# An Endogenous Growth Model with Endogenous Money Supply. Integration of Post-Keynesian Growth Models\*

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### 1. Introduction

This paper attempts to integrate the growth models presented by such authors as Kalecki, Harrod, Robinson and Kaldor, together with an introduction of monetary aspects, based on the endogenous money supply theory. By introducing monetary factors explicitly, we show that Robinson's long-run version of the paradox of thrift disappears. Capital accumulation rate function and saving function determine only the long-run rate of interest, since the growth rate of the steady growth state is basically determined by the technical progress function. In other words, the long-run interest rate is determined by the rate of profit in the steady growth state – a point made by classical economists such as Ricardo and Marx – as long as the monetary authority changes the short-run interest rate in order to keep the purchasing power of money stable.

Capital accumulation plays the central role in the process of economic growth. In the first place, although R&D investment is necessary to raise productivity, physical capital goods are indispensable for innovation to realise their fruits as production technique for marketable goods. In addition, expenditure on physical capital goods is much larger, and therefore riskier, for firms than the cost of R&D. Most of the recent endogenous growth models, especially in the

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neoclassical school, do not, however, seem to pay enough attention to these aspects of productivity growth and capital investment, which leads us to reconsider such concepts as Kaldor's technical progress function and Robinson's animal spirits function. Some post-Keynesians such as Dutt (1990, 1994) and You (1994a, 1994b) have, however, studied these aspects. The specifications by Eltis (1973) and Dutt (1994) are especially useful in relation to the technical progress function, although these differ slightly from Kaldor's original model. As we know the technical progress function changes Harrod's natural rate of growth to an endogenous variable.

Secondly, capital accumulation changes effective demand and production capacity, thereby causing fluctuation not only in the employment of labour but also in the rate of capacity utilization. In other words, capital investment causes both economic fluctuation and economic growth at the same time. It is not desirable, therefore, to formulate them separately. Unlike the neoclassical school economists, post-Keynesians such as Jarsulic (1988, 1994), Scott (1989), Dutt (1990, 1994) and You (1994a, 1994b), for example, have shown full awareness of these aspects. In the tracks of these authors, the rate of capacity utilization takes on an important role in our model, too. In the case of the capital investment function, however, we adopt Robinson's animal spirits function, with the inclusion of the real interest rate when the inflation barrier applies.

As Kurz (1992) points out, the industrial system is very flexible or elastic, which enables the system to have a wide range of capacity utilisation rates in the long-run steady growth state. This allows for a stable discrepancy between the warranted growth rate and the natural growth rate.

Even in the tradition of post-Keynesian economics, however, financial aspects have been neglected in many studies on economic growth, with the exceptions of Jarsulic (1988), Scott (1994) and Taylor (1991), for example. Investment must be financed for its realization, and the difficulty here would be represented by the level of real interest rates. In usual business conditions, this demand for finance would be satisfied almost automatically by the financial system as long as the firms concerned had enough creditworthiness. If a continuing high rate of capital accumulation had been causing inflation, the financial system would raise the interest rate, thereby reducing the rate of capital accumulation. This would be normal practice for the financial system, because its most important role is to keep the purchasing power of money stable, as Robinson (1956) pointed out. In the models by the above-mentioned authors this "inflation barrier" effect is not incorporated. We follow Kaldor (1982) regarding the money supply as an endogenous variable always equal to its demand under the interest rate which the monetary authority determines. However, the monetary authority must change the interest rate according to economic conditions – the difference between the actual inflation rate and the target one – in the long run, which works as a stabilizing factor for economic growth, thereby making the economy locally stable in the neighbourhood of the steady growth state. In other words the interest rate is an exogenous variable in the short run and becomes an endogenous one in the long run.

As for the determination of money wages, we do not adopt the varieties of Phillips curve. Instead, we concur with Kuh (1967) that the level of the unemployment rate, together with the level of productivity and the level of a value-added deflator, determines *the level of* the money wages, *not the rate of change* in them. The level of money wages, too, together with the level of employment, are determined by the level of effective demand, which seems to represent Keynes's point of view better, since he denies the so-called adjustment mechanism of supply of and demand for labour in the labour market. However, Phillips curves assume that this mechanism works in the long run, at least.

The structure of this paper is as follows: in Section 2 a short-run income determination model is presented and in Section 3 this is extended to a long-run model, without assuming full employment. In Section 4 the steady growth state is given and investigated. In Section 5 the dynamic aspects of this model are studied briefly under a further simplified assumption, using phase diagrams, and fiscal and monetary policies are investigated in a dynamic growth process. Section 6 sets out the concluding remarks.

### 2. The short-run model

The normal output  $Q_n$  at the normal capacity utilization depends on the productive capacity, and especially on the real fixed capital stock K being constant in the short run. Like Dutt (1994) and You (1994a, 1994b), we assume the following simplest relation in aggregate real value terms.

$$Q_n = \sigma K \tag{1}$$

where  $\sigma$  is the output coefficient, assumed to be constant both in the short and long run. Therefore,  $Q_n$  is also constant in the short run. The normal employment  $N_n$  for the normal output is determined as follows:

$$N_n = Q_n / q \tag{2}$$

where q stands for the labour productivity at the normal rate of capacity utilization, determined basically by the techniques embolied in the real capital stock K. It is assumed to be constant in the short run, and  $N_n$  is therefore also constant.

The actual, current output Q may fluctuate around this normal output  $Q_n$ . The capacity utilization rate x is defined, therefore, as follows:

$$Q \equiv xQ_n \tag{3}$$

For production of Q labour N is employed according to the following equation:

$$N = n(x)N_{n}; n(1) = 1; n' > 0$$
(4)

That is, when the capacity utilization rate rises beyond the normal value (x = 1), employment increases beyond the normal level, proportionately to the function n(x).

The apparent labour productivity Q/N fluctuates around q with business conditions which are shown by the change in the capacity utilization rate x. Its expression is obtained by equations (2)-(4):

$$Q/N = qx/n(x) \tag{5}$$

Its short-run change is as follows:

$$\Delta(Q/N)/(Q/N) = (1 - \varepsilon)\Delta x/x; \ \varepsilon \equiv xn'/n \tag{6}$$

As we know (on the basis of Okun's law, for instance) labour productivity fluctuates procyclically.<sup>1</sup> This implies that it increases as the capacity utilization rate rises. In other words, there are, generally, increasing returns with respect to labour in the short run.<sup>2</sup> Therefore we may assume:

$$0 < \varepsilon < 1 \tag{7}$$

As for price formation, we adopt the mark-up pricing or the full-cost principle<sup>3</sup> (see Appendix 1).

$$\mathbf{p} = \mathbf{h}(\mathbf{x})\mathbf{w}/\mathbf{q}; \ \mathbf{\varepsilon}_{\mathbf{h}} \equiv \mathbf{x}\mathbf{h}'/\mathbf{h} \geq \mathbf{0}$$
(8)

where w stands for money wages. This equation means that higher prices are set as the capacity utilization rate rises. In other words, firms set higher prices when supply-demand conditions are tighter. In fact, we often hear businessmen complain that they cannot shift cost rises on prices because of the recession.<sup>4</sup> It should be noted that this price equation implies countercyclical fluctuations of real wages *in the short run* (w/q = q/h(x)).<sup>5</sup>

<sup>3</sup> The price equation is similar to that of You (1994b). While the rate of change in price is the dependent variable in his model, however, in our formulation it is the level of price. This difference will be found in the specification of money wages equation, too.

<sup>4</sup> In addition, an economy usually includes such sectors as agriculture and mining whose prices are far more responsive to supply-demand conditions than those of the manufacturing sector. This is another reason why we may specify the general price level depending on supply-demand conditions.

<sup>5</sup> This might be questioned. In empirical studies it is not easy to abstract the short-run fluctuations because actual data of real wages include the effect of the change in the labour productivity which is hard to separate fully in many cases. Schor (1985) and Bils (1987) seem to have no relation with our specification. Schor's study does not clarify the movement of real wages around normal capacity utilization rate, which is important to the study of the steady growth state because it is based on the data at peaks and troughs. Bils shows us a countercyclical fluctuation of mark-ups. However, his study focuses on the relation between prices and marginal labour cost, not the normal, average cost. See also the comprehensive survey by Abraham and Haltiwanger (1995). We get  $\mathcal{E} \cong 0.85$ ,  $\beta' \cong -0.5$  and  $\mathcal{E}_b \cong 0.5$  from Sato (1989, ch. 3) who uses normalized data with the values at the full-employment state for labour shares and Okun's law, for the cases of Japan and the United States in 1974-84. We assume, therefore, that  $\mathcal{E}_b$  is not so large, even if negative.

<sup>&</sup>lt;sup>1</sup> Okun's law shows that the decrease in unemployment by 1%, that is, the increase in employment by 1%, increases output by 2-3%, implying a rise in labour productivity.

<sup>&</sup>lt;sup>2</sup> See Lavoie (1992, ch. 5). Keynes (1939, p. 46) noted this increasing return, for he writes: "It is, beyond doubt, the practical assumption of the producer that his price policy ought to be influenced by the fact that he is normally operating subject to decreasing average cost, even if in the short-period his marginal cost is rising".

The wage share  $\beta$  is given by equations (5) and (8).

$$1 \ge \beta \equiv wN/pQ = n(x)/xh(x) = \beta(x) > 0$$
(9)

Therefore, from (7), (8) and (9) we obtain,

$$\mathbf{x}\beta'/\beta = -(1 + \varepsilon_{\rm h} - \varepsilon) < 0 \tag{10}$$

which implies that the labour share decreases as the capacity utilization rate rises.<sup>6</sup> Therefore, the saving ratio s becomes an increasing function of the capacity utilization rate because the saving ratio is generally higher for the saving out of profits than those out of wages, and the profit share increases with the capacity utilization rate (see Appendix 1).

$$1 > s = s(x) > 0; s' \ge 0$$
 (11)

We define here the rate of profit R calculated with current capital value, not book value, for later use. Its expression is obtained by equations (1), (3) and (9):

$$R \equiv (pQ - wN)/pK = \sigma x[1 - \beta(x)] = R(x)$$
(12)

therefore,

$$\mathbf{R}' = \sigma[1 - \beta(\varepsilon - \varepsilon_{\rm h})] > 0 \tag{13}$$

where the sign is determined by (7)-(9). This implies that the rate of profit rises as the capacity utilization rate rises.<sup>7</sup>

Abstracting from changes in unintended inventory stock that implies a disequilibrium between production and expenditure, or investment and saving, the sake of simplicity we assume that the economy is always in equilibrium in the short run. Therefore, for net real investment I, given in the short run, we get the following equation, using (1) and (3):

$$sQ = I$$
, that is,  $\sigma xs(x)K = I$  (14)

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which implies that the investment generates necessary saving by changing the capacity utilization rate and distribution shares, thereby changing the saving ratio in the short run.<sup>8</sup> At the same time, we see the positive correlation between the profit share  $1 - \beta$  and the investment ratio I/Q = s(x), recalling that s(x) has a positive correlation with the profit share. This is also one of Kaldor's stylized facts (Kaldor 1961). Another implication is that in the short-run equilibrium the disequilibrium between the production capacity and the actual production generally remains.

When the investment increases by  $\Delta I$  through public investment in infrastructures, for instance, the output increases by  $\Delta Q$  from (1), (3) and (14).

 $\Delta Q = k(x)\Delta I$ , or  $\sigma K\Delta x = k(x)\Delta I$ ;  $1/k(x) \equiv s(x) + xs'(x) > 0$  (15)

where k(x) stands for the familiar investment multiplier.

When the capacity utilization rate x is determined by the real investment I, employment is given by equation (4) and the real wage w/p is determined by equation (8). To determine the price level p and the level of money wages w, we need a money wages equation. Following Kuh (1967), we specify it as follows:

$$\mathbf{w} = \mathbf{f}(\mathbf{N}/\mathbf{N}_{s})\mathbf{p}_{v}\mathbf{q}; \quad \mathbf{\omega} \equiv (\mathbf{N}/\mathbf{N}_{s})\mathbf{f}'/\mathbf{f} > 0 \tag{16}$$

where  $N_s$  stands for labour supply being constant in the short run and  $p_v$  is a value-added deflator which firms use in the determination of money wages. This  $p_v$  is different from the price level and assumed to be constant in the short run. However, its rate of change would vary as that of prices p changes.

The basic determinant of money wages seems a kind of 'norm' (Okun 1981), specified here as the value productivity of labour  $p_{v}q$ . However, firms would have to offer higher money wages as the labour market became tighter, as is shown by a higher  $N/N_s$ , to get the necessary workers. In other words, the unemployment rate or  $N/N_s$  represent the strength of labour in bargaining power for money wages.

This specification of money wages is in line with the full cost principle in price formation, differing from the neoclassical theory in

<sup>&</sup>lt;sup>6</sup> Therefore, the wage-led case in You (1994a, 1994b) is excluded in our model. <sup>7</sup> The actual relation between the rate of profit and the capacity utilization rate is

more complicated. See Jarsulic (1994). However, this result would be acceptable to use in simplified models such as ours.

<sup>&</sup>lt;sup>8</sup> Kurz (1992) comes to the same conclusion.

which prices continue to change until these changes bring about the equilibrium of supply and demand. Generally speaking, supply and demand are adjusted in quantity and prices are determined by profitability conditions, which seems the correct interpretation of the principle of effective demand (see Kalecki 1954, ch. 1; Robinson 1962, ch. 1; Weintraub 1958 and Morishima 1976). In this sense, Phillips curves, including natural rate hypothesis, are based on the neoclassical notion, for they assume that money wages continue to change as long as the rate of unemployment differs from the natural rate. In other words they assume the realization of full employment in 'the long-run' equilibrium with the labour market mechanism. In the above equation (16), however, the rate of unemployment affects the level of money wages, not their rate of change.<sup>9</sup>

Once the capacity utilization rate or output is determined, the necessary money M is automatically supplied under the nominal interest rate r given in the short run by the monetary authority. In other words, it is assumed that the money supply is endogenous, in accordance with Kaldor (1982) and Moore (1983), for example. Of course, this is also an assumption serving for simplification. However, it seems a more realistic one compared with the idea of the money stock as an exogenous policy variable, bearing in mind the existence of the open discount window (Kaldor 1982) and overdraft facilities (Moore 1983 and Lavoie 1992). Therefore, the money stock M is determined by the following, simplest equation, according to Kalecki (1954, ch. 6).

$$M = \ell(r)pQ; \quad \ell' < 0 \tag{17}$$

An instantaneous change in the interest rate by the monetary authority, therefore, has no influence on the capacity utilization rate and changes only the money demand that is, in the short run, automatically accommodated, for the smooth operation of the economy. It changes the capacity utilization rate in the long run, however, through the change in the capital accumulation rate, as we shall be seeing later.

Before going on to the long-run mechanism, let us rewrite equation (14) in another form. Defining the capital accumulation rate,

$$g \equiv K/K = I/K \tag{18}$$

where the dot indicates differentiation with respect to time, we get the following new expression for (14),

$$\mathbf{g} = \mathbf{\sigma}\mathbf{x}\mathbf{s}(\mathbf{x}) \tag{19}$$

that is, given the accumulation rate g in the short run, the capacity utilization x is determined. To realize the normal rate of capacity utilization x = 1, the accumulation rate must be Harrod's warranted growth rate,  $g_m = \sigma s(1)$ .<sup>10</sup>

### 3. The long-run mechanism

So far we have assumed that the accumulation rate g, capital stock K, labour productivity q, value-added deflator  $p_{q}$ , labour supply  $N_s$  and the interest rate r are constant in the short run. However, these variables change gradually in the long run, and are represented by differential equations with respect to time in the following pages.

<sup>&</sup>lt;sup>9</sup> Differentiation of equation (16) with respect to time yields that the growth rate of money wages depends not only on the unemployment rate but also on the rate of change in the unemployment rate, as well as the rate of changes in the labour productivity q and the value added deflator  $p_v$ . The importance of the rate of change in the unemployment has been known since Phillips curve studies began (see Bowen and Berry 1963, for instance, and Gordon 1989 as for a recent study). As empirical facts, money wages did not continue to fall in the economies between World War I and World War II despite the fact that an unemployment rate of over 10% continued for 10 years in the US and even more in the UK. Our specification has similarity to the efficient money wage theory in the sense that it is a function of the level of unemployment rate.

<sup>&</sup>lt;sup>10</sup> There is no explicit accumulation rate function in Harrod's model. He might have implicitly used the following function: g = a(x - 1); a(0) = 0; a' > 0, that is, when the capacity utilization is over unity, the accumulation rate continues to rise, which implies a knife-edge instability of the warranted growth rate. However, Harrod (1973) denies the notion of knife-edge instability. For instance, he writes: "But if they [deviations] are of moderate dimensions, I would not suppose that they would bring the instability principle into operation" (p. 33). We agree with this view and, therefore, do not regard the warranted growth rate as an important economic variable. From the point of view of empirical studies, it is the level of the accumulation rate, not its rate of change, that has a correlation with the rate of profit and, therefore, with the capacity utilization rate (see Bowles, Gordon and Weisskopf 1989, for example). Therefore, the above accumulation rate function seems wrong.

### 3.1. The capital accumulation rate function

A higher realized rate of profit implies that the investment plan carried out in the past is successful and would, therefore, enhance the optimism of firms and give them financial security in addition. This would increase capital accumulation. The increasing risk expected would, however, restrain firms from increasing the accumulation rate proportionally to the realized profit rate as it became higher. On the other hand, a higher real interest rate implies higher costs for investment and would therefore reduce investment. Above all, a higher but still moderate expected inflation rate would reduce the real burden of debts and, therefore, raise the accumulation rate. We, therefore, adopt Robinsons' (1962) animal spirits function. As a behavioral equation for the planned capital accumulation rate, it is mainly as a function of the realized rate of profit, R(x), modified by introduction of the real interest rate,  $r - \pi_e$ , as the financial constraint or the inflation barrier, which is drawn in Figure 1 for a given real interest rate.

$$a[R(x), r - \pi_{a}]; a_{1} > 0; a_{2} < 0$$
(20)

where  $a_1$  is the partial derivative with respect to the profit rate and  $a_2$  that with respect to the real interest rate, which means that in Figure 1 this function makes an upward shift when the interest rate falls. This is essentially the same specification as Eltis made (1973, p. 211). An implication of the animal spirits notion would be that the accumulation rate function takes higher locations when animal spirits are higher.

Keynes argued that capital investment is decided at the point where the marginal rate of efficiency of capital (corresponding to the profit rate) becomes equal to the interest rate, resulting in the usual specification that investment is a function of the interest rate only. However, because of risk premium factors the rate of profit must be generally higher than the interest rate. Moreover, their movements differ from each other in the dynamic growth process, because determinants of the profit rate are different from those of the interest rate. Therefore, it is better to separate these rates. Although Robinson disregarded the interest rate for the sake of simplicity in her study (1962, p. 43), its explicit introduction is needed for the study of the



inflation barrier.<sup>11</sup> Inclusion of both the profit rate and the real interest rate seems the essential and simplest approach in specification of the capital investment function.<sup>12</sup>

The actual accumulation rate g is, however, determined by the following equation, which implies that there is a gestation period in investment due to order and construction lags for capital goods, for example (Kalecki 1954, ch. 8 and Jarsulic 1988, p. 52).

$$\dot{\mathbf{g}} = \lambda \{ \mathbf{a}[\mathbf{R}(\mathbf{x}), \mathbf{r} - \boldsymbol{\pi}_{\mathbf{a}}] - \mathbf{g} \}; \lambda > 0$$
(21)

This gradual change in the accumulation rate causes the capacity utilization rate to change gradually, according to equation (19).

<sup>&</sup>lt;sup>11</sup> Robinson (1962, p. 43) writes: "A model of a closed system in which monetary policy, via the interest rate, controls the level of investment, is a kind of day-dream that economists delight in, but our model is not designed to pander to this indulgence. We therefore chose assumptions which give monetary policy a very minor role".

<sup>&</sup>lt;sup>12</sup> A similar functional form is also found in Malinvaud (1980). Dutt (1994) specifies the desired accumulation as a linear function of the capacity utilization, the difference between the profit rate and the real interest rate and the rate of technical change. You (1994b) gives us a function of the profit share and the capacity utilization. It seems to us that such variables as the capacity utilization and the rate of technical change affect the rate of desired capital accumulation indirectly through the rate of profit, bearing in mind that firms usually carry out test production and test sales before full-scale investment.

Differentiation of equation (19) yields, using (15),

 $G(x)\dot{x} = \dot{g}; G(x) \equiv d[\sigma xs(x)]/dx = \sigma/k(x) > 0$ (22)

### 3.2. The technical progress function

Kaldor (1957, p. 595) writes: "A society where technical change and adaptation proceed slowly, where producers are reluctant to abandon traditional methods and to adopt new technique is necessarily one where the rate of capital accumulation is small. The converse of this proposition is also true: the rate at which a society can absorb and exploit new techniques is limited by its ability to accumulate capital". This does not necessarily imply that technical progress, represented with the growth of labour productivity, needs the use of more capital per worker as Kaldor also states. Technical progress would be realized as long as capital was accumulated even if the capital per worker did not increase because of increase in the number of workers.

Therefore we specify technical progress as a function of the accumulation rate, different unlike Kaldor's original version in which the rate of change in capital per worker accounts for technical progress.

 $\dot{q}/q = T(g); T(0) > 0; T' > 0; T'' < 0$  (23)

That is, the rate of change in labour productivity rises as the accumulation rate rises. As the accumulation got higher, however, several factors would make firms unable to realize proportional growth in labour productivity, such as the difficulty of rapid absorption of technical knowledge, the lack of technical seeds and so on (T'' < 0). Considering the existence of disembodied technical growth, such as the learning-by-doing effect in the narrow sense that labour skill improves as the cumulative amount of production increases, even if without changes in equipment, we may assume that T(0) > 0.<sup>13</sup> More specifically, we assume a functional shape as shown in Figure 1, implying that 1 > T' in the relevant range of g. 101

This type of technical progress function is found in Eltis (1973, ch. 6) and Dutt (1994).<sup>14</sup> It is well known that the growth rate of labour productivit  $\dot{q}/q$  has positive correlation with investment share (see Landau 1992 and Levine and Renelt 1992, for example). Bearing in mind that investment share I/Q has positive correlation with the accumulation rate  $I/K = \dot{K}/K = g$  because output varies more or less with capital stock, we may conclude that the above technical progress function is backed by empirical evidence, too.<sup>15</sup>

Rapid growth in the stock of R&D knowledge, especially the rich stock of imitable production techniques in foreign countries, would cause the technical progress function for the economy to be in a higher location. As we know the latter is one of the most important reasons why late-comers achieve generally higher rates of economic growth. Competition generally forces firms into adopting the most advanced technique when they invest in capital stock.

Therefore, it would generally also depend on the competition between firms in an economy whether the technical progress function takes a higher location or a lower one. In addition, difficulty in labour management generally tends to drive firms to develop labour-saving techniques, which would potentially generate an increase in the Marxian reserve army. Therefore, the above technical progress function should be interpreted as an institutional equation.<sup>16</sup>

We notice that this technical progress yields a proportionate increase in capital-labour ratio as a result, recalling that  $q = \sigma K/N_n$  under the assumption of equations (1) and (2).<sup>17</sup> However, this does

<sup>&</sup>lt;sup>13</sup> Renewal investment also embodies technical progress, which would also support that T(0) > 0. However, to make the representation of the model as simple as possible, we disregard the depreciation.

<sup>&</sup>lt;sup>14</sup> Eltis gives us a function of investment share as the explanation variable. Dutt also includes the case, consistent with the Schumpeterian view, that T' < 0. However, empirical evidence does not seem to bear this out.

<sup>&</sup>lt;sup>15</sup> In our model,  $I/Q_{\mu} = I/\sigma K = g/\sigma$ .

<sup>&</sup>lt;sup>16</sup> The technical progress function would also depend on the rate of transference of labour to the manufacturing sector from those sectors where there are either diminishing returns, or where no relationship exists between employment growth and output growth, such as agriculture and mining. This is an implication of Kaldor's third law, by Thirlwall (1983).

<sup>&</sup>lt;sup>17</sup> In You (1994a), technical progress implies an increase in the capital-labour ratio that is caused by a rise in employment-labour force ratio, which seems to imply the neoclassical production function having the substitution between labour and capital. This relation reduces to a convex function of the accumulation rate, whereas our function is a concave function. In You (1994b), the rate of change in real wages is introduced as another explanation variable. Atkinson and Stiglitz (1969) point out inappropriateness of the notion of production function having an invariant functional form in growth processes by their notion of localized search (see also Gomulka 1990). As another example, Gallegati (1993) specifies the technical progress as a function of R&D that is linked to equity base, different from Kaldor's idea.

not imply that an increase in the capital-labour ratio causes technical progress, as apparent in equation (23). Equation (1) shows a macroeconomic relation, in models based on aggregate variables such as ours, which includes the appearance of new products, too. Therefore, an increase in the capital-labour ratio does not necessarily imply the capital deepening only in the sectors in existence.

With this technical progress function, employment N changes gradually in the long run. First, substituting (1) and (2) into equation (3) gives

$$N = n(x)\sigma K/q \tag{24}$$

Differentiating both sides with respect to time and using (18) and (23), we get

$$WN = \varepsilon \dot{x}/x + g - T(g)$$
(25)

That is, employment increases as the rates of capacity utilization rise. Capital accumulation has two opposite effects on employment. It increases employment by raising national income. On the other, it decreases employment because of the accompanying rise in labour productivity. Usually, in the relevant range of g, therefore, it would increase employment (recall that 1 > T').

Assuming here that the growth rate of the labour force is constant v, given exogenously,

$$\dot{N}_{s}/N_{s} = v$$
 (26)

Harrod's natural growth rate g, becomes an endogenous variable.

$$g_n \equiv \dot{q}/q + v = T(g) + v = g_n(g)$$
 (27)

### 3.3. The interest rate in the long run

It is often said that the ultimate goals of monetary policy are economic growth, stabilization of economic fluctuations and control of balance of payments and exchange rates. However, a more basic object would be to secure stability for the purchasing power of money, without which smooth operation is impossible for the economy. Therefore, when inflation rate  $\pi$  rises, interest rate r would be raised and *vice versa*. We assume here that the monetary authority attempts to keep the actual inflation rate  $\pi$  to a constant value  $\pi_{o}$ which may be zero, in the long run and gradually changes interest rate r for that purpose. That is:

$$\dot{\mathbf{r}} = \mathbf{Z}(\pi - \pi_0); \quad \mathbf{Z}(0) = 0; \quad \mathbf{Z}' > 0$$
 (28)

where  $\pi$  is the rate of change in prices, defined as follows:

$$\pi \equiv \dot{p}/p \tag{29}$$

Defining the rate of change in the value-added deflator  $\pi_{v}$ ,

$$\pi_{\rm v} \equiv \dot{\rm p}_{\rm v}/{\rm p}_{\rm v} \tag{30}$$

we assume:

$$\dot{\pi}_{v} = V(\pi - \pi_{v}); \quad V(0) = 0; \quad V' > 0$$
 (31)

That is, the rate of change in the value-added deflator increases as the actual inflation rate is larger than it. Therefore, this mechanism shows the possibility of the so-called wage-price spiral, together with equation (8) and (16).

The expected inflation rate is formed as follows:

$$\dot{\pi}_{e} = U(\pi - \pi_{e}); \quad U(0) = 0; \quad U' > 0$$
 (32)

The expected inflation rate rises when the actual inflation rate  $\pi$  is higher than the current expected inflation rate, and *vice versa*.

Differentiating both sides of equation (16) with respect to time and using (23), (25), (26), (30) and (31), we get the long-run change in money wages.

$$\dot{w}/w = (\dot{W}N - \dot{N}_{s}/N_{s}) + \pi_{v} + \dot{q}/q = = \omega \left[\epsilon \dot{x}/x + g - T(g) - v\right] + \pi_{v} + T(g)$$
(33)

Money wages increase with rising capacity utilization rates and rate of change in the value-added deflator, and decrease as the growth rate of the labour force rises. Capital accumulation increases money wages by the accompanying increase in employment, but it raises the rate of technical progress, which has two opposite effects on money wages. It raises the 'norm' for money wages, thereby increasing them. On the other hand, rising labour productivity increases the reserve army, thereby reducing money wages. It depends, therefore, on parameter  $\omega$ , the strength of bargaining power of firms in the sense that money wages are decided by labour market conditions, whether capital accumulation raises money wages or not. As is pointed out with the employment equation (25), however, as long as 1 > T' or capital accumulation increases employment, it raises money wages.

Differentiating both sides of equation (8) and using (23), (29) and (33), we get the equation for the rate of change in prices.

$$\pi = \varepsilon_{h} \dot{\mathbf{x}} / \mathbf{x} + \dot{\mathbf{w}} / \mathbf{w} - \dot{\mathbf{q}} / \mathbf{q} = \cdot$$
$$= (\varepsilon_{h} + \omega \varepsilon) \dot{\mathbf{x}} / \mathbf{x} + \omega [\mathbf{g} - T(\mathbf{g}) - \mathbf{v}] + \pi_{v}$$
(34)

The inflation rate rises with rising capacity utilization rates and the rate of change in the value-added deflator. Capital accumulation usually raises the inflation rate.

Thus we have all the necessary equations for the long-run mechanism.

### 4. The steady growth state

First let us review the long-run equilibrium or the steady growth state, assuming that it exists. It may be defined by the condition that

$$\dot{g} = \dot{\pi}_{e} = \dot{\pi}_{v} = \dot{r} = 0$$

i.e., the capital accumulation rate, the expected inflation rate, the rate of change in the value-added deflator and the interest rate are constant. With this definition we get the following values in the steady growth state (denoted by the symbol \*) for the actual and expected inflation rates and the rate of change in the value-added deflator, from equations (28), (31) and (32).

$$\pi^* = \pi_{\rm e}^* = \pi_{\rm v}^* = \pi_{\rm 0} \tag{35}$$

These are equal to the target inflation rate of the monetary authority.

The condition that  $\dot{g} = 0$  means that the capacity utilization rate is also constant,  $\dot{x} = 0$ , with equation (22). Therefore, equation (34) and (35) imply that

$$g^* = T(g^*) + v = g_n(g^*)$$
 (36)

in the steady growth state, i.e., the capital accumulation rate is equal to the natural growth rate. In other words, the steady growth rate of capital accumulation (therefore, output, too) is determined by the technical progress function, or as the equilibrium between the actual accumulation rate and the natural one, as shown in Figure 1, which, in turn, determines the level of the capacity utilization  $x^*$  at the steady growth state, with equation (19).<sup>18</sup>

This  $x^*$  is not necessarily equal to the normal capacity utilization rate (x = 1). Is it possible, then, that this is a long-run equilibrium value? Kurz (1992, p. 79) writes: "Reserves of capacity are seen to be carried in order to meet peak expected rates of demand; to deter potential competitors from entering the market; and because of the fear of firms losing customers in the case of an unexpected increase in demand". Therefore, even the normal rate seems to have some range, not a point.<sup>19</sup> We may assume that  $x^*$  is within the relevant range of the capacity utilization rate.<sup>20</sup>

<sup>&</sup>lt;sup>18</sup> If the curve  $g = \sigma xs(x)$  takes an extremely lower location because of small saving ratios, the long-run rate of capacity utilization determined by equation  $g^* = \sigma x^*s(x^*)$  may be beyond its maximum rate determined physically. In this case the economy has no steady growth state.

<sup>&</sup>lt;sup>19</sup> Quoting Eichner (1976) and Koutsoyiannis (1975), Lavoie (1992, p. 122) writes: "Companies, such as General Motors, plan the presence of excess capacity, operating at rates of utilization of practical capacity which oscillate between 65 and 95%, and aiming for normal rates of utilization in the 80-95% range".

 $<sup>^{\</sup>rm 20}$  Dutt (1994) considers explicitly the existence of the maximum capacity utilization rate.

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In this steady growth state, the apparent capital-output ratio becomes, with equations (1) and (3),

$$(\mathrm{K}/\mathrm{Q})^* = 1/(\sigma x^*)$$

When the technical progress function takes a lower location, the accumulation rate and the capacity utilization become smaller. Therefore, lower growth rates accompany higher apparent capital-output ratios.<sup>21</sup>

In this steady growth state, with equation (21) the actual accumulation rate  $g^*$  must be equal to planned accumulation rate a, which determines interest rate  $r^*$ , given the target inflation rate  $\pi_0$ .

$$a[R(x^*), r^* - \pi_n] = g^* = \sigma x^* s(x^*)$$
(37)

In Figure 1 this determination implies that the animal spirits function must pass through intersection  $S^{22}$ . This is possible through changes in the interest rate.

In other words, whereas the *actual investment* g generates the necessary saving for it in the short-run equilibrium, as shown with equation (19), in the long-run steady growth state the interest rate is determined so as to equate the *planned investment* a to saving, implying the loanable fund theory in a sense.<sup>23</sup> The interest rate is determined by the monetary authority in the short run, as a policy variable. In the long run, however, it becomes an endogenous variable for the monetary authority, because it attempts to keep the stability of the purchasing power of money by changing the interest rate according to the movement of the inflation rate. How, then, is it

<sup>23</sup> In Dutt (1994) and You (1994a, 1994b), there is no distinction between the planned investment and the actual one.

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determined as the value in equation (37)? Suppose that the economy is in the steady growth at first and the monetary authority lowers the interest rate a little. This raises the planned accumulation rate, which gradually increases the actual accumulation rate and the capacity utilization rate as shown in equations (21) and (22), thereby raising the inflation rate and the rate of change in the value-added deflator, too, with equations (34) and (31). Therefore, the monetary authority must raise the interest rate with equation (28), which brings the economy to the steady growth again.<sup>24</sup>

What effect, then, does a rise in the saving ratio have on the steady state? A rise in the saving ratio (a decrease in effective demand) shifts the curve  $g = \sigma xs(x)$  upwards in Figure 2, which reduces the capacity utilization rate  $x^*$  in the steady growth state and, therefore, lowers the profit rate  $R(x^*)$ , because the steady growth rate of capital accumulation g\* is determined by the technical progress function and, therefore, does not change. There must be an accompanying upward shift of the planned accumulation rate curve for the new steady growth state so that it passes through the new intersection  $(x_1^*, g^*)$ , which implies a lower interest rate. In short, a rise in saving ratio brings about a fall in the interest rate, which implies again the loanable fund theory, but only in the steady growth state. In addition, we could conclude that the (real) interest rate is basically determined by the profit rate, because a lower interest rate accompanies the lower profit rate.<sup>25</sup> It should be noted that a rise in saving ratio does not change the steady growth rate of capital accumulation, which implies that Robinson's long-run version of the paradox of thrift disappears in this model.26

Let us review here the other variables in the steady growth state. The growth rate of money wages is given as the sum of the growth rate of labour productivity and the target inflation rate by equations (33) and (36).

$$(\dot{w}/w)^* = \pi_0 + T(g^*)$$

<sup>24</sup> The steady growth state is locally stable as shown in Appendix 2.

 $<sup>^{21}</sup>$  Some authors such as Weintraub (1966) point out that the capital-output ratio changes in the long run. It seems to us that we need precise data on the capacity utilization rate for empirical studies on this ratio, although there are certainly some real factors that increase this ratio, such as enforcement adoption of equipment for pollution prevention.

<sup>&</sup>lt;sup>22</sup> When the technical progress function takes a much lower location, the steady growth state may be  $x^+$ , instead of  $x^*$ . This, however, seems unusual. Therefore, we focus mainly on  $x^*$  in what follows. On the other hand, if the accumulation rate function takes an extremely lower location because of weak animal spirits, there would be no intersection with  $g = \sigma xs(x)$  curve, even with the lowest interest rate (the liquidity trap). In this case, too, there is no steady growth state.

<sup>&</sup>lt;sup>25</sup> Robinson (1962, p. 8) writes: "The rate of profit on investment dominates the rate of interest on loans" and "Keynes was taking it for granted that the dominant reason for borrowing is the expectation of profit on investment".

<sup>&</sup>lt;sup>26</sup> You (1994b) shows this disappearance in his model, too, although his model does not include financial factors.

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The growth rate of employment is given by equations (25) and (36), which is equal to the growth rate of labour force.

 $(\dot{N}/N)^* = v$ 

This does not necessarily imply that full employment is achieved in the steady growth state, because the level of employment depends on the capacity utilization rate as is apparent with equation (4). We do not assume the so-called law of supply and demand for labour market. What then causes the growth rate of employment to become equal to the growth rate of the labour force? Let us suppose a rise in the growth rate of the labour force? Let us suppose a rise in the growth rate of the labour force in the steady growth state. This causes a decrease in the growth rate of money wages and a fall in the inflation rate, as in equation (34). Therefore, the monetary authority lowers the interest rate, with equation (28), which causes the planned rate of capital accumulation to rise, thereby gradually increasing the actual accumulation rate, as in equation (21). Thus, the growth rate of employment is raised, by equation (25) (recall that  $g > T(g) + \nu$  for  $g > g^*$  in Figure 1). In Figure 1, an increase in the growth rate of the labour force makes the curve of natural growth rate  $g_v$  shift upwards, thereby generating a higher rate of capital accumulation and, therefore, a higher rate of employment, which requires a lower rate of interest for the steady growth. In short, it is the behaviour of capital accumulation that equalizes the growth rates of the employment and labour force.

The growth rate of the capital-labour ratio in the steady growth state becomes as follows:

$$[(K/N)/(K/N)]^* = (K/K)^* - (N/N)^* = g^* - v = T(g)$$

That is, it is equal to the growth rate of labour productivity, as is easily understood with equation (1). The growth rate of money supply is equal to the sum of the accumulation rate and the inflation rate, as is easily obtained with equation (17).

$$(M/M)^* = \pi_0 + g^*$$

### 5. The dynamics of the economy

In the above equation system is hard to get the dynamic paths because of too many variables. For mathematical simplicity, we assume here that the expected inflation rate and the rate of change in the value-added deflator are equal to the target inflation rate  $\pi_0$ . This would, however, be permitted in the case where the long-run inflation rate is relatively stable. The economy, then, is described by the following three equations. Equations (21) and (22) give the second equation (38). Substituting (34) into (28) and using (23) and (38) yields the third equation (39).

$$g = \sigma x s(x) \tag{19}$$

$$G(x)\dot{x} = \lambda [a(R(x), r - \pi_0) - g]; \quad \lambda > 0; \quad G(x) > 0$$
(38)

$$\dot{\mathbf{r}} = Z \left\{ \theta \,\lambda/\mathbf{x} G(\mathbf{x}) \cdot \left[ \mathbf{a}(\mathbf{R}(\mathbf{x}), \, \mathbf{r} - \boldsymbol{\pi}_0) - \mathbf{g} \right] + \omega \left[ \mathbf{g} - \mathbf{T}(\mathbf{g}) - \mathbf{v} \right] \right\};$$
$$\theta \equiv \varepsilon_h + \omega \,\varepsilon > 0; \quad Z(0) = 0; \quad Z' > 0 \tag{39}$$

### 5.1. Global instability

With the substitution of equation (19) into (38) and (39), the economy is described with the two differential equations, (38) and (39), on the two economic variables, the capacity utilization rate x and the interest rate r. A seemingly typical case is represented as a phase diagram shown in Figure 3 (see Appendix 2). As is easily shown, this equation system has the same steady growth state  $(x^*, r^*)$  as the original one and is locally stable (see Appendix 3).<sup>27</sup> If the economy followed such a locus as c, as shown in the Figure, it would show a usual business cycle. However, it depends on the values of parameters whether it makes really cyclical movements or not (see Appendix 3).



If it falls into such a locus as d, however, the economy would collapse. In other words, the steady growth state is globally unstable. How does this collapse come about? Let us suppose that at first the economy is at point T shown in Figure 3 and that  $\dot{x} = 0$  curve makes a downward shift because of weakened animal spirits, for example. In this case, in the new phase diagram, the new locus that passes through point T may change to one of such loci as d in the old one. In this case, the economy would collapse instead of recovering. This would be an explanation for a collapse of the economy such as the Great Depression.<sup>28</sup>

Of course, functions such as animal spirits, saving and price formations are not necessarily stable through time and have parametric changes as often shown in econometric studies. This type of parametric changes would also explain, therefore, the persistence of usual business cycles; for without such changes the economy would usually reach the steady growth state after the intrinsic cyclical movements, because the steady growth state is locally stable.<sup>29</sup> Kalecki (1954, chs 11-13) points out that damping economic cycles generate persistent cycles through random shocks. If our model is correct in describing actual economies, therefore, we may conclude that persistent cycles are generated by parametric changes in behavioral equations, although economies usually show damping cyclical movements without such changes.<sup>30</sup>

# 5.2. Effects of fiscal and monetary policies on business cycles

Let us review a usual business cycle first. The locus passing through point S in Figure 4 shows the following movement. At point S the interest rate is still high and the profit rate is low and, therefore, the planned accumulation rate is lower than the actual, realized one. Thus, the economy continues to decline. At point T, it reaches the trough of the business cycle where the actual accumulation rate is low enough to begin increasing, owing to the falling interest rate. At point

<sup>30</sup> It should be noted that our model is totally different from the so-called real business cycle theory which assumes that an economy is always in full employment equilibrium and, therefore, has generally no intrinsic cyclical movements.

<sup>&</sup>lt;sup>27</sup> If the steady growth state is the lower intersection of the capacity utilization rate with  $g = \sigma xs(x)$  curve, it is not necessarily locally stable and may have limit cycles.

<sup>&</sup>lt;sup>28</sup> Dutt (1994, p. 108) gives a similar explanation for crisis.

<sup>&</sup>lt;sup>29</sup> Jarsulic (1988, ch. 3) shows us the existence of limit cycles in a cyclical growth model introducing explicitly the credit market, but without any inflation barrier mechanism. When the economy approaches asymptotically to a limit cycle, the cycle periods and its amplitude become asymptotically constant. These do not seem the characteristics of actual business cycles, for they show usually relatively irregular cycle periods and different amplitudes. In this sense, the demonstration of limit cycles is not necessarily the best way of explaining business cycles. However, parametric changes affect these limit cycles, too, thereby changing cycle periods and amplitudes. In this sense, they can also give an explanation for business cycles. Scott (1989) also gives us limit cycles in his cyclical growth model relying on the price mechanism, therefore, specifying the desired rate of capacity utilization function, rather than on the principle of effective demand.

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L, the interest rate reaches the minimum value, which urges a further accumulation. At point P, the economy reaches the peak, where the profit rate is maximum. However, the high actual accumulation rate and the rising interest rate make the accumulation rate begin to decline.

How, then, do demand control policies act upon this business cycle? Let us suppose that investment is increased by public investment in the short run when the economy is at point S in Figure 3. This causes output to increase, and thus, the capacity utilization rate to rise instantaneously (with equation (15)), as shown with the jump to point F in the Figure. The economy traces a new locus after this jump, according to equations (38) and (39), which is shown by the broken curve starting from point F. This policy therefore has the effect of reducing the amplitude of a business cycle.

Similarly, a once-and-for-all reduction in the interest rate at point S makes the economy jump to point M in the short run, which

also reduces the amplitude of business cycles. Study of short-run policy effects in the framework of dynamic growth processes would be more useful than in the short-run equilibrium models, for the short-run unemployment equilibrium cannot be a stationary state as long as positive investment increases the capital stock.

# 6. Concluding remarks

The steady growth rate is determined by equality between the actual growth rate and the natural growth rate redefined as the sum of the growth rate of labour, assumed constant, and the technical progress function which is a function of capital accumulation rate, a little different from Kaldor's original one. This steady growth rate determines the long-run rate of capacity utilization on the curve of the saving-capital ratio. In turn, this location determines the long-run interest rate by shifting the capital accumulation function that is specified with the introduction of real interest rate into Robinson's animal spirits function, although the short-run interest rate is given by the monetary authority as a policy variable. The long-run inflation rate becomes equal to the target rate of the monetary authority, which changes the short-run interest comparing the actual inflation rate with the target one.

We may conclude that the long-run rate of interest is basically determined by the profit rate, through the function of the capital accumulation rate. A rise in saving ratios causes only a reduction in the long-run rate of capacity utilization and a fall in the real and nominal interest rates, not the steady growth rate of capacity accumulation, implying the disappearance of Robinson's long-run version of the paradox of thrift.

The steady growth state is locally stable, but globally unstable, which implies that the economy has the chance of falling into collapse. This might offer an explanation for the Great Depression. The phase diagrams for growth models give us an explanation for the effects of short-run demand-control policies under dynamic growth processes.

Our model may be a kind of integration of models given by post-Keynesians such as Kalecki, Harrod, Robinson and Kaldor, having several points in common with Dutt (1994), You (1994a, 1994b) and Jarsulic (1988), for example. The most important difference is, however, in the explicit introduction of a financial sector functioning as an inflation barrier.

We assumed the expected inflation rate to be constant. If it also changes, how does the economy fluctuate? A higher inflation rate gradually generates a higher expected inflation rate, as in equation (32), thereby gradually raising the accumulation rate, as in equation (21). This results in a higher inflation rate, as in equation (34). This factor therefore, makes the economic system unstable. In fact, assuming that the interest rate is instead, constant, at the value in the steady growth state, the original equation system shows that the steady growth state becomes a saddle point, implying instability. The conclusion in Section 5 would, therefore, be correct on condition that the effect of the inflation barrier were stronger than the effect of expected inflation.

The performance of the actual financial sector as an inflation barrier may be worse, as shown by Jarsulic (1988), considering explicitly the credit market and Scott (1994), drawing on Minsky's "financial instability hypothesis". It seems to us, however, that study of the role of the monetary authority as inflation barrier is also important, especially under the controlled monetary system in the capitalist economy.

# APPENDIX 1

### 1. Price formation

Following the full-cost principle or the target-return pricing by Eckstein (1964), firms set the normal price  $p_n$  at the normal rate of capacity utilization, with the target rate of return  $R_0$ , which is assumed to be constant both in the short and in the long run.

$$\mathbf{p}_{n} = (\mathbf{w}\mathbf{N}_{n} + \mathbf{R}_{0}\mathbf{p}\mathbf{K})/\mathbf{Q}_{n}$$

where pK means that the capital stock is evaluated with the actual, current prices, taking account of inflationary conditions. For reference, Robinson (1962, p. 8) concludes: "Whatever the rate of interest charged for new borrowing may be, the opportunity cost of any one investment, from the point of view of the firm considering it, is the rate of profit obtainable on other investments. Thus, it is the rate of profit, not the rate of interest, that enters into the normal supply price of any particular commodity".

In setting the actual prices p, however, firms would consider the current supply-demand conditions, represented by the capacity utilization rate x.

$$p = H(x)p_{r}; H > 0; H(1) = 1; H' \ge 0$$

That is, the tighter supply-demand conditions are, the higher is the price level. From the above two equations and equations (1) and (2) in the text, we get equation (8) in the text.

$$\begin{split} \mathbf{p} &= \mathbf{h}(\mathbf{x})\mathbf{w}/\mathbf{q}; \quad \mathbf{h}(\mathbf{x}) \equiv \mathbf{H}(\mathbf{x})/[1 - \mathbf{R}_0\mathbf{H}(\mathbf{x})/\sigma] > 0\\ \mathbf{\epsilon}_{\mathbf{h}} &\equiv \mathbf{x}\mathbf{h}'/\mathbf{h} = \mathbf{x}\mathbf{H}'/\mathbf{H}[1 - \mathbf{R}_0\mathbf{H}(\mathbf{x})/\sigma] \ge 0 \end{split}$$

A higher target of return  $R_0$  implies a larger h(x), which suggests that h(x) may be regarded as a measure of the degree of monopoly.

#### 2. Saving ratio

With the saving ratios  $s_w$  out of wages and  $s_p$  out of profits, the total saving ratio is defined as follows:

$$s \equiv [s_w w N + s_p (pQ - wN)]/pQ = s_p - (s_p - s_w)\beta(x) = s(x)$$

With the assumption that  $1 \ge s_p > s_w \ge 0$  and inequality (9) in the text we get

### $1>s\geq 0$

Using equation (10) yields inequality (11) in the text.

# $s' = -(s_p - s_w)\beta' > 0$

APPENDIX 2

### Phase diagram

First, let us obtain  $\dot{x} = 0$  curve in Figure 2. For  $\dot{x} = 0$ , the right hand side of equation (38) in the text must be 0. When the interest rate equals  $r^*$ , a = g both at  $x = x^*$  and  $x = x^+$ . As the interest rate rises, curve a shifts downwards in Figure 1. Therefore, the two intersections with the curve g approach each other and finally become one point. For further higher interest rates, there is no point such that a = g. On the other hand, as the interest rate falls from  $r^*$ , the two intersections with the curve g go wider of each other. Thus,  $\dot{x} = 0$  curve becomes the one drawn in Figure 2. Above this curve, a < g (therefore,  $\dot{x} < 0$ ) because higher interest rates decrease a for the same x, as shown in Figure 4, meaning that x decreases there and *vice versa*, as shown on Figure 3.

Next, let us draw  $\dot{r} = 0$  curve. When  $x > x^*$  (that is,  $g > g^*$ ), g > T(g) + vand vice versa. For  $\dot{r} = 0$ , the right hand side of equation (39) must be 0. In other words, the combination must be a < g and g > T(g) + v, or a > g and g < T(g) + v. Therefore,  $\dot{r} = 0$  curve must exist in phases I and III in Figure 5. In the right hand side of this curve,  $\dot{r} > 0$ , meaning that r rises, and vice versa. Thus, we get Figure 3.

Figure 5



#### **APPENDIX 3**

Linear approximation in the neighbourhood of the steady growth state

Defining  $\Delta x \equiv x - x^*$  and  $\Delta r \equiv r - r^*$  and linearising equations (38) and (39), we get the following two equations.

 $\Delta \dot{\mathbf{x}} = -\mathbf{A}_1 \Delta \mathbf{x} - \mathbf{A}_2 \Delta \mathbf{r}; \quad \mathbf{A}_1 \equiv \lambda (\mathbf{G} - \mathbf{a}_1 \mathbf{R}')/\mathbf{G} > 0; \quad \mathbf{A}_2 \equiv -\lambda \mathbf{a}_2/\mathbf{G} > 0$ 

 $\Delta \dot{\mathbf{r}} = \mathbf{B}_1 \Delta \mathbf{x} - \mathbf{B}_2 \Delta \mathbf{r}; \quad \mathbf{B}_1 \equiv \left[ \boldsymbol{\omega} \left( 1 - \mathbf{T}' \right) \mathbf{G} - \boldsymbol{\theta} \ \mathbf{A}_1 / \mathbf{x} \right] \mathbf{Z}'; \quad \mathbf{B}_2 \equiv \boldsymbol{\theta} \ \mathbf{A}_2 \mathbf{Z}' / \mathbf{x} > \mathbf{0}$ 

where the signs are determined by equations (30), (20) and Figure 1; and the sign of  $B_1$  is not determined. Denoting the coefficient matrix as J,

tr J =  $-A_1 - B_2 < 0$ 

det 
$$I = A_1 B_2 + A_2 B_1 = A_2 \omega (1 - T') G Z' > 0$$

Therefore, the steady growth state is locally stable.

Dc J = 
$$(tr J)^2 - 4 det J = (A_1 - B_2)^2 - 4A_2B_1$$

Therefore,  $B_1$  must be positive for cyclical fluctuations. Assuming cyclical movement, Figures 2 and 3 are drawn.

Let us confirm the possibility of cyclical fluctuations with a numerical example. Assuming that  $s_w = 0.1$ ,  $s_p = 0.7$  and  $\beta = 0.75$ , then s = 0.25 as in Appendix 1. Assuming that  $\varepsilon_h = 0.5$  and  $\varepsilon = 0.85$ , then  $\beta' = -0.488$  and s' = 0.293 and, therefore, the multiplier k = 1.84. And if  $\sigma = 1$ , R' = 0.738.

With the additional assumptions that  $\omega = 0.5$ , T = 0.5,  $\lambda = 1$ ,  $a_1 = 0.7$ ,  $a_2 = -0.3$  and Z' = 0.5, then,  $A_1 = 0.049$ ,  $A_2 = 0.552$ ,  $B_1 = 0.0453 > 0$  and  $B_2 = 0.255$ . Therefore, *tr J* = -0.304, *det J* = 0.0375 and *Dc J* = -0.0986 < 0, which implies damping cyclical movements.

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