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INNOVATIVE APPROACH FOR THE DETERMINATION OF UNIT WEIGHT AND DENSITY OF SOIL AND ROCK MASSES

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EXTENDED ABSTRACT

Il peso per unità di volume e la densità rappresentano le proprietà fisiche di base e più importanti dei terreni e delle rocce. Tali proprietà determinano lo stato tensionale e l'intensità delle forze agenti su qualunque sistema naturale e, pertanto, sono direttamente impiegate in tutte le analisi numeriche e analitiche che coinvolgono terreni e ammassi rocciosi. A scala del singolo campione o nel caso di terreni e rocce intatte e omogenee, il peso per unità di volume e la densità sono facilmente ottenibili attraverso prove di laboratorio o correlazioni empiriche tra le fasi che compongono il mezzo. In natura però gli ammassi non si presentano quasi mai come mezzi continui e omogenei, in quanto risultano quasi sempre caratterizzati da discontinuità e disomogeneità. In queste condizioni l'attribuzione delle caratteristiche fisiche ottenute da un campione di laboratorio all'intero ammasso può risultare errata, in quanto non vengono prese in considerazione tutte le caratteristiche meso- e macro-strutturali del mezzo. Per gli ammassi naturali, pertanto, la determinazione dei parametri fisici è piuttosto complessa e richiede l'impiego di relazioni analitiche adeguate.

Per tali motivi, nel presente studio è stato sviluppato un approccio analitico per la definizione del peso per unità di volume e della densità. Le equazioni sono state sviluppate partendo dalle relazioni note per i materiali continui e omogenei e sono state opportunamente modificate al fine di considerare tutte le caratteristiche strutturali delle diverse tipologie di ammassi presenti in natura. Questi ultimi sono stati suddivisi in quattro categorie principali, ovvero: (i) ammassi omogenei; (ii) ammassi fratturati; (iii) ammassi stratificati; (iv) ammassi caotici. Le relazioni proposte si basano sull'analisi delle proprietà fisiche e dei volumi dei diversi elementi che costituiscono l'ammasso, quali discontinuità, strati e blocchi. Ogni singolo ammasso è stato trattato attraverso un "approccio equivalente continuo", che consente di semplificare le caratteristiche del mezzo e di applicare le stesse ad ammassi di notevole volume ed estensione. L'approccio è basato su un numero limitato di dati ingresso facilmente ottenibili attraverso le convenzionali indagini di sito e di laboratorio.

Le diverse relazioni sono state validate tramite l'applicazione a numerosi casi di studio, rappresentativi delle differenti condizioni analizzate. Gli ammassi sono stati modellati tramite software al fine di definire con precisione i volumi di ogni singolo elemento costituente l'ammasso. I risultati ottenuti sono stati quindi confrontati con quelli derivanti dall'applicazione delle equazioni proposte, in modo da analizzare statisticamente l'attendibilità delle stesse. Le relazioni sviluppate permettono di determinare sia il peso per unità di volume che la densità di qualunque tipo di terreno o ammasso roccioso naturale sulla scorta di pochi dati di semplice determinazione. Gli errori commessi nella stima dei parametri risultano sempre estremamente bassi e, comunque, ben al di sotto dell'accuratezza richiesta dai normali studi geologici e geotecnici.

Le equazioni proposte presentano un elevato grado di affidabilità e, pertanto, costituiscono un approccio standardizzato e ripetibile per l'analisi delle proprietà fisiche di ammassi reali, ferme restando le prescrizioni fornite per l'applicazione di ogni equazione. Un aspetto fondamentale è rappresentato dalla modalità di acquisizione dei dati di base, che influisce in maniera diretta sui risultati ottenuti. I parametri fisici dei materiali possono essere acquisiti mediante le convenzionali analisi di laboratorio o, in alternativa, attraverso fonti bibliografiche e correlazioni indirette. I dati sugli ammassi possono essere stabiliti in funzione dei convenzionali rilievi geologici e geomeccanici, oltre che sulla scorta dei log di sondaggio. Le equazioni sono state sviluppate per la determinazione del peso per unità di volume e della densità, ma possono essere applicate facilmente anche ad altre caratteristiche fisiche e meccaniche degli ammassi, lasciando immutata la formulazione generale e sostituendo il parametro prescelto a quelli analizzati. Ovviamente, le equazioni possono essere applicate solo a parametri non influenzati dagli effetti dell'anisotropia, ma che dipendono unicamente dai volumi degli elementi che costituiscono l'ammasso.

ABSTRACT

The present paper aims to develop an analytical approach for defining the two most important physical properties of soils and rocks at mass scale: unit weight and density. A series of equations have been developed for the calculation of the aforementioned parameters for different types of soil and rock masses available in nature; specifically homogenous, jointed, layered, and chaotic masses. The relationships were determined according to an "equivalent continuum approach" by analyzing the physical characteristics and volumetric fractions of different elements that make up the mass, such as discontinuities, layers, and blocks.

The required inputs for the proposed equations are limited in number and easily obtainable by conventional site surveys and laboratory tests. Each of the equations has been validated through the application to several case studies representative of soil and rock masses with different physical conditions. The proposed relationships show a high degree of reliability and are therefore applicable to different types of soil and rock masses, according to the developed standardized and repeatable approach.

Keywords: *physical properties, unit weight, density, soil and rock masses, equivalent continuum approach, analytical solution*

INTRODUCTION

Unit weight and density represent the basic but most important physical properties of soils and rocks. They determine the stress and strength state acting on any natural system and, therefore, are directly used in all analytical and numerical analyses involving soil and rock masses. These properties are easily obtainable at a sample scale through laboratory tests in case of intact and homogeneous soil and rock, or through empirical correlations between the phases that make up the medium. However, in nature, the materials are generally characterized by inhomogeneities and discontinuities and never appear as continuous or homogeneous medium. Therefore, the attribution of physical properties obtained for a laboratory sample to the whole mass may be incorrect since the medium's meso- and macro-structural characteristics of the medium may not be taken into account. Determining physical parameters for natural materials is somewhat complex and requires to be derived through analytical relationships, which the literature lacks in providing a suitable one.

This study addresses the above gap by developing a proper analytical approach to determine the physical properties of soil and rock masses, such as unit weight and density. The study uses the fundamental relationships known for continuous and homogeneous materials as a base followed by the development of rigorous relationships to consider all the structural characteristics of different types of masses present in nature (i.e., homogenous masses, jointed masses, layered masses, and chaotic masses). The developed equations are based on the analysis of the physical properties and volumes of the different elements that make up the mass (i.e., discontinuities, layers, and blocks). The mass has been defined according to an "equivalent continuum approach", with a limited number of parameters easily obtainable by conventional site and laboratory investigations. Finally, the developed relationships have been validated through practical application to several case studies representative of the different analyzed conditions. The masses were modeled through AutoCAD Map 3D 2022 to define the volume of each element present in the mass, followed by calculation and analysis through Excel 365. It is observed that the equations have a high degree of reliability and, therefore, constitute a standardized and repeatable approach for the analysis of the physical properties of natural masses.

STATE OF THE ART

The study of the physical and mechanical characteristics of soil and rock masses can be carried out according to two different analysis methods: the "discrete approach" and the "equivalent continuum approach" (Sitharam *et alii*, 2001; Discenza *et alii*, 2020). The first approach is mainly used for jointed rock masses, while the latter is used for both soil and rock masses. The "discrete approach" considers the masses as discontinuous media, in which each element is endowed with its own constitutive laws (Pande *et alii*, 1990; Eberhardt *et alii*, 2002). The "equivalent continuum approach", instead, considers the masses as continuous media, endowed with a single constitutive law that is derived from the characteristics of all the elements that constitute the mass (Salamon, 1968; Amadei *et alii*, 1988; Sitharam *et alii*, 2001; Hoek *et alii*, 2002; Zhang & Einstein, 2004; Sitharam *et alii*, 2007; Zhang, 2016; Ramamurthy *et alii*, 2017).

In the last decades, several "continuum equivalent approaches" were developed to define the physical and mechanical properties of jointed and heterogeneous rock masses (Discenza *et alii*, 2020). In contrary to the "discrete approaches" (Sitharam *et alii*, 2001; Stead *et alii*, 2006; Sitharam *et alii*, 2007), the "equivalent continuum approaches" are applicable to slopes of considerable extension (Hoek & Brown, 1980; Sitharam *et alii*, 2001, 2007; Khanna *et alii*, 2018; Discenza *et alii*, 2020) and are therefore widely used in the geological and geotechnical fields. Several "equivalent continuum approaches" have also been developed for the study of strength and deformability characteristics of jointed rock masses (Singh, 1973a, 1973b; Hoek & Brown, 1980; Gerrard, 1982; Fossum, 1985; Wei & Hudson, 1986; Arora, 1987; Chen, 1989; Cai & Horii, 1992; Priest, 1993; Hoek & Brown, 1997; Verman *et alii*, 1997; Ramamurthy, 2001; Sitharam *et alii*, 2001; Hoek *et alii*, 2002; Sitharam & Latha, 2002; Zhang & Einstein, 2004;

Hoek & Diederichs, 2006; Sitharam *et alii*, 2007; Singh & Singh, 2008; Bahrani & Kaiser, 2013; Ramamurthy *et alii*, 2017; Khanna *et alii*, 2018). Furthermore, many methods are also available for the analysis of the mechanical characteristics of anisotropic, layered, and heterogeneous rock masses (Jaeger, 1960; Salamon, 1968; Nova, 1980; Amadei *et alii*, 1988; Ramamurthy *et alii*, 1988; Amadei & Savage, 1989; Goodman, 1989; Parsons *et alii*, 1993; Single *et alii*, 1998; Marinos & Hoek, 2001; Zhang & Zhu, 2007; Saroglou & Tsiambaos, 2008; Marinos *et alii*, 2011; Fortsakis *et alii*, 2012; Ghazvinian & Hadei, 2012; Lee *et alii*, 2012; Zhang *et alii*, 2012; Saeidi *et alii*, 2014; Triantafyllidis & Gerolymatou, 2014; Usol'tseva *et alii*, 2017; Zhou *et alii*, 2017; Behnia *et alii*, 2018; Meng *et alii*, 2018).

Recently, based on the results of a small-scale physical-analogue laboratory modeling (Discenza *et alii*, 2013) an equivalent continuum approach was proposed for the determination of the rheological properties of jointed rock masses with a set of systematic joints (Discenza *et alii*, 2020). The approach is based on the viscosity of rock matrix and the geometrical characteristics of the discontinuities (i.e., spacing and dip). As mentioned, the studies are primarily aimed at defining the strength and deformability characteristics of the masses, but their physical characteristics (i.e., density and unit weight) have not been considered. One of the few studies that address this problem is Amadei *et alii* (1988), which analyzes transversely isotropic rock masses for the definition of gravitational stresses in the subsoil. Among the solutions proposed by Amadei *et alii* (1988), the one based on the continuous equivalent approach acknowledges that the density of the masses corresponds to the average density of the layers that constitute it. Here, the average density is a function of a dimensionless factor that expresses the relative thickness of each layer in the mass.

UNIT WEIGHT AND DENSITY

Unit weight and density are two basic physical properties of soils and rocks. These properties are directly connected to the intrinsic characteristics of the materials and their relative volumes, as well as to the structural setting of the mass, discontinuities, and inhomogeneities.

The density ρ expresses the quantity of mass m per unit of volume V and, in general, can be expressed through the following relationship:

$$\rho = \frac{m}{V} \tag{1}$$

The unit weight γ expresses the weight force produced by a mass and, therefore, is dependent on the gravity acceleration g (9.81 m/s^2) by the equation:

$$\gamma = \rho \, g \tag{2}$$

or:

$$\gamma = \frac{m}{V} \, g \tag{3}$$

The weight force W is expressed as the product of mass m and gravity acceleration g such as:

$$W = m \, g \tag{4}$$

hence, the unit weight γ can be expressed as:

$$\gamma = \frac{W}{V} \tag{5}$$

At a microscopic level, soils and rocks are not perfectly continuous and homogeneous materials but are made up of different phases (solid, liquid, and gaseous). On a scale of the laboratory sample or in the case of continuous and homogeneous materials, the unit weight and density of the medium depend, therefore, both on the relative values of the aforementioned phases and on parameters that express the volumetric percentage of the same, i.e., porosity, void index, and degree of saturation (Lambe & Whitman, 1969; Verruijt, 2001; Budhu, 2007; Peng & Zhang, 2007; Lancellotta, 2012).

At a mass scale, the physical parameters of the medium are connected to meso- and macro-structural characteristics such as discontinuities, layers and blocks. Therefore, for large volumes of soils and rocks, the determination of the physical properties of the material must be carried out by considering all the mentioned elements and specific geometrical characteristics of the mass.

METHODOLOGY AND PROPOSED EQUATIONS

In the case of continuous and homogeneous media, the determination of the unit weight and density of the mass is quite simple, as it perfectly corresponds to that of the material that constitutes it. However, deal continuous and homogeneous soil and rock masses are rare in nature and characterized mainly by discontinuities and inhomogeneities that greatly complicate their structural setting. Therefore, the determination of the physical parameters of earth masses is considerably complex than the ideal condition and requires suitable analytical relationships.

Both the unit weight and density are volumetric properties and, therefore, not affected by any anisotropies. These parameters are only a function of the properties and volume of the individual components that make up the mass:

$$\gamma_m = f\{\gamma_i \, V_i\} \tag{6}$$

$$\rho_m = f\{\rho_i \, V_i\} \tag{7}$$

where γ_m and ρ_m are respectively the unit weight and density of the mass, γ_i and ρ_i are the unit weight and density of i-th material that constitute it, and V_i is the volume of i-th material.

In a simple case, the unit weight of an earth mass γ_m can be determined as:

$$\gamma_m = \sum_{i=1}^{n} \frac{\gamma_i V_i}{V_t} \tag{8}$$

where γ_i is unit weight of i-th material, V_i is volume of the i-th material, and V_t is total volume of the mass. In the equations, 'i' and 'n' represent individual components of the mass.

Similarly, the density of an earth mass ρ_m can be determined as:

$$\rho_m = \sum_{i=1}^{n} \frac{\rho_i V_i}{V_t} \tag{9}$$

where ρ_i is density of i-th material, while V_i and V_t are same as Eq. (8).

Generally, knowing the measurements along the three axes of considered mass i.e., l_x, l_y and l_z, the total volume of the analyzed mass can be calculated as:

$$V_t = l_x\, l_y\, l_z \tag{10}$$

Through Eq. (8) the problem can be solved by knowing the values of γ_i and V_i of each individual component of the mass. Considering that the values of γ_i are determined by laboratory tests (or more rarely through empirical correlations), the accuracy of this equation is determined solely by the evaluation of different values of V_i. For simplicity, in the following reference will be made only to the unit weight. The equation can be formulated for density by substituting γ with ρ as per the relationship in Eq. (2).

Although the structural and geometric setting of soils and rocks is very complex and difficult to describe analytically, it is still possible to develop mathematical relationships that allow to define (with a sufficient degree of approximation) the volumes of each element V_i. A series of analytical correlations were then developed for the calculation of γ_m of the most common types of soil and rock masses in nature (Fig. 1): a) homogenous masses; b) jointed masses; c) layered masses; d) chaotic masses.

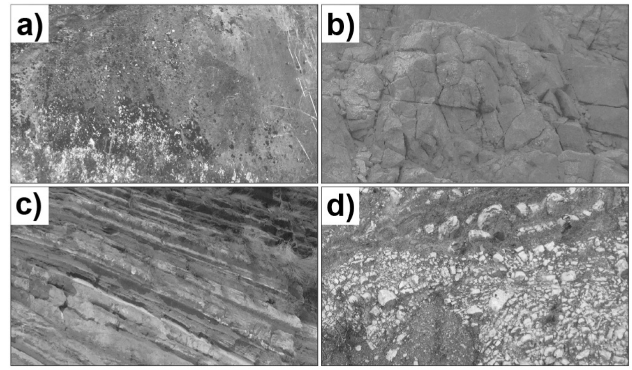

Fig. 1 - Examples of the four types of analyzed masses: a) homogenous mass; b) jointed mass; c) layered mass; d) chaotic mass

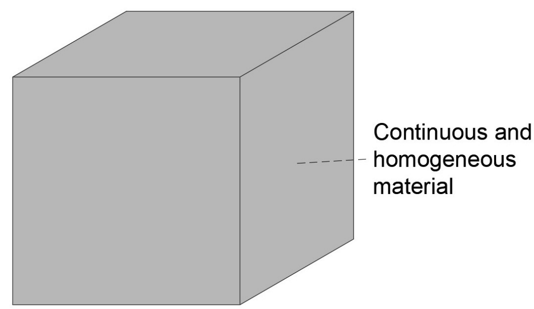

Fig. 2 - Schematic 3-dimensional representation of a homogeneous mass devoid of any meso- or macro-structural discontinuities

Homogeneous masses

Homogeneous masses are materials without discontinuity and inhomogeneity (Fig. 2). In nature, there are ideally no continuous and homogenous masses but, in certain conditions, the presence of fractures or inhomogeneities are not relevant to the scale of study and, hence, may be considered continuous. The homogeneous masses in the context of this study are made up of soils and rocks formed through a single geological process without subsequent tectonic disturbance or rearrangement phenomena that could have altered their original structure. Typical examples of these homogeneous masses are pelitic succession in a marine environment, banks of compact tuffs, and deep and non-tectonized igneous rocks.

In this case, Eq. (8) is valid, as it is general and well suited to homogeneous masses. Suppose that the material is continuous and homogeneous, the volume occupied by the intact material in question V_h is practically equal to the total volume of mass V_t and therefore:

$$\frac{V_h}{V_t} = 1 \tag{11}$$

In this hypothesis, the unit weight of a continuous and homogeneous mass γ_m can be determined according to the following relationship:

$$\gamma_m = \gamma_h \tag{12}$$

where γ_h is the unit weight of the continuous and homogeneous material.

Eq. (12) applies to all continuous and homogeneous masses present in nature. In any case, if the mass has few or irregular discontinuities, and the inhomogeneities are negligible on the mass scale, the determination of unit weight of the medium can be carried out in a manner similar to Eq. (12).

If it is assumed that a material constitutes a large part of the mass volume, the volume occupied by the material V_h will be approximately equal to the volume of the mass V_t and therefore:

$$\frac{V_h}{V_t} \cong 1 \tag{13}$$

In this condition, the unit weight of the continuous and homogeneous mass γ_m can be determined by a relationship similar to Eq. (12), that is:

$$\gamma_m \cong \gamma_v \tag{14}$$

where γ_v is the unit weight of the volumetrically most significant material.

Eq. (14) applies to jointed masses with closed and few joints or to layered and chaotic masses with a material present in a very high percentage. Although Eq. (14) shows a good degree of reliability in the above hypotheses, it is still preferable to use the following relationships proposed for each of the different types of considered mass.

Jointed masses

Jointed masses are formed by homogeneous rock blocks separated by well-defined and easily identifiable discontinuity surfaces (Fig. 3). Joints in a mass are differently oriented in space and often occur in sets. The joints in each set have similar dip and dip direction and follow a specific spacing distribution. Surface roughness of the discontinuities can be planar, irregular, or wavy, while the persistence can be variable depending on the lithology and the geological processes its formation has been subjected to. Above mentioned joint characteristics are well exhibited by competent igneous, sedimentary and metamorphic rocks and, at times, by compact and well-cemented soils. Typical examples are platform carbonate successions, igneous and metamorphic rocks exhumed by the geological processes, and well cemented tuff deposits. Highly tectonized, cataclastic, and/or mylonitic masses should not be considered in this typology, as structures in these types of rocks show strong heterogeneity in discontinuity planes and to be considered as chaotic masses.

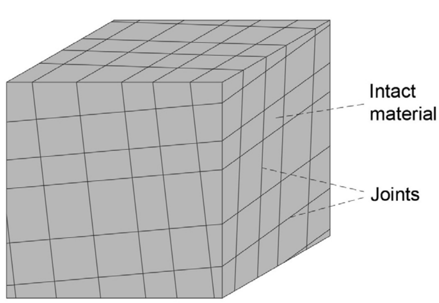

Fig. 3 - Schematic 3-dimensional representation of a jointed mass formed by an intact material with different sets of persistent joints

The detailed geometrical characteristics of joints should be collected adequately through the classic geomechanical survey methods (ISRM, 1978) or the boreholes surveys. In addition to the geometrical characteristics of the joints, it is necessary to evaluate possible filling material present within the discontinuities, which affect both physical parameters and mechanical characteristics of the mass.

At a higher depth, discontinuities are generally closed due to high lithostatic pressure and, therefore, the unit weight of rock mass is approximately equal to that of the intact rock which can be determined using of Eq. (14). Also, if the discontinuities are few or have low persistence, the jointing has little effect on the physical properties of the mass.

In all situations where discontinuities are numerous and/or very open, the physical characteristics of a mass can be quite different from those of the intact rock. Therefore, it is necessary to use specific analytical relationships to calculate the unit weight. The type of approach to be used in determining the physical parameters must be chosen according to particular context and available geomechanical data.

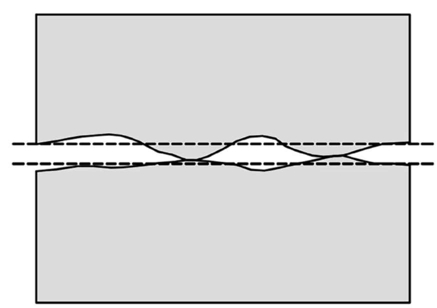

Fig. 4 - Average equivalent opening of a rough joint with rock bridges

There are few difficulties in determining the average value of aperture in the case of widely open planar fractures without rock bridges. In the case of wavy discontinuities or numerous rock bridges, the aperture value can be extremely variable and subject to estimation errors (Fig. 4). Therefore, an appropriate statistical analysis of all the data from a geomechanical survey and the use of an average or modal value would be the most representative.

In many cases, the discontinuities that characterize a mass can be divided into sets with certain physical and geometric characteristics. In this circumstance, starting from Eq. (8), the unit weight of a jointed mass γ_m can be determined as:

$$\gamma_m = \gamma_r - \sum_{j=1}^{n} \frac{\gamma_r V_j}{V_t} + \sum_{j=1}^{n} \frac{\gamma_j V_j}{V_t} \tag{15}$$

where γ_r is the unit weight of the intact rock, γ_j is the unit weight of the joint filling material in a particular joint set, V_j is the volume of each joint set, V_t is the total volume of the considered mass, and 'n' is the number of joint sets. For unfilled discontinuities, γ_j is null. In this case, the overall volume of each joint set V_j can be expressed as:

$$V_j = n_j\, a_j\, l_j^{\,2} \tag{16}$$

where a_j is the average opening of the joints belonging to a particular set, n_j is the number of discontinuities present within the considered mass volume for each set, and l_j is the average length of a certain joint set.

In first approximation, by knowing the average spacing values of the discontinuities s_j, the number of joints n_j for each considered set can be calculated using the following relationship:

$$n_j = \frac{l_m}{s_j + a_j} \tag{17}$$

where l_m is the average length of the considered mass. Then, modifying Eq. (15), γ_m is thus obtained as:

$$\gamma_m = \gamma_r - \sum_{j=1}^{n} \frac{\gamma_r\, n_j\, a_j\, l_j^{\,2}}{V_t} + \sum_{j=1}^{n} \frac{\gamma_j\, n_j\, a_j\, l_j^{\,2}}{V_t} \tag{18}$$

Ultimately, simplifying Eq. (18), the unit weight of a jointed mass γ_m with several joint sets can be calculated as:

$$\gamma_m = \gamma_r - \sum_{j=1}^{n} \frac{\gamma_r\, a_j\, l_m\, l_j^{\,2}}{V_t\,(s_j + a_j)} + \sum_{j=1}^{n} \frac{\gamma_j\, a_j\, l_m\, l_j^{\,2}}{V_t\,(s_j + a_j)} \tag{19}$$

Eq. (19) allows to determine the physical properties of any type of jointed mass characterized by discontinuities that can be grouped in sets with certain geometrical characteristics.

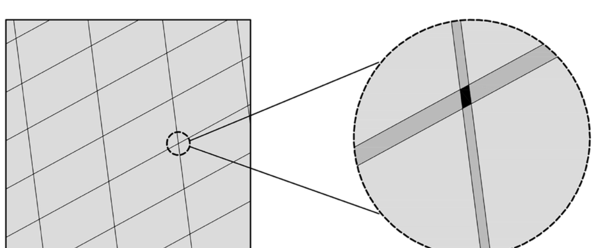

Fig. 5 - Illustration showing the number of intersections and the volume of the same as function of the geometrical characteristics of joints

In Eq. (19), the intersections between the discontinuities are considered more than once, as a function of the number of analyzed joints sets (Fig. 5). The number of intersections and their volume can vary largely in the individual portions of the mass, in relation to the specific geometrical characteristics of the discontinuities. Therefore, the determination of this parameter would require a rather complex statistical approach, in which, the dip of the joints and the variability of the opening values are also considered. However, the analyzes carried out show that although the calculated joint volume is overestimated, the introduced error is negligible and does not significantly affect the calculation of the physical parameters.

For masses having non-systematic joints or complex setting, the unit weight of the mass can be determined as a function of the number of joints present in the considered volume J_u. This parameter is easily obtainable according to the classic approaches of the geomechanical survey and can be determined either for the entire mass or for groups of joints with similar characteristics. If estimated for the whole mass considering a unit volume, J_u corresponds to the parameter J_v, which is commonly used in geomechanics.

Based on these considerations, inserting the parameter J_u in Eq. (18), the unit weight of a jointed mass γ_m with unsystematic joints can be calculated as:

$$\gamma_m = \gamma_r - \sum_{u=1}^{n} \frac{\gamma_r\, J_u\, a_u\, l_u^{\,2}}{V_t} + \sum_{u=1}^{n} \frac{\gamma_u\, J_u\, a_u\, l_u^{\,2}}{V_t} \tag{20}$$

where γ_u is the unit weight of the joint filling material in a particular joint group, a_u is the average opening value of discontinuities belonging to a group with similar characteristics, l_u is their average length, and 'n' is the number of joint groups.

To improve the estimation, the analyses conducted on the different case studies have shown a reduction factor of 0.9 to the equation reduces the error incurred due to the intersections between the joints. Ultimately, the unit weight of a jointed mass γ_m with unsystematic joints can be determined as:

$$\gamma_m = \gamma_r - \sum_{u=1}^{n} \frac{0.9\,\gamma_r\, J_u\, a_u\, l_u^{\,2}}{V_t} + \sum_{u=1}^{n} \frac{0.9\,\gamma_u\, J_u\, a_u\, l_u^{\,2}}{V_t} \tag{21}$$

Eq. (21) allows an extremely accurate estimate of the physical characteristics of jointed masses with unsystematic joints and with variable persistence. Volumetric error due to the intersections between discontinuities is considered by the reduction factor when persistence is analyzed by means of the parameter l_u. The latter must be calculated as the average extent of the discontinuities within the analyzed volume of mass.

Layered masses

Layered masses are made up of materials arranged in layers with a well-defined geometrical setting (Fig. 6). The layers can be both tabular and irregular and often appear inclined or folded, especially in the case of masses affected by certain tectonic forces. Generally, these masses are made up of alternating soils and rocks, mainly of sedimentary origin and sometimes affected by tectonic deformation that alter original sedimentary structures. Classic examples are the alternative clay and sand deposits of marine or alluvial environment, and the pelitic-arenaceous and calcareous-marly flysch successions.

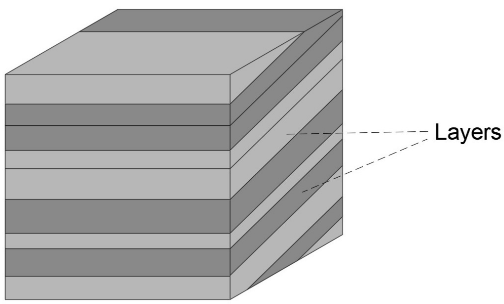

Fig. 6 - Schematic 3-dimensional representation of a layered mass formed by tabular layers of different materials

In analogy with what is proposed by Amadei *et alii* (1988) for density, the unit weight of a stratified mass is given by the weighted average of the weights of all the layers that constitute it. Therefore, assuming that the layers are approximately parallel, it is possible to simplify the problem and consider their thickness instead of the volume.

According to this assumption it is possible to modify Eq. (8) and calculate the unit weight of a layered mass γ_m, such as:

$$\gamma_m = \sum_{s=1}^{n} \frac{\gamma_s\, S_s}{S_t} \tag{22}$$

where γ_s is the unit weight of s-th layer, S_s is its thickness, S_t is the total thickness of considered mass, and 'n' is the number of layers. In the more general case, the thickness S_t can be calculated from the thickness of the individual layers S_s according to the following relationship:

$$S_t = \sum_{s=1}^{n} S_s \tag{23}$$

Therefore, replacing the latter relationship in Eq. (22), the unit weight of a generic stratified mass γ_m can be determined as:

$$\gamma_m = \frac{\sum_{s=1}^{n} \gamma_s\, S_s}{\sum_{s=1}^{n} S_s} \tag{24}$$

In order to correctly use Eq. (24) it is necessary to consider all the layers that make up the mass, or a single portion of the analyzed mass (where there is difficulty in evaluating the structure of the entire mass) in determining their relative thickness and unit weight. In this way, the determination of the unit weight is extremely precise and reliable since the characteristics of every single layer constituting the mass are considered here. In most cases, however, the data are insufficient for the above equation, and therefore, necessary to resort to simplifications as below.

Most of the masses and sedimentary successions can be schematized through a limited number of lithotypes with precise physical characteristics. These lithotypes are arranged in a series of regular layers with a precise average thickness, which alternate between them with a defined number of layers within the sequence. If the sequence is regular and systematic enough to be schematized with the repetition of n-layers having thickness S_s, Eq. (8) can be changed as follows:

$$\gamma_m = \sum_{s=1}^{n} \frac{\gamma_s\, S_s\, n_s}{S_t} \tag{25}$$

where n_s is the number of layers within the considered sequence of a particular lithotype. In this case, the total thickness S_t can be calculated as:

$$S_t = \sum_{s=1}^{n} S_s\, n_s \tag{26}$$

Ultimately, by replacing this relationship in Eq. (25), unit weight of a layered mass γ_m within a regular sequence can be determined through the following relationship:

$$\gamma_m = \frac{\sum_{s=1}^{n} \gamma_s\, S_s\, n_s}{\sum_{s=1}^{n} S_s\, n_s} \tag{27}$$

Eq. (27) is useful because it allows to determine unit weight of a mass without knowing the exact arrangement of all the layers, but only with precise average thickness and number of layers within a sequence. The equation is valid, as a first approximation, for any type of stratified mass, i.e., tabular, irregular, dipping, or folded.

In the case that a sequence is perfectly regular (Fig. 7a), determination of number of layers is rather simple and can be carried out by referring to the basic pattern of layers that constitutes the mass (e.g., in Fig. 7a n_s = 3 for the layer A and n_s = 1 for the layer B). If the sequence is irregular (Fig. 7b), it is impossible to refer to a basic pattern of layers that repeats itself regularly and therefore, the calculations can be more complex. In such conditions, it is recommended to use Eq. (24) or to consider a representative volume of the mass and to take as reference the total number of particular layers that make up this portion (e.g., in Fig. 7b n_s = 8 for layer A and n_s = 3 for layer B).

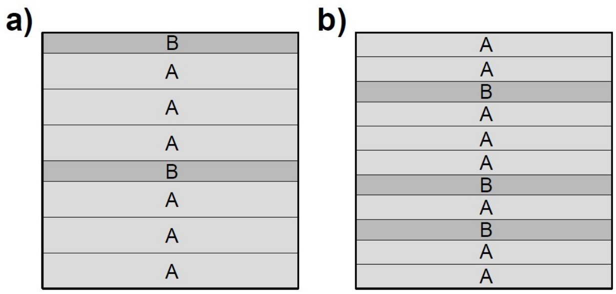

Fig. 7 - Examples of how to select the number of layers within a sequence: a) a sequence with regular layers arrangement; b) a sequence with irregular layer arrangement

Chaotic masses

Chaotic masses are made up of a set of elements lacking definite geometrical structure and repetitive sequences (Fig. 8). In this case, it is not possible to define precise structural characteristics and, therefore, necessary to make general assessments for the estimation of volumes. From a geological point of view, chaotic soil and rock masses are heavily tectonized and/or weathered, to a point that the original geological structures are no longer detectable within the mass. These types of masses are often constituted of accumulated sedimentary or detrital materials, mainly due to mass movement. Typical examples are cataclastic or mylonitic bands, landslides accumulations, and alluvial or detritic deposits.

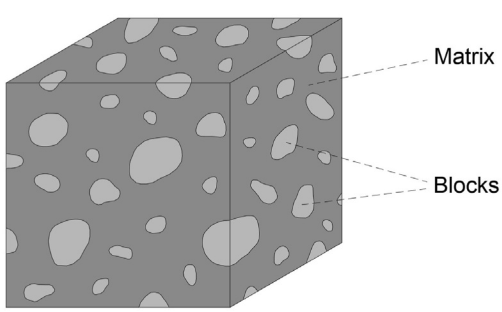

Fig. 8 - Schematic 3-dimensional representation of a chaotic mass formed by matrix with irregular blocks

Since a geometric treatment of the structural setting is not possible for these types of masses, it is necessary to consider the volumes of different materials without precise equations. In this case, it is essential to define the volumetric ratio of each material making up the mass V_c through the following relationship:

$$V_c = \frac{V_i}{V_t} \tag{28}$$

where V_i is volume of i-th material and V_t is total volume of the mass. The volume V_c must be expressed in values between 0 and 1, although it can also be reported as a percentage by suitably modifying the following equations.

Replacing this relationship in Eq. (8) it is possible to determine the unit weight of a chaotic mass γ_m such as:

$$\gamma_m = \sum_{c=1}^{n} \gamma_c V_c \tag{29}$$

where γ_c is the unit weight of the c-th material and 'n' is the number of discrete materials constituting the mass. Using Eq. (29), the accuracy at which the unit weight of a mass can be determined depend on the precision with which the volumetric ratio of each constituting components can be measured.

When the chaotic mass consists of blocks of regular size that can be divided into classes or groups of defined block volume, it is possible to assess the relative volumes of the various components accurately. In this case, the total volume of the blocks V_k can be determinable as:

$$V_k = \sum_{b=1}^{n} n_b V_b \tag{30}$$

in which V_b is the average volume of the blocks belonging to a certain class and n_b is the number of the blocks within the considered volume. Substituting Eq. (30) in Eq. (8) and considering the various elements that make up the medium, the unit weight of a chaotic mass γ_m can be modified in the following way:

$$\gamma_m = \gamma_p - \sum_{b=1}^{n} \frac{\gamma_p n_b V_b}{V_t} + \sum_{b=1}^{n} \frac{\gamma_b n_b V_b}{V_t} \tag{31}$$

and therefore:

$$\gamma_m = \frac{1}{V_t}\left(\gamma_p V_t - \gamma_p \sum_{b=1}^{n} n_b V_b + \sum_{b=1}^{n} \gamma_b n_b V_b\right) \tag{32}$$

where γ_p is unit weight of the matrix, γ_b is unit weight of the blocks, V_t is the total volume of the considered mass, and 'n' is the number of block classes.

Finally, simplifying the previous equation, unit weight of a chaotic mass γ_m consisting of regular blocks can be calculated as:

$$\gamma_m = \gamma_p + \frac{1}{V_t} \sum_{b=1}^{n} (n_b V_b)(\gamma_b - \gamma_p) \tag{33}$$

Eq. (33) demands an improvement in the estimation of the volumes of the various elements that make up the mass. Therefore, a fundamental aspect in the calculation of unit weight is proper subdivision of blocks inside a mass into homogeneous volumetric classes.

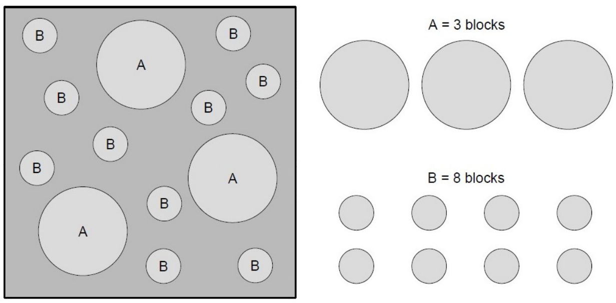

Fig. 9 - Example of how to select the volumetric classes of regular blocks

In general, it is suggested to divide the blocks into a limited number of classes, in which the elements of each class should be of quite similar average volume and homogeneous with respect to their physical characteristics (e.g., in Fig. 9 n_b = 3 for the blocks class A and n_b = 10 for the blocks class B). In fact, the division of the blocks into a large number of classes do not greatly improve the estimation of the physical parameters, instead, it can lead to significant errors in the evaluation of the number of elements within the analyzed mass volume.

VALIDATION OF THE PROPOSED METHODS

It is necessary to analyze the precision, reliability, applicability, and repeatability of the proposed equations to estimate the physical characteristic of different types of masses. For this purpose, 274 number of case studies were chosen to represent different types of considered masses (i.e., homogeneous masses, jointed masses, layered masses, and chaotic masses). The case studies are based on natural masses, for which geological and geomechanical surveys were conducted. Again, artificial masses were specially reconstructed to assess the variability of the geometric characteristics of each element (Fig. 10). For each mass, a geometrical model was reconstructed with AutoCAD Map 3D 2022 in order to accurately evaluate the volumes of every single component of the mass. In this way, it was possible to determine the unit weight of the model and compare the results with the values derived from the various proposed equations (Table 1).

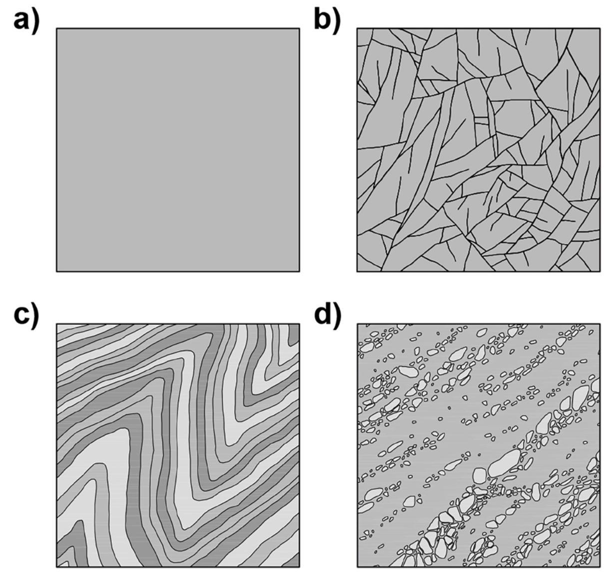

Fig. 10 - Examples of geometric models of the different types of masses used for the validation of the proposed equations: a) homogenous mass; b) jointed mass; c) layered mass; d) chaotic mass

			n. models	equations (y = ax)	r^2
Homogeneous masses	**General**		**63**	**1.0003x**	**0.9993**
	Perfectly continuous and homogeneous	*H1*	*6*	*1.0000x*	*1.0000*
	Poorly jointed with close or poorly opened joints	*H2*	*24*	*1.0033x*	*0.9995*
	Layered with thin and infrequent intercalations	*H3*	*18*	*1.0004x*	*0.9997*
	Chaotic with few blocks	*H4*	*15*	*0.9913x*	*0.9993*
Jointed masses	**General**		**76**	**0.9995x**	**0.9999**
	Sets of persistent joints	*J1*	*48*	*0.9996x*	*0.9999*
	Sets of non-persistent joints	*J2*	*8*	*0.9994x*	*0.9994*
	Non-systematic persistent joints	*J3*	*9*	*0.9995x*	*0.9995*
	Non-systematic non-persistent joints	*J4*	*11*	*0.9992x*	*0.9997*
Layered masses	**General**		**76**	**0.9999x**	**0.9999**
	Regular tabular	*S1*	*51*	*0.9997x*	*0.9999*
	Irregular tabular	*S2*	*7*	*1.0003x*	*0.9998*
	Regular folded	*S3*	*14*	*1.0002x*	*0.9999*
	Irregular folded	*S4*	*4*	*1.0010x*	*0.9999*
Chaotic masses	**General**		**59**	**1.0003x**	**0.9995**
	Irregular blocks	*C1*	*20*	*1.0005x*	*0.9989*
	Tectonized	*C2*	*6*	*0.9997x*	*0.9983*
	Regular blocks	*C3*	*33*	*1.0003x*	*0.9998*

Tab. 1 - Summary of the analyzed case studies, with indication of the number of models and the statistical parameters

For simplicity, the analyses were carried out on models in plane conditions, i.e., considering depth of the masses as a unit ($l_z = 1$). In this way, it is possible to simplify both the real models and the application of the equations. Since the analyzed physical parameters are not subject to anisotropy, all the proposed relationships can easily be transposed from a three-dimensional to two-dimensional model without affecting the accuracy of the obtained results. In order to consider the variability of the geometrical conditions of the different elements making up the mass, case studies with surface area of 5m x 5m were considered for the analysis.

Initially, 274 case studies were analyzed through the relationships generally used in the geological and geotechnical fields (Fig. 11). In the case of homogeneous and jointed masses, as well as for layered and chaotic ones where one lithotype is clearly predominant over the others, the values of the volumetrically most important medium were taken as reference. In the other cases, a simple average of all the unit weight of the lithotypes constituting the mass was considered. In general, the classic relationships for the determination of the physical characteristics of the masses have a fair correlation with the actual value of the same parameter ($r^2 = 0.9362$). The estimation is extremely accurate in the case of homogenous masses ($r^2 = 1.000$) and reliable enough for jointed ($r^2 = 0.9537$) and layered ($r^2 = 0.9634$) masses, while it exhibits comparatively a low r^2 (0.8597) in case of chaotic masses.

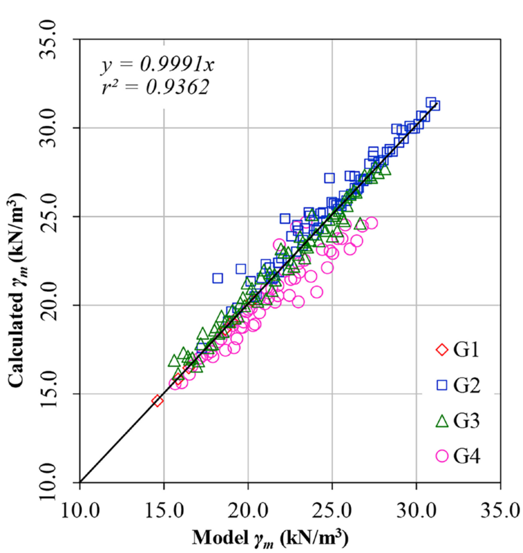

Fig. 11 - Correlation between real values and calculated values according to the relationships normally adopted in geotechnics: (G1) homogeneous masses; (G2) jointed masses; (G3) layered masses; (G4) chaotic masses

Despite the statistical values determined through the linear regression of the calculated parameters, preliminary analysis clearly highlights a few problems related to the rough estimation of the unit weight of studied masses. Although on average the errors remain within acceptable ranges of variation, for some masses the difference between the actual value and the estimated one can even reach 3.45 kN/m^2. As demonstrated below, the proposed equations are able to minimize both the standard deviation of the unit weight distribution and the difference between the actual and calculated values.

A total of 63 models were analyzed to evaluate the proposed equations for homogeneous masses (Fig. 12a). Out of these, some models are representative of perfectly continuous and homogeneous masses (n.6) while the rest represent jointed (n.24), layered (n.18), and chaotic (n.15) masses, which are assimilable at least in the first approximation to a continuous and homogeneous medium. The perfectly continuous and homogeneous masses were analyzed using Eq. (12), while the remaining masses were analyzed using Eq. (14). The analyses have shown that, under the described conditions, Eq. (12) and Eq. (14) guarantee a high degree of reliability and correctness in the results ($r^2 = 0.9993$). In fact, as visible from the graph, the estimates of the unit weight are perfectly congruent with those of the models analyzed and the maximum difference between the estimated and actual values of the whole series is equal to 0.24 kN/m^3. The proposed relationships are perfectly congruent in the case of continuous and homogeneous masses ($r^2 = 1.0000$). The relationships also show a high accuracy in the case of jointed ($r^2 = 0.9995$), layered ($r^2 = 0.9997$) and chaotic ($r^2 = 0.9993$) masses. Apparently, for truly continuous and homogeneous masses there are no discontinuities and inhomogeneities that can lead to a discrepancy between the estimated and the actual values. However, in the case of other masses, the measured differences are negligible if applied only to the types described above.

With regard to jointed masses, 76 models were analyzed (Fig. 12b). Out of these, some refer to masses with different sets of joints both persistent (n.48) and non-persistent (n.8), while others refer to masses with non-systematic joints, both persistent (n.9) and non-persistent (n.11). Jointed masses in which discontinuities can be grouped into families were analyzed using Eq. (19), while masses with unsystematic joints were analyzed using Eq. (21). The conducted analyses show that, for the described conditions, Eq. (19) and Eq. (21) provide a high degree of reliability and correctness in the results ($r^2 = 0.9999$). In this case, the graphs show that the estimates of the physical parameters are extremely congruent with those of the models and the maximum difference between the estimated and the real value of the whole series is equal to 0.14 kN/m^3. The proposed relationships are very accurate both for masses with different persistent ($r^2 = 0.9999$) and non-persistent ($r^2 = 0.9994$) joints systems. The equations are reliable also for masses with non-systematic persistent ($r^2 = 0.9995$) and non-persistent ($r^2 = 0.9997$) joints. In all cases, the orientation of the discontinuities does not affect the determination of the

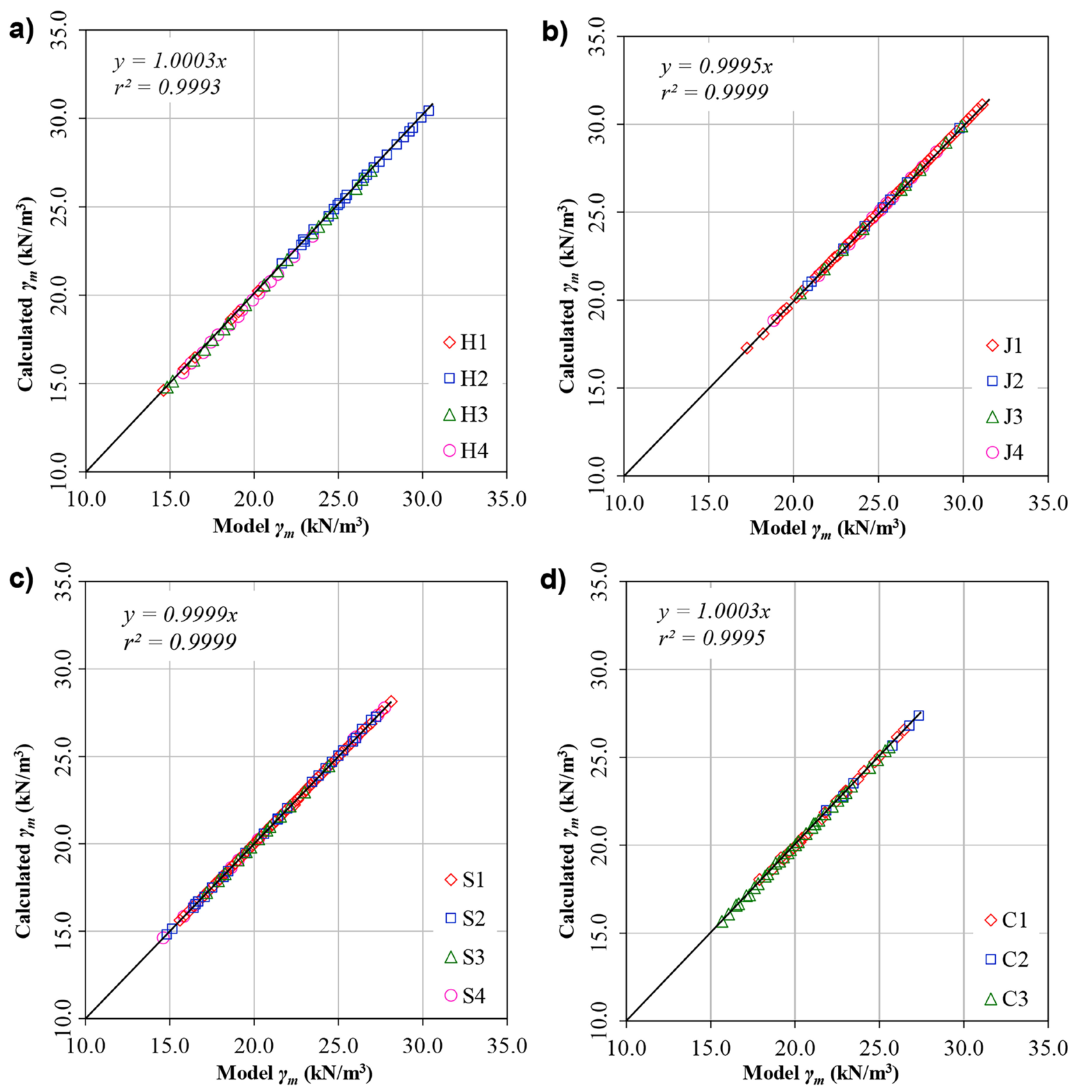

Fig. 12 - Correlation between real values and values calculated according to the equation determined for the different types of soil and rock masses: a) homogenous masses, b) jointed masses, c) layered masses, and d) chaotic masses. In a): (H1) perfectly continuous and homogeneous masses, (H2) slightly jointed masses with closed or slightly open joints, (H3) layered masses with thin and infrequent intercalations, (H4) chaotic masses with few blocks. In b): (J1) masses with sets of persistent joints, (J2) masses with sets of non-persistent joints, (J3) masses with persistent non-systematic joints, (J4) masses with non-persistent non-systematic joints. In c): (S1) regular tabular masses, (S2) irregular tabular masses, (S3) regular folded masses, (S4) irregular folded masses. In d): (C1) masses with irregular blocks, (C2) tectonized masses, (C3) masses with regular blocks

parameters in question, while it is essential to accurately estimate the average length and the persistence of the joints.

For the volume of intersections between different discontinuities introduced generically in the proposed equations, it is observed that they have negligible influence on the values of unit weight calculated. Obviously, in the case of masses with numerous and widely open discontinuities (greater than 10-11 cm), it may be appropriate to consider the intersection elements and rectify the calculated values. Given the complexity of the analytical treatment, it is suggested to determine the number of intersections and the volume of the same through careful observation and measurement of natural rock exposures.

A total of 76 models were analyzed for the layered masses (Fig. 12c). Of these, many are representative of regular tabular masses (n.51), while the rest are made up of irregular tabular masses (n.7), regular folded masses (n.14), and irregular folded masses (n.4). The tabular and folded masses with irregular sequence were analyzed by Eq. (24), while the tabular and folded masses with regular sequence were analyzed by means of Eq. (27). For the described conditions, Eq. (24) and Eq. (27) provide a high degree of reliability and correctness in the results (r^2 = 0.9999). The graphs show that the estimates of unit

type	*description*		*equation*
General	All type of masses	(8)	$\gamma_m = \sum_{i=1}^{n} \frac{\gamma_i V_i}{V_t}$
Homogeneous masses	Perfectly continuous and homogeneous masses	(12)	$\gamma_m = \gamma_h$
	Jointed, layered and chaotic masses assimilable to continuous and homogeneous masses	(14)	$\gamma_m \cong \gamma_v$
Jointed masses	Jointed masses with several sets of persistent and non-persistent joints	(19)	$\gamma_m = \gamma_r - \sum_{j=1}^{n} \frac{\gamma_r a_j l_m l_j^2}{V_t (s_j + a_j)} + \sum_{j=1}^{n} \frac{\gamma_j a_j l_m l_j^2}{V_t (s_j + a_j)}$
	Jointed masses with non-systematic and non-persistent joints	(21)	$\gamma_m = \gamma_r - \sum_{u=1}^{n} \frac{0.9\, \gamma_r J_u a_u l_u^2}{V_t} + \sum_{u=1}^{n} \frac{0.9\, \gamma_u J_u a_u l_u^2}{V_t}$
Layered masses	Layered masses with irregular sequence	(24)	$\gamma_m = \frac{\sum_{s=1}^{n} \gamma_s S_s}{\sum_{s=1}^{n} S_s}$
	Layered masses with regular sequence	(27)	$\gamma_m = \frac{\sum_{s=1}^{n} \gamma_s S_s n_s}{\sum_{s=1}^{n} S_s n_s}$
Chaotic masses	Chaotic masses tectonized or with irregular blocks	(29)	$\gamma_m = \sum_{c=1}^{n} \gamma_c V_c$
	Chaotic masses with regular blocks	(33)	$\gamma_m = \gamma_p + \frac{1}{V_t} \sum_{b=1}^{n} (n_b V_b) \left(\gamma_b - \gamma_p\right)$

Tab. 2 - Summary of the proposed equations for the different types of soil and rock masses, with indication of the methods and fields of use

weight are congruent with those of the models. The maximum difference between the estimated and actual value of the whole series is equal to 0.17 kN/m^3. The proposed relationships are extremely accurate both for regular (r^2 = 0.9999) and irregular (r^2 = 0.9998) tabular masses, as well as for regular (r^2 = 0.9999) and irregular (r^2 = 0.9999) folded masses. Also in this case, the orientation of the layers and the relative regularity do not affect the determination of the physical parameters of the medium, while it is particularly important to accurately estimate the average thickness of each layer and its relative frequency in the series.

For the evaluation of chaotic masses, a total of 59 models were analyzed (Fig. 12d). Of these, many are representative of masses with irregular blocks (n.20) or tectonized (n.6), while the remaining ones refer to masses with regular blocks (n.33). The masses with irregular blocks and the tectonized ones were analyzed with Eq. (29), while the masses with regular blocks were analyzed through Eq. (33). The conducted tests show that, for the described conditions, Eq. (29) and Eq. (33) provide a good degree of reliability and correctness in the results (r^2 = 0.9995). In fact, for these masses the graphs show that the estimates of the physical parameters are congruent with those of the models and the maximum difference between the estimated and the real value of the whole series is equal to 0.15 kN/m^3. The proposed relations are accurate for masses with irregular blocks (r^2 = 0.9989), for tectonized masses (r^2 = 0.9983), and also for regular blocks (r^2 = 0.9998). For all chaotic masses, the determination of the unit weight is always very reliable as long as it is feasible to make a precise estimation of the volumetric percentages of the elements or the dimensions of the blocks.

SUMMARY AND DISCUSSION

As a part of the present study, a series of analytical equations have been developed for the determination of unit weight and density of different types of natural soil and rock masses (Table 2). In particular, these natural materials were classified as homogeneous masses, jointed masses, layered masses, and chaotic masses. As visible from the analyses carried out from the results of the case studies, all the proposed equations show high precision and accuracy. The standard deviation of the different series of relations is not very high ($r^2 \geq 0.9983$), and the difference between the estimated and real value never exceeds 1.3%. The equations can determine respective parameters at a higher accuracy for masses with simpler geometries described with less complicated analytical models. Under the defined conditions, the equations provide reliable and sufficiently accurate results.

Considering the effectiveness of the proposed equations, a fundamental aspect is represented by the data acquisition methods used for the calculations. The latter directly affect the obtained results and, therefore, are of primary importance in studying the physical characteristics of these materials. For all types of masses, the unit weight and density of the intact materials can be acquired through conventional laboratory tests. Alternatively, these parameters can be estimated by means of bibliographic sources or by indirect correlations with other characteristics of the media (i.e., elastic wave velocity, penetrometer tests, index parameters, etc.).

The geometric characteristics and the relative volumes of each element constituting the mass, on the other hand, can be determined through field surveys. The proposed equations facilitate direct use of the data derived from the geomechanical surveys of rock fronts, conducted according to the classical survey standards (ISRM, 1978). For other types of masses, the data of a classical geological survey are more than sufficient to provide all the parameters necessary for the application of the equations. An alternative way to obtain the data is by analyzing the borehole logs, which are always fundamental in the study of natural masses. In this case, it is generally sufficient to estimate the volumetric fractions of the different components and determine the parameters in question using the general Eq. (8).

All the proposed equations have been developed for the determination of the main physical properties of soils and rock masses, such as unit weight and density. However, they are applicable in a similar form to any type of physical and mechanical characteristics of these media depending only on the volumetric percentages of the elements that make up the masses and not directly influenced by the effect of the anisotropy (i.e., porosity, void index, degree of saturation, etc.).

CONCLUSIONS

In the present study, a series of equations have been developed for the determination of the major physical properties of natural soil and rock masses, namely, unit weight and density. The proposed equations are based on the relationship known for continuous and homogenous materials, which are then properly modified to consider the structural characteristics of the natural soils and rocks. These masses were divided into four categories: (i) homogenous masses; (ii) jointed masses; (iii) layered masses; (iv) chaotic masses.

The relationships developed in this study are based on the analysis of the physical properties of the different elements that make up the mass and their relative volumes within it. In particular, the medium was treated through an "equivalent continuum approach", which allows to simplify the characteristics of the material and to apply the same to masses of considerable volume and extension. The approach is based on a limited number of input data and is therefore easily applicable both for the purpose of research purpose and practical applications.

Every single equation was validated through the application to several real cases, representative of the different analyzed conditions. The case studies were modeled by software in order to accurately determine the volumes of each single element constituting the mass. To analyze the statistical reliability, the obtained results were then compared with those derived from the application of the proposed equations.

In all the studied cases, the proposed relationships are extremely accurate and precise. Therefore, they allow determining both the unit weight and density of any type of natural soil and rock masses based on a few basic data. The errors in the estimation of the parameters are always extremely low and well below the accuracy required in normal geological and geotechnical studies.

The developed approach in standardized and repeatable for any type of natural medium, without prejudice to the prescriptions provided in the application of each equation. A fundamental aspect is represented by the basic data acquisition method, which directly affects the obtained results. The physical parameters of the materials can be obtained through conventional laboratory tests or, alternatively, through bibliographic sources and indirect correlations. The data on the masses and the relative structural characteristics can be established according to the conventional geological and geomechanical surveys, as well as on the basis of the borehole logs.

The equations have been developed for the determination of unit weight and density but can also be easily applied to other physical and mechanical characteristics of the masses (i.e., porosity, void index, degree of saturation, etc.). This can be achieved by leaving the general formulation unchanged and replacing the chosen parameter with those that are to be analyzed. Evidently, the relationships can be applied only to parameters not influenced by the effects of anisotropy, but which depend solely on the volume of the elements that make up the mass.

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