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Steindl on Stochastic Processes

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"*Economics consists of theoretical laws which nobody has verified and of empirical laws which nobody can explain*"
(epigram of Michal Kalecki, quoted by Steindl, 1965, p.18)

1. Introduction

Josef Steindl's work takes to heart Schumpeter's dictum that *good economics* must encompass history, economic theory and statistics, and therefore does not generally take the form of elegant formal models that are applicable to all and everything. He shares the view – expressed, among others, by Sylos Labini (2002) – that theoretical models, econometric and statistical analyses are crystallizations that enable us to order and compare alternative developments and keep us in touch with reality; but they should not be allowed to take on a life of their own or to dominate discourse, which has to make allowances for institutional factors and dynamic developments.[1]

Steindl's interest in the stochastic approach is somehow at the crossroads of his two careers: his career at Oxford when he worked with

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[1] Cf. Corsi (2007). Paolo Sylos Labini loved to quote this quip from Bruno De Finetti, the famous Italian mathematician: "I have often thought that not always, but often, applying mathematics to economics means making the easy difficult by means of the useless" (Sylos Labini, 2002, p. 11). Steindl, in his *Reflections on the Present State of Economics*, writes: "The role of mathematics in economics has been a most unfortunate one. Instead of being a tool of the economist it has developed a life of its own. […] General equilibrium economics certainly exemplifies the dangers of atrophy resulting from an isolation against outside stimulus and irritation, an economics withdrawn into itself and contemplating its own navel." (Steindl, 1984, p. 9) Moreover, "there is a misguided idea that since proficiency in mathematics can be judged more objectively than creative ideas in economics, selection of economists is best based on the former." (p. 5)

Michal Kalecki and his Austrian career at WIFO. His work is definitely consistent with Kalecki's interest for the application of stochastic processes to the analysis of the distribution of national income as well as with Keynes' views on the logic of probability.[2] At the same time, it may be connected to "the wish to establish a relation between the macro and the micro world in economics" (Steindl, 1984, p. 11), so crucial for studying the cyclical behaviour of the economy, in empirical terms.

Steindl has adopted a stochastic approach in different fields of analysis, always "wide off the beaten track" (Steindl, 1984, p. 11).[3] In Steindl's words,

> "[…] I felt that random processes, if the stress is put on *process*, have a good chance of making our theory more dynamic. Their basic concept is the transition probability, that is, the chance of moving from a given state one day to a certain different state the next day. A steady state can be derived on certain conditions and can be compared with the observed data. […] The empirical materials suitable for estimating transition probabilities are the panel data" (Steindl, 1984, p. 11).

In this paper I shall concentrate my attention on the stochastic approach to the distribution of personal income, not only because the majority of Steindl's works on stochastic processes take into consideration the Pareto distribution, but also because this is a clear-cut example of how the stochastic approach can drive economists in the direction that Steindl aimed to, i.e. "to show the influence which the past has wrought on the present, and how past growth is expressed in the patterns and formations of the present which bear the imprints of events gone by" (Steindl, 1990, p. 320).

This is equivalent to saying that all societies evolve in history, which consists of irreversible processes, so that all interpretive schemes in the

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[2] On Keynes and probability see Roncaglia (2009).

[3] *Random Processes and the Growth of Firms* (1965), "The Pareto Distribution" (1978), "Pareto Distribution" (1987), "The Distribution of Wealth after a Model of Wold and Whittle" (1972), "The Personal Distribution of Income" (1990), "The Dispersion of Expectations in a Speculative Market" (1990).

social sciences are historically determined. This is, of course, equally true for the theoretical models of an economy.[4]

In what follows, after having revised Steindl's analysis of the Pareto distribution and its implications for a stochastic approach to the distribution of personal income, I draw the reader's attention to the crucial methodological issue raised by Steindl, i.e. the objection that stochastic models are only able to introduce some 'elegant' explanation for observed empirical distributions, but without being *realistic*. In order to find an answer to this objection Steindl refers on several occasions to Champernowne's analysis of the distribution of earnings, as do I. Champernowne's contribution to the stochastic phenomenology of income distribution is indeed striking for its capacity to highlight pros and cons of the stochastic approach, thus it is extremely useful in order to conclude, using Steindl's words, that although stochastic models have often been criticized for their lack of economic content, perhaps "it has been overlooked that they only represent the first steps in a new and exceedingly difficult terrain" (Steindl, 1987, p. 810).

On this terrain, as is well known, Steindl has come a long way in dealing with "a kind of *equilibrium* exemplified by the size-distribution of firms and its statistical law, which is, in fact, the law of Pareto" (Steindl, 1965, p. 5).[5] In my own way, I tried to follow in his steps when drawing on stochastic analysis to formalize the random character of the various forms of *division of labour* (i.e. *technical change*), according to a Classical point of view (Corsi, 1991). However, for sake of simplicity (and hopefully of clarity) I do not deal here with these applications,

[4] Quoting Becattini *et al.*, "The logical validity of the models (granted that there is one) will persist, but their interpretive efficacy is relatively short-lived, as it is conditioned by the realism of the hypothesis. So there are no immutable laws in economics […]. Whenever we set out with a few axioms to interpret certain aspects of economic reality, we may identify regularities, which have a probabilistic nature, based on large number series. These regularities are historically determined, in the sense that they are true as long as certain structural characteristics of the society under study persist – when these characteristics change over time, so do the regularities." (Becattini *et al.*, 1989, pp. 9-10)

[5] From the pioneering work of Gibrat (1930), Kalecki (1945) and Simon (1955), stochastic theories of the size distribution of firms have tried to explain observed size differences among firms as a consequence of random growth rate differences, accumulated over time. Steindl (1965) goes in the same direction, not without originality.

although I am aware that further discussion in these directions would be much required for a development of economic theory according to a stochastic approach.

2. The Pareto distribution

An inquiry into the nature of the personal income distribution is at the core of political economy at least since Adam Smith. In his *Wealth of Nations*, Smith seems to link personal income distribution to the institutional aspects of the economy, more than to economic trends; in this sense, as an effect of economic growth, individual incomes rise more or less in *proportional terms* (Smith, 1776, pp. 80 and 159). By contrast, in his *Principles of Political Economy*, J.S. Mill is worried by income inequalities that arise as an output of economic growth boosting the middle classes without improving the economic conditions of the poorest part of the population (Mill, 1849, p. 699). But it is especially with David Ricardo – and Karl Marx after him – that income distribution became one of the main economic issues, focusing on *social antagonism* in the context of the distribution of income within the society and among social classes.[6]

Within the Ricardian milieu J.B. Say is considered the first economist who explicitly considers the distribution of personal income as a *must* (i.e., an unavoidable issue) in the analysis of the demand for goods, developing, at the same time, a graphical analysis of the distribution itself.

> "[…] The fortunes of private individuals in all countries rise by immeasurable amounts, from the smallest to the greatest. The smaller they are, the more commonly they are found, and they become rarer and rarer the greater they are. In this way, they could be compared to vertical lines in a pyramid. If a horizontal line were used at a certain height to represent the price of production for a particular product, the number of vertical lines it intersected with at this height would be the number of fortunes able to reach this price and, therefore, the number of consumers of the product. The higher this horizontal line is, the fewer fortunes will be able to make the sacrifice of that expenditure. By the same token the lower the line is, the lower production costs will be and

[6] Cf. Kaldor (1956), Lombardini and Quadrio Curzio (1972), Asimakopulos (1988), Screpanti (1990).

> greater will be the number of fortunes able to purchase the product. […] This picture shows how a product, the lower its price is, the more consumers can buy it and how the number of consumers lessens the greater its price is”[7] (Say, 1840, pp. 168-169, my translation).

This is also the starting point of the analysis of Vilfredo Pareto, when presenting for the first time his income curve, as the basis of a collective demand curve (Pareto, 1895).

Using data on personal income from various sources,[8] Pareto plotted income (y) on the abscissa and the number of people earning more than that (N_y) on the ordinate of logarithmic paper and found a roughly linear relation. This Pareto distribution or “Pareto Law,” may be written as:

$$N_y = Ay^{-\alpha} \qquad (1)$$

or

$$\log N_y = \mathrm{A} - \alpha \log y \qquad (2)$$

where α (the negative slope of the straight line) is called the Pareto coefficient.[9] The coefficient α may be used as a measure of inequality (for high income range): it takes only positive values and the larger α, the

7 “Les fortunes des particuliers, en tout pays, s’élèvent, depuis les plus petites jusqu’à la plus grande. Elles sont d'autant plus nombreuses qu’elles sont moindres, et deviennent d’autant plus rares qu’elles sont plus grandes. De sorte que l’on pourrait les comparer à cette multitude de lignes verticales qui remplissent une pyramide. Si l’on représente, par une ligne horizontale tracée plus ou moins haut, la hauteur des frais de production d’un produit quelconque, le nombre des lignes verticales qui attendra cette section, représentera le nombre des fortunes capables d’atteindre à ce prix, et par conséquent le nombre des consommateurs du produit. Plus la section sera haute, et moins il y aura de fortunes capables de faire le sacrifice de cette somme de frais. Plus au contraire la section sera basse, plus le frais de production seront réduits, et plus seront nombreuses les fortunes qui pourront faire l’acquisition du produit. […] Ce tableau rend sensible à l’œil comment, à mesure qu’un produit baisse de prix, il rencontre plus de consommateurs; et comment il en rencontre d’autant moins qu’il est plus cher.”

8 He used income tax figures from various States and times: Prussia, Basel, Britain, Augsburg (in the Fifteenth century), Peru, Perugia, Saxony, and Florence.

9 The relationship between y and N_y may also be expressed in other ways. Corrado Gini has reformulated the Pareto Law, by taking into account the cumulative income Y earned by N_y individuals (cf. Gini, 1922, 1936). He empirically obtains:

$$N_y = BY^{\beta}$$

Gini finds this law empirically, just as Pareto did his; it has since been proved that the two functions can be transposed into each other.

less unequal is the distribution. In other words, the Pareto law amounts to saying that the number of income recipients earning at least a given income decreases by a fixed percentage if that income increases. This percentage is α. Suppose that in a given country $\alpha = 2$, then we can state the law as follows: imagine a certain income – the selected income level – and count how many people earn this or a higher income; now perform the same operation with an income that is, for example, 1% higher; the number of income recipients will have become 2% smaller. Whichever selected income level we start with, the same percentage always emerges.

It goes without saying that Pareto and other authors after him attached deep importance to this regularity, "naturally so since regular patterns are felt to be a challenge to the mind" (Steindl, 1990, p. 322).[10]

The emergence of such regularity is at the basis of the stochastic approach to the distribution of personal incomes.[11]

There are different theories to explain the Pareto distribution as an effect of stochastic growth processes, all of them equally plausible, such as the multiplier effect (Roy, 1950), the proportional effect (Gibrat, 1930; Champernowne, 1953, 1973) and their variants (Kalecki, 1938; Rutherford, 1955). All these theories, taken with the laws of probability, lead us to the structure of earnings empirically observed. In other words, the explanation for the Pareto Law depends ultimately on the apparent magic of probability theory, by which order in mass is produced out of individual chaos, by the very fact of the chaotic or random character of individual action.

In particular, Champernowne (1953) (and later Wold-Whittle, 1957) explains the characteristic pattern as the steady state of a stochastic process that has evolved in time, so that the pattern reflects something that has been going on in the past.[12]

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[10] Several contributions have been presented in the literature, in different research directions; for a survey – updated at different times – see, in chronological order, Mincer (1970), Sahota (1978), Atkinson (1983), Lambert (2001).

[11] According to Champernowne (1953) α is useful as a measure of inequality for the high-income range, whereas for medium and low incomes other measures are preferable.

[12] Champernowne's pioneering work (1953) in essence goes back to his fellowship dissertation of 1936, *The Distribution of Income between Persons*, at King's College, Cambridge, finally published in 1973. In the Introduction, he states that the choice of the

As Steindl (1965) explains well, the idea at the back of Champernowne's approach is that certain economic distributions (e.g. the distribution of personal income) are stable, at least up to a point, though not altogether, while he is aware of a continuing movement of the elements that make up the population in question. This suggests the idea of a steady-state equilibrium that can be described as "a state of macroscopic equilibrium maintained by a large number of transitions in opposite directions" (Feller, 1968, p. 395).

This is a central concept in the theory of random processes that can be best explained by reference to a stationary human population.[13] It is in a state of continual flux: people are born, age, and die. Individual births and deaths are unpredictable; to common mankind, birth and death are random events *by definition*. Yet the total population and its age structure remain relatively stable, determined by the probabilities of birth and death for various ages. We may say that the steady-state equilibrium, to which many (though by no means all) random processes tend, is independent of the initial conditions.[14]

Economic populations can be compared with the human one.[15] We note at once a difference: it is not only age (or sex) which interests us, but certain variables depending upon it (e.g. employment status, career, etc.). A person's employment status, as a function of time, may be considered

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topic for his dissertation was strongly influenced by the suggestion from one of his supervisors, J.M. Keynes, "to try to explain the conformity to the Pareto's Law shown by several data referring to the distribution of incomes, published by the tax offices in different countries" (Champernowne, 1973, p. 1).

[13] In mathematical terms, a random (stochastic) process can be defined as an arbitrary family of random variables X_t, where t is a parameter running over a suitable index set T (that can be either continuous or discrete). The process moves from one state to the next as time goes on, and each transition is, or may be in principle, influenced by chance; but it is no less influenced by bias, that is, by systematic influences. Cf. Feller (1968).

[14] Such a process is called *ergodic* (cf. Feller, 1968, ch. 17).

[15] The use of biological analogies is typical of a Marshallian approach to economics, shared by Champernowne as cultural heritage. As Ridolfi (1979) stresses, Marshallian equilibrium theory is developed along a biological analogy taking as a point of reference the sort of equilibrium studied in demography. Evolution is modelled using a methodology that A.J. Lotka adopted at first, soon after the death of Alfred Marshall, applying a mathematical analysis similar to the one used in biology. Cf. Ridolfi (1979) and his references.

as a stochastic variable. Thus we have two interdependent stochastic processes: the birth-death process in individuals, and another birth-death process that determines the ups and downs of an earner according to his/her employment status. The main characteristic of stationary equilibrium is that its structure reflects the probable outcomes of the individuals who comprise it.

The structure of the human population by age is determined by life expectancy and birth rates in different periods. Job-based income distribution is determined by the probability of individuals being promoted or sacked at different stages during their lives, and the expectation of new job opportunities. The equilibrium structure shows individuals at various stages of their transition through the structure. As Steindl (1965) stresses, a comparison may be made with the stratification of rock sediments, which represents a process of evolution by a system of layers, or with tree rings, which represent growth over a certain number of years.

Additionally, transition across a structure may affect a certain number of generations. If we consider a population of manual labourers or of office workers, we can see that they represent different jobs, different skill levels, or different stages in economic and social progress. Considering a complete transit from entry to exit, we may include the transition from farm labourers to factory workers, from manual labourers to office workers, or the complete progress through the social hierarchy. The structure will show us an historic process of differentiation between jobs, and changes to individual capacities that, to a certain degree, affect such progress.

The objection has been made that stationary equilibrium is of little use to economists because most populations tend to grow. However, population studies show that a population in exponential growth at a stationary rate will also have a stable age structure, dependent upon the growth rate. This is encouraging and shows there can indeed be correspondence between development in life and layers of sedimentation, even in a growing population.

Thus it is clear that when explaining the Pareto distribution one may be surprised to find connections to risk theory, in a very general sense, including greater growth and promotion probability (negative risk).[16] Naturally, the concept of risk considered in this case is objective: the relative frequency of various destinies, the objective probability that someone in a certain population may have when beginning a career to move upwards and/or downwards in earnings terms at different stages of their life, because one's destiny can never be forecast. Such unpredictability is implicit in the very concept of risk: one cannot even use the term to describe people unless their destiny is unknown to them.

3. Pros and Cons of the Stochastic Approach

According to Steindl, the stochastic explanation of the Pareto distribution shows that there are no unchanging, universal laws in economics, since, when we begin considering empirical observable phenomena, we will always see certain underlying rules of probability which have been determined in the past. Indeed, it must be pointed out that, according to Steindl, such rules are seen every time some phenomena dependent upon large numbers occur in a society, whether or not these phenomena involve economics.

However, the objection has been made that stochastic models are all to a greater or lesser degree unilateral; they all introduce some 'elegant' explanation, but this stops them being realistic. Such criticisms are perfectly justified, according to Steindl. Champernowne, who was the first to explain the Pareto distribution as the result of a stochastic process, admitted of his most well-known article that,

> "[…] it was found necessary to concentrate on the mathematical *skeleton* of the theory and to dispense with most of the *flesh* concerning conjectures about the effects of particular measures and of relaxing the extreme simplifying assumptions associated with static equilibrium. As a result the article gave a false impression of what had been the main purpose of the original theory, namely to provide a theoretical apparatus for determining the effects of particular economic influences upon the distribution of incomes between persons." (Champernowne, 1973, p. 5)

[16] Cf. Kanbur (1979).

Let us now try to understand the factors behind this 'lack of economic content' by examining the nature and purpose of stochastic models. This is indeed the crucial issue according to Steindl.

Following Steindl (1965), we can compare stochastic processes with deterministic ones by taking an example from physics. We can consider an electrical circuit, as representing a deterministic process; it contains valves, switches, etc. at certain points, and the flows passing through the system elicit at those points automatic responses which influence the flow, this in turn leading to automatic responses at the next point. Were we to represent the same system using a stochastic model, we would understand the nodes as being made of some inferior material: in this way, for example, a switch does not always respond in the same way to a flow of current (sometimes the light comes on, and sometimes it does not). This can invalidate the consistency of the system, which now functions according to casual factors in addition to systematic ones.

In this example, it is suggested that the casual elements correspond to defects within the system, i.e. to errors in the instruments used. In economics, however, this is only partially true. Decisions made by individuals are often subject to error, but the asymmetry of income distribution seems to show, on the other hand, that the casual elements implicit in this distribution are the effect of those conditions in which an economic system works. To go back to our example, to make it more applicable to the economy we would need to wire certain devices into the circuit to generate casual signals (either when pushed, or by themselves). These, in addition to reactions by other parts of the circuit, would then modify the flow of electrical current.

A more abstract portrayal of the stochastic process can be seen in terms of a model for drawing numbers out of a hat. A Markov chain,[17] for example, can be represented as follows: imagine several hats, each of

[17] By Markov chain, I refer to a stochastic process that undergoes transitions from one state to another, between a finite or countable number of possible states. It is a random process characterized as *memoryless*: the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of '*memorylessness*' is called the Markov property. Markov chains have many applications as statistical models of real-world processes.

them bearing a number, and each of them containing several balls with numbers on them. Each hat represents one possible state for the system. Taking the balls from one hat determines the next hat. The choice of the first hat is completely independent, and this is the *sine qua non* of the process. The choice of the second hat depends upon what we find in the first one, and so on, and thus the probability for transiting from one state to another is determined. If we continue to take out balls for long enough, we will be able to make a reasonably reliable forecast of what balls to expect next. This means we have a stable probability distribution over the various states in the system, and it is influenced by how the process is carried out.

With reference to Champernowne's model, which Steindl always takes as a reference, we can imagine each hat representing one income band in a particular year, and the number on a particular ball representing the difference in income over the previous year. Thus differences between income levels will be partly due to chance and partly due to systematic factors (the balls in the hat). Actually, the law of proportional effect says that the probability of a particular difference between one income level and another depends exclusively upon the difference, and has nothing to do with the starting income.

Having given these examples let us now explain the double meaning of the term stochastic (or random). We intuitively use this adjective to describe events, which occur unsystematically, such as choosing a number from the telephone directory at random. However, when we consider stochastic processes or the random variables which give rise to them, we cannot exclude the influence of systematic factors on such phenomena: all the effects on a process must be taken into account.

Let us look at the purpose of the stochastic approach, going back to the personal income distribution. Champernowne says, "[…] the main purpose of the original theory [is] to provide a theoretical apparatus for determining the effects of particular economic influences upon the distribution of incomes between persons" (Champernowne, 1973, p. 5). It may be that by trying to define the frame of reference too precisely, Pareto and the authors mentioned above have not been able to give a proper economic explanation. The hypotheses in the models they use tend to over-simplify, and this prevents them from analysing the problem of inequality properly. According to stochastic theory, instability is an endogenous factor connected to the difficulty people have in giving an appraisal of all situations, and to fundamental uncertainty.[18] As Champernowne stresses,

> "[…] the degree of inequality of the personal income distribution is the outcome of a struggle between two sets of forces:
> i. forces causing inequality: a) institutions and social norms, which give the wealthy and their heirs the monopoly of certain types of employment and of property, and b) unsettled conditions which offer opportunities of large proportional gains and losses of income to many individuals.
> ii. forces limiting inequality: those such as progressive taxation and death duties and the social services, which provide better opportunities for some of the poor to become richer and which limit the tendency for the rich and their offspring to become richer still." (Champernowne, 1973, p. 190)

The forces of change alter income distribution. However, "under their action, the properties of the distribution move towards equilibrium, but never reach it, because they are shaken away by impulses of change, and also because the forces of change are themselves smoothly altering." (Champernowne, 1973, p. 9)

According to stochastic theory, thus the economy is usually in a state of imbalance, its starting conditions are an accident of history, and nobody has a clue what tomorrow will bring. Generally speaking, this interpretation only identifies the forces underlying economic processes and declares various types of imbalance as a dynamic force. It speculates on the chain of quantitative causes, which may be understood. This does not mean that we cannot imagine a balanced distribution – when the various forces are constant – as a way of pointing out certain fundamental characteristics. However it is not the job of an economic analyst to study

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[18] Taking into account the relationship between Champernowne and Keynes, it is worthwhile recalling the following sentence by Keynes (1937): "By *uncertain* knowledge […] I do not mean merely to distinguish what is known for certain from what is only probable. […] The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth-owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know." (Keynes, 1937, pp. 113-114, italics added)

the conditions for equilibrium in income distribution, but rather to work on how to bring about change. Such a purpose is compatible with the use of stochastic models, because these also entail the study of systematic phenomena. What we often describe as 'random' is actually the result of lots and lots of independent forces acting together on a particular phenomenon.

4. Conclusions

In the last section of the collection of his papers (1990), Steindl introduces his work on random processes by writing:

> "Why do I use an approach that offers such overwhelming technical difficulties for most economists and yet requires even more simplifications than ordinary deterministic models? […] I want the economists to take a new look, with different eyes, at the statistical material that so far they have very often interpreted with a disarming naïveté. […] The stochastic approach is nearer than most other economics to the kind of questions asked in science. It is inspired by the observation of regularities which are a challenge to the intellect while most other economic questions are more or less directly inspired by social problems, by criticism of the existing society, by utopias and by problems of economic policy. The difference is one of approach but the subject matter is ultimately the same." (Steindl, 1990, pp. 319-320)

The stochastic approach to income distribution, taken here as an example of the possible applications of the stochastic approach in economics, has made an important contribution to our understanding of income differences by trying to find empirical laws for distribution. The starting point in all models recalled is the observation that inequality must be understood as the asymmetrical distribution of income, where a large portion of overall income is held by a tiny portion of the population.

This phenomenon can be explained simply by saying that people earn money according to certain characteristics, and that income distribution depends upon the distribution of the characteristics required to earn an income. Income differences between individuals are due to differences in characteristics. The characteristics usually considered are individual skills, personal savings, and one's job. These are considered as freely interchangeable between earners.[19] However, there are characteristics that cannot be interchanged, and yet they significantly affect the opportunity of an individual to receive a certain income: age, gender, class, ethnicity, and physical handicap (which is always understood to be negative).

Steindl was aware that individual characteristics could not be taken into consideration without considering the social structure in a particular community and the contingent economic conditions. In particular, economic factors may influence income distribution either by changing the distribution of those characteristics necessary to earn money or by changing the value given to each characteristic. Indeed, certain inertial phenomena (traditions, conventions, and institutions) are much more important today than they have been at various times in the past.

I hope that this *excursus* has shown that none of the models taken into consideration here can give a full, proper explanation for income inequality. However, from a methodological point of view, this does not mean that they have not provided us with a few guidelines to help us reach a more pragmatic view of society. In this sense, as Steindl would say, “they only represent the first steps in a new and exceedingly difficult terrain” (Steindl, 1987, p. 810).

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[19] “A man’s income is determined by his total *hand* of qualifications, and so are his prospects of change in income. This was taken as justification, as a first approximation, for regarding a man’s total income as a single parameter summarising his prospect of change of income in the future. But the author did admit that this approximation was only a good one for people who were fairly well off, since it was these people who most frequently sold and bought qualifications (such as well-situated house or good education) for themselves or their families” (Champernowne, 1953, p. 6).

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