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Complexity: the Role of Information in Organizing Chance

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Abstract

This paper reviews some of the more relevant results arising from the crisis of reductionism, which have lead to complexity theory and the problem of foundations in different contexts of sciences, starting from the late 19th century. It focuses on the relation between complexity and mathematical non-linearity, and suggests a comparison between Chaitin's compressible/incompressible strings and the philosophical notion of universal. It also suggests a comparison between the notion of information, especially in its algorithmic version, and the Aristotelian-Thomistic concept of form. It shows how chance can generate stable order and organization only if some information drives the process dynamics. Starting from random initial conditions, suitable information leads the evolution trajectories of each cell (element) of some system (either simple or complex) towards a precise attractor. Examples of fractals, galaxies, and models of a biological organ in a living system, like a human heart, generated by stem cells are proposed. The fact that ancient Aristotelian-Thomistic logic/ontology appear to be more close to contemporary science than in the past centuries may contribute to both philosophy and science.

Keywords: crisis of reductionism, complexity, chance and order, fractals, cellular automata, stem cells, organ generation

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1. Complexity: the Role of Information in Organizing Chance

Where is science going? From a practical point of view, this is a hard question because of the philosophical, ideological, and economical-political constraints imposed by several (or unified?) powers on concrete scientists. A sort of unpredictable bifurcation between dangerous scenarios and marvelous perspectives appear on the horizon of the future.

From a theoretical point of view, it is easier to guess what the new paradigms—emerged in the 19th century and developed along the 20th and the 21st century—are requiring for scientific disciplines to advance.

This paper will provide a small contribution in trying to answer the theoretical question. Which positive or even unavoidable *scenarios and perspectives* seem to emerge for science to widen its own subject of investigation and renew its own methods?

In §2 something is said about the paradigm of *complexity*, arisen from the so called "crisis of reductionism."

§3 is devoted to emphasize some relevant steps in science theories—from Einstein's Relativity to the more recent approaches to complexity—and their philosophical interpretations. These show a drive toward some logical and metaphysical notions by Aristotle and Aquinas rather than the conceptions of modern philosophers.

In §4 an §5 we attempt to demonstrate that a stable order and organization may arise from either order or chance—a system is given random initial conditions in the evolutive trajectories of its constitutive parts (cells), where some leading information (a law) drives the entire process. Chance (randomness) alone only generates chance. Computer simulation examples involving 3D-fractals and a model of a spiral galaxy are presented.

In §6 an application to biology is proposed. This shows the stem cell generation modelling of an organ in a living system, such as the human heart. We offer a rough example, based on algorithmic information coded into a *compressible* string, and a more realistic model, where information is coded into a string considered as *incompressible*. The Conclusion offers some remarks and perspectives.

2. Complexity from the Crisis of Reductionism

An educational sentence describes complexity as follows: *the whole is not equal to the sum of its parts* (Polkinghorne, 2002). The scientific effectiveness of this sentence relates to the mathematical concepts of *linearity* and *non-linearity*. These allow to represent the "philosophical" concept of complexity into mathematical equations. In fact, it is well known that only *linear differential equations* feature the following property: the sum (more properly, a linear combination) of two or more of their solutions (which we may interpret as *parts*) provides a new solution (which we may interpret as a *whole*) of the same differential equation. In wave theory, such a property is equivalent to the superposition principle of wave amplitudes. This allows for a simple solution of any linear system of differential equations. Hence, before the discovery of complexity, physics and mathematics' theorists were supposed to find the *linear equations* that govern natural phenomena or, at least, to approximate any problem to a linear one. Even the so called *perturbative methods* aim at progressively approximating solutions of non-linear equations by means of linear techniques—the only ones people could manipulate at the time. The introduction of computer has enormously improved the power of numerical methods. However, it has also highlighted odd instability phenomena, such as *deterministic chaos* (Lorenz 1963). Linearization techniques resemble the attitude of a traveler who joints the pieces of information acquired in each neighborhood from different points of observation along the road.

Unexpectedly, the emergence of complexity has revealed that some *global properties* of things exceed the capabilities of these extensions of linear methods.

During a confidential conversation, several years ago, the well-known astrophysicist Nicola Dallaporta (1910-2003) said that, in the end, the *harmonic oscillator* and the *two-body problem* are the only two genuine physical problems we can solve with exactness. Both are governed by the same linear differential equation!

Reductionism is the scientific method that attempts to reduce all our scientific knowledge to linear problems, decomposing any *wholeness* into a sum of simpler *parts*. Thus, the universe is a set of different kinds of galaxies; a galaxy is stars and planets and other celestial bodies; a living body is cells; a cell, or any kind of physical body, is molecules; a molecule is atoms, and an atom is elementary particles (*fermions*). Everything is interacting thanks to fundamental fields, which in turn are carried by another kind of elementary particles (*bosons*).

The adventure of this science resembles the exploration of the sole portion of a plane confined in the neighborhood of the tangency point with a wider non-linear manifold (the complex real world).

Now, it is legitimate and unavoidable for a human being to adopt a reductionistic approach to reality, since our mind cannot know everything in a single, all-embracing act, as divine mind does. Our mind needs to put together a sequence of acts of knowledge partially localized in space and time, step by step.

However, in some lucky circumstances, through attempts, one can reach one or more exact solutions of a non-linear equation, even if no general method can obtain a general solution that encompasses, as a family, all the solutions to that equation.

After exploring the entire neighborhood of the point of contact between the tangent plane (the range of linear investigation) and the manifold (the non-linear world or reality), scientists seem to have discovered that the great majority of real phenomena need to be represented and must be governed by non-linear laws. The sum of their solutions (provided that they are individually known) generally is not a new solution. Indeed, the discovery of *complexity* has gained not only new problems about the *structure of things*, but also surprises about the *dynamics* of their behavior in time, such as unpredictability. This depends on the high sensibility toward the initial conditions of the solutions (Poincaré 1892-1899; 1905-1911). Stability theory is just concerned in such topics, involving also *qualitative analysis* of the *topology* of trajectories when a quantitative one is no longer fully possible. Among the first pioneering and most relevant studies on these topics we remember (Thom 1972) and (Arnold 1992).

A new approach to reality seems to require a sort of enlargement of mathematics so that it becomes capable of dealing not only with number and extension (i.e., quantity), function, equation and inequality (i.e., relation), but also with quality and other properties (De Giorgi 2013). This means approaching a more comprehensive ontology formalized by symbols like usual mathematics, but widened respect to its object of investigation (Strumia 2012). As a consequence all mathematized sciences will result widened.

– *Set theory*, e.g., represents a first step in that direction, enlarging mathematics from numbers to collections of any kind of things (Tapp 2005).

– *Topology*, formerly (significantly) known as analysis *situs*, within set theory, is another step in attempting to rediscover the old Aristotelian category of situs (Poincaré 1895), catching global (*topological*) properties of sets and classifying them into different irreducible species, depending on the number of holes and handles (Wall 2016).

– *Biology, informatics, cognitive sciences*, and the so-called *artificial intelligence* are discovering unexpected, tight relationships between the notions of Aristotelian form and information (Marks II 2014).

– Furthermore, the new branch of the so-called *humanities*, according to some authors, is interested in following a demonstrative approach in dealing with humanistic disciplines, similar to what Aristotle did, so that the old opposition between hard science and human disciplines is overridden (Burguete-Lam 2008).

In that sense, we can say that contemporary science is progressively approaching an Aristotelian method to investigate reality. The predicted formal approach to ontology is now seen as a sort of rediscovery and re-building of metaphysics as a *theory of foundations* of science. The most recent interest—documented by several articles in literature recalling the names of Aristotle and Aquinas—is a symptom of such a change of mind in scientific methodology (Modrak 1990; Simpson-Koons-Teh 2019).

3. Relevant Steps in Science towards Aristotle and Aquinas

3.1 General Relativity and Aristotelian Space

Einstein himself (Einstein 1969) maintained that in his theory of gravitation, space-time is no longer a sort of pre-existing box that contains the bodies in the old Cartesian-Newtonian fashion. On the contrary space-time *is made* by bodies, i.e., matter and fields (energy-momentum distribution), since the latter determine, as a source, the metric, the connection, and the curvature of space-time. This alternative conception of space (including time) is rather close to the Aristotelian idea of space defined by the intimate contact relations between bodies (Koyré 1971).

3.2 Proper Classes in Set Theory and Analogy

Since *sets* (in *set theory*) are more generic entities than numbers, being collections of any kind of things and not of just numbers, several paradoxes and contradictions emerged while dealing with them. These paradoxes recall the ones of the Aristotelian-Thomisitic notion of *entity* (Latin, *ens*). Georg Cantor (1845-1918) discovered the contradiction arising in attempting to introduce the notion of universal set, or set of all sets (Cantor 1932). Later, Bertrand Russell (1872-1970) solved the paradox (Russell 1938) introducing *non-univocal definitions* of sets (theory of *types*). More simply, Kurt Gödel (1906-1978) introduced the distinction between *proper* and *non-proper classes* or sets in the usual sense (Gödel 2001, orig. 1938). Both made a relevant step in rediscovering the Aristotelian-Thomisitc notion of *analogy* (Bochenski 1961, Strumia 2010).

3.3 Gödel's Undecidability and its Informatic Implications (Turing *et al.*)

With his *undecidability theorem* (1931), Kurt Gödel showed that not all the propositions one is able to formulate with the symbols and rules of an axiomatic system as that of Russell's *Principia Mathematica*, can be demonstrated either as consistent or non-consistent with the axioms. So, they are *undecidable* (Gödel 1931). More, very ingeniously, he was able to relate bi-univocally any proposition to a number (*Gödel's number*), and any demonstration procedure to a computation, labelled in its turn, bi-univocally, by a number. Therefore, if there exist undecidable propositions, then the corresponding numbers are non-computable, i.e., they cannot be evaluated by any formula. They can be obtained only by listing sequentially their digits, one after the other.

3.4 Turing and the Halting Problem

Applying Gödel's theorem to computer science, Alan Turing (1912-1954) discovered the halting problem. If not, any number is computable through a computational procedure, a computer should not halt when attempting to evaluate such a number through any possible program (Turing 1937).

3.5 Wiener and Information Theory

In the meanwhile (1948) Norbert Wiener (1894-1964), one of the fathers of information theory, realized that information would have played, in science, the role of a new principle of explanation of reality, being irreducible to the known principle of *matter* and *energy*. He claimed: "Information is information, not matter or energy. No materialism which does not admit this can survive at the present day" (Wiener 1965).

Later information acquired a more and more relevant role in science, which was largely exceeding the field of telecommunications, within which the main problem was that of minimizing the noise disturbing radio broadcastings (Shannon 1948). The role played today by information in biology strongly resembles that of the Aristotelian *form*, as the governing principle of living organisms' structure, organization, and *dynamics* (*individual behavior* and *species evolution*). In philosophical terms, the scientific notion of *form/information* as organizing principle of the *structure* remembers the Aristotelian-Thomistic notion of *essence*, and the scientific notion of *dynamics* recalls the Aristotelian-Thomistic notion of *nature*, philosophically defined as *essence*, *principle of action* (Strumia 2012).

3.6 Chaitin's Irreducible Strings and the Problem of Universals

Gregory Chaitin (born in 1947), applied the aforementioned results to the programming languages and computer algorithms. He discovered that if we deal with a computer program like a *string* that sequentially collects all the characters required to write down the instructions, then the strings corresponding to a computable number could be compressed into shorter ones, corresponding to more efficient programs. However, the strings corresponding to a non-computable number, could not be compressed, and only the sequence of their digits listed one after the other could identify them (Chaitin 1992, p. 141; Strumia 2020).

A similar situation suggests a direct comparison with the philosophical topic of the *universals* as developed by Aristotle and Thomas Aquinas (followed by many other philosophers).

A universal, in fact, is a *concept/name* under which one collects a lot of individuals, in order to avoid to list them one after the other. From the cognitive point of view, Aristotle and Aquinas discovered that our mind, being finite, can know the world only through universals (*immaterial information*), since it cannot know everything as singulars caught by the same cognitive act. Only the divine mind, being infinite, can know all singulars together by one act. On the contrary we need to compress information into shorter strings (*the universals*).

3.7 Cognitive Sciences, Human and Artificial Intelligence, and Self-Consciousness

Another intriguing and wide topic where science meets the Aristotelian-Thomistic tehory of knowledge is the field of cognitive sciences, in its different branches. Here the immaterial character of information, which allows to transfer it from one matter support to another one (as it happens, e.g., from the hardware peripherals of our computer toward the network), plays a crucial role. The same information, which organizes the structure of an observed material body, is caught by our external senses thanks to electro-chemical signals, and travels across our neural system to reach our brain. Being immaterial, according to Aquinas, it is extracted by our mind and fixed in it immaterially, as a universal. According to Aquinas the immaterial operation of such extraction (Latin, *abstractio*), requires an immaterial self-subsisting operating subject, which is the immortal soul of man. Our science, at present, is just investigating the lower levels of that process, concerning the five senses, the neural system, and the brain. It is barely discovering that the mind seems to be irreducible to the sole material brain (Basti 2002).

3.8 Heisenberg and the Aristotelian Potency

In a completely different context, i.e. quantum mechanics, and several years before, another fundamental principle of Aristotelian metaphysics had been tangentially taken in consideration by Werner Heisenberg (1901-1976). According to his first interpretation of the "new physics", at that time, the probabilistic character of the wave function suggested a correlation with the Aristotelian notion of potency. In his own words: "It was a quantitative version of the old concept of 'potentia' in Aristotelian philosophy." (Heisenberg 1962, p. 12, Strumia 2021)

Here, we will rather focus on simple examples (mainly taken from Strumia 2020) of the role of information as a driving principle in generating order and organization in both non-living and biological systems.

In particular we aim to show how algorithmic information plays a leading role in generating any ordered structure starting either from already sequentially ordered initial conditions, or from random initial conditions. The final organized and ordered structure emerges as an attractor toward which the trajectories—solutions to the dynamics driven by some law (information)—converge.

In living systems, a contiguity constraint is to be imposed to random multiplication of cells that reproduce one near to the other, in order to generate the organs of a living body. The simple scheme of *cellular automata* (Wolfram 1982, Wolfram 2018) will be used to model such a behavior.

4. Order from Order and Information (Generation of a Spherical Shell, Fractals, and Galaxies)

There is no surprise if we generate an ordered structure starting from an already sequentially ordered distribution of initial conditions, either in the case of a simple structure (like a spherical shell, see Figure 1) or in the case of a complex structure such as a fractal set (e.g., a *3D-Julia set*, see Figure 2).

Generation of *order from order* and organization assigning a *law* (*information*) is quite a usual experience. Animations linked to Figures 1 and 2 show clearly the possible dynamics of the processes.

4.1 Order from Order: Spherical Shell

The information is now given by the parametric equations of the shell:

$$x = r\cos\theta\sin\varphi,\ \ y = r\sin\theta\sin\varphi,\ \ z = r\cos\varphi,$$

where the parameters θ, φ are assigned in sequential order, within the respective intervals:

$$0 \le \theta \le 2\pi;\ \ 0 \le \varphi \le \pi.$$

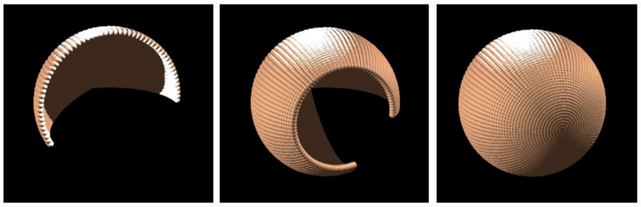

Figure 1: Generating a non-complex structure (spherical shell) starting from sequentially ordered initial conditions. Computer simulation. https://tinyurl.com/5n8r8p9u

4.2 Order from Order: A 3D-Julia Set

Here the information is provided by the recursion rule: $q_{n+1} = q_n^2 + c$, which generates each term of the series $\sum_n q_n$ of the quaternions q_n, the convergence domain of which defines the set. For details see (Strumia 2017, chapter 8).

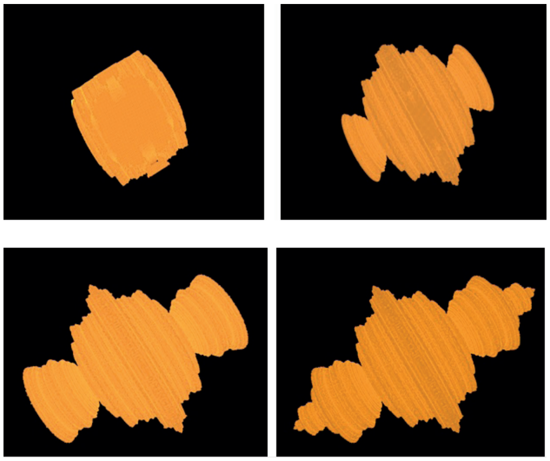

Fig. 2. Generating a complex fractal structure (*3D Julia set*) starting from sequentially ordered initial conditions - Computer simulation https://tinyurl.com/2tcf7hkx

4.3 Order from Order: A Spiral Galaxy

The same process generating order from order through an assigned law can be implemented also in the case of a more physical system, like the generating model of a spiral galaxy.

Here, the simplest law (playing the role of driving information) is provided by the Geodesic equations of motion of particles:

$$\frac{du^{\mu}}{ds} + \Gamma^{\mu}_{\alpha\beta} u^{\alpha} u^{\beta} = 0$$

in Schwarzschild metric:

$$g_{00} = 1 - \frac{r_s}{r},\ g_{11} = -\frac{1}{1-\frac{r_s}{r}},$$
$$g_{22} = -r^2,\ g_{33} = -r^2 \sin\theta,$$

which is a static solution to Einstein's equations of General Relativity.

Figure 3 compares a computer-generated image with an astronomical photo of the spiral galaxy Arp 273-d3d1cbb96446.

In our computer simulation the initial conditions are assigned in a sequential order, at regular identical angles.

Fig. 3. Generating a spiral galaxy structure starting from sequentially ordered initial conditions.
https://tinyurl.com/yc352rhx

5. Order from Chance and Information (Generation of a Spherical Shell, Fractals, and Galaxies)

In Figure 4, 5, and 6 the same structures are generated by computer programs starting from random initial conditions. The information involved in a suitable mathematical law drives the trajectories toward *attractors* that lead to final ordered structures.

The animations show initial random distributions of points that step by step reveal an emergent organized and ordered structure.

5.1 Order from Chance & Information: A Spherical Shell

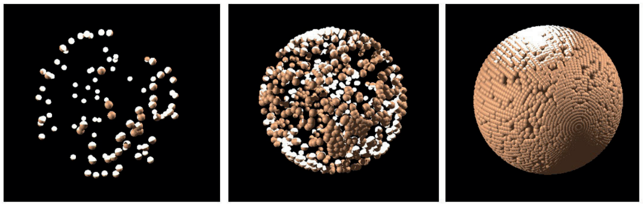

Figure 4: Generating a non-complex structure (a spherical shell) starting from random initial conditions. Computer simulation. www.youtube.com/watch?v=NBiTiJsPqLo&list=PLwSewYMk4YzGHORxIVvHrQlQ-uwIGsDuO&index=3

5.2 Order from Chance & Information: A 3D-Julia Set

Figure 5: Generating a complex fractal structure (3D-Julia set) starting from random initial conditions. Computer simulation. https://tinyurl.com/59c6atk8

5.3 Order from Chance & Information: A Spiral Galaxy

Figure 6 shows the same spiral galaxy as in Figure 3, yet this is generated starting from initial conditions at random angles and random distance from the center.

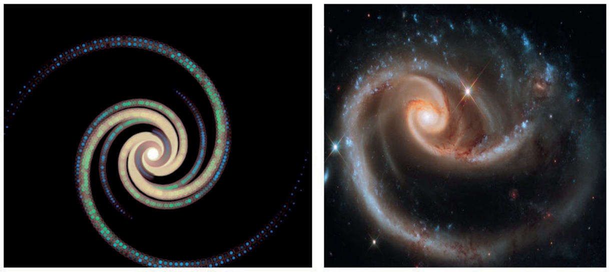

Figure 6: Generating a spiral galaxy structure starting from random initial conditions. https://tinyurl.com/yvz3aacf

6. Random Cellular Automata: Stem Cells and Heart Models Generation

What we have shown in the previous sections has relevance in biological applications. The methodology that implies random initial conditions and a driving information leads to an organized and ordered structure, such as a living body's organ. This model of an organ emerges as an attractor. In order to respect the dynamics of cell generation, we need to impose the constraint of space contiguity between each mother cell and its immediate daughter cell. A simple useful way to model such a constraint is provided by *cellular automata*, which we have suitably adapted to our problem. It is remarkable that when chance is left alone in governing the dynamics, the probability to get an ordered and organized system at the end of the generation process is so small that the result looks more similar to a cancer rather than an operating organized system. On the contrary, when a driving information is added to chance, even starting from random initial conditions, within the suitable basin of attraction, the system shows emergent organization.

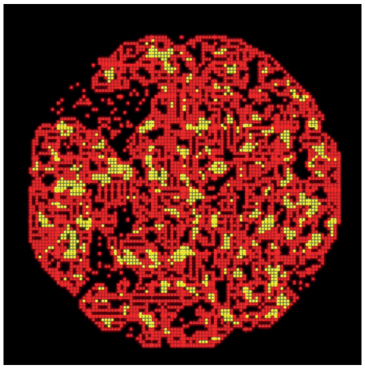

Figure 7: Generating a disordered cell formation by random cellular automata. Computer simulation. http://albertostrumia.it/sites/default/files/Animations/MeteorGuns.m4v

6.1 A 3D-Julia set generated by random cellular automata

A simple mathematical example is offered by the same Julia set examined before, which has been generated by random cellular automata constrained by contiguity condition.

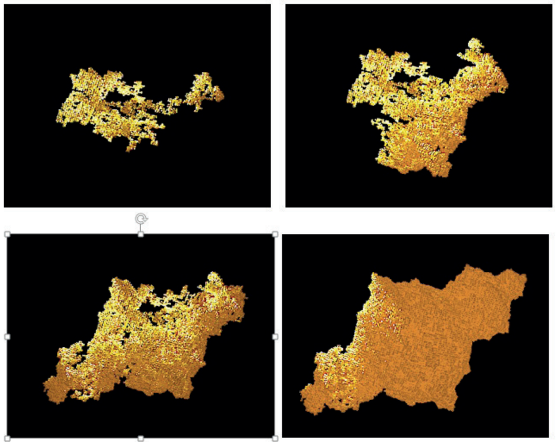

Fig. 8. Generating a 3D-Julia set by random cellular automata. Computer simulation. https://tinyurl.com/2mp2d8et

We can apply the same method of governing chance by some leading information driving cellular automata, in order to model the generation of a biological system, like an organ of a living body.

For the sake of simplicity, we limit ourselves to the generation of the exterior surface of the organ. We choose two heart-like models.

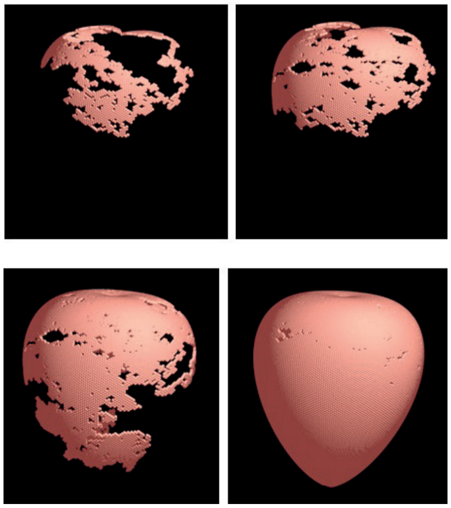

Figure 9: Generating a rough model of a heart by random cellular automata. Computer simulation. https://tinyurl.com/2p9xxad5

6.2 A "Compressible" String for a Rough Model of Heart-like External Surface

A first, very rough scheme tries to start from the idea of "compressing" the required information into a simple set of parametric equations of the organ (*compressible string*), as follows:

$$x = \pm R \sin\theta \sin\varphi + ay^2,$$
$$y = \pm b\, R \cos\theta \sin\varphi, \qquad \theta, \varphi \in [0, \pi],$$
$$z = c\, R \cos\varphi\, e^{d\varphi},$$

with a, b, c, d parameters to be suitably adjusted (see Figure 9).

An 6.3 An "Incompressible" String for a Realistic Model of a Human Heart External Surface

A more realistic shape of an external surface of a human heart can be obtained starting from a true anatomic model such as the one that Bob Hughes elaborated by means of vectorialization (Strumia 2020, chap. 9) and that we have suitably adapted.

In such model, the coordinates of each small block of cells (not each single cell, for evident reason of visual scale) are mapped one by one, into a sort of "*incompressible*" string of information, and painted as random *cellular automata* (see Figure 10).

We ignore if the information string within the DNA of a biological living system is compressed or incompressible. At present, perhaps nobody still knows.

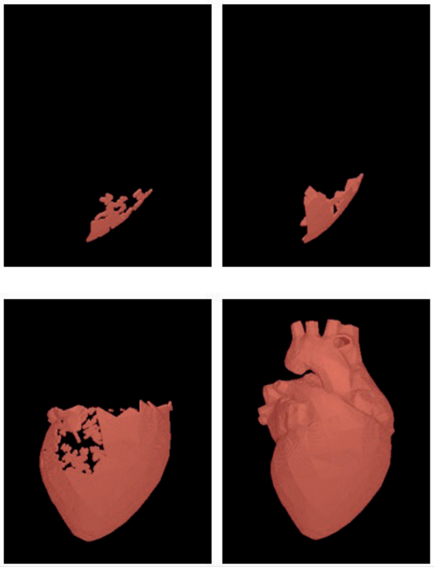

Figure 10: Generating an anatomic heart model by random cellular automata. Computer simulation. www.youtube.com/watch?v=9gZi87yojJ8

Conclusions

We have sketched how information plays a determinant role in generating order and organization in complex systems. In doing so, it resembles the ancient Aristotelian-Thomistic *form*.

Today's approach to the role of algorithmic information in driving chance towards organized and ordered structures as attractors is a sort of philosophically poorer, mathematized version of the ancient form.

The models presented in pictures and animations are enough illustrative.

All the mathematical and programming details (definitions, equations, and program lists) can be found in (Strumia 2020).

Future research should focus on modeling, beside the structure of complex systems, also their dynamics, corresponding to the Aristotelian-Thomistic notion of nature (*the essence as principle of action*).

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