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Chess as a Model of Collective Intelligence: Analyzing a Distributed Form of Chess with Piece-wise Agency

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Abstract

Chess is a much-studied virtual world in which human and artificially-intelligent players move pieces toward desired ends, within established rules. The typical scenario involves top-down control where a single cognitive agent plans and executes moves using the pieces as its embodiment within the chess universe. However, ultimately both biological and engineered agents are composed of parts, with radically differing degrees of competency. The emerging field of Diverse Intelligence seeks to understand how coherent behavior and goal-directed navigation of problem spaces arises in compound agents from the interaction of their simpler components. Thus, we explored the world of chess rules from the perspective of collective intelligence, and characterized a bottom-up version of this classic game in which there is no central controller or long-term planning. Rather, each individual piece has its own drives and makes decisions based on local, limited information and its own goals. We analyzed the behavior of this distributed agent when playing against Stockfish, a standard chess algorithm. We tested a few individual policies designed by hand, and then implemented an evolutionary algorithm to see how the individuals' behavioral genomes would evolve under selection applied to the chess-based fitness of the collective agent. We observed that despite the minimal intelligence of each piece, the team of distributed chess pieces exhibit Elo of up to ~1050, equivalent to a novice human chess player. And, compared to advanced chess engines like Stockfish, the distributed chess pieces are significantly more efficient in computing. Distributed chess pieces select their next move approximately 7 times faster than the Stockfish Engine with a search depth of 8. Investigating different local policies for the distributed agents, we found that policies promoting offense, such as swarming the opposing king and opposing highest valued piece, moving less cautiously, and a radius of vision of 4 spaces yields optimal performance. Comparisons between centralized and distributed versions of familiar minimal environments have the potential to shed light on the scaling of cognition and the requirements for collective intelligence in naturally evolved and engineered systems.

Keywords: decentralized intelligence, emergence, behavior, minimal models, distributed systems

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Introduction

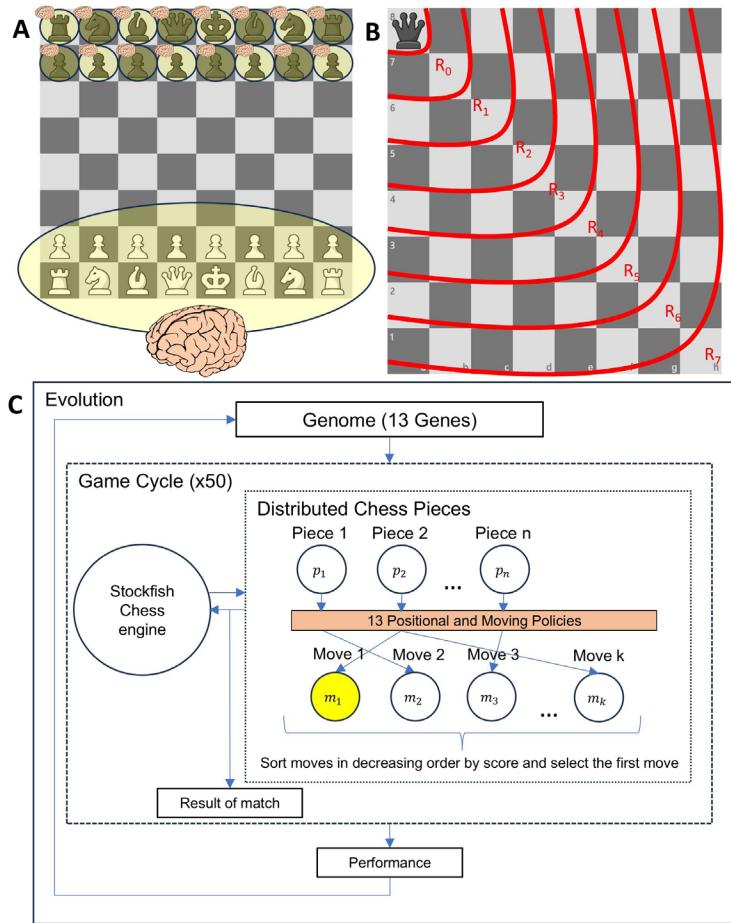
Intelligence, both natural and artificial, has fascinated thinkers for millennia. One especially interesting emerging field is that of Diverse Intelligence, which includes efforts to understand problem-solving competencies in unconventional embodiments outside of familiar brainy animals (ranging across minimal chemical models, unicellular organisms, plants, tissues, robots, hybrid cyborgs and hybrots, and software AI's) (Sole, Moses, & Forrest 2019; Lyon 2006; Lyon *et al.* 2021; Levin *et al.* 2021; Lyon 2020; Lyon 2015; Vallverdu *et al.* 2018; Baluška & Levin 2016). Another component of this effort is the goal of understanding the scaling of collective intelligence (Berdahl *et al.* 2018; Couzin 2007; Couzin 2009; Couzin 2018; Couzin 2002; Deisboeck & Couzin 2009; Levin 2022; Trianni & Campo 2015; Gomes, Urbano, & Christensen 2013): how do the capabilities of swarms relate to the competencies of the subunits of which they are made (Levin 2022)? How much foresight, sensing, and memory is needed in the components to endow a composite agent with a specific level of competency in a given problem space? The latter is not only of concern to swarm roboticists (Trianni & Campo 2015; Gomes, Urbano, & Christensen 2013; Brambilla *et al.* 2013; Barca & Sekercioglu 2013) and ethologists studying ant colonies and bird flocks (Letendre & Moses 2019; Gordon 2016a; Gordon 2016b; Gordon 2016c; Reid *et al.* 2016; Reid *et al.* 2015a; Reid *et al.* 2015b), but is also a central issue for understanding human cognition. While many think of themselves as unified, individual agents, the reality is that we too are a collective of neural cells, and start life as a single cell which proliferates into a collective that navigates anatomical space long before we can navigate 3-dimensional behavioral space and linguistic spaces (Fields & Levin 2022). Even after embryogenesis, the neuropsychology of split-brain patients and dissociative identity states reveal that our cognitive system is far from a monolithic, unified controller (Miller & Triggiano 1992; Putnam 1992; Braude 1995; Gazzaniga 2005; Montgomery 2003). Elucidating the functional policies that enable highly complex cells to work together toward the emergence of a high-order Self, which has memories, goals, preferences, and capabilities that belong to it and none of its parts (Dewan 1976; Solms 2018; Paulson *et al.* 2017; Ramstead *et al.* 2019; Badcock *et al.* 2019; Friston & Buzsaki 2016; Pezzulo, Rigoli, &

Friston 2015; Friston & Frith 2015), is an essential part of understanding what we are and how minds like ours arise. In a sense, all real-world intelligence is collective intelligence (composed of parts), underscoring the importance of understanding how the properties of subunits give rise to system-level problem-solving behavior.

1. Multiscale Competency Architecture

Our goal is to understand biology's multiscale competency architecture (Pio-Lopez *et al.* 2023; Levin 2023a; Levin 2023b; Levin 2023c). We seek to understand the "cognitive glue" that enables collective intelligence in living tissue, including neural systems as well as non-neural ones (which in turn drives regenerative medicine research programs by exploiting the information-processing capabilities of cells and tissues [Lagasse & Levin 2023; Mathews *et al.* 2023]). We characterize the biological policies for communication, cooperation, and competition between parts (Gawne, McKenna, & Levin 2020; Boddy *et al.* 2015) that engineering may want to emulate, in creating robust intelligences. Here, we take a minimal model approach, using the game of chess as a highly simplified universe, with well-defined and much studied dynamics, in which we can ask questions about how problem-solving competencies can emerge from extremely sparse bottom-up capabilities. We use principles of agent-based modeling (Griffin 2006; Steinbacher *et al.* 2021) to implement a chess player as a collection of individual pieces with their own perspective and agency. That is, instead of a top-down human or computer controlling all a player's moves, we let the pieces decide and move themselves.

The popular game of chess has been enjoyed by millions of people throughout history (Hearst & Knott 2009; Sharples 2017; Davidson 2012; Dangauthier *et al.* 2007). With an exorbitant number of games played, different openings, tactics, and theories have been developed and analyzed. When playing a traditional game of chess, an integrated "player" creates strategies with the ultimate goal of checkmating the opponent player. While in the past these have been human minds, over the last few decades it has become clear that artificial intelligences excel at navigating the space of behaviors in the chess universe (Fujita 2022; Maharaj, Polson, & Turk 2022; Schmid *et al.* 2022).



The traditional mode requires a player to exploit counterfactual thought in order to choose moves, long-term planning to consider the large tree of possible moves, and the ability to make choices that often require sacrifice (temporary reduction of advantage, or delayed gratification), as well as modeling of their opponent and their likely actions. We wondered: how much of this is actually necessary to play a game of chess, and how crucial is the centralized architecture that directs all the pieces?

Here, we compare the traditional top-down mode with a more biological scenario, in which each component has its own goals (“goals” are meant in the cybernetic sense, not requiring human-level second-order metacognition, i.e. not requiring “I know that I have goals”). Specifically, we sought to implement many extremely simple piece-players, who had no ability to plan and very limited knowledge of the board. Each piece-player could only see its local environment, had no direct control over any other pieces, no memory, and had no capability for counterfactual projections into the future. In this scheme, each piece

had its own agency, and was seeking to maximize the “nutritional value” of other pieces it captures. Under these conditions, with no central planner or controller provided, would a passable game of chess emerge? What would be its observable characteristics? How would its efficiencies compare to the conventional version? What properties of the tiny individual protominds would most impact the quality of play of the collective? And, how would evolution work, if each piece-player acted independently, but the selection took place on an entire team of piece-players? We investigated those questions using the system shown in Figure 1.

2. Methods

We used two experimental methods (in both of which, all pieces followed the same policy): manual design of perception-action policies for the pieces, to test specific hypotheses of what aspects might improve the collective agent’s ability to win, and an evolutionary strategy for determining optimal policies.

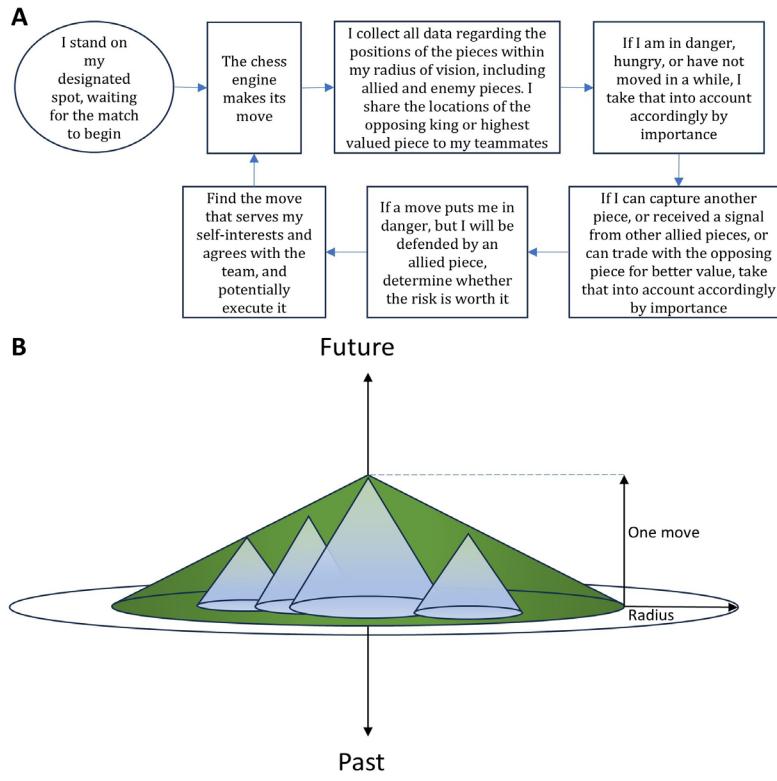


Figure 2: The cyclical selection process for the next move (the decision-making of an individual piece). (A) Perspective of an individual Distributed Piece (playing as Black). Each piece-player underwent a decision-making process to determine its desired move based on its “biological needs”. The piece-player with the strongest desire to move got to play, determined by its genetic makeup. (B) An individual piece-player’s cognition was limited to its radius of vision (represented schematically by the blue cones). Piece-players transferred information about the positions of opposing pieces (specifically the king and the highest valued piece in view) to one another, expanding their spatial awareness (represented by the green cone). Nevertheless, piece-players were limited to analyzing a maximum of one move when determining the risk of their next move. In addition, they had no recollection of previous moves per se; rather, they only knew how many turns ago they last played.

2.1. The Umwelt of a Single Chess Piece Agent

Without a central host controlling all the pieces, every piece-player made decisions on its own (the proto-cognitive world of such simple agents have been studied previously [Beer 2014; Albantakis *et al.* 2014; Edlund *et al.* 2011]). We roughly mapped the agents’ goal—to capture other pieces—to a biological imperative to catch and eat others in order to maintain metabolic status and survive. The pieces also had rudimentary sensory capacities, that allowed them to get information about their local neighborhood. Thus, each chess piece had four fundamental characteristics: value, radius of vision, hunger level, and a turn counter. A piece was assigned a relative value according to the standard chess piece value convention (Capablanca 2006; Chess.com 2024a): Pawns a value of 1, Knights and Bishops a value of 3, Rooks a value of 5, and Queens a value of 9. (Kings did not have a value as they cannot be captured.) The radius of vision was a whole number between 0 and 7 inclusively that described the number of adjacent squares it could detect (including diagonal squares), i.e., $R_0, R_1, R_2, \dots, R_7$ (see Figure 1).

Hunger levels reflected how many moves ago a piece-player last captured another. All piece-players

in the manual experiments started with a controlled hunger level of 0 (i.e. no hunger). In the evolution experiments, the hunger levels were distributed through a Gaussian function amongst the piece-players, to increase diversity of the population and reflect a natural environment. The Gaussian distribution used a mean of 7.5 and a standard deviation of 3, and the hunger levels were rounded to integers. With each turn on the chess board a piece-player made, its hunger level was incremented by one. When a “predator” piece-player captured a “prey” piece-player, the predator’s hunger level reset to 0, indicating that it has previously eaten 0 moves ago (i.e. the current move).

The turn characteristic represented how many moves ago a piece-player’s last move occurred. With each turn on the chess board, the turn value was incremented by one. When a piece-player moved, its turn parameter reset to 0, indicating that it has previously moved 0 moves ago (i.e. the current move).

As an agent, the primary motivator for a piece-player was to fulfill its nutritional needs by capturing another piece of the opponent, preferably of higher value. In accordance with typical chess strategies, a piece-player might also be interested in improving its position to capture other pieces. At the same time, piece-players

might have the goal to prevent themselves from being captured, either by staying protected in the range of a same-side piece (minimizing chances of being captured) or moving away from the targeted path of an opposing piece. Apart from a piece's self-interest, it can be motivated to attack the opposing king and defend the same-side king, as well as capture the opposing highest-valued piece.

A piece-player's sensory horizon was limited by its radius of vision. A piece-player could observe other pieces surrounding it and differentiate which side it belonged to. Also, a piece-player could detect the type of a piece near it, as well as the latter's distance away and location. A piece-player was apprised of how each piece moves (including itself) according to the official chess rules (FIDE 2023). Using the information about "moves", a piece-player thus had information about what squares other pieces could attack or defend (within the piece-player's field of vision).

Pieces here did not have memory of previous moves and could not think multiple moves ahead. However, before making the next move, a piece-player was programmed to disclose information about the locations of opposing pieces within its field of view to other same-team pieces (long range communication) by providing the coordinates of the opposing pieces. For example, if the opposing king was within the radius of a piece-player, then the piece-player would communicate this information to the other piece-players who also opposed that king. Conversely, if the same-side king detected it was in danger (i.e. that there was an opposing piece within its radius), the king would communicate this information to the other piece-players on that king's team so they could potentially advance closer to protect the king. Every piece-player was programmed to communicate accurate information to other pieces (by relaying from one piece to another), and to receive accurate information automatically without conflicts or errors. Information was received simultaneously and in parallel, before any move was made. While the distributed pieces took their own well-being into account, they could also determine their course of action based on information transferred from other pieces. In addition, each piece knew its own value and the value of other pieces. A piece's decision to capture was therefore influenced by the comparison of the value of itself and of an opposing piece (see Figure 1 and Figure 2).

Furthermore, as each piece had its own perspective on its outside world, it was faced with the natural limitation of not being able to see the entire board, similar to the local perspectives of cells within a body and the need for biological systems to form their own perspectives, interpretations, and models of their microenvironment (Levin 2023a; Bongard & Levin 2023; Levin 2024).

2.2. Life in Chessworld: The Algorithm of a Single Game

In biological scenarios, multiple agents can effectively act at the same time. In a standard chess game, a centralized algorithm decides which of their pieces will be moved at a given turn. In our case, the distributed agents would all try to act in each time step, due to their self-interests. We implemented our simulation on a linear computer architecture by providing a turn-based scheme that regulated the order of operations for the pieces as follows. Each piece-player initially had a radius of vision R_2 . It first identified whether any pieces could be captured, and if there were, the piece-player with the highest sum of hunger level and turn would capture, and if there was a tie, it would be broken in favor of the piece-player in position to capture the highest valued piece. If a piece-player could capture multiple pieces, it would capture the higher valued one. If no pieces could be captured, then the following predefined steps occurred, varying in accordance with the experiment being performed (described in subsections below): the scheme selected the next piece-player to be moved based on necessity (a rough analogy to how cognitive systems use attention and prioritization drives to choose among actions [Bongard & Levin 2023]):

1. If a piece-player was in danger (i.e., risk of being captured) and not defended, then that piece-player was prioritized first to move.
 - a. Among the piece-players that were in danger, the scheme prioritized piece-players who had the highest hunger value and turn.
 - b. Among the remaining pieces that were not in danger, the scheme prioritized piece-players who had the highest hunger value and turn. This step is done as backup for step 2 in case the pieces in danger have no safe square to move to.
2. If a piece-player's move would place it in danger, that specific move was prioritized last.
3. Once the most prioritized move had been executed,

and the opposing team responded, this process would repeat until a checkmate or stalemate occurred.

We described how these rules were applied in detail in our different experiments in the following sections. While many possible methods could be explored, we chose this logic flow as an analog to “motivation” (i.e., it models the idea that agents with the most opportunity or the most threat would be the most active and most likely to act quicker than others).

2.3. Evolution in Chessworld: How Genomes Change over Time

In the evolutionary algorithm, 13 different genes were encoded into one chromosome, described in Tables 1 and 2. The chromosome contained five positional genes that controlled a piece-player’s radius of vision and how a piece-player’s state (e.g. its position, hunger level, turn) would influence its next move. Additionally, each chromosome included eight action genes that control the favorability of each potential move for the piece-player. The complete move selection process is described in Figure 2. There was no developmental process here (genotype directly encodes phenotype with no generative layer or complex mapping between them).

Once the game started and the chess engine made its move, each piece-player analyzed all the information within its radius of vision. A piece-player examined its current state by considering factors such as its position, hunger level, and turn. Before a piece made a turn, it was assigned an initial move score of zero. Then, the score was updated in accordance with positional genes based on information of itself and its neighbors. The positional genes and their range of values are summarized in Table 1 [Appended at end].

To calculate the score of each piece-player’s next possible moves, we took the current positional score of each piece-player and assigned it to all its possible moves, giving all possible moves a value. In addition, every action gene updated a prospective move’s score based on information of the move’s outcome. The action gene and their range of values are summarized in Table 2 [Appended at end]. For example, if a rook had a positional value of 50, all its possible moves get a value of 50. If a possible move would place the piece-player into danger, the move’s score updated, according to Positional Gene #4 in Table 2.

The piece with the highest move score executed their move. The chess engine responded, and the cycle repeated until the game ended in checkmate or stalemate.

2.4. Implementation Details

All simulations for both portions were built using Python 3.12.0. The opposing side to the distributed piece-players was controlled by Stockfish 15.1, winx64 avx2 version (The Stockfish developers, n.d.). To integrate the Stockfish engine with Python, the Python library (Zhelyabuzhsky 2022) was used. To ensure the engine selects moves quickly and accurately, the Stockfish engine was set to a depth of 8, and a skill level of 20. The Stockfish engine selected the third best move (average ELO [Elo 2008] of 300) for the manual portion, and randomly selected either the second best move or the third best move for the evolution portion to increase difficulty (average ELO of 500). This arrangement matched the skill level of the piece-players to ensure no side overpowers the other. We used a standard 8x8 chess board with standard chess rules, except for the En Passant (Chess.com 2024b) rule for simplicity. One game took approximately 0.8 seconds to compute. One trial consisted of 50 chess matches between the Stockfish chess engine and the distributed pieces and lasted around 40 seconds to be completed. One generation for the evolution component took on average 3 minutes. For the manual experiments, 10 trials were conducted for every experiment setting, taking 40 seconds for each. Trial results were recorded, and the mean along with standard deviation was taken for each experiment.

In the evolutionary algorithm, the skill level of the engine gradually increased every 25 generations by 1 from level 0 to level 20. The winning percentage of the piece-players for each trial served as the fitness value and was expected to approach a fitness value of 1 (100%). PyGAD 3.2.0 was used for the genetic evolution (Gad 2023). We used a population size of 200 trials per 500 generations and calculated the fitness of each trial in parallel. The genetic evolution was conducted on a Linux server, running on 2x AMD EPYC 7532, 32x2 core, 64x2 thread, 512GB RAM. In the genetic algorithm, the random mutation probability parameter was set to 10% and the single-point crossover parameter was enabled. Piece-players in each trial were characterized with a chromosome of 13 genes, controlling the influence of each policy in its decision-making.

3. Results

3.1. Minimal Distributed Agents Can Play Chess

We first established a baseline in which pieces guided by totally random legal actions played against the Stockfish chess engine for 50 matches. As might be expected, the random moving pieces had a 0% winning percentage against the Stockfish engine out of 50 chess matches, playing at an Elo of around 0 (Chess.com 2024b)—roughly the level of a person who only knows how pieces move and knows no strategy, but all moves were legal.

To determine whether distributed agents could play chess against a classical (unified) player, we then had the distributed chess pieces play against the Stockfish engine using the logic described in the manual policies portion of the Methods (see subsection “Life in Chessworld: The Algorithm of a Single Game”) and a radius of R_2 . The distributed pieces were able to obtain an average win rate of $20\% \pm 6\%$ in the manual experiments alone. After four evolutions running in parallel, the pieces were able to achieve a peak winning percentage of $44\% \pm 2\%$. The distributed pieces played at a maximum Elo of 1050 and an average Elo of 750 (Chess.com 2024b), equivalent to a casual chess player who knows the rules and is familiar with basic strategy. The pieces were successfully able to form defensive structures and avoid active dangers (Figure 3A). However, the pieces performed poorly near

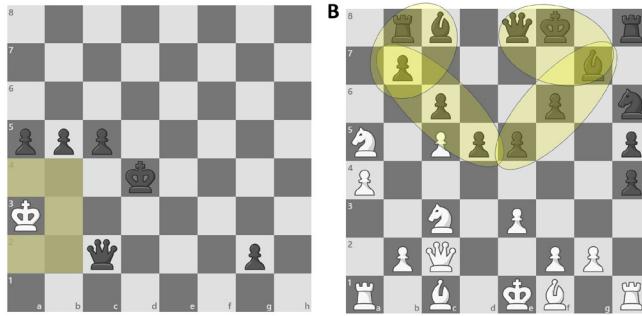


Figure 3: Scenarios of distributed agents in action. A) The black pieces, played by the distributed pieces, have a significant material advantage over the white pieces. However, the distributed pieces inadvertently force a stalemate by eliminating any squares the white king can move to. B) The black pieces, played by the distributed pieces, create a defensive structure, forming two pawn chains of length three. A black rook and black bishop defended a pawn at the b7 square, a black bishop defends the black pawn on the f6 square. The black queen protects three pawns, and a bishop, creating a defensive front.

the end of the game when encircling and checkmating the king (Figure 3B). Often, the large quantity of pieces eliminated any paths for the opposing king to go, resulting in a stalemate. In addition, an inadvertent blunder of a significant piece like the queen (because of the limited radius of vision, or poor risk taking) at the beginning of the matches would make the distributed pieces’ chances of winning significantly worse. Computationally, the Stockfish engine at a depth of 8 determined the next move in around 5.8 milliseconds, while the distributed piece-players collectively decided on their next move in a total of 2.0 milliseconds.

3.2. Radius of Vision Positively Correlates with Winning Percentage

We next sought to understand whether, and to what degree, being able to see further along the board would enable the collective to play better. Thus, to analyze the correlation of radius of vision and winning percentage in the manual experiments, we altered the radius settings (8, between R_0 and R_7) to analyze which was most optimal.

The mean and standard deviation of the data are shown in Figure 4A [Appended at end]. We found that the winning percentage sharply increased from radii R_0 to R_2 , but plateaued for radii R_2 – R_7 with minor fluctuations. The highest winning percentage became apparent from a radius of R_4 , with a $21\% \pm 3\%$. We conclude that a larger radius of vision improves performance for all pieces, but the gains drop off and having information about distant regions of the board does not add much to the efficacy of play under these conditions.

We then tested the evolutionary approach, setting possible radius bounds for alleles in the population to range from R_2 to R_7 inclusively. Initially, the frequency of each radius was randomized (Figure 4B). Throughout the evolution, the radius of the best trial in each generation fluctuated between R_3 – R_7 . At the end of the evolution, R_4 appeared the most frequently in the final population (as seen in Figure 4C), with $27\% \pm 15\%$ of trials in the final population having an R_4 radius. The evolution had a higher winning percentage ($44\% \pm 2\%$) than the manual experiments’ results because the evolution balanced all the policies together, complementing the optimal radius (R_4). Comparing Figure 4B and Figure 4C, there is a contrast between the frequency of radii at the beginning of the evolution, and at the end [Appended at end].

3.3. Long Range Information Transfer Positively Correlates with Winning Percentage

To determine the benefits of information transfer between piece-players, a long-range communication rule was implemented in the manual experiments during a turn only when the opposing king was within the radius of a piece-player. We utilized the base scheme described in Methods section. Before moves were sorted based on whether a move would place a piece-player in danger, moves that increased the distance between the piece and the opposing king were prioritized last. If the opposing king was not in the radius of a piece, the pieces would play according to the base scheme. Keeping the radius to R_2 , we compared the winning percentages of the winning performance without the new rule and with the new rule. We observed that R_2 had a winning percentage of $20\% \pm 4\%$. The winning percentage of the experiment with the new rule increased to $24\% \pm 6\%$ (Figure 5A) [Appended at end]. (1-tailed, two-sample equal variance [homoscedastic] T-test, $p=0.066$). Thus, we observed that the long-range communication group's winning percentage was 4% more than the control group.

We then tested the evolutionary approach with the same goal, by comparing the frequency of genes in the initial and final population. Genes related to long-range information transfer had parameter values between -100-100 (see Policy 6, 7 and 8 in Table 2). The parameter values of each trial in a generation were grouped in ten bins, each bin of size 20 (as seen in Figure 5B-5G) to capture the main features of the data. The results of the genetic algorithm revealed that genes related to long-range information transfer were reliably selected for in the population with the best fitness.

Starting with the gene that controlled whether a piece-player prioritized moving closer to the opposing king (see Policy 6 in Table 2), we observed that on average, $29\% \pm 14\%$ of parameter values in the final population were concentrated between 20 and 40, while $18\% \pm 13\%$ of values were concentrated between 0 and 20 in the four evolutions (Figure 5C), deviating from the random trend in the initial population (Figure 5B). The most successful individuals in each of the four runs had parameter values of 7, 21, 23, and 40, showing the prioritizing moving closer to the opposing king (Policy 6 in Table 2) was favorable in the decision-making of a piece.

Then, we analyzed the gene that controls whether a piece-player prioritized moving closer to the defending king if an opposing piece is in the defending king's radius of vision (see Policy 7 in Table 2). We observed that on average, $32\% \pm 15\%$ of parameter values in the final population were concentrated between -40 and -20 (Figure 5E). The most successful individuals in each of the four runs had parameter values of -8, -9, -29, and -38, deviating from the random trend in the initial population (Figure 5D). This implies that prioritizing moving closer to the same-side king (Policy 7 in Table 2) was unfavorable in the decision-making of a piece.

Then, we analyzed the gene that controls whether a piece-player prioritized moving closer to the highest-valued piece (see Policy 8 in Table 2). We observed that on average, $22\% \pm 11\%$ of parameter values in the final population were concentrated between 20 and 40 (Figure 5G), deviating from the random trend in the initial population (Figure 5F). The most successful individual within the population in the four runs had parameter values of -5, 34, 49, and 96, demonstrating that prioritizing moving closer to the highest-valued piece (Policy 8 in Table 2) was favorable in the decision-making of a piece.

From the results, the ability to transfer long-range information about the opposing king and the highest valued piece proved to be highly favorable. Defending the same-side king proved to be unfavorable, as the parameter values were largely negative, also showing that the pieces fared better on the offensive.

3.4. Courage is More Favorable than Caution in Certain Scenarios

We next sought to understand the contribution of risk-taking and risk aversion to the quality of play by the collective agent. To determine whether pieces should act more courageously or cautiously for the best winning percentage, we analyzed the performance difference between these two strategies. A piece was cautious when it deliberately avoided or prevented the risk of being captured by choosing an alternative safe move. A piece was courageous when exposing itself to risk.

To implement these strategies in the manual experiments, we utilized the base scheme described in Methods section. However, we changed how pieces made decisions based on danger in two experiments. The first experiment examined when pieces acted cautiously, following the base scheme. The second

experiment showcased when a piece acted courageously for accepting the risk of being captured. This experiment is the same as the first one, except a piece-player is not considered to be in danger if it was defended by another same-side piece-player. In addition, a piece-player's move is not considered dangerous if a piece would be (after the potential move) defended by another same-side piece. We observed (Figure 6A) [Appended at end], that the winning percentage was $18\% \pm 6\%$ when the piece had only a sense of danger. When the piece could detect that it was defended and in danger at the same time (resulting in no action), the winning percentage was $15\% \pm 5\%$. We see that the more cautious pieces have a mildly higher winning percentage than the courageous ones.

To have a more accurate understanding of which specific courageous and cautious strategies are optimal, we allowed evolution to set the risk-taking level for the pieces. Genes related to courage and caution had parameter values between -100-100 (see Policy 2, 5 in Table 1 and Policy 4, 5 in Table 2). The parameter values of each trial in a generation were grouped in ten bins, each bin of size 20 (as seen in Figure 6B-6I) to capture the main features of the data. The results of the genetic algorithm revealed that genes that encouraged more risk were more favorable. Starting with the gene that controls how motivated a piece is to escape imminent danger (see Policy 2 in Table 1), we observed that $62\% \pm 14\%$ of parameter values in the final population were between 20 and 40, (Figure 6C), deviating from the random trend in the initial population (Figure 6B). The most successful individual within the population in the four runs had parameter values of 12, 12, 20, 22, demonstrating that the gene had a large influence on the decision-making of a piece.

We then analyzed the gene influencing the decision-making for a piece when the piece is defended by another same-side piece (see Policy 5 in Table 1). $76\% \pm 4\%$ of parameter values in the final population were concentrated around 0-20 (Figure 6E), deviating from the random trend in the initial population (Figure 6D). The most successful individual within the population in the four runs had a weighting of 3, 7, 9, and 18, indicating that the gene had some influence in the decision-making.

We then analyzed the gene that controls voluntary decision of a piece to put itself in danger (see Policy 4 in Table 2). We observed that $83\% \pm 1\%$ of the parameter

values in the final population on average were between 0-40 among four evolutions (Figure 6G), deviating from the random trend in the initial population (Figure 6F). The most successful individual within the population in the four runs had a weighting of 1, 2, 2, and 3, highlighting that this policy had some influence in the decision-making of a piece.

We then analyzed the gene influencing the decision-making for a piece when the piece's next move would be defended by another same-side piece (see Policy 5 in Table 2). $81\% \pm 5\%$ of the parameter values in the final population were between 0-20 among four evolutions (Figure 6I), deviating from the random trend in the initial population (Figure 6H). The most successful individual within the population in the four runs had a weighting of 5, 23, 24, and 34, highlighting that this policy had some influence in the decision-making of a piece, and making it more cautious.

The results show that piece-players that were at risk of being captured were more eager to move than piece-players that were not at risk of being captured, meaning piece-players were cautious. In addition, piece-players in general acted courageously and moved to squares that could lead them to be captured (Figure 6J). Genes depending on whether a piece-player is protected and whether a piece's move would be protected by other pieces had minor influence on the decision-making about the next move, however still favoring protection over risk. Thus, piece-players were cautious about their position when they were at risk of capture, and were courageous when making a move.

3.5. Having More Patience when Hungry, and Less Patience when Not Having Moved in a While, is Optimal

The ability to perform “delayed gratification” in a problem space-making moves that temporarily take the agent further from its goals in order to recoup gains later—is one metric of basal intelligence (James 1890). It is interesting to ask what kind of policy should be used among the components of a collective intelligence to determine which ones get to act at what time, for optimal adaptive performance. Thus, we next sought to determine what was the optimal move order for the pieces, we analyzed which strategy is more optimal—making decisions based on the hunger level, turn, or both. We compared the results to when neither of the attributes are applied. A piece-player was considered to

be more patient when its hunger level and/or turn was high, but the piece refrained from moving. A piece was considered to be less patient if its hunger level and/or turn was high, and the piece prioritized its self-interests and moved.

For our first experiment, we prioritized pieces with the highest hunger value and turn (as described in the base scheme). For the second experiment, we prioritized neither of these values instead. For the third experiment, we prioritized pieces with the highest hunger value, and for the fourth experiment, we prioritized pieces with the highest turn instead. Keeping the radius to R_2 , we compared the winning percentages of the winning performance of each of the moves' ordering methods. From the four experiments, we observed that the control had a $14\% \pm 5\%$ winning percentage, the hunger based moving pieces had a $10\% \pm 3\%$ winning percentage, and the alternating order moving pieces had a $16\% \pm 4\%$ winning percentage (Figure 7A) [Appended at end]. It appears the collective did best when emphasizing a strict turn order for its members.

To analyze which strategies were most optimal and their magnitude of impact, we conducted the evolutionary by comparing the frequency of genes in the initial and final population. Genes related to hunger and turn had parameter values between 0.0-5.0 (see Policies 3 and 4 in Table 1). The parameter values of each trial in a generation were grouped in ten bins, each bin of size 0.5 (as seen in Figure 7B-7E) to capture the main features of the data. The results of the genetic algorithm revealed that having more patience when hungry and less patience when not having moved in a while is optimal.

For the gene controlling how motivated a piece is based on their hunger (see Policy 3 in Table 1), we observed that on average, $77\% \pm 6\%$ of the parameter values in the final population were concentrated around 0-0.5 (Figure 7C), deviating from the random trend in the initial population (Figure 7B). The most successful individual within the population in the four runs had parameter values of 0.05, 0.1, 0.1, and 0.2, demonstrating that a piece ignoring their hunger completely is optimal.

Then, we analyzed the gene that controls how motivated a piece is based on their turn (see Policy 4 in Table 1). We observed on average, $36\% \pm 14\%$ of the parameter values in the final population were concentrated around 3.50-4.00, and ~60% of values

were above 3.5, with the maximum possible parameter value being 5 (Figure 7E), deviating from the random trend in the initial population (Figure 7D). The most successful individual within the population in the four runs had parameter values of 2.7, 3.4, 3.6, and 3.6, indicating that the policy had a large impact on the decision-making of a piece.

The hunger level multiplier clearly indicates that the hunger level was almost negligible in the decision-making process, meaning it is best when piece-players are patient with their hunger when making decisions. Conversely, the turn multiplier shows that prioritizing to make a turn after not doing so in a while is deemed to be more beneficial.

3.6. Adding a “Threatening” Drive Significantly Improves Performance

In the basic scheme, the only drive that guides pieces' behavior is the ability to consume another piece. We next sought to examine the consequences of giving them a motivation to threaten another piece. Moves were prioritized based on whether they would place an opposing piece in danger. We varied the radius of vision from R_0 to R_7 and compared the winning percentage to the original base scheme (Fig. 4A). The winning percentages of this experiment were significantly higher than the base scheme (except for R_0 and R_1), with higher radius of vision performing better. For example, the winning percentage for R_7 with the new rule was $42\% \pm 5\%$, while the winning percentage for R_7 without the new rule was $20\% \pm 6\%$ (1-tailed, two-sample equal variance (homoscedastic) T-test, $p < 0.01$). The ELO improved by ~50.

4. Discussion

We tested the hypothesis that a passable game of chess could be played without a central planner, memory, training in prior games, forethought, or consideration of the consequences of specific actions. By implementing a bottom-up, distributed player where the pieces had their own agency, we created an alternative to conventional chess AI (Duca Iliescu 2020; K.B. 2021)—one based on the concepts of collective intelligence (Couzin 2007; Couzin 2009; Couzin *et al.* 2002; Deisboeck & Cousin 2009; McMillen & Levin 2024; Witkowski & Ikegami 2019; Pinero & Sole 2019; Sole *et al.* 2016; Heylighen 2013; Wheeler 1911; Ward

et al. 2008; Bazazi *et al.* 2008). We found that this very minimal system operates at the level of a human novice when each piece is computing its own desired moves. Using evolutionary algorithms with only 13 possible parameters, a ragtag group of player-pieces can in aggregate increase its score to an ELO of 1050, which is comparable to the score needed to be competitive against a beginner.

4.1. Parameters Impacting Play Quality

A critical component of any collective intelligence is the set of policies which regulate their actions and interactions. As with many examples of collective problem-solving, the parameter values that improve the functionality of the group are not obvious and hard to predict from first principles (McMillen & Levin 2024; Rahwan 2019). Our exploration of the parameter space and evolutionary search found several ways to optimize the performance; First, that the optimal radius of vision was R4. The reason why it was not significantly higher (like R6 or R7) was most likely because a too large radius of vision can hamper a piece's decision-making: with too much awareness of opposing pieces across the board, a piece may move to aggressively, stranded in enemy territory and ultimately sacrificed. Conversely, a lower radius of vision that 4 performed significantly worse, presumably because unaware of the whereabouts of opponent's pieces, therefore diminishing captures and diminishing the winning percentage of the collective. The radius of R4 allows for the balance between controlled risk taking and capture.

We found that the best performance came when pieces were more cautious about their current position, i.e. escape imminent danger if an opposing piece is attacking them, but more courageous (i.e. take more risks) when selecting their next move. They avoided imminent threats, regardless of whether they were defended by a same-side piece. The pieces were content with placing themselves at risk while moving to squares where they would be protected, thus expanding the position and working as a collective. This configuration allows offensive attacks, while preventing passive play and takes into consideration the present danger/ defense set up.

For long-range communication, the policy of having knowledge of the opposing king's position and the opposing highest valued piece allowed for swarming.

This ability was influential, which is reflective by the increase in winning percentage in the manual experiments. The ability to defend the same-side king had a negative influence in the decision-making process. This is because defending the same-side king promotes passive play, allowing the chess engine to take down the distributed agents. As a result of these policies, pieces were able to collectively surround the opposing king to deliver checkmates more consistently, and attack the opposing highest valued piece, all while playing offensively.

Moves driven by hunger level were seen to be suboptimal. In the manually-parametrized experiments, moving based on hunger level decreased the winning percentage compared to the other moves' ordering strategies. Moreover, the hunger level multiplier in the evolution portion was almost at its minimum, because hunger level provided little input in the decision-making for the next move. On the other hand, having the desire to move when having not moved in a while (based on turn) was favored significantly more in the evolution, and moving based off turn boosted the winning percentage. In classical chess theory, it is typically suggested to move a wide range of pieces to develop and strengthen the chess position, which is what the distributed agents confirmed.

4.2. Emergent Collective Goals

One formalism for the study of collective intelligence is the notion of the cognitive light cone—the spatio-temporal radius of the largest state that an agent can actively work towards (Levin 2019). Here, pieces are limited by their radius of vision. However, when pieces are able to communicate with one another, they are able to expand their spatial cognition and receive information about pieces across the board when applicable. An example of this is knowing the location of the opposing king. If a piece were to attempt to target an opposing king in their radius of vision, the probability of the king stepping into the radius of vision (e.g. R₄) is not high, making the policy ineffective, with few pieces being able to know the location of the opposing king at a given time. When pieces are able to communicate to other pieces about the location of the opposing king, pieces are able to create consistent pressure throughout the match and checkmate the opposing king. This expansion of cognition is significant, in that

a piece does not require its own large radius of vision like R_g to successfully apply pressure. A smaller radius of vision is sufficient for a piece's needs, because the collective communication provides a larger effective sensory radius—like that seen in group sensing in weakly-electric fish who can effectively “see” through each other's senses (Pedraja & Sawtell 2024).

Each piece-player has its immediate goal to survive by working to capture valuable opposing pieces. From pawn to queen, every piece player moves and positions themselves in such a way that satisfies its metabolic instincts. However, transcending the level of the individual, the functional purpose of the collective is ultimately to checkmate the opposing king. This is an emergent outcome, not specifically encoded in the algorithm. Despite having desires and constraints that occasionally hamper play, the individuals' primary motivations (e.g. capturing opposing pieces) align with the collective's overarching goal. These motivations converge into one effort, thereby boosting the resiliency of the collective and allowing them to reach their goal in the face of internal adversity.

One fascinating question, bearing on discussions of whether intelligence is intrinsic or observer-dependent (Bongard & Levin 2023), is: what does an external observer, who knows nothing about the inner construction of each player, think of the games that our swarm plays? Would a chess-savvy observer see game-level goals being pursued—emergent long-term strategies in the eye of the beholder that do not exist in the ground truth of the algorithms being pursued by the agents (Heider & Simmel 1944; Scholl & Tremoulet 2000)? We recently showed a similar phenomenon in sorting algorithms (Zhang, Goldstein, & Levin 2024), which were exhibiting several behavioral problem-solving traits that had not been baked in to their algorithm directly (Zhang, Goldstein, & Levin 2024). In our dataset, what could be observed were: pawns marching forward despite not having the knowledge of queen promotion, especially the center pawns, the queen and pieces in the center of the board were active at the very beginning of a chess game, and pieces went on the attack (offensive), venturing to the opposite side of the board. They did not play passively.

Central to the function of collectives are the balances of cooperation and competition among their members (Gawne, McKenna, & Levin 2020; Strassmann & Queller 2010). The distributed pieces in the Chessworld

might have two types of conflict, intra-pieces conflict, and inter-pieces conflict. Intra-piece conflict encapsulates the conflict of self-interests between the pieces. More notably, factors like nutritional needs, patience, and protection (all ingrained in each piece) may cause conflict among pieces (only emergent in gameplay) in the decision to move or avoid moving. Inter-pieces conflict expresses the individual piece's disunity with the group's goals. A piece's desire might not be in accordance with the team's goals. For example, a piece might be used as a sacrifice, or be prevented from moving due to the strategic position on the board. These inter-pieces conflicts appear in gameplay depending on the game dynamics.

4.3. How Does Bottom-up Chess Play Compare to Human Players? An Informal Analysis

One of the authors (GC) is a former chess player (max. Elo rating = 2270) with 15 years of chess coaching experience from absolute beginners to international masters, and a prolific researcher in chess expertise (Bilalić, McLeod, & Gobet 2007; Campitelli & Gobet 2008; Campitelli, Gobet, & Bilalić 2014). GC has played several games against the distributed piece player and observed its behavior. He made the following observations. The distributed piece engine plays like a clever 6-year-old child who has just learned the rules of the game. The engine is excellent at detecting when it can capture an opponent's piece, which is common in novice players with some experience in chess playing, but not so common in children who have just learned the rules of the game. Individual differences are typically observed, with more intelligent children detecting they can capture pieces faster than other children (Campitelli *et al.* 2007). Another characteristic of the distributed piece engine is that of following the concept of development. In chess, is a strategic concept by which a player moves several pieces at the beginning of the game; rather than moving the same piece several times (Capablanca 1921). Development is one of the first strategic concepts taught to novices (Rozman 2023), with intelligent children learning this concept faster than other children. Another characteristic of the engine that resembles. The development applied by the distributed chess engine is not optimal (i.e., it does not move the pieces to the best positions) but, again, it reflects a smart kid who, instead of moving

the same piece several times discovers that it is better to develop several pieces.

An important characteristic of the distributed chess engine is its difficulty to check mate the opponent. It is capable of capturing opponent pieces, but its behavior denotes it does not have the concept of check, let alone that of check mate. Therefore, it either stalemates the opponent or check mates the opponent by chance. Again, this is a very common occurrence in children's games in which one of the players is much better than the opponent and captures all their pieces (except the King) but they struggle to checkmate the opponent.

The version of the distributed engine that contains a "threatening drive", which improved performance relative to the original engine as shown in Figure 8 [Appended at end], shows a very different "human style". The "threatening drive" version does not do piece development well because it is very keen on attacking, moving a piece to attack an opponent's piece and in the next move it moves the same piece again to capture the opponent's piece. This version does not look clever any longer as it sometimes moves the Queen to attack a pawn and captures the pawn, allowing the opponent to capture the queen. Rather, it is a very aggressive player. On the other hand, this version is a much better player at endgames in which all the pieces of the opponent are captured and the engine has to checkmate the opponent. The "threatening drive" version behaves as a child who has just learned how to check and checks the opponent all the time. Given that it checks the opponent, this version is more likely to check mate the opponent than the version without the threatening drive. Summing up, the previous version is a better and more conservative player in the opening stage, and the current version is an aggressive player during the whole game, and better player in the endgame, given that it is more likely to check mate the opponent.

4.4. Limitations of the Study

There are several aspects of the current system and dataset which will be developed and explored in subsequent work. These include additional analysis of the games to uncover novel emergent features of strategy, allowing more individual identity to the different types of pieces (specialization), and a deeper investigation of the role of scheduling in this process.

In current digital architectures, it is very difficult to truly implement simultaneous actions by a swarm—while possible in the (macroscopically) continuous 3D world, standard architectures must break down the moves into atomic operations, preventing truly independent activity. The role of these dynamics in the outcomes must be studied more deeply and examined in parallel architectures. Likewise, the implementation of negotiation among the pieces could enrich game-theoretic perspectives and evolutionary dynamics.

Additional future work will be focused around finding ways that improve play further while maintaining the minimal nature of the agents. For example, we recently suggested the role of stress sharing as another kind of cognitive glue (Shreesha & Levin 2024); this and other biological dynamics will be explored. Finally, it will be important to extend this approach to other classic games (checkers, Go, etc.) beyond our analysis of chess, to see where it is successful and what game conditions are or are not ideal for a distributed approach.

Conclusions

It is tempting to draw categorical distinctions between swarms and "true unified beings" like human beings and other brainy organisms. However, all of us are made of parts and all intelligences are, in a sense, collective intelligences. Even human beings are made of components which must work together to result in a degree of unified performance (Sole, Moses, & Forrest 2019; Pinero & Sole 2019; Seoane 2019; Martinez-Corral *et al.* 2019; Manicka & Levin 2019)—collective dynamics which exhibit occasional breakdowns, resulting in cognitive dissociation or morphogenetic dissociation disorder known as cancer (Braude 1995; Levin 2019; Levin 2021). Thus, there are no truly unified, monolithic, monadic chess players, and our individual neurons likely do not know about the strategies of chess any more than our *in silico* virtual players do. However, neurobiological studies of novices and chess grandmasters have revealed differences—specifically, increased whole-brain functional connectivity patterns (Song *et al.* 2022; Liang *et al.* 2022; Amidzic *et al.* 2001). Thus, using information theory to understand the relationship between parts and whole (Kolchinsky *et al.* 2014; Sporns 2011; Bullmore & Sporns 2009; Tononi, Edelman, & Sporns 1998; Tononi, Sporns, & Edelman

1994; Albantakis *et al.* 2017; Hoel *et al.* 2016; Hoel, Albantakis, & Tononi 2013) in minimal model systems and strongly constrained virtual worlds are likely to enable rich comparison between artificial life and natural biological beings. It is also interesting however that while in our simulation, both the individual agents and the collective intelligence both live in the same world (Chessworld), biological collective intelligences project themselves into new worlds, as evolution pivots the tools needed to navigate physiological and gene expression spaces into anatomical morphospace, 3D behavioral space, linguistic space, and many others (Fields & Levin 2022; Levin 2023).

We believe it is essential to develop a science not only of emergent complexity (Adami 2002; Prokopenko, Boschietti, & Ryan 2009), but of emergent cognition: to be able to predict the appearance of, and characterize the problem-solving competency and effective goals of, novel unconventional agents such as swarms of robots or minimal active matter (Blackiston *et al.* 2023; Strong, Holderbaum, & Hayashi 2024; Adamatzky, Chiolerio, & Szacilowski, 2020; Cejkova *et al.* 2017; Hanczyc 2014), of large-scale financial and

political structures constructed in societies, and in the collective intelligence of our own brains, composed of large numbers of competent cells which nevertheless give rise to problem-solving, forward-thinking beings (Chater 2018; Seth 2013) (Tononi, Edelman, & Sporns 1998; Friston 2013; Ramstead *et al.* 2022) with many unanswered questions about our nature, our capabilities, and ways in which those supervene on the biochemistry and physiology of our components.

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Table 1. Positional genes and parameters

Policy #	Name	Description	Value Range
1	Radius	Determines the radius of vision of a chess piece.	2-7
2	Imminent Danger	Increments the positional score by the value of this policy multiple times for every opposing piece in radius that is attacking it	-100-100
3	Hunger Level Multiplier	Multiplies the hunger level by the value of this policy.	0-5
4	Turn Multiplier	Multiplies the turn by the value of this policy.	0-5
5	Protection	Updates the positional score by the value of this policy for every same-side piece that is defending it.	-100-100

Table 2. Action genes and behavioral parameters their loci determine

Policy #	Name	Description	Value Range
1	Capture	Increments the move score by the value of this locus only if a piece can capture another piece.	-100-100
2	Favorable capture	Increments the move score by the value of this locus only if a piece can capture an opposing piece of higher or equal value.	-100-100
3	Unfavorable Capture	Updates the move score by the value of this locus only if a piece can capture an opposing piece of lower value.	-100-100
4	Dangerous Move	Updates the move score by the value of this locus if the move places a piece in danger. The score is updated multiple times for every opposing piece in radius that would be attacking it.	-100-100
5	Defended Move	Increments the move score by the value of this locus if a same-side piece will defend the moved piece. The score is updated multiple times for every same-side piece that would be defending.	-100-100
6	Approach Opposing King	If the opposing king is in the radius of any distributed piece and the move brings the piece closer to the opposing king, it increments the move score with the value of this locus.	-100-100
7	Move Closer to Same Side King	If the same-side king is in danger (an opposing piece is within its radius) and the move brings the piece closer to the same-side king, the value of this locus increments the move score.	-100-100
8	Approach Highest Valued Opposing Piece	If the move brings the piece closer to the opposing highest-valued piece, the value of this locus increments the move score.	-100-100

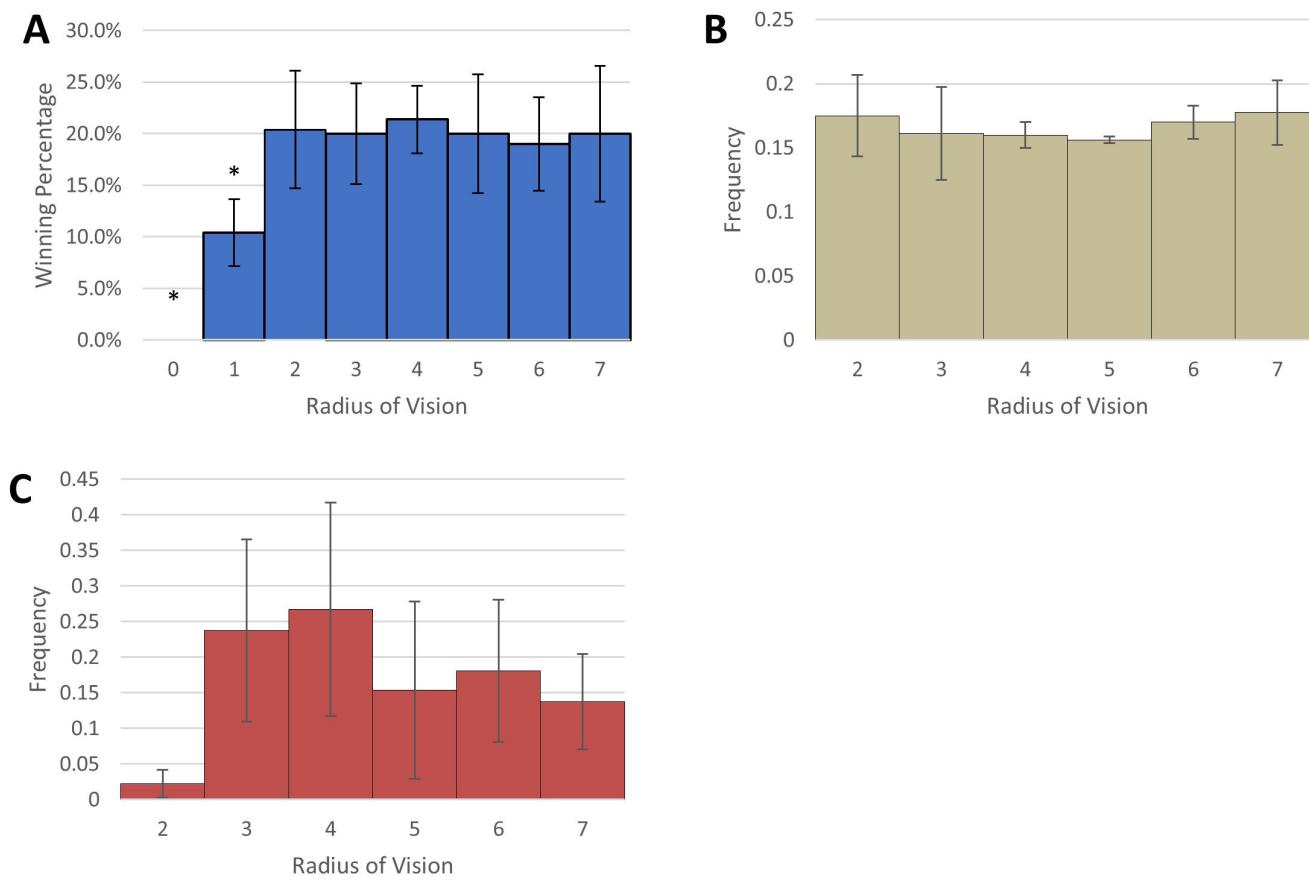
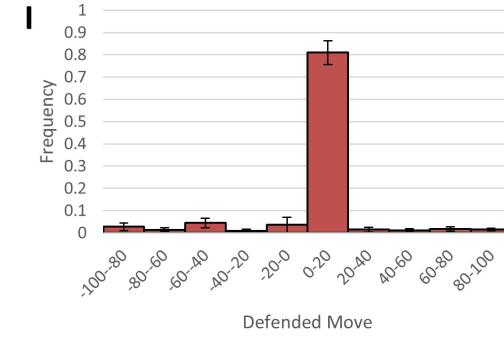
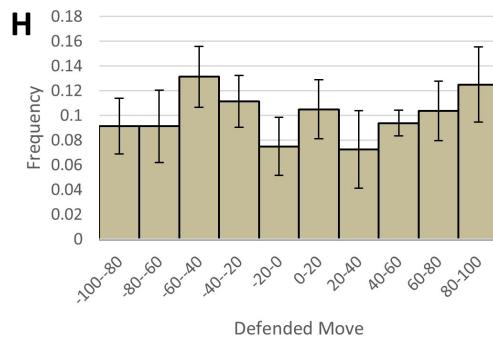
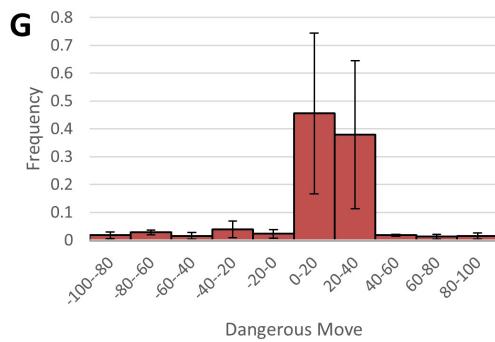
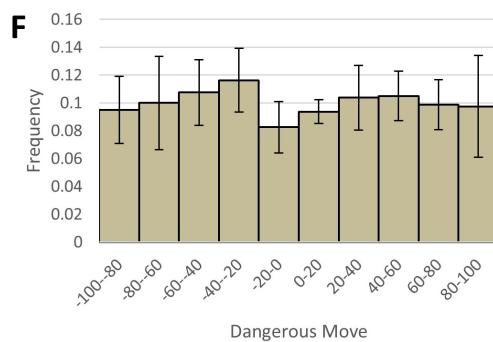
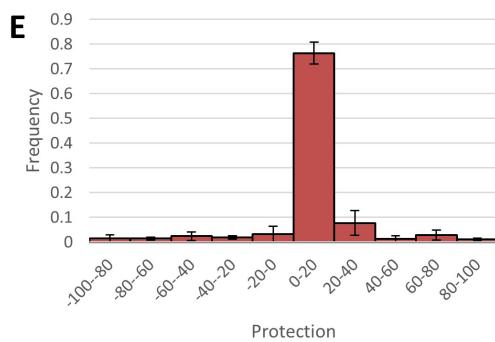
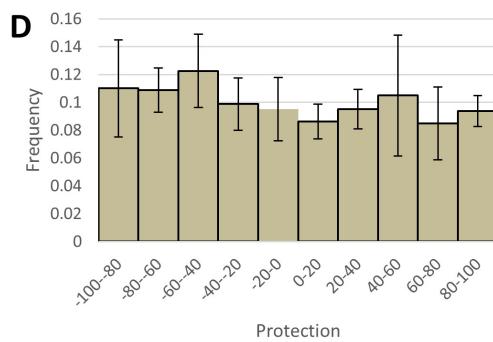
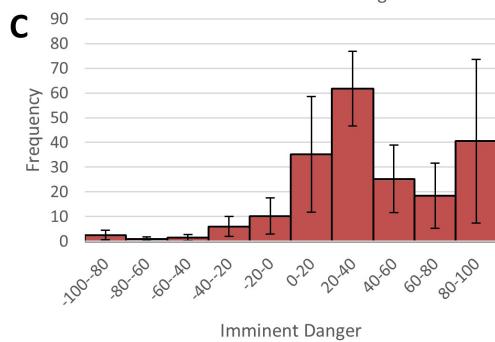
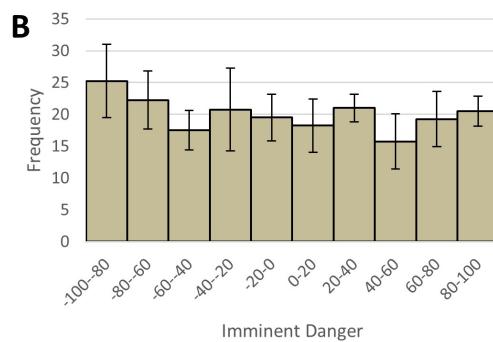
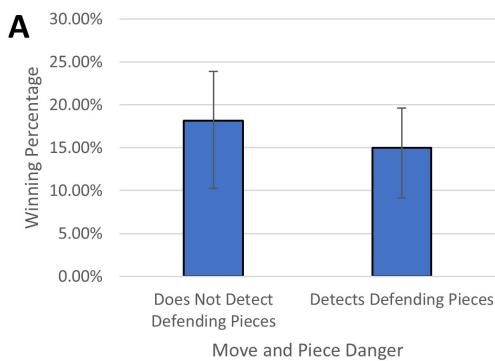


Figure 4: Effect of range of vision on play quality. (A) Winning percentage by players using each radius of vision ($N=50$). R_0 and R_1 winning percentages were significantly lower than R_2 . (B) Initial frequency distribution of radius values ($N=200$) across the population before evolution for Radius of Vision (Policy 1 in Table 1) between R_2 and R_7 . (C) Final frequency distribution of radius values ($N=200$) across the population before evolution for Radius of Vision (Policy 1 in Table 1) between R_2 and R_7 .

Figure 5 (next page): Long Range Communication among pieces increases winning percentage. (A) Comparison between the control group, and the group with long range communication ($N=50$). The control group not including long range communication had a winning percentage of $20\% \pm 4\%$ while the group including long range communication had a winning percentage of $24\% \pm 6\%$. T-test resulted in a 0.066 significance value. (B) Initial frequency distribution of parameter values ($N=200$) across the population before evolution for the gene that controls whether a piece moves closer to the opposing king (Policy 6 in Table 2). Parameter values were randomized, resulting in bars of similar size (C) Final frequency distribution of parameter values ($N=200$) across the population after the evolution for the gene that controls whether a piece moves closer to the opposing king (Policy 6 in Table 2). $29\% \pm 14\%$ of parameter values in the final population were concentrated between 20 and 40, while $18\% \pm 13\%$ of values were concentrated between 0 and 20. (D) Initial frequency distribution respectively of parameter values ($N=200$) across the population for the gene controlling whether a piece moves closer to the defending king if an opposing piece is in the defending king's radius of vision (see Policy 7 in Table 2). (E) Final frequency distribution respectively of parameter values ($N=200$) across the population for the gene controlling whether a piece moves closer to the defending king if an opposing piece is in the defending king's radius of vision (Policy 7 in Table 2). (F) Initial frequency of parameter values ($N=200$) respectively for the gene controlling whether a piece moves closer to the highest-valued piece (Policy 8 in Table 2). (G) Final frequency of parameter values ($N=200$) respectively for the gene controlling whether a piece moves closer to the highest-valued piece (see Policy 8 in Table 2).



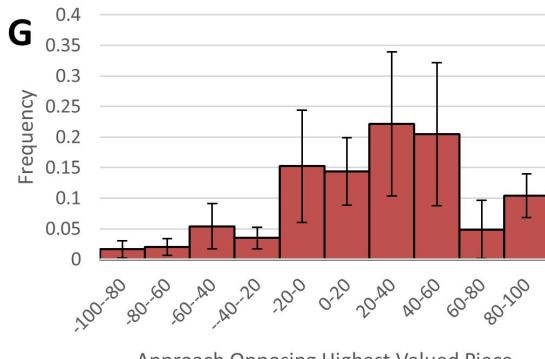
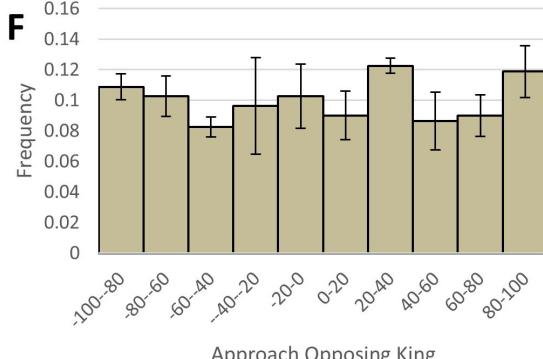
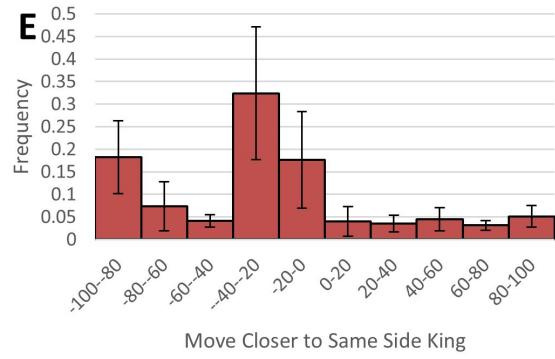
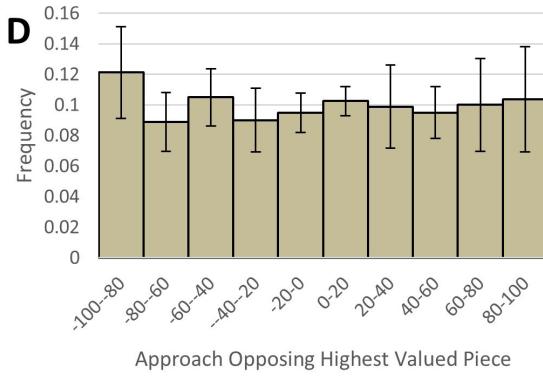
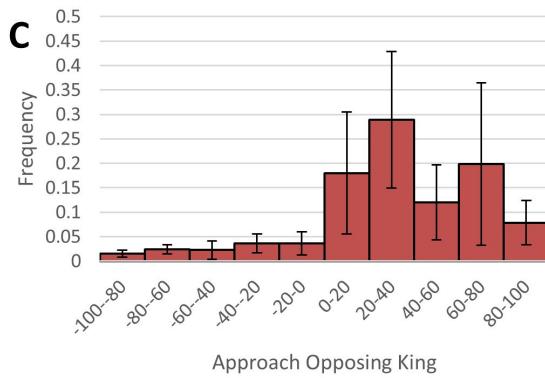
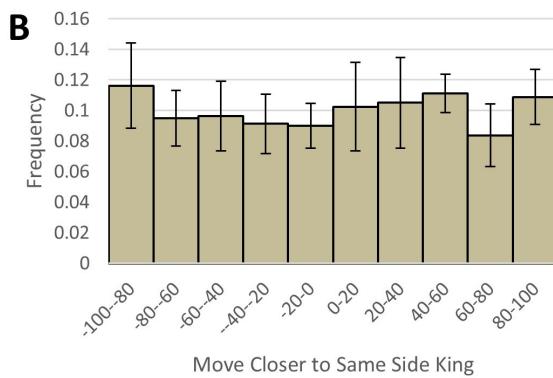
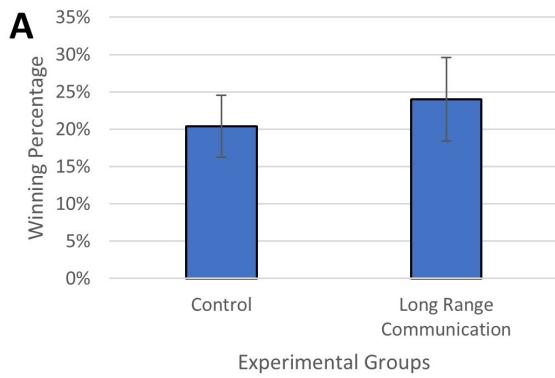


Figure 6: Courage is more favorable than caution in certain scenarios. (A) Pieces that are more cautious (denoted as 'Does Not Detect Defending Pieces') have a higher winning percentage than those that have more courage (denoted as 'Detects Defending Pieces') (N=50). (B) Initial frequency distribution of parameter values (N=200) across the population before evolution for the gene that controls how motivated a piece is to escape imminent danger (see Policy 2 in Table 1). (C) Final frequency distribution of parameter values (N=200)

across the population after the evolution for the gene that controls how motivated a piece is to escape imminent danger (see Policy 2 in Table 1). (D) Initial frequency distribution respectively of parameter values ($N=200$) across the population for the gene influencing the decision-making for a piece when the piece is defended by another same-side piece (see Policy 5 in Table 1). (E) Final frequency distribution respectively of parameter values ($N=200$) across the population for the gene influencing the decision-making for a piece when the piece is defended by another same-side piece (see Policy 5 in Table 1). (F) Initial frequency of parameter values ($N=200$) respectively for the gene controlling the voluntary decision of a piece to put itself in danger (see Policy 4 in Table 2). (G) Final frequency of parameter values ($N=200$) respectively for the gene controlling the voluntary decision of a piece to put itself in danger (see Policy 4 in Table 2). (H) Initial frequency of parameter values ($N=200$) respectively for the gene influencing the decision-making for a piece when the piece's next move would be defended by another same-side piece (see Policy 5 in Table 2). (I) Final frequency of parameter values ($N=200$) respectively for the gene influencing the decision-making for a piece when the piece's next move would be defended by another same-side piece (see Policy 5 in Table 2). (J) The black pieces are played by the distributed pieces. In this position, the black light squared bishop is under attack by the white queen and is inclined to move out of danger. The black bishop accepts the risk of moving to the square at the end of the green arrow because a same-side piece-player (black pawn) is defending that square, despite the white queen and white bishop guarding that square.

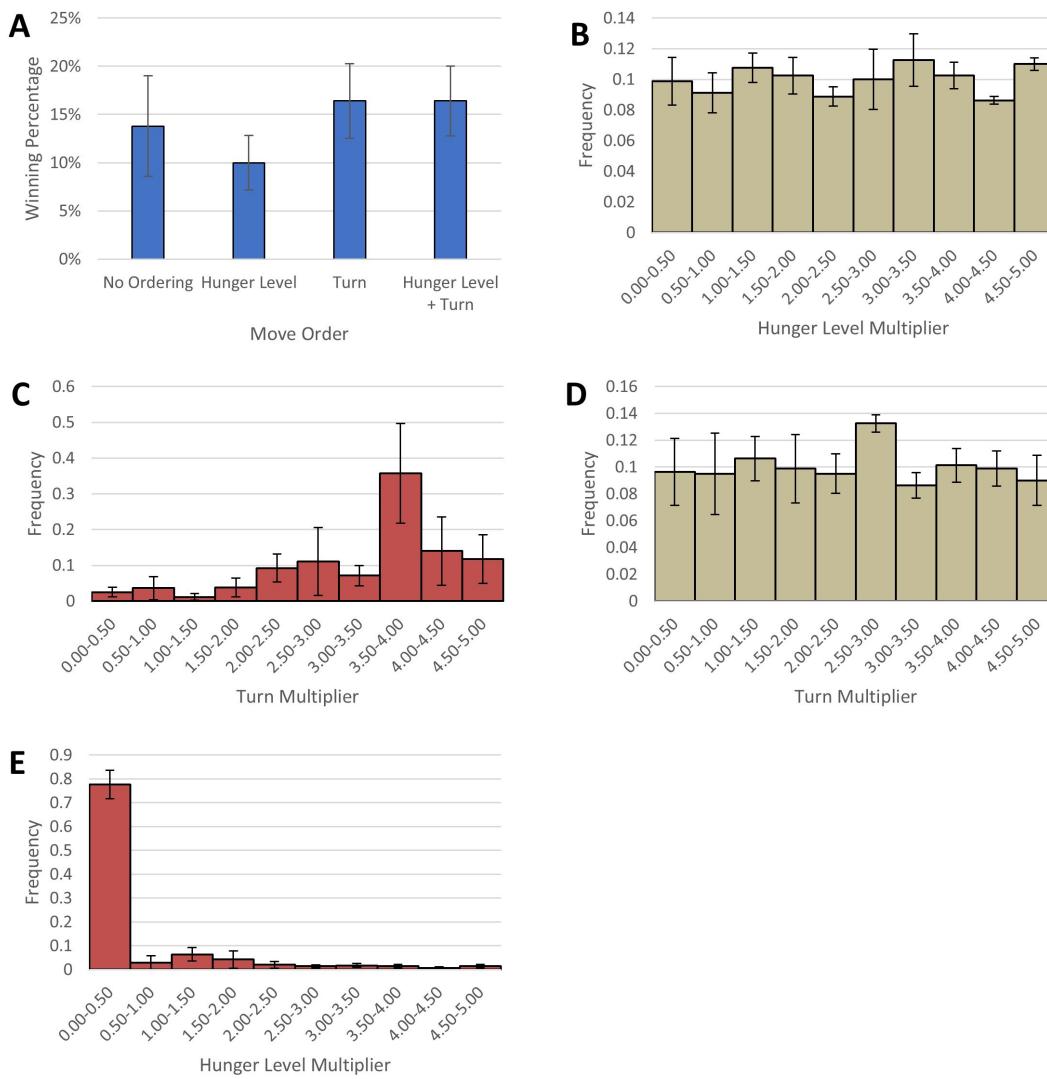


Figure 7: Quality of performance is best optimized when using strict turn order to determine activity of the agents. (A) Different ordering strategies of moves when no captures are available ($N=50$). (B) Initial frequency distribution of multiplier values ($N=200$) for the gene controlling how motivated a piece is based on their hunger (Policy 3 in Table 1). (C) Final frequency distribution of multiplier values ($N=200$) for the gene controlling how motivated a piece is based on their hunger (Policy 3 in Table 1). (D) Initial frequency distribution of multiplier values ($N=200$) for the gene that controls how motivated a piece is based on their turn (Policy 4 in Table 1). (E) Final frequency distribution of multiplier values ($N=200$) for the gene that controls how motivated a piece is based on their turn (Policy 4 in Table 1).

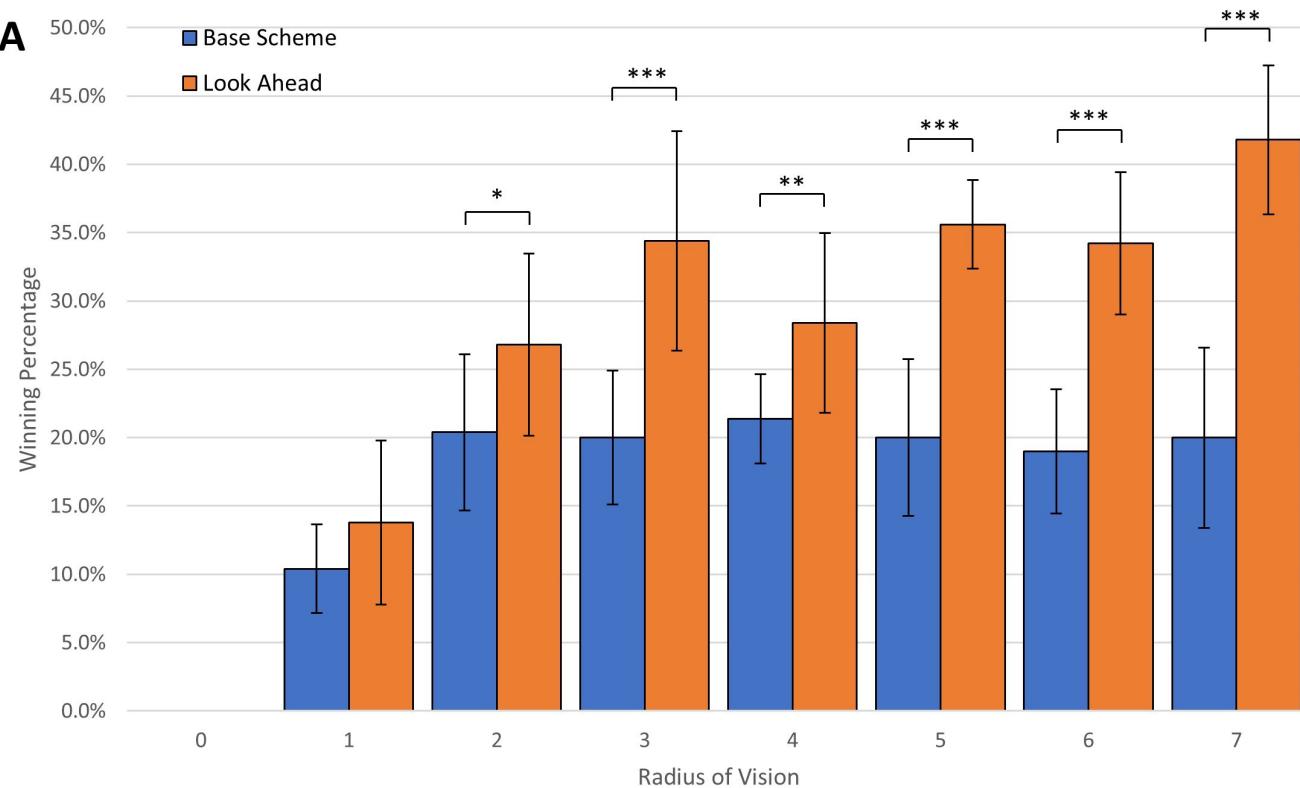


Figure 8: Performance is significantly better when pieces look to attack one move ahead, compared to the base scheme. A radius-by-radius performance comparison between the base scheme, and the base scheme with pieces looking one move ahead (N=50).

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