpawe Shale	82	64	21	0.65	0.633	0.623	0.615	0.717	0.600	0.609	0.642	0.388	Singh *et alii* (1973)
44	Drammen Clay 1	60	31	31.7	0.49	0.498	0.485	0.389	0.487	0.457	0.468	0.475	0.332	Berre & Bjerrum (1973)
45	Drammen Clay 2	33	10	30	0.49	0.517	0.505	0.419	0.518	0.478	0.488	0.500	0.341	Berre & Bjerrum (1973)
46	Drammen Clay	55	27	30.7	0.49	0.509	0.497	0.407	0.505	0.469	0.480	0.489	0.337	Brown *et alii* (1977)
47	New York Varved Clay	65/35	39/12	20.9	0.67	0.635	0.624	0.617	0.719	0.601	0.610	0.643	0.388	Leathers & Ladd (1978)
48	Hackensack Valley Varved Clay	65/40	35/25	19	0.65	0.662	0.652	0.668	0.769	0.630	0.639	0.674	0.398	Saxena *et alii* (1978)
49	South African Clay	-	-	28.7	0.48	0.533	0.521	0.444	0.544	0.494	0.505	0.520	0.347	Knight & Blight (1965)
50	Portsmouth Clay	35	15	32	0.47	0.494	0.482	0.384	0.481	0.454	0.465	0.470	0.331	Simon *et alii* (1974)
51	Beaumont Clay	67	41	24	0.55	0.592	0.581	0.543	0.644	0.556	0.566	0.593	0.372	Mahar & Ingram (1979)
52	Boston Blue Clay	41	21	26.8	0.54	0.556	0.545	0.482	0.582	0.518	0.529	0.549	0.357	Kinner & Ladd (1973)
53	Goose Lake Flour	32	16	27.5	0.50	0.547	0.536	0.468	0.568	0.509	0.520	0.538	0.354	Brooker & Ireland (1965)
54	Albuquerque Clay-Sand	25	11	30.5	0.56	0.512	0.499	0.410	0.509	0.472	0.482	0.492	0.338	Calhoun & Triandafilidis (1969)
55	Backebol Clay	90	60	30	0.49	0.517	0.505	0.419	0.518	0.478	0.488	0.500	0.341	Massarsch & Broms (1976)
56	Bombay Clay	115	70	24	0.63	0.592	0.581	0.543	0.644	0.556	0.566	0.593	0.372	Kulkarni (1973)
57	Khor-Al-Zubair Clay	55	35	27.3	0.49	0.550	0.538	0.472	0.572	0.512	0.522	0.541	0.355	Hanzawa (1977a)
58	Fao Clay	39	20	36.9	0.44	0.441	0.428	0.306	0.400	0.399	0.410	0.400	0.306	Hanzawa (1977b)
59	Norwegian Clay	26	8	10	0.75	0.806	0.799	0.961	1.054	0.785	0.791	0.826	0.446	Bjerrum (1961)
60	Moose River Muskeg	-	-	47.7	0.30	0.340	0.326	0.176	0.257	0.296	0.308	0.260	0.254	Adams (1965)
61	Portage Peat	-	-	53.8	0.30	0.292	0.278	0.122	0.195	0.247	0.259	0.193	0.226	Edil & Dhowian (1981)
62	Kyoto Clay	88	57	32.5	0.45	0.488	0.476	0.375	0.472	0.448	0.459	0.463	0.328	Akai & Adachi (1965)
63	Lagunillas Clay	61	37	26.8	0.53	0.556	0.545	0.482	0.582	0.518	0.529	0.549	0.350	Lambe (1964)
64	New England Marine Clay	-	20	32	0.50	0.494	0.482	0.384	0.481	0.454	0.465	0.470	0.331	Ladd (1976)
65	Haney Clay	-	-	30	0.55	0.517	0.505	0.419	0.518	0.478	0.488	0.500	0.341	Campanella & Vaid (1972)
66	Newfield Clay	31	13	28.6	0.50	0.534	0.522	0.446	0.546	0.495	0.506	0.521	0.348	Singh (1971)
Average value					0.52	0.532	0.520	0.453	0.550	0.494	0.504	0.514		

Tab. 4 - Measured and predicted $K_{0(NC)}$ values and predicted ν values

and (9'), as well as Jaky's well-known approximate Eq. (12), and the predicted values for Poisson's ratio, although the latter are dealt with later on.

As pointed about, the measured values of $K_{0(NC)}$ (Tab. 4) can be slightly lower − in principle − than the true values, considering the following factors: even very small amounts of lateral movement can produce some decrease in the apparent value of $K_{0(NC)}$ (Bishop, 1958); the effect of side wall friction (Wroth, 1972); and the difficulty in ensuring that strain gauges are sensitive enough to detect the strain in the metal ring under the relatively small radial

$K_{0(NC)}$ Equation		Mean Predicted Value $K_{0(NC)}$	Mean Discrepancy Ratio (Eq. 16) $\overline{x}$	Mean Absolute Percentage Error (Eq. 17) $\overline{p}(\%)$	Coefficient of Determination R^2	Standard Deviation S_d	Coefficient of Variation CV
Proposed Equation	Eq. (20)	0.532	0.989	6.54	0.83	0.076	0.143
Federico *et alii* (2008)	Eq. (10')	0.520	1.013	6.54	0.83	0.080	0.154
Abdelhamid & Krizek (1976)	Eq. (6')	0.453	1.247	18.82	0.61	0.297	0.656
Bolton (1991)	Eq. (7')	0.550	0.994	13.39	0.69	0.164	0.298
Simpson (1992)	Eq. (8')	0.494	1.071	8.22	0.75	0.093	0.188
Hayat (1992)	Eq. (9')	0.504	1.047	7.24	0.79	0.087	0.173
Jaky (1948)	Eq. (12)	0.514	1.043	9.15	0.78	0.132	0.257

Tab. 5 - Accuracy of the $K_{0(NC)}$ prediction using various $K_{0(NC)}$ equations

stresses imposed by the soil (Briaud, 2013).

The effect of what was pointed out by the above three authors, especially for some rather "old" experimental $K_{0(NC)}$ data, could be confirmed by the value (0.989) of the Mean Discrepancy Ratio (Tabs. 3 and 5) or by the small difference between the average $K_{0(NC)}$ value (0.532) predicted by the proposed Eq. (20) and its average experimental value (0.524) (Tab. 4).

This consideration and the statistical results summarised in Tab. 5 infer that Eq. (20) has a slightly better predictive capability than Eq. (10') and the other equations, including Jaky's approximate Eq. (12).

CORRELATION FOR POISSON'S RATIO

Finally, with regard to the Poisson's ratio ν, the same connection between Φ' and Φ'_{mob}, combined with the link between K_0 and ν for elastic materials, makes it possible to obtain, following Federico & Elia (2009), an interesting correlation between the Poisson's ratio ν of a normally consolidated soil and its shearing resistance angle Φ'.

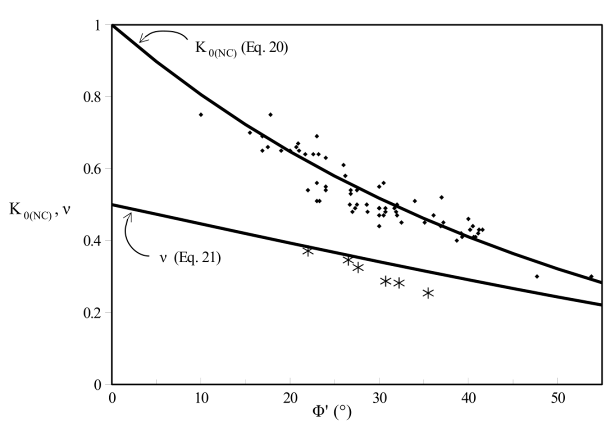

Fig. 5 - Measured $K_{0(NC)}$ (Tab. 4) and measured ν values versus Φ' compared with their prediction as a function of Φ' using Eq. (20) and Eq. (21), respectively

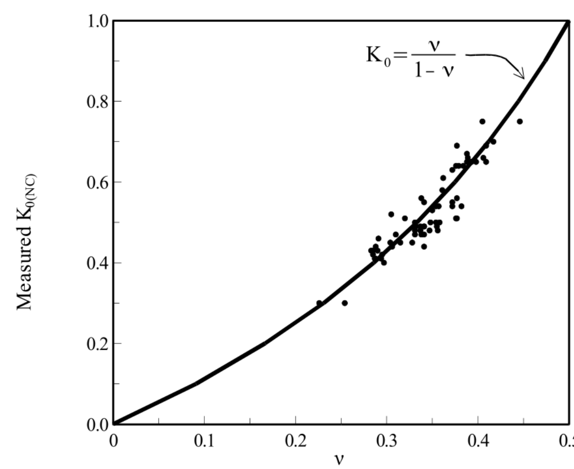

Fig. 6 - Measured $K_{0(NC)}$ values (Tab. 4) versus predicted ν values. The curve is the previously discussed graph of the function $K_0 = K_0(\nu)$, Eq. (3)

To this end, the substitution of K_0 with Eq. (20) in Eq. (4), gives:

$$\nu = \frac{1-\sin\frac{\Phi'}{\tau}}{2} = \frac{1-\sin 0.618\Phi'}{2} \tag{21}$$

Note that, strictly speaking, the validity of this equation is limited to the assumption of linear elasticity.

No genuine laboratory dataset in terms of pairs (Φ',ν) is found in the literature. However (see Federico & Elia, 2009), some experimental data for ν was obtained by Wroth (1975) for several lightly overconsolidated soils and plotted against the Plasticity Index *PI*. The corresponding Φ' angles were obtained by relying on the empirical correlation (Muir Wood, 1990) between *PI* and Φ', i.e.

$$\sin\Phi' = 0.35 - 0.1\ln PI \tag{22}$$

although the considerable scatter around the average line in this correlation makes the resulting Φ' values a very rough estimate only.

The trend of the few measured ν values versus the derived approximate *Φ'* values is shown in Fig. 5, where the predictions based on Eq. (21) are also shown for comparison, while Fig. 6 shows the measured $K_{0(NC)}$ data (Tab. 4) versus the corresponding ν data predicted via Eq. (21).

It should be pointed out that the relevant dots in the latter figure are scattered around the upper portion of the theoretical curve $K_0 - \nu$ (Eq. 3), but that Eq. (21) – which, as already reported, assumes elastic soil behaviour (Eq. (4)). This is, however, a borderline case for normally consolidated clays and granular soils and, therefore, it requires adequate experimental validation.

CONCLUSIONS

The Golden Ratio appears to emerge as the link (or, more precisely, as the ratio) between the effective peak shearing resistance angle *Φ'* and its mobilised proportion Φ'_{mob} in the one-dimensional virgin compression of soils. This link was used for the prediction of the at-rest earth pressure coefficient $K_{0(NC)}$ and the Poisson's ratio *ν* of normally consolidated soils.

The predictive capability of the corresponding equation for $K_{0(NC)}$ – derived simply from an experimental correlation between Φ'_{mob} and *Φ'* and without any theoretical assumption – was tested via a comparison between predicted and experimental values made on a significant body of disparate literature data. This data, chiefly related to reconstituted samples, was obtained by using a variety of experimental techniques with varying degrees of accuracy, particularly as regards the control of the crucial condition $\varepsilon_r = 0$ during the virgin consolidation phase.

With the use of better equipment and a higher quality of testing, the difference between measured and predicted $K_{0(NC)}$ values is likely to diminish further, thereby confirming the role of the Golden Ratio as a natural link between the peak shearing resistance angle *Φ'* and its mobilised proportion in the process of virgin consolidation of soils, as well as the link between the force of gravity and its horizontal effect in normally consolidated soils.

The same link between *Φ'* and Φ'_{mob} (i.e. the Golden Ratio), combined with the link between the coefficient K_0 and the Poisson's ratio *ν* for elastic materials, elicited a further interesting correlation between *Φ'* and *ν*. This is, however, a borderline case for normally consolidated soils and, therefore, it requires experimental validation.

The fact that the Golden Ratio proved to be a good link between $K_{0(NC)}$ and *Φ'* may appear as incidental; however, there may be an underlying physical reason. In this respect, further investigations are required. It is worth noting that even Jaky's popular formula (Eq. 12) – which is perfectly acceptable for engineering purposes – lacks physical consistency, since it was derived under questionable assumptions (Tschebotarioff, 1951 *apud* Feda, 1978; Michalowski, 2005), and has no obvious connection with soil behaviour. Despite this deficiency, it is largely accepted for practical applications, to such an extent that it is recommended in EUROCODE 7 and, *en passant*, it is probably the most famous and most used formula in Geotechnical Engineering.

APPENDIX I

As is known, for a linearly elastic material, the value of Poisson's ratio is constant, and, in principle, it may be obtained through a triaxial test, if the radial strains are measured. Thus:

$$\nu = \frac{-\varepsilon_3\sigma_1 + \varepsilon_1\sigma_3}{\varepsilon_1\sigma_1 + \varepsilon_1\sigma_3 - 2\varepsilon_3\sigma_3} \quad (1\text{-I})$$

Only if σ_3 is zero (unconfined compression test), Poisson's ratio is given by:

$$\nu = -\frac{\varepsilon_3}{\varepsilon_1} \quad (2\text{-I})$$

The negative sign indicates that, when ε_1 is in compression, ε_3 is in tension and Poisson's ratio is positive.

Although this elastic parameter may be readily obtained from tables for most materials, it is somewhat problematic for soils, in spite of the availability of many such tables (e.g., McCarty, 1947; Barcan, 1962; Harr, 1966; Kezdy, 1974; Carter, 1983; Whitlow, 1983; Hunt, 1986; Cernica, 1995; Bowles, 2002; Budhu, 2010; Das, 2010; Rajapaksu, 2011; Briaud, 2013), as shown, for example, by comparing the three tables in Fig. 1-I .

Soil Type		ν
Clay		
	Soft	0.35-0.40
	Medium	0.30-0.35
	Stiff	0.20-0.30
Sand		
	Loose	0.15-0.25
	Medium	0.25-0.30
	Dense	0.25-0.35

a)

Soil Type		ν
Clay		
	Soft	0.15-0.25
	Medium	0.20-0.50
Sand		
	Loose	0.20-0.40
	Medium	0.25-0.40
	Dense	0.30-0.45
	Silty	0.20-0.40

b)

Soil Type		ν
Clay		
	Soft	0.40
	Medium	0.30
	Hard	0.25
	Sandy	0.25
Sand		
	Loose	0.20
	Dense	0.30
Sand and Gravel		
	Loose	0.20
	Dense	0.30

c)

Fig. 1-I - Typical Poisson's ratio ν values: a) according to Budhu (2010), b) according to Das (2010) and c) according to Cernica (1995). These ν values are effective values

In addition to being very limited in terms of quantity, the experimental results in this regard vary widely and are rather inconclusive (Cernica, 1995), while the bulk of the *ν* values found in the literature are "assumed values".

Moreover, unlike K_0, very little information is available in the literature regarding the correlation with the Poisson's ratio *ν*, and the data that is provided (e.g., Kulhawy *et alii*, 1969; Trautmann & Kulhawy, 1987; Duncan *et alii*, 1991; Dysli, 2001) appears to be of little or no use.

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