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DETERMINING THE ROCK BRITTLE INDEX (BI) USING MULTIVARIATE REGRESSION (A CASE STUDY)

Mojtaba RAHIMI SHAHID(*) & Fariba KARGARANBAFGHI(**)

(*)Ferdowsi University of Mashhad - Department of Geology - Faculty of Sciences - Mashhad, Iran
(**) Yazd University - Department of Geology - Faculty of Sciences - Yazd, Iran
Corresponding author: fkargaran@yazd.ac.ir

EXTENDED ABSTRACT

Il calcolo dell'indice di fragilità delle rocce (*BI*) è di fondamentale importanza nei progetti di geo-ingegneria, inclusi quelli relativi alle strutture sotterranee e allo smaltimento delle scorie nucleari. La fragilità della roccia ha un effetto significativo sul suo processo di fratturazione. Ad esempio, il grado di fratturazione della roccia, che influisce sulla produzione di petrolio, è controllato dalla pressione di iniezione dei fluidi che, tramite il processo di fratturazione idraulica, regola il livello di estrazione in funzione della fragilità della roccia. Allo stesso modo, il fenomeno di collasso della roccia, che si verifica principalmente in miniere profonde e tunnel, è un chiaro esempio di un processo di fratturazione fragile in cui vengono rilasciate grandi quantità di energia (Meng *et alii*, 2015). La conoscenza della relazione tra perforabilità e fragilità della roccia è una delle componenti più importanti dal punto di vista degli ingegneri durante le operazioni di perforazione. Inoltre, c'è una crescente richiesta di stime dei parametri rocciosi, che sono i dati più importanti in fase di progetto e pianificazione dello scavo sotterraneo (Altindag & Guney, 2010; Yarali, Soyer, 2011; Yarali & Kahraman, 2011; Özfirat *et alii*, 2016 ; Karrari *et alii*, 2022). Sebbene la *BI* sia una delle principali proprietà meccaniche della roccia, non esiste un parere condiviso da parte della comunità dell'ingegneria geotecnica su come descriverla o misurarla (Altindag & Guney, 2010; Kaunda & Asbury, 2016).

L'uso della regressione multivariata per la stima della *BI* è stato relativamente poco considerato dai ricercatori. Tuttavia, data la semplicità dei calcoli e l'inclusione di vari parametri nel processo di analisi, questo metodo può essere adatto ed efficiente nella determinazione del BI. Pertanto, nel presente studio è stato determinato l'indice di fragilità (*BI*) dell'argilla marnosa utilizzando le proprietà fisiche e meccaniche delle rocce e il metodo di regressione multivariata. I campioni sono stati prelevati dalla formazione argilloso-marnosa dell'Amiran nell'ovest dell'Iran (diga di Havasan).

I dati geotecnici utilizzati in questo studio includono: resistenza a compressione uniassiale (*UCS*), modulo di elasticità (*E*), velocità delle onde *p* (V_p), velocità delle onde di taglio (V_s), porosità (n) e densità (ρ). Tali valori sono stati ottenuti eseguendo vari test su campioni prelevati dall'area della diga di Havasan nella Formazione argilloso-marnosa dell'Amiran. I valori di *BI* sono stati dapprima calcolati sulla base dell'*UCS*. Nella fase successiva, la *BI* è stata calcolata utilizzando la regressione univariata e multivariata sulla base di diversi parametri. Analisi statistiche quali: stima dell'R quadro, analisi della varianza (ANOVA), stima dei coefficienti, analisi della distribuzione *Beta* e *VIF* sono stati utilizzati nella prima fase per valutare le diverse regressioni calcolate, dopodiché è stata eseguita l'analisi dei residui di ciascuna regressione. Infine, è stata studiata la correlazione tra i valori di BI calcolati e quelli previsti utilizzando ciascuna regressione. A seguito delle regressioni, i risultati sono stati confrontati con quelli di studi simili.

Questo studio presenta alcune relazioni per la previsione della BI della roccia utilizzando metodi di regressione univariata e multivariata. In queste relazioni, le proprietà meccaniche (*UCS*, *E*, V_p e V_s) e fisiche (ρ e n) dell'argilla marnosa sono state utilizzate come variabili indipendenti. La sintesi dei risultati di questo studio è la seguente:

1. L'indice di fragilità della formazione argilloso-marnosa dell'Amiran ha un valore massimo di 6,31 MPa a causa della presenza di minerali argillosi. Tale valore suggerisce una bassa fragilità secondo la classificazione Hoek (1983).
2. Esiste una relazione diretta tra *BI*, *UCS* e n nella regressione univariata. La relazione tra *BI* con V_p e V_s è di tipo inverso, principalmente a causa dell'anisotropia nelle porzioni più argillose. Non risulta alcuna relazione significativa tra *BI* e ρ in questo studio.
3. Le proprietà meccaniche mostrano una maggiore rispondenza alla *BI* rispetto alle proprietà fisiche della formazione dell'Amiran.
4. Nell'analisi di regressione bivariata, l'uso di *UCS* ed *E* o di *UCS* e V_s si traduce in una previsione del *BI* più affidabile.
5. Nell'analisi di regressione a tre variabili, l'uso di *UCS*, *E* e V_s fornisce i risultati di previsione del *BI* più affidabili.
6. Nelle regressioni multivariate, all'aumentare del numero di variabili, si ottiene una previsione più accurata di *BI*.

ABSTRACT

One of the geotechnical properties of rocks, which is particularly important in sensitive projects such as oil and gas extraction, nuclear waste disposal, and underground drilling, is their brittleness. Currently, there are no standards methods for direct measurement of rock brittleness. Different studies have used various indirect methods to predict rock brittleness index (BI).

However, researchers have paid less attention to the prediction of BI using multivariate regression. Accordingly, this research has used the multivariate regression method to determine BI considering mechanical characteristics. Specifically, we used uniaxial compressive strength (UCS), modulus of elasticity (*E*), pressure wave velocity (*Vp*), and shear wave velocity (*Vs*), and physical characteristics, including porosity (*n*) and density (ρ) to determine the BI.

Statistical indicators, R square, results of ANOVA (Analysis of Variance) test, coefficients, beta statistics, and VIF were used in the first step to evaluate the regression relationships. Then, residual analysis of each regression were performed. Finally, the correlation between the calculated and predicted BI values was investigated using each regression. The best results were obtained using UCS and *E* or UCS and *Vs* in the bivariate regression and UCS, *E*, and *Vs* in the three-variable regression. According to the results, increasing the number of variables in multivariate regressions leads to more accurate predictions of BI.

Keywords: *correlation, dam, Iran, physical and mechanical properties, regression, analysis of residual, shale*

INTRODUCTION

Calculation of rock brittleness index (BI) is of fundamental importance in rock engineering projects, including underground structures and nuclear waste disposal. The rock brittleness has a significant effect on the fracture process. Hydraulic fracturing forms complex hydraulic fracture networks in shale reservoirs and significantly improves the permeability of shale reservoirs. Rock brittleness is a major factor in determining whether a shale reservoir can be fractured or not. Similarly, the phenomenon of rock burst, which occurs mainly in deep mining and tunneling, is a clear example of a brittle fracture process in which large amounts of energy are released (Meng *et alii*, 2015). Knowledge of the relationship between drillability and rock brittleness is one of the most important components for engineers in drilling operations. Besides, there is an increasing demand for the estimation of rock parameters, which are the most important data in project estimations and planning for underground excavation (Özfirat *et alii*, 2016). Although BI is one of the main mechanical properties of rock, there is no comprehensive consensus to describe or measure it in the geotechnical engineering community (Altindag & Guney, 2010; Kaunda & Asbury, 2016). In rock mechanics BI is not defined unequivocally. There are different definitions for rock brittleness. Brittleness has been described as the lack of ductility (Hetenyi, 1950) or the destruction of internal cohesion (Ramsay, 1967). The ability for a rock to deform and fail with a low degree of inelastic behavior has also been used to define brittleness (Andreev, 1995), along with the process by which sudden loss of strength occurs with little or no plastic deformation (Jaeger *et alii,* 2007), and the rock's capability to self-sustain fracturing (Tarasov & Potvin, 2013).Therefore, brittleness is a comprehensive response of a rock's combined properties (physical and mechanical). The brittleness index (BI) is utilized to indicate if the formation rocks are brittle, which are preferable to form a complex network of fractures (Grieser & Bray, 2007), or ductile, which would be more resistant to fracture growth and failure. However, the existence of various methods of calculating the brittleness index such as the mineral-based brittleness index, the log-based brittleness index (LBI), and the elastic-based brittleness index lead to inconclusive estimations of the brittleness index.

Since the 1960s many researchers have indirectly calculated BI using different methods. The role of modulus of elasticity (*E*) and Poisson ratio (υ) in determining the brittleness index of rock has been investigated, indicating that the increase in *E* and υ leads to an increase and a decrease in the rock brittleness, respectively (Rickman *et alii,* 2008). Li & Li (2018) used a quantitative seismic prediction method to determine BI on shale, using modulus of elasticity, Poisson ratio, and mineralogical compositions. Samaei *et alii* (2018) also used uniaxial compressive strength (UCS), Brazilian tensile strength (BTS), specific gravity, and rock material to calculate brittleness. In a review study, Xia *et alii* (2019) evaluated the brittleness index of rocks. They examined the effect of various parameters in determining the rock BI and the initial problems of determining the brittleness index of rocks. A bivariate linear regression was established between BI with UCS and BTS in the study of Ghadernejad *et alii* (2019). Li *et alii* (2020) calculated the shale BI using the Energy Evolution Theory. Sun *et alii* (2020) used the neural network method to determine the rock BI. Ye *et alii* (2020) examined the relationship between modulus of elasticity (*E*), Poisson ratio (υ), and mineralogical compositions with shale BI. Lashkaripour *et alii* (2018) and Karami *et alii.* (2021) presented a direct linear relationship between Vp and BI in the dry and saturated states. They also established a significant and valid relationship between UCS, *Vp*, and BTS with BI using multivariate linear regression. The effect of anisotropy on the strength and BI of a laminated sandstone was also investigated by Jamshidi *et alii* (2021). The multivariate regression uses for BI estimation has been less considered by researchers. However, this method can be suitable and efficient in determining the BI due to simplicity in calculations and consideration of various parameters. Hence, the present study has determined the brittleness index (BI) of shale using the physical and mechanical properties of rocks and multivariate regression method. The samples were selected from Amiran shale-marl formation in west of Iran (Havasan Dam site).

HAVASAN DAM

Location

Havasan Storage dam is a clay core rockfill dam with a height of 54 meters from the foundation, a crown length of 1200 meters, and a reservoir volume of 70 million cubic meters. This dam is using to supply water for farming and electricity generation on the Havasan River 39 km to the northwest of Sarpol-e Zahab city, Kermanshah province in the west of Iran (fig 1).

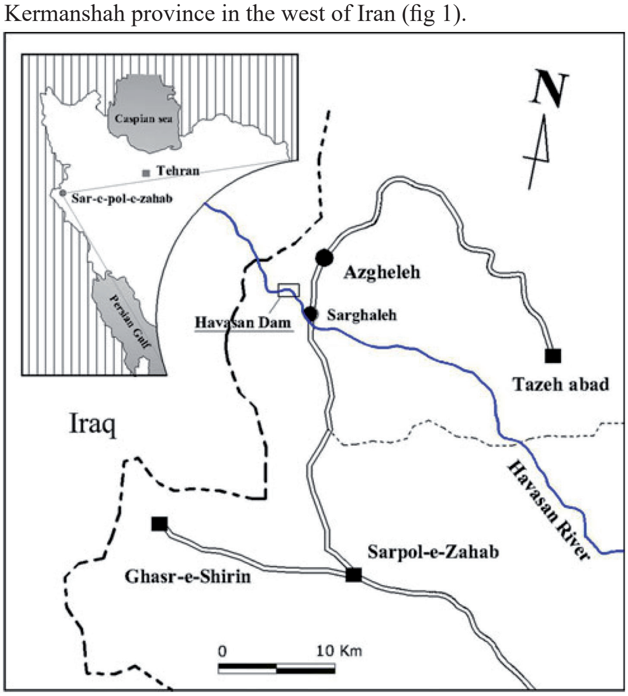


Fig. 1 - Location of the Havasan Dam in Iran

Geology

The stratigraphic units of the Havasan Dam site include calcareous and marl-shale rock units of the Upper Cretaceous and Paleocene, along with young deposits.The Talezang Formation, consisting of light gray calcareous sand to marl stones which are sometimes crystalline, form the left abutment and foundation of the Havasan Dam. The Talezang Formation is a unit of carbonate of Palaeocene to Middle Eocene age in the type section and includes 870 m of gray limestone, which is typically located on the Amiran Detrital Formation and below the Kashkan Formation (Aghanabati, 2004). The right abutment of the dam is formed by the shale-marl Amiran formation, which is often composed of alternating dark-gray shale and marl layers. The most important structural complication close to the site is a branch of the main Zagros fault, which is calling the mountain front fault (MFF). This branch is divided into several sub-branches, one of which passes very close to the left abutment of the dam. The water diversion system is located in the initial and final parts of the Amiran Formation and the middle part of the Telezang Formation due to the arched shape of the entire system path. Figure 2 shows the three-dimensional model of the dam and the location of the Talezang and Amiran Formations of the Havasan Dam site. Figure 3 also shows a geological section of through the dam axis and the position at the boreholes.

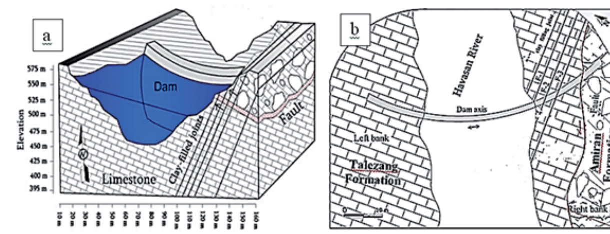


Fig. 2 - a) 3D model of Havasan Dam, b) location of Talezang and Amiran Formations in the Havasan Dam site (Maleki, 2011)

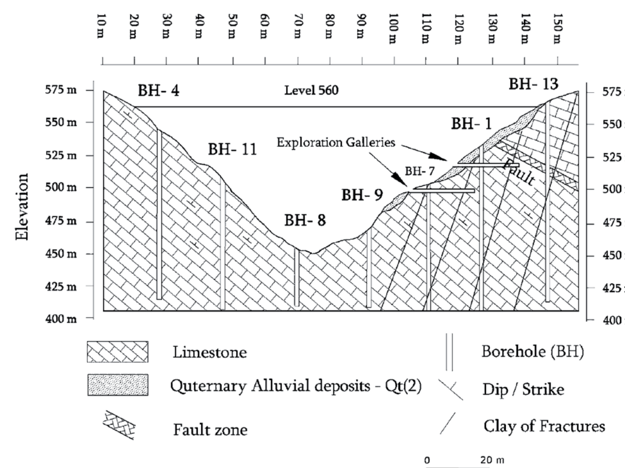


Fig. 3 - Geological cross-section and location of boreholes in the dam axis (Maleki, 2011)

MATERIALS AND METHODS

This research used literature, field, and laboratory studies to collect basic information. Geological maps, available reports, and related sources were also used for preliminary study. The geotechnical data used in this study included uniaxial compressive strength (UCS), modulus of elasticity (E), longitudinal wave velocity (V_p), shear wave velocity (V_s), porosity (n), and density (ρ) obtained by performing various tests on samples extracted from the area of Havasan Dam in the shale-marl Amiran Formation. BI values were calculated using UCS. In the next step, BI was calculated using univariate and multivariate regression and different parameters. The statistical indicators of R square, Result of ANOVA (Analysis of Variance) test, Coefficients, Beta statistic, and VIF were used in the first step to evaluate the different regressions calculated, after which the residuals analysis of each regression was performed.

Finally, the correlation between the calculated and predicted *BI* values using each regression was investigated. In the last section, the results were compared with those of similar studies.

Uniaxial compressive strength (UCS) and Young's modulus (*E*) parameters were estimated according to (ASTMD7012, 2014). Cores with a diameter of 54 mm and a length of 110 to 130 mm were used to perform the uniaxial compressive strength *(UCS)* test. The tests were performed under saturation conditions after the two ends of each sample were cut completely smooth and parallel using a saw. The pressure wave velocity (V_p) and shear wave velocity (V_s) were also determined according to the ASTM D2845 standard and in saturated conditions. Samples of rock with a diameter of 54 mm and a length of 25 to 30 mm was prepared to evaluate the wave velocity. The physical properties of the samples were measured according to the ISRM (1981) standard. These properties included density (ρ) and porosity (% *n*). Figure 4 shows the frequency histogram, normal curve, mean, and standard deviation of each of these parameters.

RESULTS

Calculating BI

Numerous studies on different rock materials show that high strength of the rock leads to high brittleness and increases BI (Altindag, 2010-a and b; Mews *et alii*, 2019; Yang *et alii*, 2020; Ye *et alii*, 2020). The brittleness test provides a reliable measure of the strength of the rock due to frequent impacts. Yilmaz *et alii* (2009) defined material brittleness as the ability to fracture without appreciable permanent deformation in the tension or compression test. Once the maximum strength is attained, extremely small strains lead to a dramatic strength drop. This behavior is then coupled to small overall strain before the maximum strength is reached. Furthermore, the concept of strain localization is involved. In this study, the Rock Brittleness Index is calculated by using the uniaxial compressive strength (UCS) and rock type.

Goktan & Yilmaz Gunes (2005) equation was used to calculate the brittleness index (BI) of shale samples of the Amiran Formation (Relationship 1). In this relation, σc is the uniaxial compressive strength (UCS), and *K* is a coefficient varying from 0.170 to 0.659 and is equal to 0.231 for shale (Hoek, 1983; Johnstone, 1985). Figure 5 shows the frequency histogram, normal curve, mean, and standard deviation of BI calculated using this relation. As can be seen in this figure, the mean and maximum BI are 5.73 and 6.31 MPa, respectively. Therefore, according to Hoek classification, shales of the Amiran Formation are considered as rocks with low brittleness.

$$BI = 2.065 + K\,(\log \sigma c)^2 \qquad (1)$$

Prediciton of BI using univariate regression

The use of regression, particularly multivariate regression, to estimate various geotechnical parameters is a simple, valid, and widely

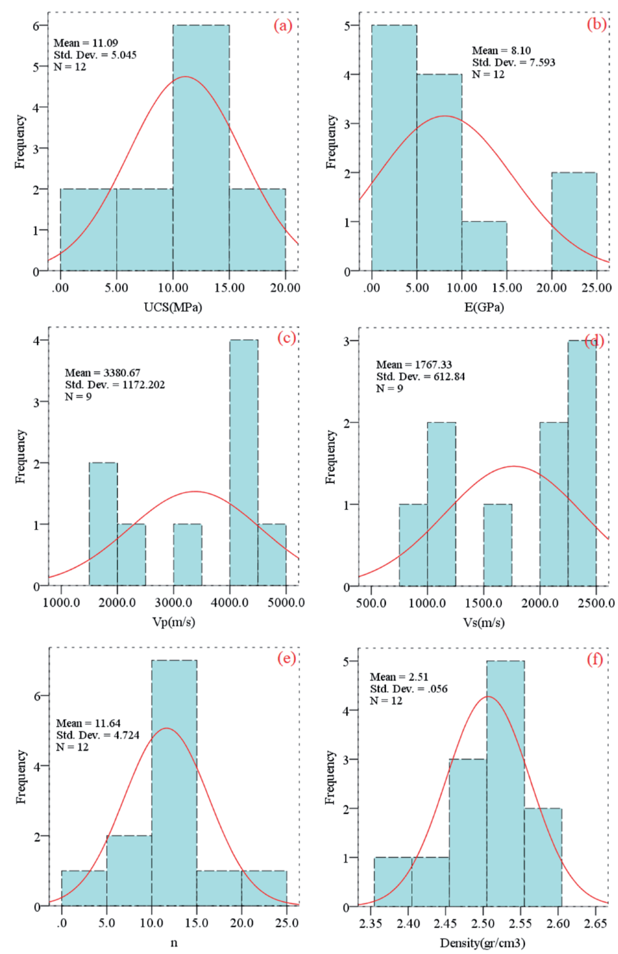


Fig.4 - Frequency histogram (a) univariate compressive strength, (b) modulus of elasticity, (c) pressure wave velocity, (d) shear wave velocity, (e) porosity, (f) density

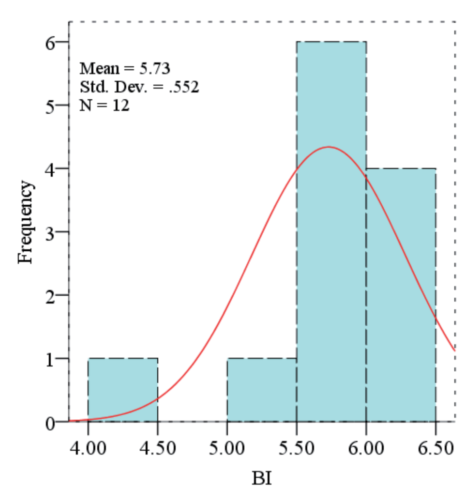


Fig. 5 - BI frequency histogram

used tool that has found a special place in geotechnical engineering in the last decade. This tool is used in new studies in the field of geotechnics, especially in determining engineering characteristics (Rahimi Shahid, 2015; Chamanzadeh *et alii*, 2016; Rahimi Shahid *et alii*, 2021) and brittleness index (Yagiz *et alii*, 2018; Ghadernejad *et alii*, 2019; Jamshidi *et alii*, 2020; Karami *et alii*, 2021). In this research, a linear relationship (with 95% confidence interval) between BI with different physical and mechanical parameters of the Amiran Formation rock mass is calculated after removing the wild values (Figure 6). Table 1 indicates regression relationships and different statistics of each regression. One way to check the significance of the regression relationship is to determine the Sig element. There are the following assumptions for this statistic.

H_0: There is no correlation.

H_1: There is correlation.

At the 95% significance level ($\alpha = 0.05$), if the value of zero is rejected, it means that there is significant relationship between the variables.

$$\{ Sig = 0.0 < \alpha \rightarrow RH_0, \text{ with } \alpha = 0.05 \qquad (2)$$

Univariate linear regression analysis shows that there is no significant relationship between BI and ρ (Sig > 0.05). While there is a significant relationship between n and BI with high accuracy ($R^2 = 0.654$). Also, the range of changes of n values compared to ρ is much larger in this study. The most accurate relationships for prediciton BI include the use of UCS and *Vp* (Table 1).

Regression Equation	Std. Error of the estimate	R square (R^2)	Result of ANOVA test	
			F	Sig.
$\mathbf{BI = 0.075\ UCS + 4.96}$	0.069	0.958	204.966	0.000
$\mathbf{BI = -0.019\ E + 6.171}$	0.133	0.582	9.728	0.017
$\mathbf{BI = -10^{-4} V_p + 6.63}$	0.163	0.661	11.705	0.014
$\mathbf{BI = -3.6 * 10^{-4} V_s + 6.60}$	0.176	0.604	9.133	0.023
$\mathbf{BI = 0.061\ n + 5.214}$	0.158	0.654	11.358	0.015
$\mathbf{BI = -2.57\ \rho + 12.17}$	0.559	0.068	0.731	0.413

Tab. 1 - A summary of univariate regression analysis results

The results show that the relationship of BI is direct with UCS and *n* and inverse with *E*, *Vp* and *Vs* (Table 1). According to Sig statistics, all regression equations presented in Table 1 (except ρ) are significant and Sig values are less than 5%.

Many rock types have naturally occurring inherent anisotropic planes, such as bedding planes, foliation, or flow structures. Such characteristic induces directional features and anisotropy in rocks' strength and deformational properties. The existence of weak planes increases both the heterogeneity and the anisotropy of stress distributions within the transversely isotropic rock, with the degree of influence varying with the foliation orientation (Heng *et alii*, 2014; Ismael *et alii*, 2014; Shuai *et alii*, 2015).

The results obtained indicate that for sedimentary

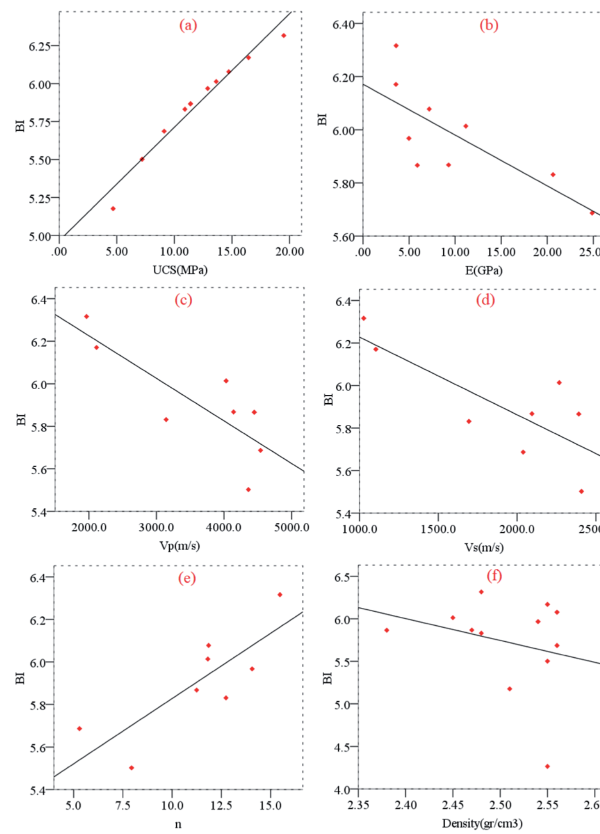


Fig. 6 - The relationship of BI and (a) uniaxial compressive strength, (b) modulus of elasticity, (c) pressure wave velocity, (d) shear wave velocity, (e) porosity, (f) desity

rocks, a higher Young's modulus reduces the brittleness of rock Ye *et alii* (2020).

In fact, in an isotropic rocks BI should not inversely correlated to E, Vp and Vs. In fact, in anisotropic rocks such as shale, the brittle index decreases with increasing elastic properties (such as *E*, *Vp* and *Vs*). Residual analysis (difference between the Calculated (Relationship 1) and predicted values) is one of the methods for the estimation of the predicted values, used widely in recent studies (Jabinpour *et alii*, 2018; Lashkaripour *et alii*, 2018; Rahimi Shahid & Hashemian, 2021; Karami *et alii*, 2021). In this type of analysis, the closer the residual mean is to zero, and the closer the residual distribution is to the normal distribution, the more reliable the predicted values will be. As shown in Figure 7, the residual means of the univariate regressions presented are approximately zero with an almost normal distribution. In most relationships, the frequency of zero residuals is maximal. Therefore, according to the presented results, there is a significant

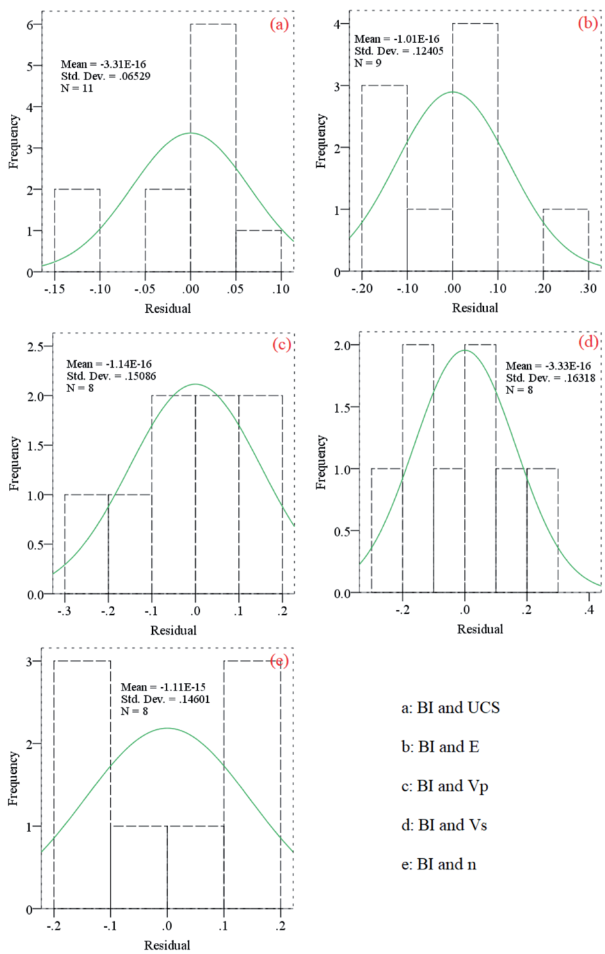


Fig. 7 Frequency histogram of univariate regression residuals

and valid relationship between BI and other characteristics studied (except ρ) in this study.

BI Prediction Using Multivariate Linear Regression

This section examined the linear relationship between BI and two different variables (at a 95% confidence level), and finally, 5 significant relationships were obtained between BI and different parameters (Sig < 0.05). Figure 8 shows the three-dimensional diagram of each of these relationships. Tables 2 and 3 indicate the regression relationships and different statistics of each regression. BI regression has the highest values of R^2 and F with UCS and E and the lowest values of R^2 and F with Vp and E (Table 2). A standardized regression coefficient (Beta) compares the strength of the effect of each individual independent variable to the dependent variable. Higher values of Beta lead to more importance of the coefficients in the regression model. Thus, it is concluded that the UCS variable is the most effective in the prediction of BI. Vp has the greatest effect in the BI regression with Vp and

Equation	Std. Error of the estimate	R square (R^2)	Result of ANOVA test	
			F	Sig.
$BI = 0.076\ UCS + 0.005\ E + 4.902$	0.590	0.973	141.601	.000
$BI = 0.07\ UCS + 2.461 * 10^{-5} V_p + 4.945$	0.040	0.982	140.306	.000
$BI = 0.069\ UCS + 3.701 * 10^{-5} V_s + 4.981$	0.041	0.982	135.057	.000
$BI = 0.062\ UCS + 0.005\ n + 5.081$	0.043	0.978	112.510	.000
$BI = -0.013\ E - 7.327 * 10^{-5} V_p + 6.339$	0.129	0.705	5.969	.047

Tab. 2 - A summary of bivariate regression analysis results

Model		Unstandardized Coefficients		Standardized Coefficients	Collinearity Statistics
		B	Std. Error	Beta	VIF
BI	(Constant)	4.902	.064		
	UCS (MPa)	.076	.005	.997	1.024
	E (GPa)	.005	.003	.122	1.024
BI	(Constant)	4.945	.184		
	UCS (MPa)	.070	.007	1.075	3.594
	Vp (m/s)	2.461E-5	.000	.100	3.594
BI	(Constant)	4.981	.167		
	UCS (MPa)	.069	.007	1.054	2.935
	Vs (m/s)	3.701E-5	.000	.079	2.935
BI	(Constant)	5.081	.061		
	UCS (MPa)	.062	.007	.931	2.674
	n	.005	.008	.072	2.674
BI	(Constant)	6.339	.142		
	Vp (m/s)	-7.327E-5	.000	-.434	1.423
	E (GPa)	-.013	.007	-.520	1.423

Tab. 3 - Statistics of Equation coefficients, Beta statistic, and VIF in bi-variate regression model

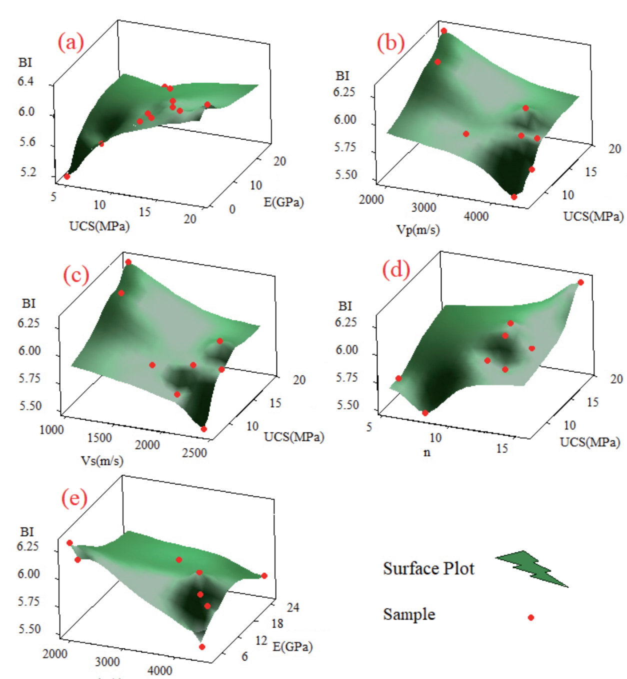


Fig. 8 - Three-dimensional diagram of BI with different variables

E (Table 3). If the VIF is greater than 10, the regression model suffers from the problem of collinearity. As shown in Table 3, the VIF value for all relationships is less than 4. Figure 9 shows the distribution of standardized residuals of bivariate regressions with their normal curves. The mean of the residuals tends to zero for most relationships. Also, the Std Deviation of standardized residuals is greater than 0.845 and close to one for all regressions. Comparison of the Calculated (Relationship 1) and predicted BI values shows that the results of BI regression with *Vp* and *E* are

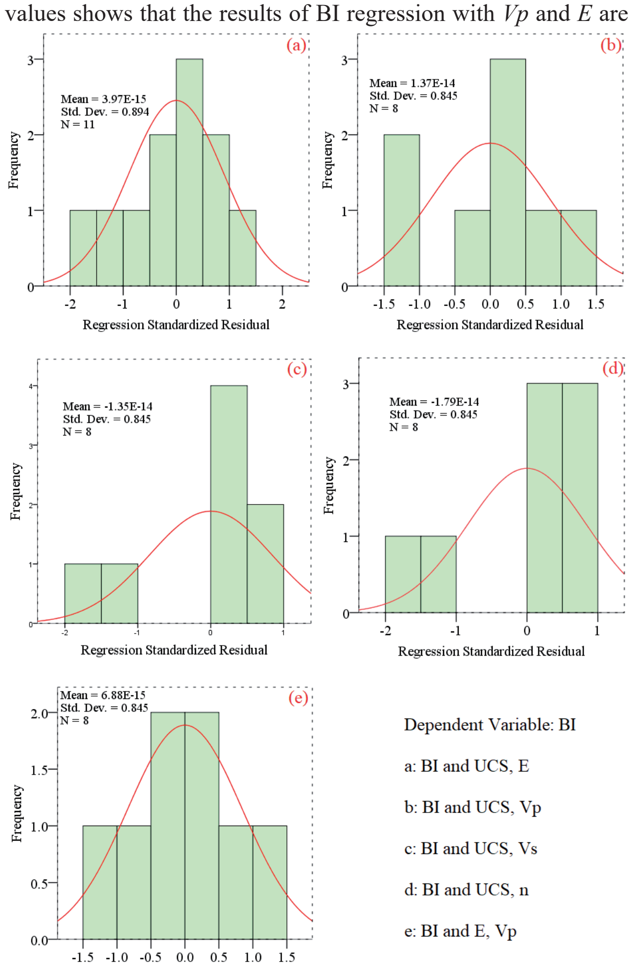


Fig. 9 - Frequency histograms of standardized residuals (bivariate regression)

less valid (R^2 = 0.392) while the results of other regressions are more valid (R^2> 0.9). BI predicted using two variables of UCS and Vs shows the highest correlation coefficient (R^2 = 0.946) with real values (Figure 10). Table 4 shows the results of BI three-variable linear regression with different parameters. As can be seen, BI regression has R^2 = 0.994 and F = 221.164 with UCS, *E*, and *Vs*, indicating the highest validity among other regressions. Table 5 presents the different statistics of each regression. According to Beta Statistics, UCS has the highest impact on BI prediction as expected in all regressions, and there is no collinearity (VIF <10) in any regression (Table 5). Figure 11 shows the frequency

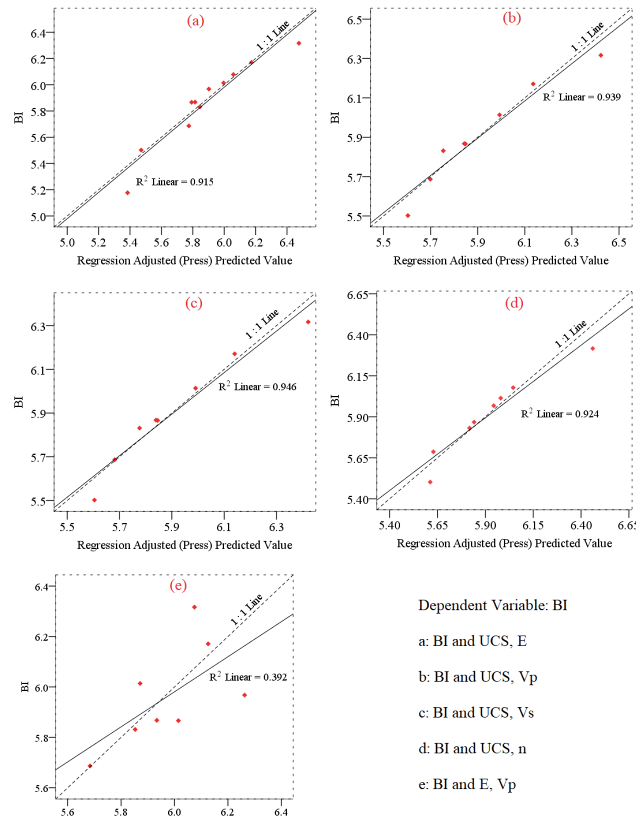


Fig. 10 - Comparison of the relationship between Calculated (Relationship 1) and predicted values of BI using bivariate regression

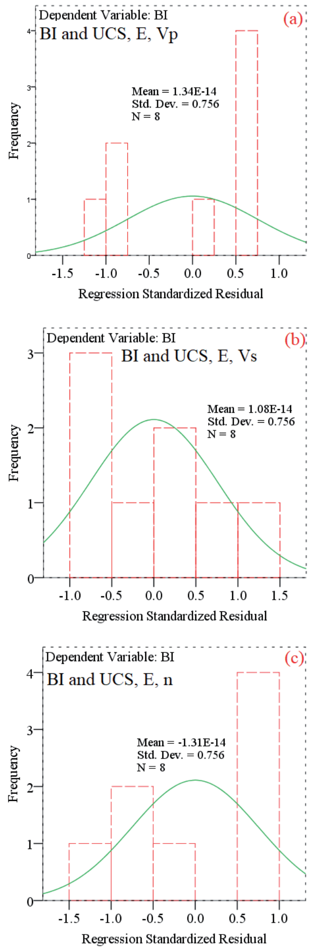
Equation	Std. Error of the estimate	R square (R^2)	Result of ANOVA test	
			F	Sig.
$BI = 0.072\ UCS + 0.002\ E + 2.609 * 10^{-5} V_p + 4.891$	0.041	0.985	89.534	.000
$BI = 0.055\ UCS - 0.002\ E + 6.378 * 10^{-7} V_s + 5.206$	0.021	0.994	221.164	.000
$BI = 0.062\ UCS + 0.003\ E + 0.009\ n + 5.007$	0.042	0.984	79.618	.001

Tab. 4 - A summary of the Three-variable regression analysis results

Model		Unstandardized Coefficients		Standardized Coefficients	Collinearity Statistics
		B	Std. Error	Beta	VIF
BI	(Constant)	4.891	.198		
	UCS (MPa)	.072	.008	1.109	4.002
	E (GPa)	.002	.002	.061	1.298
	Vp (m/s)	2.609E-5	.000	.106	3.606
BI	(Constant)	5.260	.078		
	UCS (MPa)	.055	.004	.926	2.767
	E (GPa)	-.002	.001	-.098	2.031
	V_S (m/s)	6.378E-7	.000	.002	1.680
BI	(Constant)	5.007	.088		
	UCS (MPa)	.062	.007	.934	2.676
	E (GPa)	.003	.002	.088	1.464
	n	.009	.009	.119	3.093

Tab. 5 - Values of Equation coefficients, Beta static, and VIF in three regression model

histogram of the standardized residues of the three-variable regressions. As can be seen, the mean standardized residuals of all three-variable regressions tend to zero, and the Std Deviation



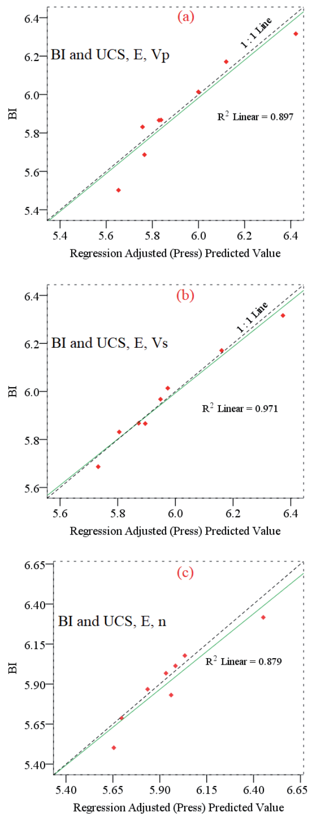


Fig. 12 - Comparison of the relationship between Calculated (Relationship 1) and predicted values of BI using three-variable regression

Fig. 11 - Frequency histograms of standardized residuals (three-variable regression)

of all regressions is greater than 0.75. The normal curve of standardized residuals of BI regression with UCS, *E*, and *Vs* has a more appropriate distribution and greater compliance with the normal state. Also, residuals equal to zero show more frequency in this curve (Figure 11.b). Figure 12 compares the Calculated (Relationship 1) and predicted BI values using three-variable regression. As can be seen, the results of BI regression with UCS, *E*, and *Vs* have the highest validity ($R^2 = 0.971$). Hence, different evaluation methods show that BI regression with UCS, *E*, and *Vs* gives the best results compared to other regression relationships.

DISCUSSION

The maximum value of Calculated (Relationship 1) BI for the Shale-Marl Amiran Formation in this study was 6.31 MPa, typical of slightly brittle rocks according to Hoek (1983) classification. The main reason for the decrease in brittleness in the saturated state is the presence of clay minerals, in this formation particularly

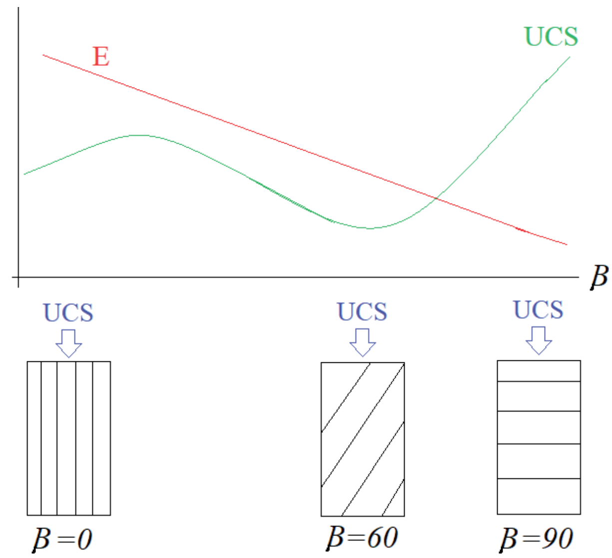


Fig. 13 - Impact of layering angle on E and UCS

montmorillonite. According to Safari Farrokhad *et alii* (2019), increasing clay minerals reduces brittleness and increases rock ductility. In a study conducted by Lou *et alii* (2016), rock brittleness decreased with the increasing clay content of samples. In the shale-marl Amiran Formation at saturation state, there is a significant and direct relationship between BI with UCS and *n*. Also, Safari Farrokhad *et alii* (2019) showed that the brittleness of limestone increased with increasing *n* in the saturated state. There was no significant relationship between BI and ρ in this study. The range of changes of ρ values (2.47-2.55 gr/cm^3) is much more limited than the range of changes of n values, which can be due to the small variety of shale rocks under study. Mineralogy changes in shale rocks also cover a wide range. Nevertheless, an overall trend shows that abundant quartz and carbonates content yield high brittleness values, while the high clay content and porosity lower the rock brittleness (Mews *et alii*, 2019). Therefore, the different trends of ρ and *n* with BI are related to the lithological and mineralogical properties of the samples.

In the present study, the value of BI decreased with increasing *E*, *Vp*, and *Vs*, which can be due to the fissility of shale rocks. In slate rocks such as shale, the angle between the direction of UCS application and the foliation surface plays a key role in the values of UCS and *E* (Figure 13). As the angle between the normal to the foliation surface and the loading direction (β angle) decreases from 60° to zero, the inverse relationship between UCS and *E* is observed. In this case, brittleness increases with decreasing *E*. The results of research conducted by Ye *et alii*. (2020) on shale rocks also showed an inverse relationship between *E* and BI (Table 6). Yang *et alii* (2020) showed that β=0 and Jamshidi *et alii* (2021) showed that β=90 led to the highest brittleness in

Equation	R^2	Reference
$BI = 0.075\ UCS + 4.96$	0.958	This Study
$BI = -0.019\ E + 6.171$	0.582	
$BI = -10^{-4} V_p + 6.63$	0.661	
$BI = -3.6 * 10^{-4} V_s + 6.60$	0.604	
$BI = 0.061\ n + 5.214$	0.654	
$BI = -2.57\ \rho + 12.17$	0.068	
$BI_{sat} = 1.7 * 10^{-4} V_{p-sat} + 5.078$	0.78	Karami *et alii.* (2021)
$BI_{dry} = 3.756\ V_{p-dry} + 3.04$	0.875	Lashkaripour *et alii.* (2018)
$BI_{sat} = 4.14\ V_{p-sat} + 0.328$	0.853	
$BI = 1.022e^{0.026SRH}$ (SRH: Schnidt Rebound Hammer)	0.700	Ghobadi & Rasouli Farah (2012)
$BI = 0.2427\rho_{Dry} + 3.3198$	0.633	Ghobadi *et alii.* (2018)
$BI = 0.0011\ V_p - 1.6347$	0.642	
$BI = -0.013\ E + 0.942$	0.650	Ye *et alii.* (2020)
$BI = 0.076\ UCS + 0.005\ E + 4.902$	0.973	This Study
$BI = 0.07\ UCS + 2.461 * 10^{-5} V_p + 4.945$	0.982	
$BI = 0.069\ UCS + 3.701 * 10^{-5} V_s + 4.981$	0.982	
$BI = 0.062\ UCS + 0.005\ n + 5.081$	0.978	
$BI = -0.013\ E - 7.327 * 10^{-5} V_p + 6.339$	0.705	
$BI = \frac{E_n + \upsilon_n}{2}$	-	Rickman *et alii.* (2008)
$BI = 0.201UCS - 1.942BTS + 17.05$	0.860	Ghadernejad *et alii.* (2019)
$BI = 0.072\ UCS + 0.002\ E + 2.609 * 10^{-5} V_p + 4.891$	0.985	This Study
$BI = 0.055\ UCS - 0.002\ E + 6.378 * 10^{-7} V_s + 5.206$	0.994	
$BI = 0.062\ UCS + 0.003\ E + 0.009\ n + 5.007$	0.984	
$BI = 0.011\ UCS_{sat} + 3.411 * 10^{-5}\ V_{p-sat} - 0.01\ BTS_{sat} + 5.132$	0.96	Karami *et alii.* (2021)
$BI = 0.009\ UCS_{Dry} + 2.59\ V_{p-Dry} + 0.349\ BTS_{Dry} + 4.95$	0.89	Lashkaripour *et alii.* (2018)
$BI = 0.198\ UCS + 0.913\ \rho - 2.174\ BTS - 3.807$	0.88	Yagiz (2009)
$BI = 2.02\rho + 0.008V_P - 0.007V_S - 61.12\upsilon - 2.62(\text{Rock type}) - 21.76$	-	Kaunda & Asbury (2016)
$BI = 0.299\rho + UCS - 0.214BTS + 0.1455$	0.914	Yagiz *et alii.* (2018)
$BI = 0.232\rho + 1.835UCS - 0.916BTS + 0.1839$	0.851	

Tab. 6 - Comparison of the results of this study with previous studies

shales and laminated sandstone, respectively. It is obvious because failure occurs within the intact shale layers (foliation does not play a significant role). The relationship between BI and *Vp* has been direct in other rocks such as limestone (Lashkaripour *et alii*, 2018; Karami *et alii*, 2021) and peridotites (Ghobadi *et alii*, 2018) (Table 6). The mechanical properties of the Amiran Formation show more compliance with the brittleness index (BI) of the rock compared to the physical properties, which is also consistent with the results of Ghobadi *et alii* (2018). Similarly in isotopic or marginally anisotropic rocks, UCS and *Vp* or *Vs* are inversely correlated to porosity. A multivariate relationship using mechanical properties was proposed to predict BI in shale rock, which provides valid results (Li & Li, 2018).

For the Kong-2 member shale of the Guandong block, as the confining pressure increases, the brittleness index decreases significantly when the confining pressure is less than 25 MPa, and the brittleness index decreases slightly when the confining pressure is greater than 25 MPa (Li *et alii*, 2020). Table 6 presents the results of this study and previous studies. According to this table, multivariate regression has been used limitedly to estimate brittleness index (BI) in different studies, while a higher number of variables in regression equations leads to the more accurate prediction of BI (Table 6). In determining different geotechnical parameters such as BI, different properties of rock mass are effective and simultaneous study of the effect of these properties in determining different geotechnical parameters can provide more realistic results. One way to achieve this goal is to use multivariate regressions (Kaunda & Asbury, 2016; Lashkaripour *et alii*, 2018; Yagiz *et alii*, 2018; Li & Li, 2018; Karami *et alii*, 2021; Moradi *et alii*, 2021). Simultaneous use of UCS and *Vp* variables has provided acceptable results in predicting BI values in this study and previous studies (Lashkaripour *et alii*, 2018). The results of the present study also show that the simultaneous use of UCS and *E* or UCS and *Vs* provides reliable results in predicting BI values. In the multivariate regression equation presented, the effect of UCS in determining BI is greater than the other properties studied (Ghadernejad *et alii*, 2019 & Lashkaripour *et alii*, 2018). In Li & Li (2018) study, the predicted values of BI were compared with the values measured in exploratory wells using multivariate regression. Field application shows that this technique is reliable, since its prediction results coincide with the calculated brittleness index of exploratory wells, with a relative error margin below 4% (Li & Li, 2018). In the Ye *et alii* study (2020) the use of Young's modulus (*E*) to predict the fragility index provides more reliable results than Poisson's ratio. The results obtained indicate that for sedimentary rocks (Shale), a higher Young's modulus reduces the brittleness of rock, and Poisson's ratio weakly correlates with brittleness. Furthermore, the most suitable fracturing layers possess a high brittleness index and low minimum horizontal stress (YE *et alii*, 2020). Therefore, the results of these studies confirm the results of the present study.

CONCLUSION

This study presented some relationships for the prediction of rock BI using univariate and multivariate regression methods. In these relationships, mechanical (UCS, *E*, *Vp*, and *Vs*) and physical properties (ρ and *n*) of shale were used as independent variables. The summary of the results of this study is as follows. The brittleness index (Calculated with Relationship 1) of the shale Amiran Formation is a maximum of 6.31 MPa due to the presence of clay minerals, indicating low brittleness according to the Hoek (1983) classification. There is a direct relationship between BI with UCS and n in univariate regression. In fact, in an isotropic rocks BI should not inversely correlated to *E, Vp* and *Vs*. The relationship between BI with *Vp* and *Vs* is inverse, mainly due to the anisotropy in the shales. There was no significant relationship between BI and ρ in this study. The range of changes of ρ values (2.47-2.55 gr/cm^3) is much more limited than the range of changes of n values, which can be due to the small variety of shale rocks under study. The mechanical properties show more compliance with the BI compared to physical properties in the Amiran Formation. In bivariate regression analysis, the use of UCS and *E* or UCS and *Vs* leads to the most reliable BI prediction.

In three-variable regression analysis, the use of UCS, *E*, and *Vs* provides the most reliable BI prediction results. In multivariate regressions, as the number of variables increases, that obtained the most accurate prediction of BI. According to the results of the recent research, there is a significant and valid relationship between BI and other characteristics studied (except ρ).

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