# An Image of Vanitas: Geometrical Optics and Shakespearean Points of View 

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An image concerning mirror-imaging

Vanitas as well as Superbia are clearly referenced in an intriguing illustration to Der Ritter vom Turn (Fig. 1), which was Marquard vom Stein's 1493 German translation of Chevalier Geoffroy de La Tour Landry's Livre pour l'enseignement de ses filles (c. 1372). This woodcut, quite possibly made by Albrecht Dürer, shows a young woman standing in front of a small mirror grooming her hair while, behind her, a naked horned demon deliberately crouches in order to display his anus and scrotum.


Fig. 1

At the moment captured in this illustration, the girl is clearly viewing her own reflected image with equanimity and pleasure, quite oblivious to the diabolical companion standing behind her ${ }^{1}$. Conversely, as seen from the point of view of the observer implied by the illustration (I will call this "our" point of view), the small mirror on the wall shows only the demon's posterior parts and does not show the girl at all. Thus, in the same mirror, at the same moment, what the girl sees reflected is satisfactory and appealing, and what we see reflected is diabolical and disgusting. Importantly, we as onlookers are able to see the demon as well as the girl and thus are placed so as to see not only that which the girl cannot see, but also to see that she cannot see it. Eventually, I will attempt to bring such a configuration in line with theatrical gestures that offer Shakespeare's audiences insights into the incomplete or distorted perceptions of dramatic characters.

To prepare for this, I will examine in detail some material aspects of the scene portrayed in the Ritter vom Turn woodcut, and

1 Both text and the woodcut illustration provide no support for the view that "[t]he girl in the engraving from the Ritter vom Turn does not seem to be afraid of the horrible vision, almost as if she were familiar with this image of herself. Perhaps a witch, she holds the mirror aloft while waiting for the sabbath dance of spirits to begin again, a mobile and inconstant figure of an unstable world" (Sabine MelchiorBonnet, The Mirror: A History, trans. Katharine H. Jewett, London, Routledge, 2014, p. 208). Rather, as will be argued, the girl simply does not (yet) see the demon.
in particular how geometrical optics make its peculiar arrangement credible. I will also mention that the optical principles supporting this plausibility, although understood since classical times, acquired a deeper significance in Shakespeare's period. The intellectual and cultural conditions that helped to enable that increment in scientific knowledge may have also influenced artistic practices - such a notion is of course not provable but is perhaps still worthy of consideration.

First, however, before considering the geometry of the mirroring depicted in our woodcut, we may note how its topic, female mirror-use, accumulated complex and multiple cultural valences. One polarity is emphasized in the source of the Ritter vom Turn text, a chapter of Landry's Livre pour l'enseignement de ses filles that is headed "Of a lady that dispended the fourthe parte of the day for to araye her capitulo" (quoted here from Caxton's 1484 translation). This chapter explains that "She was alweye acustomed for to be long to araye her And to make her fresshe and gay" which "annoyed and greued moche the parson of the Chirche and the parysshens" because she repeatedly asked them to "tarye for her". Therefore, "somme said softely. god sende to her an euyll syght. in her myrroure that causeth vs this day and so oftymes to muse \& to abyde for her. \& thene as it plesyd god for an ensample. as she loked in the Myrroure she sawe therin the fende whiche shewed to her his hynder parte so fowle and horryble that the lady wente oute of her wytte". Finally, "god sente to her helthe And after she was not so long in arayeng but thanked god that had so suffred her to be chastysed" ${ }^{2}$.

However, later in the very same text, Landry represents mirroring as morally exemplary (which indeed was an extremely common figurative usage). Thus chapter 41 in Caxton's edition describes the "good wymmen" of the "byble" who "were the myrrour and exemplary to alle other of that tyme that now ben \& to them that ben yet to come" ${ }^{3}$.

[^0]Here we encounter a contrast between women using mirrors unworthily and women serving as mirrors worthily (even the Virgin Mary was so represented in some traditions) ${ }^{4}$. However, other traditions contrast two kinds of mirror use, as we can see in Thomas Salter's 1579 A mirrhor mete for all mothers, matrones, and maidens, where a kind of mental mirroring is associated with selfcontemplation:

In my iudgemente there is nothyng more meete, especially for yong Maidens then a Mirrhor, there in to see and beholde how to order their dooyng, I meane not a Christall Mirrhor, made by handie Arte, by whiche Maidens now adaies, dooe onely take delight daiely to tricke and trim their tresses, standyng tootyng twoo howers by the Clocke, lookyng now on this side, now on that, least any thyng should bee lacking needefull to further Pride, not sufferyng so muche as a hare to hang out of order, no I meane no suche Mirrhor, but the Mirrhor I meane is made of an other maner of matter, and is of muche more worthe then any Christall Mirrhor; for as the one teacheth how to attire the outwarde bodie, so the other guideth to garnishe the inwarde mynde, and maketh it meete for vertue, and therefore is intituled a Mirrhor, meete for Matrones and Maidens, for Matrones to knowe how to traine vp suche young Maidens as are committed to their charge and tuission, and for Maidens how to behaue them selues to attaine to the feate of good fame. ${ }^{5}$

Near the climax of Purgatorio XXVII, Dante, who "speaks of mirrors explicitly in thirty separate passages" ${ }^{6}$, offers yet another valuation of females employing mirrors. Here the Biblical Leah appears to the narrator Dante in a dream vision saying that she "go[es] moving my fair hands around to make me a garland. / To please me at the glass [specchio] here I deck me". Leah adds next, "but Rachel my sister ne'er stirs from her mirror, and sitteth all day, / She is fain to behold her fair eyes, as I to deck me with my hands:

4 See Mark Pendergrast, Mirror Mirror: A History of the Human Love Affair with Reflection, New York, Basic Books, 2003, pp. 121-23.
5 Transcribed from Chadwyck Healey EEBO page images 6-7 of A mirrhor mete for all mothers, matrones, and maidens, intituled the Mirrhor of Modestie no lesse profitable and pleasant, then necessarie to bee read and practiced [STC (2nd ed.) / 21634]. This text is partially discussed in Carol Banks, "'The Purpose of Playing...': Further Reflections on the Mirror Metaphor in Shakespeare's Plays", Signatures, 2 (2000), pp. 1-12.
6 H. D. Austin, "Dante and Mirrors", Italica, 21:1 (1944), pp. 13-17: p. 13.
her, contemplation; me, action, doth satisfy" (ll. 97-108)7. I do not believe that there is any implicit condemnation here of Leah's active-life artistic use of a mirror to create beauty, even to adorn herself. The contemplative Rachel also pursues beauty (in "fair eyes", not a garland), and there may be a hierarchy implied favouring her mirror use for meditative retreat ${ }^{8}$. Yet Dante states explicitly in Convivio that "it is to be known that we can have in this life two happinesses by following two different roads, both good and excellent which lead to them; one is the Active life and the other is the Contemplative life" 9 . For further confirmation, we may consider the physical experiment proposed by Beatrice in Paradiso II (91-105) ${ }^{10}$, in which three mirrors are arranged with the third further away from the viewer than the other two. The light source set up for this experiment will be reflected as smaller in the more distant mirror but will not be less bright in that refection - this purports to explain the true cause of variations of brightness on the surface of the moon but also may be read to suggest that the material world of action, located at a greater distance from the heavenly source of light, is no less bright than the nearer world of contemplation.

Dante's account of Leah's and Rachel's differing mirror usage, juxtaposing artistic and spiritual practices that are pursued in the material and immaterial realms respectively, does not denigrate either. In similar ways, many Renaissance artists, including Shakespeare, strove to convey complex impressions involving differing, or even contrary, visions of matters at hand. Let us next investigate how the depiction of mirror-imaging in the Ritter vom Turn woodcut brings that motif to life.

## The geometry of the illustration

Mirrors produce what are known as 'virtual' optical images, which is to say images giving rise to the visual impression that an object

[^1]is located in a place other than where it is actually situated. Such displaced images may be produced by refractions due to nonuniformities of transparent media or by various sorts of reflection. Here we will consider only the least complex of such alternatives, situations where virtual images are formed by plane (flat) mirrors. We will also leave for consideration elsewhere the artistic and other impacts of circumstances in which mirrors are significantly imperfect in terms of flatness, reflectivity, or colouration ${ }^{11}$.

Shakespeare's age spoke of "mirror images" and inherited a long tradition of catoptrics explaining their functioning but did not yet have the terminology of "virtual images". Thus we find in the second edition of the OED, under "image", subsection 2.a, a citation from Hall's 1548 Chronicle of Richard III (a text certainly familiar to Shakespeare): "As perfectely as I sawe my awne Image in a glasse" (III 34 v$)^{12}$. The same edition of the OED, under "virtual a.", subsection 4.c., headed "Optics", dates its earliest citation for a "virtual [...] image or focus" to 1704 . The third edition of the OED quotes exactly the same passage from Hall's Union under its subsection "image n. 3.a." but cites a slightly earlier text illustrating "virtual a. 5" (a subsection headed "Physics"), noting the use of the term "virtual image" in William Molyneux's 1692 Dioptrica nova.

The earliest mentions of "virtual images" post-dated Shakespeare's age probably because the understanding of the distinction between a virtual image and the contrary sort of optical images that can be projected on a screen was not yet well formed ${ }^{13}$. However, in precisely Shakespeare's time, the understanding of the

[^2]production of virtual images did take a great step forward. Such matters are worth examining in detail.

First, we should note that, because our topic will be geometrical "ray" optics, we will be free to overlook classical theories holding that visual perception arises when eidola or simulacra of seen objects proceed through the air ${ }^{14}$. Likewise, we need not consider distinctions between "extramission" and "intromission" (or mixed) theories of vision, that is to say, between theories proposing that eyes send out visualizing rays versus theories proposing that seen objects send light rays to eyes. Such much-discussed distinctions do not impact on ray geometry because the geometrical relations will be identical regardless of the direction in which the visual or light rays are thought to progress.

For convenience, we will assume the validity of the intromission model that was widely adopted by Shakespeare's time ${ }^{15}$. In fact, there was a great increment of understanding in Shakespeare's age regarding visual intromission, for, in 1604, Kepler established (following others' hints) that the boundary between the (geometrically traceable) light rays producing vision and the further physiological and psychological functions that enable visual perception lies precisely at the retina at the back of the eye. The realization that an interface between mechanism and organism lies exactly at a retina, which acts as a screen upon which material optical forces impinge, arguably constituted a great epistemological shift ${ }^{16}$. For then the light rays focused on the retina by the lens of the eye (upside down and reversed) will constitute the only and entire external stimulus to the visual system.

[^3]After considering the optical processing of physical inputs by the lens of the eye, Kepler's analysis leaves the remainder of human visual perception unexplained. However, Kepler does provide the crucial insight that whatever we see is produced by, and only by, light rays entering our eyes, so that, however much those rays are steered or deflected by external conditions, we will perceive them exactly as if they had proceeded directly (in straight lines) from apparent sources. This understanding served Kepler's initial aim, which was to analyse how atmospheric distortions of the paths of light rays impact on astronomical observations ${ }^{17}$. Pursuing this aim made Kepler into a profound optical scientist as well as a profound astronomical one.

The technical starting point for the geometrical optics of intromission is the principle that the visible surface of an illuminated object (or illuminating light source) is made up of innumerable bright points, each of which sends out light rays in straight lines in all directions. Such rectilinear rays may then be analysed using Euclidean geometry. The crucial principle that rays of light or vision follow straight-line paths in free space is enunciated in Euclid's Optics, is reiterated in the Euclidean Catoptrics (which deals with mirrors) ${ }^{18}$ and is assumed by all geometrical optical writers.

We should appreciate that this straight-line principle was always understood in sophisticated ways. It could be taken to express how Platonic ideals are approximated when partaken of in an imperfect world, and, crucially for us, it was also used to provide an invaluable tool for making analysis and calculations possible. Such sophistication is made explicit in John Dee's Mathematicall Praeface to Billingsley's 1570 translation of Euclid's Elements of Geometrie: "there are other (very many) Methodicall Artes, which,

[^4]declyning from the purity, simplicitie, and Immateriality, of our Principall Science of Magnitudes: do yet neuertheles vse the great ayde, direction, and Method of the sayd principall Science" ${ }^{19}$.

Dee's "principall Science", Euclidean geometry, deals with pure immaterial points and lines that do not correspond exactly with that which can be actually seen and measured. For Euclidean lines may have orientations and locations, but no other dimensions except length, and Euclidean points have no dimensions at all, but only positions. However, certain objections heard that dimensionless points cannot emit light, or that an ensemble of discrete thin rays cannot delineate a continuous surface, are misconceived ${ }^{20}$. On the contrary, the visible surface of an illuminated or illuminating object can be quite exactly modelled as if it were made up of an infinite collection of infinitesimal points, each one emitting infinitely many infinitesimally thin rays. The exact logical justification for such long-used methods of analysis was at last fully expounded in the nineteenth century, but the methods were applied long before and found wholly adequate for use.

Indeed, the techniques of geometrical ray tracing have been pursued from Euclid's time to the present. Conceptualising and plotting the paths of infinitely narrow rays of light has not at all been replaced by more recent concerns with "pixels", which are tiny but not infinitely small areas of illumination; modern technologists do not shun Euclidean geometry (which is still of vital importance to them) but refer to discrete and countable light sources and receptors because current-day light sensing, light emitting, and light analysing devices and software are by nature constrained to dealing with finite numbers of positions or directions in space. Yet the aim of modern optical technologists is

[^5]still to approximate (by means of discrete elements) the infinite continuum of an ideal Euclidean space.

Next, we come to the question of what happens when light rays encounter partial impediments such as mirrors or lenses that may alter their direction but will not stop their progress. Here, as stated above, we will concern ourselves only with reflections by plane mirrors. Again of great antiquity is the principle that a ray will bounce off a mirrored surface (one capable of "specular reflection") so that the angle between a line "normal" (that is perpendicular in all directions) to the mirrored surface and the original ray (the socalled "angle of incidence") is equal to the angle between the same normal and the reflected ray (the so-called "angle of reflection"). This principle was demonstrated upon one basis in proposition one of Euclid's Catoptrics ${ }^{21}$ and upon another basis in propositions one and two of Hero of Alexandria's Catoptrics ${ }^{22}$. It was also the third principle of Ptolemy's Optics, where an ingenious experimental setup for its verification was proposed ${ }^{23}$. A different "systematic empirical verification" of this principle was also described in book four of Alhacen's De aspectibus, where the experiments proposed were arranged with "exquisite care" ${ }^{24}$.

Showing that, long ago, the "two cultures" were not disjoint, a simile in Dante's Purgatorio XV provides an exact description of the equal-angle principle:

As when a ray of light leaps from the water or from the mirror to the opposite direction, ascending at an angle similar to that which it descends, and departs as far as the line of the falling stone in an equal space, even as experiment and science shows. (ll. 16-22)

Here Dante envisions the reflecting surface of a body of water or of a mirror placed horizontally, so that "the line of the falling stone" will be normal to the plane of reflection, and he accurately describes the equality of the angles of incidence and reflection.

[^6]In addition, as Ptolemy showed explicitly ${ }^{25}$, the incident and reflected rays and a normal meeting them on the surface of a mirror will all lie in one single plane. If we are considering a planar (flat) mirror, this plane containing the incident and reflected rays must be plane perpendicular to the plane that contains the mirror's surface (because it also contains a normal to the mirror's surface). In consequence, when viewing towards this plane containing the two rays and the normal, we will see the (flat) mirror's plane edgewise, and it will appear as a straight line segment. In Fig. 2, the line segment $\mathrm{MM}^{\prime}$ is the edge view of a mirror, and the line PN is the normal to the mirror's surface at the point P , where a light ray emanating from the illuminated object O is reflected toward a viewer at position $V$. The incoming light ray OP and the normal PN thus form the angle of incidence OPN, and the reflected light ray PV and the normal PN form the angle of reflection VPN. These two angles have been shown to be equal, as Dante put it, "even as experiment and science shows". Additionally, crucially, the reflected light ray PV will be projected onto the retina of an observer at V exactly as if it came from the direction of the line PV and its dotted extension:
figure 002


Fig. 2

Now let us add some additional dotted lines, the first being the extension in both directions of the mirror surface line $\mathrm{MM}^{\prime}$ to the long line AB - this line represents an edgewise view of an extension
of the plane surface of the mirror (we may for convenience call this extended plane the "mirror-wall plane", imagining it to be the edge view of an indefinitely large wall upon which the mirror is hung). Add also the line OR perpendicular to AB and continuing through it (in three dimensions, this perpendicular line will be normal to the mirror-wall plane because it is included in a plane that is perpendicular to the mirror-wall plane). Place the label Q on the point of intersection of OR with AB. Also extend the line VP to the right, carrying it beyond AB (that will correspond, in three dimensions, to an extension of the viewer's line of sight beyond the mirror-wall plane), and place the label I on the intersection of this line with line OR. It will be shown that OQ = QI. This is to say, the point I lies just as far behind $A B$ (and thus behind the mirror-wall plane) as the illuminated point O stands in front of it (Fig. 3).


Fig. 3

The proof that $\mathrm{OQ}=\mathrm{QI}$ is quite straightforward. Triangles OQP and IQP are congruent because they share the side PQ, while the angles PQO and PQI are equal (both being right angles) and the angles QPO and QPI are equal because of the rule that, in mirroring, the angle of incidence equals the angle of reflection (both angles are a right angle less the angle of incidence or reflection). The congruence then arises because the two triangles have two pairs of angles equal and the sides between these angles also equal. In consequence, the corresponding sides OQ and QI must be equal.

This means that point I lies exactly as far behind the mirror-wall plane (measured along the normal OR) as the bright point O is in front of the mirror-wall plane. Next, let us consider the significance of the fact that the point I occupies the same point in space regardless of the vantage point of the viewer of reflections.

Because (as long as the location of point O remains fixed) the location of point I behind the mirror-wall plane will be the same for any viewer-positioning point V , if a single observer sequentially moves to differing vantage points (or uses their two separate eyes at the same time), they will always view the mirror image of point O as if it lay along a line (not always the same line) that passes through the point $\mathrm{I}^{26}$. This behaviour of the various light rays that reach various observers, that all such rays point towards the fixed-in-space point $I$, is the exactly same as would obtain if an actual bright point were located at I. In Keplerian terms, and on the assumption that a plane mirror is perfect ${ }^{27}$, the retinas of an observer or observers cannot distinguish between viewing a reflection of a point apparently at I and seeing an actual bright point located at I. Finally, because each reflected point will have these same properties regardless of the position from which it is viewed, "[entire] images in plane mirrors appear to be the same size as their objects and also appear to lie the same distance from the reflecting surface as their objects" (as Euclid and Claudius Ptolemy showed) ${ }^{28}$.

26 Any viewer's line of sight toward the mirror image of a given point O will always join their vantage point V to the point I (which is fixed in position in relation to the mirror and O ). This means that, if two different vantage points, say V and W , are considered, all the angles (of incidence, of reflection) in the diagram corresponding to our Fig. 3 may differ from one another, and yet the two (or any number of) lines of sight will intersect at point I. The intersection of the two reflected rays reaching a single person's two eyes must therefore also be point $I$, and that is the basis of that person's perception of the distance between their vantage point and point I on the image that they see.
27 As Jonathan Miller explains in great detail, a perfect plane mirror will not betray its presence by revealing its surface, yet, in many artistic representations, as in life, either imperfections or circumstantial clues make mirrored surfaces apparent (see On Reflection, London, National Gallery Publications, 1998, pp. 57-133).
28 Miller thus summarizes proposition nineteen of Euclid's Catoptrics (p. 61). Miller also summarizes an identical conclusion in Ptolemy's Catoptrics which is claimed to be demonstrated "with somewhat greater mathematical rigor" (p. 98).

To this we must append a significant caveat: all who gaze in a flat mirror - regardless of where they stand - will perceive the reflection of object point O in real space at the same position I in virtual (behind the mirror) space, provided they can see the point I in the mirror at all. And there is the catch: if a mirror is of finite size (as all real mirrors must be), it will present only a bounded window into virtual mirror space - this finitude is represented in our planar diagrams by the limited length of the line segment $\mathrm{MM}^{\prime}$. A consequence is that not all observers placed in all positions are be able to see all the image points visible to observers standing in other positions. To be exact, any observer standing outside of the shaded area in Fig. 4 below will not be able to see the image point I corresponding to the object point O when they look into the mirror $\mathrm{MM}^{\prime}$. Instead, when they look in the direction where they might see the virtual image point I , their eye will meet a blank section of wall where no reflection can be seen:


Fig. 4
But are we justified in applying to all mirror observers the above analysis showing the existence of unshaded areas filled with what automobile drivers call "mirror blind spots"? For, admittedly, the plane in our planar diagram contains some but not all possible observers. The answer is "yes", because, for any new observer, a new planar diagram can be drawn containing that observer's viewpoint and the normal OI. That new plane will therefore still
contain the object point O and the image point I , and these will remain in the same places in three dimensional (real or virtual) space as before. When looking toward our new plane, the flat mirror will still be seen edgewise (because this plane contains a normal to the mirror plane). So the mirror will still be represented in the new plane as a line segment, although one of perhaps a different length than the line segment $\mathrm{MM}^{\prime}$. Yet, if the mirror has a finite overall size, this must still be a line segment of finite length. Hence, by doing the same analysis in this new plane as before, we can show that the reflection of an object visible from one point of view can still be invisible from other points of view ${ }^{29}$.

To summarize, we may enunciate the dual principles that: 1) from any vantage point, any image of an illuminated point that is visible in a plane mirror will be seen exactly as if it lay at a distance behind the plane containing the mirror equal to the distance in front of the plane of the mirror of the point reflected; and 2) mirror-images that are visible from some places may be invisible from other vantage points. These two principles allow us to illustrate exactly how an onlooker, a girl, a demon and a mirror may be arranged to produce the appearances shown in the Ritter vom Turn woodcut. In Fig. 5 below, G is a representative point on the girl (for simplicity, we will assume it is also the place from which the girl views the mirror), D is a representative point on the demon, V is the location of the observer, and $\mathrm{MM}^{\prime}$ represents the extent of a flat mirror hanging on the flat wall AB. Let G' be the fixed position (in virtual behind-the-mirror space) of the mirror image of G and $\mathrm{D}^{\prime}$ be the fixed position (in the same virtual mirror space) of the mirror image of D . In our illustrative diagram, the line of sight $\mathrm{GD}^{\prime}$ meets the wall where

[^7]there is no mirror so that the girl peering in her mirror will not see a reflection of the demon crouching behind her. However, the line of sight $\mathrm{VD}^{\prime}$ does pass through the mirror so that the reflection of the demon in the mirror is visible to the observer at V . The demon and girl are both directly visible to the observer at V along the lines of sight VD and VG. The line of sight $\mathrm{VG}^{\prime}$ strikes the wall where there is no mirror, so the observer cannot see a reflection of the girl in the mirror, but the line $G^{\prime}$ does pass through the mirror so that the girl can see her own reflection. In summary, our diagram shows it possible for the girl to see her own reflection in the mirror, but not the demon's, and for the observer to see the demon's reflection in the mirror, but not the girl's, while, at the same time, the observer can see both the girl and the demon directly.


Fig. 5

Those who can easily envision three dimensional spatial relations may find the configurations diagrammed above obvious. However, the variability of mirror images according to a viewer's vantage point has elicited wonderment even in the extremely observant Leonardo da Vinci. For Leonardo presents as surprising the outcome of an experiment that he proposes involving two observers standing beside one another in front of the same flat mirror. After diagramming the equal-angle principle and showing how rays of vision can be traced interchangeably in either direction,

Leonardo concludes: "if you touch the eye of the other man in the mirror it will seem to him that you are touching your own [eye]"30.

Many other Renaissance figures wondered at or found mystification in the effects of mirrors deliberately arranged to confound vision ${ }^{31}$. A. Mark Smith explains that the founding figures of geometrical optics seemed obsessed with how "mirrors could be manipulated in various ways to create startling illusions", and so reports that "[e]ven Euclid felt compelled to discuss such illusions in propositions thirteen to fifteen of the Catoptrics", while "the concluding eight" of the eighteen propositions in Hero of Alexandria's Catoptrics "are concerned how to arrange plane mirrors to produce startling visual effects". Smith cites the last of Hero's propositions as "a prime example" of his point. It "promises to show how 'to put a [flat] mirror in a given place so that everyone who approaches will see neither himself nor someone else, but only whatever picture someone has chosen in advance'"32. This and similar "fun-house" illusions, as Smith calls them, depend on magicians not revealing their arrangements of trick mirrors. However, in our woodcut above and also in certain Shakespearean scenes to be addressed presently, spectators are positioned so that they can actually perceive the processes whereby vision is distorted or deluded thanks to oblique outlooks. In such cases, the artist-

[^8]magician takes pains to reveal, rather than to conceal, the bases of false vision.

## Why mirrors?

I believe that mirrors and mirror-imaging were obsessive topics for so long, and increasingly so in the late Renaissance ${ }^{33}$, largely because they are capable of defamiliarizing the visual realm by drawing attention to the place of perceivers in perception. Thus, according to Sabine Melchior-Bonnet, a 1567 treatise finding the mirror "a master of illusion" demonstrates
> how relationships between objects could be made deceptive by the diversity of points of view and positions that mirrors made possible. Without a fixed, unique, and objective reference point, that embraces the totality of perspectives, the spectator could never verify the preciseness or accuracy of his point of view. ${ }^{34}$

However, a different often-heard claim is that mirrors became increasingly fascinating during the later European Renaissance because people increasingly viewed their own reflections, thereby "discovering" that they had selves. I have some doubts about this oddly Eurocentric position, for mirror uses of various sorts have been noted in nearly all cultures and throughout all eras of recorded history ${ }^{35}$. Moreover, mirrors can be used to reflect objects quite apart from a mirror-gazer's self and are indeed represented doing just that in many fascinating Renaissance works of visual $\operatorname{art}^{36}$. Deborah Shuger even asserts that "before the late seventeenth century, [mirror-viewing's] objectification of the viewing subject, allowing one to watch oneself, elicits virtually no interest", and that rather "[i]n the Renaissance, the self's internal mirror angles outward". She concludes that "early modern selfhood was not

See Herbert Grabes, The Mutable Glass: Mirror Imagery in Titles and Texts of the Middle Ages and English Renaissance, trans. Gordon Collier, Cambridge, Cambridge University Press, 1982. engineer Jacques Besson (1540-1576)" whose work followed "numerous treatises on optics published earlier in the sixteenth century" (p. 129).
See Pendergrast.
For examples, including famous works of Van Eyck and Velázquez, see Miller.
experienced reflexively, but, as it were, relationally" ${ }^{37}$. These remarks attach to controversies that are not our central concern here, yet it might be noted that, in the Ritter vom Turn woodcut, a mirror provides the girl with a view only of herself, while it also provides an onlooker with a view of a signifying demon (the former view may not be, but the latter is in line with what Shuger suggests).

To return to our main point, mirror-imaging in the Ritter vom Turn woodcut conveys a deliberately confusing or defamiliarizing vision because it explicitly demonstrates that what is seen may alter radically with the perspectives of viewers. To unroll this further, we may add that, because the onlooker's position precludes seeing the girl in the mirror, the girl equally cannot see the onlooker in the mirror (since ray-tracing produces equal results regardless of the direction in which the rays go). Because she has her back turned to him and seems entranced with her own image, she likely does not even know that the onlooker is there. Also because of reversible rays, we know that the demon cannot see the girl in her mirror, although he can see her directly. The onlooker (we) can see the demon in the mirror and also directly. The girl cannot yet see the demon at all.

It is because of such asymmetries that the Ritter vom Turn illustration of gazing in a mirror is much more complex than would be a depiction of gazing out of a window of identical size and shape as the mirror. It is true that for both window-gazers or mirrorgazers the same rules apply that sightlines cannot turn around corners unaided or pass through obstructions (be those the edges of a mirror or a window frame), and also that in virtual mirror spaces as well as in out-of-window spaces the rules of perspective apply. However, because both direct and reflected sightings of the same objects are made simultaneously in our Ritter vom Turn woodcut ${ }^{38}$, something extra arises. This "something" is the
${ }^{37}$ Debora Shuger, "The ' I ' of the Beholder: Renaissance Mirrors and the Reflexive Mind", in Renaissance Culture and the Everyday, eds Patricia Fumerton and Simon Hunt, Philadelphia, University of Pennsylvania Press, 1999, pp. 21-41: p. 37.
${ }^{38}$ Neither the viewer of objects nor the objects viewed in a mirror (or equally through a window) need be directly in front of that window or mirror. Thus the shaded area in our Fig. 4 is a planar section of the interior of a truncated pyramidal threedimensional segment of space with its apex at the object (or image) viewed and its outline determined by the limiting shape and size of the mirror. A looker-into a flat
possibility of a verification of the actual presence of objects that are at the same time invisible in a mirror. Thus, for example, the onlooker can verify by direct vision the presence of the girl although he cannot see her in her mirror. Even perceptions of affect may provide verifications of how mirror gazing may distort or fail to reveal the "whole truth" of a visible realm; thus the onlooker (we) may conclude from the girl's unperturbed expression that she does not see the demon in her mirror, although we cannot know this directly since we do not stand where she does.

The Ritter vom Turn woodcut presents only relatively simple perspectival enigmas, and there are more complex ways as well in which analyses of mirror-imaging may throw light on the processes of depiction and perception of images ${ }^{39}$. Nonetheless, considering how this illustration allows its spectators to identify defects or distortions of seeing may be instructive when we meet analogous misapprehensions in Shakespearean drama.

## Crooked vision in two Shakespeare scenes

Finally, we will consider two particular Shakespearean scenes in which differing perspectives are seen to alter perceptions radically when, as Hamlet counselled the visiting players, a dramatic "mirror" is held "as 'twere [...] up to nature".

Our contention in general will be that, by means of cunning dramatic and scenic construction, Shakespeare often gives audiences access both to what there is to be seen, and also to how this may be invisible (or seen quite variously) from differing dramatized standpoints. Sometimes such constructions possess actual optical aspects, and sometimes the multiple perspectives at issue are metaphorically optical. But here, when saying

[^9]"metaphorically optical", I do not mean rhetorically or poetically metaphorically but rather cognitively so. Deploying such a usage, a colleague who was blind from youth typically responded to gaining a new insight with the remark: "I see".

In fact, the two specific examples to be examined here do involve actual optical enigmas. One is located in a scene from the early play Titus Andronicus and the other in a scene from the midperiod Troilus and Cressida ${ }^{40}$.

In Titus, the lustful empress Tamora first tries to tempt her reluctant lover Aaron into a sexual encounter in a forest glade, describing it thus:

My lovely Aaron, wherefore look'st thou sad
When everything doth make a gleeful boast?
The birds chant melody on every bush,
The snakes lies rolled in the cheerful sun,
The green leaves quiver with the cooling wind
And make a chequered shadow on the ground.
Under their sweet shade, Aaron, let us sit,
And whilst the babbling echo mocks the hounds,
Replying shrilly to the well-tuned horns,
As if a double hunt were heard at once,
Let us sit down and mark their yellowing noise,
And after conflict such as was supposed
The wand'ring prince and Dido once enjoyed
When with a happy storm they were surprised,
And curtained with a counsel-keeping cave,
We may, each wreathed in the other's arms,
Our pastimes done, possess a golden slumber
Whiles hounds and horns and sweet melodious birds
Be unto us as is a nurse's song

40 The examples of such configurations chosen here are not unique; several scholars have provided similar commentaries on other Shakespearean contexts. Thus Keir Elam discusses mirroring in Twelfth Night, and analyses "I am ready to distrust mine eyes" (IV.iii.13) and also the play's culminating "optical illusion" (William Shakespeare, Twelfth Night, ed. Keir Elam, London, The Arden Shakespeare, Third Series, 2008, p. 29). See also Allan Shickman, "The 'Perspective Glass' in Shakespeare's Richard II", Studies in English Literature, 1500-1900, 18:2 (Spring 1978), pp. 217-28, which treats Richard II, and Stephen X. Mead, "Shakespeare's Play with Perspective: Sonnet 24, Hamlet, Lear", Studies in Philology, 109:3 (Spring 2012), pp. 225-57, which treats Sonnet 24, Hamlet and Lear.

Of lullaby to bring her babe asleep. (II.iii.10-29) ${ }^{41}$

But Aaron denies her request. His preferred alternative is to pursue vengeful schemes to murder the emperor's brother Bassianus, to lay the blame for this on Titus's two sons, and to arrange for the rape and murder of Titus' daughter Lavinia. Thus, just before exiting, Aaron reveals his plan to incriminate the boys by means of a forged letter (II.iii.30-50). It appears, however, that Bassianus and his wife Lavina have spied Aaron and Tamora consorting together in the forest, and so, when they enter just after Aaron's departure, they reproach Tamora for having intended to commit adultery ("your sport", II.iii.80). Lavinia's chiding of Tamora offers an alternate description of that locale: "This valley fits the purpose passing well" (II.iii.84). Next, when Tamora's two bestial sons enter, she claims that Bassianus and Lavinia have lured her to the present "vale" (II.iii.93), which she re-describes as dire and terrifying:

These two have 'ticed me hither to this place.
A barren detested vale you see it is;
The trees, though summer, yet forlorn and lean, Overcome with moss and baleful mistletoe.
Here never shines the sun, here nothing breeds
Unless the nightly owl or fatal raven,
And when they showed me this abhorred pit
They told me here at dead time of the night
A thousand fiends, a thousand hissing snakes,
Ten thousand swelling toads, as many urchins
Would make such fearful and confused cries
As any mortal body hearing it
Should straight fall mad or else die suddenly.
No sooner had they told this hellish tale
But straight they told me they would bind me here
Unto the body of a dismal yew
And leave me to this miserable death. (II.iii.92-108)
${ }^{41}$ All quotations from Titus Andronicus refer to the edition by Jonathan Bate, in The Arden Shakespeare Complete Woks, eds Richard Proudfoot, Ann Thompson and David Scott Kastan, London, Thomson Learning, 2001.

Tamora concludes by urging her not-unwilling sons to "Revenge it as you love your mother's life, / Or be ye not henceforward called my children" (II.iii.114-15). The sons reply by killing Bassianus and throwing him into the pit, and threatening to rape Lavinia. Lavinia begs to be simply murdered and not defiled and "tumbl[ed]" into "some loathsome pit" (II.iii.176). Tamora denies Lavinia this boon and allows her sons to drag her offstage to be raped atop her husband's corpse. After that, they mutilate her. Next, Tamora exits, and Aaron enters leading Titus' two sons, whom he has drugged, to the same locale as the rest of the scene. This he misleadingly identifies as "the loathsome pit / Where I espied the panther fast asleep" (II.iii.193-94). The drugged brother Martius falls into what his brother Quintus, still above, describes as a "subtle hole [...] Whose mouth is covered with rude-growing briars / Upon whose leaves are drops of new-shed blood / [...] A very fatal place" (II.iii.198-202). Martius asks to be helped out of "this unhallowed and bloodstained hole [...] this den [...] this detested, dark, blooddrinking pit" (II.iii.210, 215, 224), having seen therein the corpse of Bassianus. When asked "If it be dark, how dost thou know 'tis he?" (II.iii.225), he replies the corpse wears "A precious ring that lightens all this hole, / Which, like a taper in some monument, / Doth shine upon the dead man's earthy cheeks / And shows the ragged entrails of this pit" (II.iii.227-30). Martius' specification of the light source needed to allow visibility indicates that he adheres to an intromission, rather than an extramission theory of sight. His description of seeing "the ragged entrails of the pit", which he equates with a "fell receptacle / As hateful as Cocytus' misty mouth" (II.iii.235-36) brings in, beside optical observation, additional fantasies of anatomical orifices.

Again in an optical mode, the two drugged brothers remark "My sight is very dull, whate'er it bodes" and "And mine, I promise you" (II.iii.195-96). In consequence of that ${ }^{42}$, or some other unsteadiness, while attempting to rescue his brother, Quintus himself finally falls into what he describes as "the swallowing womb / Of this deep pit" (II.iii.239-40). This fully completes a series

42 It may well be implied that the effects of Aaron's poison impair especially the faculty of depth perception which is enabled by stereopsis - a topic to be explored in Sokol, forthcoming.
of optical and Freudian descriptions. Then Aaron arrives bringing the emperor Saturnius to view what Saturnius calls more neutrally "this gaping hollow of the earth" (II.iii.249), whereupon Saturnius discovers Titus' two sons trapped in the pit together with the murdered body of his brother Bassianus. Finally, Tamora re-enters bearing the forged letter incriminating the innocent boys.

We find in this scene of horrors several descriptions of the same forest "vale" that differ according to contrary viewpoints. So Tamora's first description is coloured by lustful intent; Lavinia's description is sarcastic and censorious; Tamora's second description intends to motivate murder; Titus' two sons' descriptions derive from corporal fantasies and dread provoked by drugging; the Emperor at first benignly notes the proximity of the same "hollow of the earth" to a "pleasant chase" (II.iii.255). That these variations of outlook might be not only psychological is implied by Marjorie Hope Nicholson's classic study Mountain Gloom and Mountain Glory which shows that Renaissance perceptions of wild nature were self-divided ${ }^{43}$ (as indeed they are in As You Like It, where the forest of Arden allows an idyll and is also made dangerous by resident snakes).

I would suggest that the frustrated Tamora actually sees two different places when she views the vale first as an outdoor boudoir and later as an apt venue for murder. The two boys' increasingly Freudian descriptions of the fringed pit show how drugging uncovers hidden terrors. Saturnius' at-first neutral description of the same "hollow of the earth" proceeds from a perspective noting a nearby "lodge / Upon the north side of this pleasant chase", where he thinks his (now murdered) brother and "his lady both" (II.iii.254-55) are happily ensconced. We the audience cannot see the variously described "pit", but are able to verify that none in the playworld see or describe it aright.

Troilus and Cressida (V.ii) ${ }^{44}$ provides a still more complex depiction of how several onlookers have very different perceptions

[^10]when looking from differing optical and psychosexual positions. In this scene, the onlookers are hidden from a woman who is currently concerned about her own sexual allure and, in that way, resemble the onlooker in the illustration to Der Ritter vom Turn, who is invisible to the self-aware girl. However, the onlooker in the woodcut has apparently comprehensive vision, whereas the perceptions of all three of the unseen onlookers in Troilus and Cressida are severely distorted in ways that we the audience, the ultimate onlooker, can verify.

A basis for this verification has been detailed in a brilliant study by the psycho-analyst Angela Sheppard ${ }^{45}$, part of which informs the discussion below. Sheppard's essay suggests that the male onlookers represented in Troilus variously distort or curtail their perceptions of what is in plain sight on account of inner obstructions or limitations that restrict their vision. Sheppard's essay further suggests that Shakespeare's dramatization of Cressida's plight allows readers or playgoers insight into what those three male spies cannot or will not see. Thus Sheppard details a perspective from which audiences can both take note of and wonder at the delusions of these dramatized characters.

The scene in question depicts a painful interaction between the Trojan captive Cressida and her Grecian captor and soon-to-be new lover Diomedes, and simultaneously presents commentaries on this interaction made by the three covert male witnesses. Chief among those spies is Cressida's former lover, the young Trojan prince Troilus. He is so overwhelmed by sexual jealousy that he at first denies that he has actually witnessed Cressida's half-reluctant allowance of Diomedes' advances. Thus Troilus at first bluntly proclaims that "this is not Cressida" (V.ii.135), denying the witness of his eyes, but then refines this to "This is and is not Cressida" (V.ii.149).

Troilus actually asserts the unreliability of his own visual and aural senses, insisting that his unshakable belief in Cressida's faithfulness: "doth invert the attest of eyes and ears" (V.ii.124). Observing this remark, the always scurrilous Thersites, another of

[^11]the spies, comments sarcastically: "Will a swagger himself out on's own eyes?" (V.ii.138-39). But Thersites himself displays distorted vision by characterizing the scene between Cressida and Diomedes in only ribald terms. Thus he sees Cressida's evident reluctance to betray her former lover only as a tease and come-on to sharpen Diomedes' desire, a "whetstone" (V.ii.77), and he concludes, sarcastically, that "A proof of strength she could not publish more / Unless she said, 'My mind is now turned whore'" (V.ii.115-16).

Ulysses, the third covert witness to the same encounter, is less gross in the presence of the distressed Troilus, whom repeatedly urges to leave off viewing the scene. Yet Ulysses himself previously misread Cressida's attractive liveliness as slutishness (IV.vi.55-64) and continues to regard her as easy with her affections (V.ii.10).

Ulysses shows himself particularly coarsely uncomprehending when Troilus concludes regarding Cressida's behaviour:

> Think: we had mothers. Do not give advantage
> To stubborn critics, apt without a theme
> For depravation to square the general sex
> By Cressid's rule. Rather, think this not Cressid. (V.ii.132-35)

To this Ulysses responds: "What hath she done, Prince, that can soil our mothers?" (V.ii.136), and it is from this point that Sheppard's interpretation takes its impetus or origin.

Paraphrasing Sheppard briefly, with much left out, and using the terms of psycho-analytic object relations theory, we see Troilus reacting to his disappointment by "splitting" his "internal good object", which is to say, cleaving in two his primal image of the perfect woman, the descendant of an infant's phantasized allgiving mother. This internal splitting accounts for Troilus' paradoxical remarks in which he denies his own sight: "This, she? No, this is Diomed's Cressida. / If beauty have a soul, this is not she", and then "This is and is not Cressid" (V.ii.140-41, 149).

Obtuse Ulysses, only puzzled, has no grasp of the process whereby Troilus' internal reflection of what he sees effectively splits Cressida into two. But, Sheppard implies, audiences may grasp this. Interestingly, the Ritter vom Turn woodcut may also be perceived to represent a splitting, by means of reflections, between a lovely lady and a hyper-sexual demon. Indeed, one of

Shakespeare's Thesites' obscene remarks made while viewing the Cressida-Diomedes scene can be read to imply very similar repugnant anatomical imagery to that seen in the woodcut ${ }^{46}$.

Sheppard contends that the integration of Cressida as a "whole object" - as a woman who, to preserve her creativity, must make the best of a bad or sad situation and, for that reason, may not fully satisfy the demands made by others - may well be implied in Shakespeare's presentation of Cressida's difficult situation. But this re-integration is clearly not perceived by any of the spying men who witness her enigma from varied perspectives. Troilus in particular is shocked to encounter imaginatively a malign "part object", which he calls "Diomed's Cressida". His reaction is to intend bloody revenge against Diomedes on the battlefield (V.ii.166-79).

Indeed, Troilus soon after courts ruthlessness and rails against his elder brother Hector's chivalrous treatment of enemies, saying:

For th' love of all the gods,
Let's leave the hermit pity with our mother
And, when we have our armours buckled on,
The venomed vengeance ride upon our swords,
Spur them to ruthful work, rein them from ruth. (V.iii.46-50)

To this Hector, who says he prefers "fair play", replies "Fie, savage, fie!" (V.iii.51). Yet, despite Shakespeare's Troilus' rejection of mercy or genuine ruth, this being associated by him with his "soiled" mother, his ensuing furious entry into the battlefield produces a peculiarly muted outcome. For, in Shakespeare's revision of the often told medieval Troilus and Cressida story, Troilus is not destroyed in battle by Achilles' Myrmidons or by any others. That his intended violence fails to be either effectual or fatal may well indicate the evanescent nature of furious so-called "part-object phantasies".

[^12]Here Troilus' perceptions of it reveal only part of a scene, belying a wider reality. Similar occlusions of whole vision often fascinated Shakespeare, as is seen in his Sonnets 93, 137 and 138. However, the speakers in these poems actually reveal an ironic awareness of the partial or splitting nature of their own inner reflections - and this, in turn, overthrows Ulysses' reductive judgment in Troilus and Cressida that man in general "feels not what he owes but by reflection" (III.iii.94).

The several audience-apparent imaging errors or omissions explored here involve either different Shakespeare characters having differing points of view or else a single Shakespeare character adopting different views at different times. These share with the girl-demon illustration to Der Ritter vom Turn the characteristic that we, as unseen viewers of the whole scene, can assess those varied figures' partial seeing.

There are also instances among Shakespeare's works where one and the same character takes simultaneous but contradictory views of the same scene or situation; these will be the topic of a further study where defamiliarization via mirrors will again be instructive when considered in relation to physical optics ${ }^{47}$.

47 The single viewer with two points of view will be explored in Sokol, forthcoming.


[^0]:    2 Transcribed from Chadwyck Healey EEBO document image 27 of Here begynneth the booke which the knyght of the toure made and speketh of many fayre ensamples and thensygnementys and techyng of his doughters [STC (2nd ed.) / 15296].
    3 Transcribed from Chadwyck Healey EEBO document image 57 of Here begynneth the booke which the knyght of the toure made and speketh of many fayre ensamples and thensygnementys and techyng of his doughter.

[^1]:    7 All references from Purgatorio are to Dante Alighieri, The Purgatorio of Dante Alighieri, London, J. M. Dent and Sons, 1941.
    8 See the note in Alighieri, Purgatorio, pp. 345-46.
    9 Alighieri, Convivio, IV.xvii.85ff, quoted in the note in Alighieri, Purgatorio, p. 345. The passage goes on to claim "supreme happiness" for the contemplative life.
    ${ }^{10}$ Alighieri, The Paradiso of Dante Alighieri, London, J. M. Dent and Sons, 1941.

[^2]:    11 Imperfect mirror-imaging will be considered in B. J. Sokol, "Mirrors, Pictures, Optics, Shakespeare", in The Art of Picturing in Early Modern English Literature, eds Camilla Caporicci and Armelle Sabatier, forthcoming. I will also reserve for discussion there considerations in detail of how stereoscopic vision locates the reflections of objects behind the plane of a flat mirror, and will simplify here by assuming that we are dealing with monocular vision.
    12 This same subsection of OED explains in its headnote that a "virtual image" arises "when the rays from each point of the [seen] object [...] diverge as if from a point beyond the reflecting or refracting body".
    13 See A. Mark Smith, "Reflections on the Hockney-Falco Thesis: Optical Theory and Artistic Practice in the Fifteenth and Sixteenth Centuries", Early Science and Medicine, 10:2 (2005), pp. 163-85, especially pp. 164-70, 174-76, which explains why, although the mechanisms of virtual mirror-imaging were very well understood from long before, the earlier Renaissance lacked an appreciation of the other sort of optical imaging that produces images that can be focussed and projected on a screen.

[^3]:    14 See Smith, From Sight to Light: The Passage from Ancient to Modern Optics, Chicago, University of Chicago Press, 2015, pp. 30-31.
    15 This equivalence was long appreciated, for "[ $t]$ he identity of luminous and visual radiation seems to have been taken for granted throughout antiquity. It was specifically defended by Hero, Damianos, Theon, and apparently Ptolemy" (David C. Lindberg, Theories of Vision from al-Kindi to Kepler, Chicago, University of Chicago Press, 1981, note 16, p. 223).
    16 Lindberg maintains that Kepler's achievement extended already well-established themes of geometrical optics (see especially pp. 202-8), but Smith maintains, on the contrary, that it constituted a profound shift of understanding (see From Sight to Light). Lindberg does allow for that shift but perhaps understates it. Smith strongly emphasises it.

[^4]:    Kepler's aim was to understand the bending of light rays by the atmosphere that makes astronomical objects appear to be wrongly located. That makes it incomprehensible to me why Melchior-Bonnet asserts that: "Descartes, like Kepler, still ignored the optical notion of the 'virtual image', a fictive extension of rays of light received by the eye differing from the real image" (p. 131). In fact, Kepler showed that virtual images are anything but "fictive" and indeed enter the perceptual process just exactly as if they had come from "real" sources.
    Smith discusses controversies over the authorship of "Euclid's" Catoptrics, which some attribute to "the fourth-century thinker Theon of Alexandria" (From Sight to Light, note 89, pp. 55-56).

[^5]:    John Dee, The Mathematicall Praeface to Elements of Geometrie of Euclid of Megara, in Euclid's Elements of Geometrie, trans. Henry Billingsley, London, 1570, b.1. See also Smith, From Sight to Light, p. 169, which attributes to the ninth-century scholar Abu Yūsuf Ya'qūb ibn 'Ishāq aṣ-Ṣabbāḥ al-Kindī a realization that "the geometrical ray is an analytic convention".
    20 When discussing Euclid's Optics, the generally excellent work by Smith seems to fall into such error (From Sight to Light, p. 53); or Smith may be paraphrasing (without noting it) the objections to Euclidean optics made by the ninth-century al-Kindī. Those objections are discussed in Lindberg, pp. 24-26, and Smith, From Sight to Light, pp. 166-67.

[^6]:    See Smith, From Sight to Light, pp. 55-56.
    See Smith, From Sight to Light, p. 66; the result is derived by Hero of Alexandria from the principle that light rays will travel "the shortest possible distance".
    See Smith, From Sight to Light, pp. 93-97.
    See Smith, From Sight to Light, pp. 195-200.

[^7]:    29 A more dynamic way of stating this is to say that, if the viewing plane used for the analysis in our Fig. 4 is rotated around the normal line OI, then points on the series of planes thus formed will sweep out all of the space in front of the mirror wall $A B$ (on its left in our diagrams). Therefore, at some point in its rotation, this sweptaround plane will contain any place where an observer could be situated. Stop the rotation when the analysis plane contains any chosen observer's viewpoint and the diagram in Fig. 4 can be drawn anew to produce the same conclusion as ever - save that the line segment MM' will not be exactly the same. But the segment will still be finite and will therefore still produce the same result. By the way, the segment $\mathrm{MM}^{\prime}$ will remain the same under all rotations of our swept analysis plane if and only if the frame of the mirror is circular and the normal OI passes through the centre of this circle.

[^8]:    30 Leonardo da Vinci, The Literary Works, eds Jean Paul Richter and Irma A. Richter, London, Phaidon Press, 1970 ( 3 rd edition), 2 vols, vol. I, p. 201. Because of stereopsis, this experiment will only be fully sucessfully if both persons regarding the mirror keep one eye shut and the first person touches the mirror-image of the other's open eye.
    31 These included Giambattista della Porta, Giordano Bruno and, in England, John Dee, who, in his 1570 Introduction to Billingsley's Euclid, mentions the "part" of Naturall Philosophie "which dealeth with Glasses (which name, Glasse, is a generall name, in this Arte, for any thing, from which, a Beame reboundeth) is called Catoptrike: and hath so many vses, both merueilous, and proffitable: that, both, it would hold me to long, to note therin the principall conclusions, all ready knowne: And also (perchaunce) some thinges, might lacke due credite with you" (b.1). Here Dee does not allude to the "marueilous Glasse" or curved mirror that he has described just above, but rather to the scrying glasses used by several mediums, including Edward Kelley, that so fooled him. Pendergrast gives a sensational account of this (pp. 4-51), and R. Julian Roberts a more moderate one ("John Dee", in Oxford Dictionary of National Biography, eds H. C. G. Matthew and Brian Harrison, Oxford, Oxford University Press, 2006).
    ${ }_{32}$ On Euclid and Hero, see Smith, From Sight to Light, p. 64; Smith outlines Hero's last proposition in detail (From Sight to Light, pp. 64-65).

[^9]:    mirror and a looker-out of a window of the same size and shape as the mirror will see the object or image only if they are positioned inside that pyramid. But one difference is that a mirror gazer may also see the same object both in a reflection and directly. Another difference is that front and back are reversed in mirror-imaging so that moving a reflected object north, say, toward an east-west oriented plane "wall" containing a mirror, will make its reflected image in the mirror move south; or, for a mirror mounted on the ceiling, moving an object upward toward it will cause the image to move downward.

[^10]:    ${ }^{43}$ See Marjorie Hope Nicolson, Mountain Gloom and Mountain Glory: The Development of the Aesthetics of the Infinite, Ithaca, Cornell University Press, 1959; on the connection of women with wild nature, see Jeanne Addison Roberts, The Shakespearean Wild: Geography, Genus and Gender, Lincoln, University of Nebraska Press, 1991.
    ${ }^{44}$ All references to this play are from William Shakespeare, Troilus and Cressida, ed. Kenneth Palmer, London, The Arden Shakespeare (Second Series), 1982.

[^11]:    45 See Angela Sheppard, "Soiled Mother or Soul of Woman?: A Response to Troilus and Cressida", in The Undiscover'd Country: New Essays on Psychoanalysis and Shakespeare, ed. B. J. Sokol, London, Free Association Books, 1993, pp. 130-49.

[^12]:    46 Kenneth Palmer suggests that, when Ulysses remarks aside on Cressida "She will sing any man at first sight" and Thersites adds "And any man may sing her, if he can take her clef" (V.ii.9-11), the word "clef", spelled Cliff in the Quarto, puns on "cleft = female pudendum" (William Shakespeare, Troilus and Cressida, note 11, p. 270).

