The EMU Debt Criterion: An Interpretation *

Ron Berndsen

1. Introduction

The decision as to which countries are to move on to the third stage of the Economic and Monetary Union (EMU) will be taken in 1998 as soon as reliable, actual data for the year 1997 become available. So much was decided by the European Council in December 1995 at its Madrid meeting. The convergence criteria specified in the Maastricht Treaty (on government deficit and debt, inflation, the exchange rate and the long-term interest rate) will play an important, if not decisive, role in determining which countries join the monetary union.¹ The formulation of some of these criteria in the Treaty is to varying degrees open to interpretation. Although the main elements of the criteria are clear, some of the details have yet to be specified. This is especially the case with respect to the exchange rate criterion which is based on the concept of a "normal fluctuation margin".² In 1992 when the text of the Treaty was finalized, the normal fluctuation margin amounted to 2.25%. However, the widening of the bands to 15% in August 1993 renders interpretation of the exchange rate cri-

² The relevant part of Article 109j reads: "[...] the observance of the normal fluctuation margins provided for by the exchange-rate mechanism of the European Monetary System, for at least two-years, without devaluing against the currency of any other Member State".

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De Nederlandsche Bank, Monetary and Economic Policy Department, Amsterdam (The Netherlands).

^{*} Any views expressed in the paper are the author's only and do not necessarily reflect the position of De Nederlandsche Bank.

¹ At the same time, it must be noted that – strictly speaking – satisfying all convergence criteria is neither a sufficient nor a necessary condition for entering the Monetary Union as the Treaty provides some room for manoeuvre. However, in this paper it is assumed that complying with the criteria is a necessary condition for entering the Monetary Union.

terion unclear. For this reason, the exchange rate criterion will not be considered here.

The aim of this paper is to provide a possible interpretation of the EMU debt criterion. We investigate the government debt criterion, which has a considerable scope for interpretation, as Article 104c(2b) of the Treaty shows:

"[...] whether the ratio of government debt to gross domestic product exceeds a reference value, unless the ratio is sufficiently diminishing and approaching the reference value at a satisfactory pace".

The first part is clear. The debt criterion is met if the level of the ratio of government debt to GDP is smaller than or equal to the reference value (in a Protocol to the Treaty specified at 60%). The second part - only applicable if the level of debt exceeds 60% GDP - refers to the change in government debt. In order to meet the debt criterion the sign of the change must be negative. However, the magnitude of the change and the time period over which to calculate the change are open to interpretation. Prior to the third stage, performance of the EU-countries with respect to fulfilling the fiscal criteria (debt and deficit) is already monitored in the so-called "excessive-deficit procedure". This procedure is carried out annually as from the start of the second stage (1 January 1994). Although the excessive-deficit procedure was completed in full in 1994, 1995 and 1996, only two cases of "positive" jurisprudence with respect to the second part of the debt criterion (i.e. a decision in which the decline in the debt ratio of a country has been decided to fulfill the criterion) are known. In each year the Ecofin Council decided that the decline of the debt ratio in Ireland was sufficient to meet the debt criterion. The second case is the decision of the Ecofin Council in 1996 to sanction the decline of the debt ratio in Denmark. Howéver, the precise grounds on which those decisions were taken are vague and not publicly available. It is therefore interesting to investigate whether it is possible to provide a general interpretation which also covers these cases of positive jurisprudence. Another reason for this investigation is that although the subject has been discussed extensively, e.g. by Buiter, Corsetti and Rubini (1993), Holzmann, Harvé and Demmel (1996) and Masson (1996), relatively little work has been done to develop an interpretation of the EMU debt criterion. Some earlier work can be found in Wolswijk (1995) and Gros (1995a, 1995b). In the former a relation is

postulated between deficit and debt such that the more highly indebted countries need correspondingly lower deficits (in any case below 3% GDP) in order to qualify. The latter interprets the second part of the debt criterion by introducing a minimum decline in debt that is consistent with a deficit of 3% GDP and in the (very) long run a debt of 60% GDP. Here, we adopt a more flexible approach in which parts of the relevant Treaty text are characterized using two parameters. Choosing suitable values for these parameters it is possible to encompass the earlier work.

2. Decomposition of the change in government debt

2.1. The decomposition framework

As argued above, the formulation of the *change* in government debt is one of the parts in the formulation of the debt criterion which is left open to interpretation. In order to analyse the evolution of the debt, we present a decomposition of the change in debt into underlying factors. The decomposition is derived from a well-known method³ and can be based on the definitional equation in which the nominal value of the outstanding stock of debt at the end of year t (denoted by S_i) equals the stock of debt at the end of the previous year (S_{t-1}) , the deficit in year t (D_t) and a component representing the stockflow adjustment (SF_i). The deficit can be split into two components: the actual interest payments on the debt (IP_t) and the primary deficit (PD_i):

$$S_t = S_{t-1} + IP_t + PD_t + SF_t$$
⁽¹⁾

Expressing variables as ratios to GDP (denoted in lower case), the decomposition of the change in the debt ratio in year t (denoted by Δs_t) can be written as follows:⁴

$$\Delta s_{t} = \frac{y_{t}}{1+g_{t}} s_{t-1} - \frac{py_{t}}{1+g_{t}} s_{t-1} + \frac{i_{t}}{1+g_{t}} s_{t-1} + pd_{t} + sf_{t}$$
(2)

³ See e.g. EMI (1995) and European Commission (1996). ⁴ See the Appendix for full details.

The first two terms on the right-hand side indicate the influence of the growth of the denominator gross domestic product (GDP) (g,) which can be decomposed in the growth of the volume of GDP (y,)and the GDP-deflator (py_t) . The first term is called the growth effect and the second term is called the inflation effect. Both effects are usually negative (i.e. they lower the debt ratio). However, in the case of a recession ($y_t < 0$) or deflation ($py_t < 0$), the growth or inflation effect becomes positive in year t. The third term is the effect of the interest payments on the change in the debt ratio which is always positive (assuming a positive rate of interest on debt, i.e. $i_t > 0$). The fourth term measures the effect of the primary balance on government debt (the balance excluding the interest payments). In the case of a primary deficit (deficits are defined positively) the contribution to Δs_t is positive and in the case of a primary surplus ($pd_{1} < 0$) the effect becomes negative. The fifth component, s_{f_t} , represents all other influences on the debt ratio not captured by the other components. These include financial transactions which do not influence the deficit, such as differences in the time of registration (cash- or transactions-basis) of various items and any other statistical discrepancies. Another important part of s_{f_t} is the effect of exchange rate fluctuations should the debt be issued in foreign currency. An important debt-lowering part of sf_i are the receipts from privatisation and – more generally – other financial transactions which shorten the government balance sheet (but leave the net asset position unchanged).

2.2. Decomposition in practice

To illustrate the notions introduced above, decomposition of the changes in debt in the Netherlands and the EU as a whole is shown in Figures 1 and 2. In the Netherlands (Figure 1) the evolution of debt in the last twenty years is characterised by a sharp increase of debt to about 80% of GDP in 1988, stabilisation around that level until 1995 and a small decrease later on.

The steepest increase in debt took place in 1981-82. In those years, in addition to the interest payments effect, three other components contributed positively: the growth effect (because of the recession), the primary balance (which was in deficit) and the stock-flow adjustment (mainly because of new credits granted by the central government). An example of negative stock-flow adjustment can be seen in the years 1986-87 when housing corporations repaid in advance loans granted by the government of over 1% of GDP. The year 1987 is also noteworthy because of the positive contribution of the inflation effect in 1987 as a consequence of the deflation in that year. From 1984 onwards the contribution of the primary balance has been negative and increasing, reflecting the slow but steady fiscal consolidation process, while the primary surplus increased from 1,4% of GDP (average 1985-1990) to on average 2.7% in recent years.

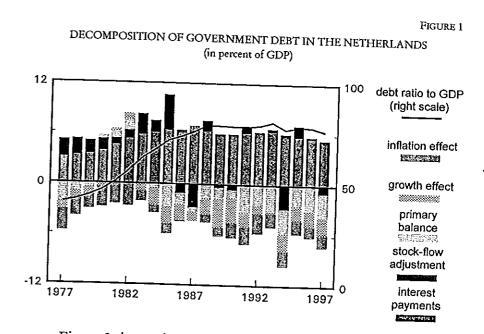
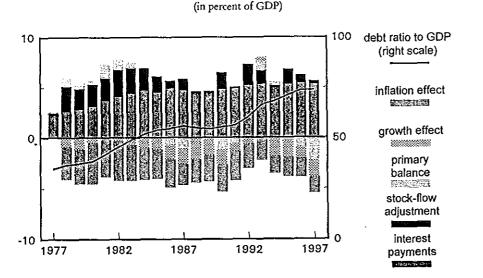


Figure 2 shows the evolution of debt at the EU-level. In the period under consideration the debt ratio has steadily increased from 35% of GDP in 1977 to around 75% of GDP in 1997, primarily because of the interest payments and – in contrast to the Netherlands – a positive stock-flow adjustment. The increase in debt is especially large in 1992-93 following a deterioration of the primary balance which started in 1990 and went into deficit in 1993, in concomitance with the recession of that year. However, from 1993 onwards fiscal discipline in Europe improved as is evidenced by the return to a primary surplus.





3. A possible interpretation of the debt criterion

The key idea of our interpretation of the debt criterion (henceforth called a Non-Redundant Interpretation, NRI) is to take into account the (implicit) constraints placed on the evolution of debt as a consequence of satisfying (or trying to satisfy) the other convergence criteria (see Section 3.1). Furthermore, as is evident from Section 1, the room for interpretation lies in the second part of the debt criterion: "the ratio is sufficiently diminishing and approaching the reference value at a satisfactory pace". We tackle that phrase by splitting it into two parts: 'sufficiently diminishing' (Section 3.2) and 'approaching the reference value at a satisfactory pace' (Section 3.3). The full procedure is recapitulated in Section 3.4.

3.1. Taking the other convergence criteria into account

One aspect in which the debt criterion differs from the other convergence criteria is that debt is a stock variable rather than a flow variable. It therefore seems natural to view the stock of debt as the *re-sult* of the impact of the other variables. Continuing in this fashion, satisfying the other convergence criteria can – in economic terms – be viewed as a prerequisite to satisfy the debt criterion, since if a country wants to join the monetary union all criteria must be satisfied (in legal terms no convergence criterion takes precedence over another criterion). Otherwise, a country could in principle satisfy the debt criterion by, for example, reducing the debt ratio through the imposition of a considerable inflation tax. This would, however, be incompatible with the goal of price stability and would result in a violation of the inflation criterion (and also the interest rate criterion, assuming that the real rate of interest does not fall). Given the stability-oriented goal of the Treaty, it therefore seems logical that the decline of the debt ratio should be attained in an environment of relatively low interest rates, low inflation and low deficits.

A way to take these considerations into account is to employ the decomposition of the change in debt into its underlying components, as introduced in Section 2. To this end, we identify the redundancies between the debt criterion and the other convergence criteria by considering these components:

1) Inflation effect (IE)

The inflation effect indicates the direct influence of inflation on the debt ratio, which implies a link with the inflation convergence criterion. The precise interpretation of this criterion has yet to be settled.⁵ However there is consensus that, at least as a starting point, the reference value can be calculated as the average inflation of the three countries with the lowest inflation plus 1.5 percentage points. The Treaty specifies the harmonised consumer price index (CPI) as the relevant measure of inflation. In our interpretation we consider the contribution of IE to the change in the debt ratio to come into line with the 'sufficiently diminishing' requirement if the inflation (denoted by CPI) does not exceed the average inflation of the three countries with the lowest inflation plus 1.5 percentage points (denoted by

⁵ See EMI (1995, p. 5). For example, the formulation in the Treaty "a rate of inflation which is close to that of, at most, the three best performing Member States in terms of price stability" leaves room to consider one, two or three countries with the lowest inflation.

CPI*). From equation (2) it is clear that IE is based on the GDPdeflator py rather than the CPI. It is therefore important to note that while assessment as to whether *IE* is in accordance with the inflation criterion is carried out on the basis of the CPI, the quantitative impact of IE on the debt ratio is determined by the GDP-deflator.⁶ Let py^* denote the GDP-deflator compatible with CPI* such that $py^* =$ $py + (CPI^* - CPI)$ and IE* is the component IE evaluated with py^* . *IE** is the maximum value of IE which is still compatible with the maximum allowable inflation (CPI*). Since IE is usually negative, it is considered in our interpretation to be compatible with the inflation criterion if IE \geq IE*.

2) Growth effect (GE)

The growth effect represents the effect of real GDP on the debt ratio. As such there is no direct link between GE and any of the other convergence criteria. There is however an indirect link because a higher level of inflation will lead to a higher GE, through the denominator effect. However, this indirect effect is usually negligible: even with high debt ratios and a large discrepancy between py and py^* the difference between GE and GE* is small. For example, if $s_{t-1} = 100$, $y_t = 0.025$, $py_t = 0.05$ and $py_t^* = 0.025$, it follows from (2) that the difference GE – GE* amounts to only $(0.025/1.075 - 0.025/1.05) \cdot 100 = 0.06$. The GE component is therefore not explicitly considered in our interpretation.

3) Interest payments (IP)

IP represents the effect of the interest rate on the debt ratio, revealing a connection with the interest rate criterion. Compared to the link between IE and the inflation criterion, this connection is less strong since the relevant interest rate measures differ considerably. The interest rate used to calculate the actual interest payments, i_i in the third term of (2), differs from the long-term interest rate as defined in the Treaty.⁷ The former represents the *average* rate of interest with respect to the total amount of outstanding debt (also termed the effective interest rate) while the latter is the interest rate on new debt and hence represents a *marginal* interest rate on government debt. Depending on the term structure of the stock of debt, relatively large discrepancies between the two interest rates may occur. Therefore, component IP will not be used in assessing whether the change in the debt ratio is compatible with the interest rate criterion.

4) Primary deficit (PD)

The Treaty defines the deficit criterion as "whether the ratio of the planned or actual government deficit to GDP exceeds 3%".⁸ In our decomposition framework, reference is to the primary deficit, which corresponds to the definition of the deficit stated in the Treaty excluding interest payments (IP) on debt. Therefore the equivalent reference value for the primary deficit would equal 3%-IP. This implies that if the interest payments are greater than 3% GDP (this is the case for all member states except Luxemburg), the reference value becomes negative, signalling that a primary surplus is required to meet the deficit criterion.

Although it is possible to proceed in this way, the concept of a reference value for the primary deficit is not very illuminating. Moreover, it would make the reference value dependent upon IP, which is not a constant. Therefore, PD and IP are assessed jointly through direct relation with the well-known 3% deficit criterion. Hence, the deficit criterion $(d \le 3)$ is used in a direct way in our interpretation.

5) Stock-flow adjustment (SF)

SF does not directly relate to any of the other convergence criteria and is therefore not treated specifically here. A possible conver-

⁶ An alternative approach would be to consider the GDP-deflator as the relevant measure of inflation. This could, however, lead to discrepancies between our interpretation and the official interpretation of the inflation criterion.

⁷ The reference value for the interest rate can be calculated much like the reference value for inflation: the average long-term interest rate of the three best performing member states in terms of price stability plus two percentage points.

⁸ In addition, two exceptions have been formulated: either the ratio has declined substantially and continuously and reached a level that comes close to the reference value; or, alternatively, the excess over 3% is only exceptional and temporary and the ratio remains close to the reference value.

gence criterion would be the exchange rate criterion since in countries with debt issued in foreign currency of EU member states, the effect of devaluation emerges in this component. However, given the difficulty of interpretation, as argued in Section 1, we do not consider the exchange rate criterion.

The above assignment of convergence criteria to debt components enables us to specify the first requirement of our interpretation:

Requirement 1 (Compliance with other criteria)

The change of the debt ratio is in compliance with the other convergence criteria if $IE \ge IE^*$ and $d \le 3$.

Hence, a country meets Requirement 1 (R1) if the inflation component of the change in debt is compatible with the inflation criterion and the deficit does not exceed 3% of GDP. The corresponding change in the debt ratio is denoted by Δs_i^* ($\Delta s_t^* = \Delta s_t$ if R1 is met; otherwise Δs_i^* is calculated as the change in the debt ratio that would occur if IE_t = IE_t* and d_t = 3). This naturally raises the question: Which policy changes are needed if a country fails to meet R1?

In that case a country has to lower the rate of inflation (i.e. increase IE) by exerting a disinflation effort E_{di} such that $IE_t \ge IE_t^*$, and/or a fiscal effort (denoted by E_{f1}) such that $D_t \le 3$. Calculation of E_{di} is straightforward: E_{di} is defined as the rate of inflation, measured by the CPI, in excess of the reference value ($E_{di} = CPI - CPI^*$; $E_{di} = 0$ if CPI $\le CPI^*$). Calculation of E_{f1} comes down to determining the excess deficit when $E_{di} = 0$ ($E_{f1} = d_t - 3$; $E_{f1} = 0$ if $d_t \le 3$). If $E_{di} = 0$ and $E_{f1} = 0$, it is said that R1 is met *unconditionally* (otherwise R1 is met conditionally upon E_{di} and E_{f1}).

3.2. The 'sufficiently diminishing' requirement

Having established the links between the two relevant debt components and the requirement that these debt components be in line with the relevant convergence criteria, we can now go on to present our definition of sufficiently diminishing debt: Requirement 2 (sufficiently diminishing)

The change in the government debt ratio of a country in year t is sufficiently diminishing if $\Delta s_t < 0$ and $\Delta s_t^* \leq \sigma$ (σ is a negative threshold parameter).⁹

Hence, a change in the debt ratio is defined as sufficiently diminishing if it is in fact declining ($\Delta s_i < 0$) and if the decline in the debt ratio compatible with R1 (Δs_i) is smaller than or equal to a prespecified parameter, σ . The actual value for σ should be constant over time and country independent, but is otherwise to some extent arbitrary. In Section 4, we set σ to -0.5 indicating that for a country to receive the 'sufficiently diminishing' predicate in our interpretation, the debt compatible with low inflation and a low deficit should be declining by a half percentage point or more in one year. Should a country fail to meet R2, the following question is relevant: Which policy changes are needed if a country fails to meet R2?

In answering this question, it is assumed that R1 is met (either conditionally or unconditionally). The fiscal effort E_p needed to meet R2, and additional to E_{fl} if $E_{f1} > 0$, can be located in two debt components: D and SF. This implies that reducing the deficit or lowering the stock-flow adjustment (through e.g. proceeds from privatisation) can be used to meet R2. How big should E_p be? We assume that the time scope of R2 is a single year. From the definition of sufficiently diminishing above, it follows that E_p is either equal to the highest of the following two expressions Δs_t (if $\Delta s_t > 0$) or $\Delta s_t^* - \sigma$ (if $\Delta s_t^* > \sigma$) or is equal to zero otherwise.¹⁰

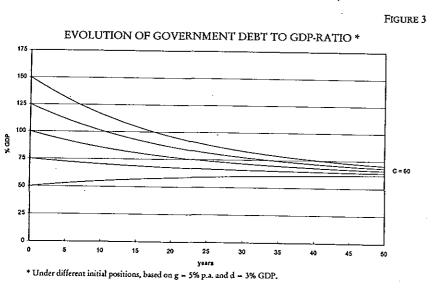
3.3. The requirement of 'satisfactory pace'

The other requirement of the debt criterion links the change in government debt to the famous 60% GDP reference value: is the debt ratio approaching that reference value at a *satisfactory pace*? A simple

⁹ In theory, it is possible to consider a longer time-span than a single year in our interpretation since the concepts used in Requirement 2, especially the debt decomposition, can be generalized to a multi-year period (see Appendix, Section 2). In practice, on only one occasion, the Ecofin Council, has a longer time-span been taken into account (see Section 4.1).

¹⁰ More precisely, E_{f_2} should be infinitesimally larger i.e. $E_{f_2} = \varepsilon + \max(\Delta s_t, \Delta s_t^* - \sigma, 0)$ (with ε an infinitesimal > 0).

and straightforward interpretation could be to specify a number of years in which the debt should be reduced to 60% GDP. However, such an approach would neglect the essential non-linear nature of debt evolution because the time needed to reach the reference value obviously depends on the initial level of debt. Given the same values for deficit and nominal GDP-growth, the speed of convergence is larger for higher debt-levels than for lower debt-levels, and the evolution of debt thus proceeds in a non-linear way. This is apparent from Figure 3 in which several initial debt positions are drawn. Starting from 150% GDP, it takes 7 years to obtain a reduction of 25 percentage points while it takes approximately 10 years to reduce the ratio another 25 percentage points and about 23 years to go from 100% GDP to 75% GDP. These calculations are based on a deficit of 3% GDP and an annual nominal GDP-growth of 5% (but the qualitative results do not depend upon the particular values chosen).



The two essential parameters underlying Figure 3 are the longterm value on which the debt ultimately converges (denoted by C) and the time it takes to reach the reference value of 60% GDP (denoted by τ). The calculations for C and τ are presented in the Box. From the discussion in the Box, it follows that for a country to exhibit a debt ratio approaching 60% GDP at a satisfactory pace, C should be smaller than 60% GDP. This leaves the issue of how much smaller C should be, which is resolved by choosing suitable values for d and g (on which C depends) in such a way that the time to reach 60% GDP (t) is not 'too long'. Fifty years seem a plausible maximum time-span for the most highly indebted EU countries (which have actual debt ratios in the order of 125% GDP). To investigate this further, nine alternative pairs of d and g are presented in Table 1 in a systematic way. Table 1 (first row) reads as follows: if the deficit is 3% GDP and the nominal rate of GDP growth is 6% per annum, it takes 40 years to reach 60% GDP if the initial debt ratio equals 125% GDP. Since we are interested in a rule for the *minimal* required decline of debt still considered approaching the reference value at a satisfactory pace, low values for d are left out of Table 1. Likewise, high values for g are left out because such rates are in general not compatible with low inflation.

Which of these alternatives provides a reasonable rule for the minimal required decline? Given that there is room for interpretation in the Treaty, it necessarily follows that every choice is to some degree arbitrary. However, the following considerations could play a role in deciding among the alternatives. Firstly, the reference value of 60% GDP should at least be attainable. This rules out the four alternatives in Table 1 in which the theoretical long-term value of the debt ratio (C) exceeds 60% GDP.

TABLE 1

HOW MANY YEARS TO 60% GDP?

d (%GDP)	g (% p.a.)	C (% GDP)	75% GDP	100% GDP	125% GDP
3	6	53.0	20	33	40
	5	63.0	N ⁶	N	N
	4	78.0	N	N	N
2.75	6	48.6	14	26	33
	5	57.8	42	60	70
	4	71.5	N	N	N
2.5	6	44.2	11	22	28
	5	52.5	23	38	46
	4	65.0	N	. N	N
2.25	6	39.8	10	19	25
	5	47.3	16	29	37
	4	58.5	61	85	97
2	6	35.3	8	17	22
	5	42.0	12	24	31
	4	52.0	27	46	56

* Rounded figures. Under the assumption of zero stock-flow adjustment and constant deficit (d) and nominal GDP growth (g) figures.

^b N = Never, i.e. C > 60% GDP

Secondly, the objective of price stability in the Monetary Union and the inflation convergence criterion implies that the rule should not be based on a relatively high level of nominal growth. In addition, a value of 5% is more or less compatible with the reference values for debt and deficit (see the Box) and is close to the average EU nominal growth in recent years. Taking 5% as the 'reference value' for parameter g leaves two alternatives in Table 1. Given also the constraint that the time-span should not be 'too long', we are left with the alternative in Table 1 (in italics) in which g = 5% and d = 2.5% GDP. The debt curve corresponding to those values for g and d will be termed as the 'reference curve' and the corresponding 'reference time' needed to reduce the debt ratio to 60% GDP along that curve will be denoted by τ_{nf} is the maximum time-span in which a decline of the debt ratio to 60% GDP is still considered 'a satisfactory pace'. Likewise, τ is the actual time-span needed to reach 60% GDP keeping the actual values of g and d constant.

Note that the implication of the above considerations is that even a *minimum* decline in the debt ratio implies a deficit *below* 3% GDP (assuming that g = 5%). Hence, in the NRI, an almost necessary condition for meeting the second part of the debt criterion is a deficit below 3% GDP.

We are now in a position to define the 'satisfactory pace' requirement:

Requirement 3 (satisfactory pace)

The change in the government debt ratio of a country in year t is approaching the reference value at a satisfactory pace if $\tau \leq \tau_{ref}$ where τ_{ref} is the time needed to reach 60% GDP along the debt evolution curve with parameters d = 2.5% and g = 5%.

Should the debt ratio of a country fail to meet R3, it is possible – using the equations in the Box (see also Appendix, Section 3) – to calculate the fiscal effort E_{f3} (additional to E_{f1} and E_{f2}) which reduces τ such that it becomes compatible with R3.

3.4. Recapitulation of the NRI procedure

In Figure 4, we present a flow chart of the complete procedure with respect to the non-redundant interpretation. The procedure Box

THE ARITHMETIC OF DEBT EVOLUTION

Under the three assumptions of a constant deficit (d), a constant nominal GDP growth (g) and zero stock-flow adjustment, the debt converges asymptotically towards a constant, C. It can be shown that C = [(1+g)d)/g] (see Appendix, Section 3). Hence, C is only dependent upon parameters d and g (which are assumed to be constant). This relationship can be employed to argue that to a certain extent the two reference values for deficit and debt in the Treaty imply a "reference value" for nominal GDP growth of approximately 5% per annum. This can be seen by substitution of the reference values (d = 3 and C = 60) in the expression for C which yields the 'reference' nominal GDP growth g of 1/19 \approx 5%.

It is also possible to derive an expression for calculating the time, τ (expressed in number of years), which is needed to reach a certain debt level in year $t(s_t)$ starting from an initial level, s_t (see Appendix, Section 3 for details):

$$\tau = [\ln(1+g)]^{-1} \ln\left(\frac{s_1 - C}{s_t - C}\right)$$
(3)

This equation shows that τ , the time needed to reduce the debt ratio from s_i to s_i , is only defined if C is not between s_i and s_i (the numerator and denominator must agree on signs, otherwise the ratio becomes negative). Secondly, it states that $s_i \rightarrow C$ implies that $\tau \rightarrow \infty$ which indicates that since C is an asymptotic value, s_i can never reach C in finite time. This, in turn, implies that the evolution towards s_i at a satisfactory pace is possible only if C is sufficiently below s_i . Given the fact that the Treaty specifies $s_i = 60\%$, it follows that C must be sufficiently below 60% GDP in order to approach 60% GDP at a satisfactory pace. Combining that constraint with the "reference value" for g (5%), it follows from the above expression for C that d must be at least below 2.85% GDP. Hence, the requirement that 60% GDP should at least be attainable in finite time (which seems to be the most relaxed interpretation of "approaching at a satisfactory pace" possible), implies a deficit lower than 3% GDP.

starts by examining whether gross government debt and deficit are respectively equal to or below 60% and 3% of GDP (for the sake of completeness called Requirement in Figure 4). There are two terminal points in the procedure:

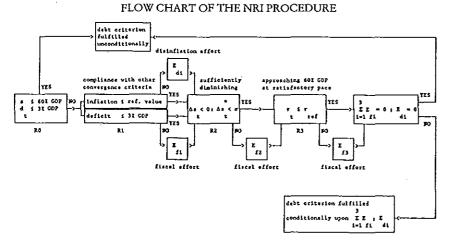
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1) the debt criterion is fulfilled unconditionally. This point can be reached in two ways. Directly via R0 or indirectly if the total effort sums to zero;

2) the debt criterion is fulfilled conditionally. In this case the total effort sums to a positive value indicating that an additional fiscal effort E_{fi} and/or a disinflation effort E_{di} is required.

Furthermore, Figure 4 shows successive determination of requirements R1 through R3. If a requirement is not met, the necessary effort is calculated and the procedure continues under the condition that the necessary effort has been implemented. Therefore, it should be noted that the fiscal effort E_{β} is additional to E_{β} which in turn is additional to E_{β} .

FIGURE 4



The NRI procedure takes two parameters, σ and τ_{ref} which correspond to R2 and R3 respectively and must be specified beforehand: σ denotes the (negative) threshold value below which a decline of the debt ratio (in one year) is considered as "sufficiently diminishing" and τ_{ref} denotes the number of years in which the gap between the current value of the debt ratio and 60% GDP should be closed in order to "approach the reference value at a satisfactory pace". Calculation of τ_{ref} , as explained in Section 3.3, depends on parameters d and g. We denote τ_{ref} (d, g) to refer to specific instances of this parameter such as in the case described above τ_{ref} (2.5, 5). The degrees of freedom for specification of the parameters reflect the subjective element in the non-redundant interpretation. As stated in Section 1, every interpretation of the EMU debt criterion remains to some extent subjective. However, by parametrizing the non-redundant interpretation, we are to a large extent able to indicate in a transparent way how the subjective parts of the EMU-debt criterion are handled.

4. Application of the NRI to the years 1994-1997

4.1. Analyzing the excessive deficit procedure 1994-1996

Since ratification of the Treaty in November 1993, the excessive deficit procedure has been completed three times in full. The first time (1994), the Ecofin Council decided that only two (out of the then twelve) EU countries did not have an excessive deficit. From those two countries, Luxembourg and Ireland, only the latter provides a positive test case for our non-redundant interpretation, i.e. a country without an excessive deficit predicate (according to the Ecofin Council) while the deficit is below the reference value and the debt above the reference value.¹¹ The second time (1995), Germany was the only country which, according to the Ecofin Council, had reversed its status with respect to the excessive deficit procedure. However since both deficit and debt in Germany were below the respective reference values, this did not provide us with an additional test case. The third time (1996), Denmark joined the group of countries without an excessive deficit, providing us with an additional test case, while Germany did receive the predicate "excessive deficit" because the deficit exceeded the reference value. In the 1994-96 period no negative test cases emerged, i.e. a country with an excessive deficit predicate (according to the Ecofin Council) while both the deficit and the debt are below the respective reference values. Hence, from experience with excessive deficit procedure so far, only four test cases remain: Ireland in 1994, 1995 and 1996, and Denmark in 1996.

¹¹ In Luxembourg both deficit and debt were below the respective reference values in all three years. In Ireland, the deficit was below and the debt above the respective reference values in all three years, providing us with three positive test cases.

The results of applying the NRI procedure to these cases are presented in Table 2. The NRI procedure is able to reproduce the outcomes of the excessive deficit procedure except for the first case (Ireland in 1994). Although the Irish debt ratio in 1993 was rising (which violates R2 in our interpretation), the Ecofin Council decided that over the longer time-span 1987-94 the decline in the debt ratio was satisfactory. Under our non-redundant interpretation, the longer term is considered under R3 (which produces the correct result, Ireland would reach 60% of GDP in only eleven years while $\tau_{ref} = 37$ years).

TABLE 2

TEST CASES FOR THE NRI FROM THE EXCESSIVE DEFICIT PROCEDURE

	Δs _{t-1} *	Compatible with R1	Compatible with R2	τ ^b	τ _{ref}	Compatible with R3	Criterion met under NRI
IRL '94	4.5	yes	no	11	37	yes	no
IRL '95	-7.2	yes	yes	9	33	yes	yes
IRL '96	-5.6	yes	yes	8	30	yes	yes
DK '96	-4.1	yes	yes	6	19	yes	yes

The excessive deficit procedure completed in year t takes primarily into account the change in the debt ratio (measured in % GDP) in year t-1. The figures in this Table are based on the official data which were available to the Ecofin Council at the time of applying the excessive deficit procedure (Commission's Spring Forecasts 1994, 1995, 1996).

r and rare measured in number of years. Calculated on the basis of equation A.19 in the Appendix.

4.2. The crucial year 1997

The composition of the group of countries to start Monetary Union will be determined in early 1998, based on the then actual, reliable 1997 data. Here, the concepts developed in the previous Sections are applied to the currently available, preliminary 1997 data of the Commission, published in the Autumn 1996 forecast. In Table 3 we have listed the results for each EU member state with respect to the fiscal data. From Table 3 it can be concluded that five countries (Denmark, France, Ireland, Luxembourg and Portugal) meet both fiscal criteria unconditionally. For three countries (Denmark, Ireland and Portugal) it was necessary to use the non-redundant interpreta-

tion. According to the NRI, in addition to the present situation for Denmark and Ireland, also Portugal shows a decline of the debt ratio compatible with the EMU debt criterion.

TABLE 3

WHICH COUNTRIES MEET THE FISCAL CRITERIA IN 1997 UNCONDITIONALLY?"

	Debt < 60 (R0)	Compliance other criterion (R1)	Sufficiently diminishing (R2)	Approaching at satisfactory pace (R3)	Criteria met
В	no	yes	yes	по	no
DK	no	yes	yes	yes	yes
D	no	yes	no	-	no
GR	no	no	- `	-	по
E	no	yes	yes	no	no
F	yes	yes	-	· -	yes
IRL	по	yes	yes	yes	yes
I	no	по	-	_ ·	no
L	yes	yes	-	-	yes
NL	no	yes	yes	no	по
Α	no	yes	no .	-	no
Р	no	yes	yes	yes	yes
SF	no	yes	no	-	no
s	no	yes	yes	no	no
UK	yes	no	-	-	no

* Both fiscal criteria are fulfilled unconditionally if R0 and R1 are met unconditionally or R1, R2 and R3 are met unconditionally.

^b Based on $\tau_{\tau} = -0.5$ and σ (2.5, 5). ^c - = not necessary to determine.

The ten EU countries not meeting the fiscal criteria under the NRI will need to undertake a (further) fiscal effort. In Table 4, the necessary fiscal effort is decomposed into three parts. In the column E_{ij} the necessary deficit reduction is shown; only three countries have at present planned deficits for 1997 exceeding 3% of GDP. The two columns E_{ρ} and E_{ρ} show the additional necessary effort in order to meet R2 and R3 successively. The relatively high effort for Germany (1.6% points) results from the fact that the debt ratio is in 1997 above 60% GDP (albeit slightly) and increasing by 1.1% point. And, in

TABLE 4

REQUIRED STRUCTURAL EFFORT TO MEET THE FISCAL CRITERIA	
in % of GDP	

B 0.0 0.0 1.0 1.0 DK $-^d$ - - - D 0.0 1.6 0.0 1.6 GR* 3.5 0.0 0.0 3.5 E 0.0 0.0 0.1 0.1 F - - - - IRL - - - - I 0.3 0.0 1.0 1.3 L - - - - NL 0.0 0.0 0.2 0.2 A 0.0 1.0 0.5 1.5 P - - - - SF 0.0 0.7 0.0 0.7			III /0 01 01/1		
DK $-d$ $ -$ D0.01.60.01.6GR*3.50.00.0.3.5E0.00.00.10.1F $ -$ IRL $ -$ I0.30.01.01.3L $ -$ NL0.00.00.20.2A0.01.00.51.5P $ -$ SF0.00.70.0S0.00.00.7		E _{f1} ⁴	Ef2 ^b	Efis	Total effort
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	В	0.0	0.0	1.0	1.0
GR* 3.5 0.0 0.0 3.5 E 0.0 0.0 0.1 0.1 F - - - - IRL - - - - I 0.3 0.0 1.0 1.3 L - - - - NL 0.0 0.0 0.2 0.2 A 0.0 1.0 0.5 1.5 P - - - - SF 0.0 0.7 0.0 0.7	DK	_ª	-	-	-
E 0.0 0.0 0.1 0.1 F - - - - IRL - - - - I 0.3 0.0 1.0 1.3 L - - - - NL 0.0 0.0 0.2 0.2 A 0.0 1.0 0.5 1.5 P - - - SF 0.0 0.7 0.0 0.7	D	0.0	1.6	0.0	1.6
F - - - - IRL - - - - I 0.3 0.0 1.0 1.3 L - - - - NL 0.0 0.0 0.2 0.2 A 0.0 1.0 0.5 1.5 P - - - SF 0.0 0.7 0.0 0.7	GR*	3.5	0.0	0.0	. 3.5
IRL - - - - I 0.3 0.0 1.0 1.3 L - - - NL 0.0 0.0 0.2 0.2 A 0.0 1.0 0.5 1.5 P - - - - SF 0.0 0.7 0.0 0.7	E	0.0	0.0	0.1	0.1
I 0.3 0.0 1.0 1.3 L - - - - NL 0.0 0.0 0.2 0.2 A 0.0 1.0 0.5 1.5 P - - - - SF 0.0 0.7 0.0 0.7 S 0.0 0.0 0.7 0.7	F	-	-	-	_
L NL 0.0 0.0 0.2 0.2 A 0.0 1.0 0.5 1.5 P SF 0.0 0.7 0.0 0.7 S 0.0 0.0 0.7 0.7	IRL	-	-	-	-
NL 0.0 0.0 0.2 0.2 A 0.0 1.0 0.5 1.5 P - - - - SF 0.0 0.7 0.0 0.7 S 0.0 0.0 0.7 0.7	I	0.3	0.0	1.0	1.3
A 0.0 1.0 0.5 1.5 P - - - - SF 0.0 0.7 0.0 0.7 S 0.0 0.0 0.7 0.7	L	-	-	-	-
P SF 0.0 0.7 0.0 0.7 S 0.0 0.0 0.7 0.7	NL	0.0	0.0	0.2	0.2
SF 0.0 0.7 0.0 0.7 S 0.0 0.0 0.7 0.7	А	0.0	1.0	0.5	1.5
S 0.0 0.0 0.7 0.7	Р	-	-	-	-
	SF	0.0	0.7	0.0	0.7
UK 0.5 0.0 0.0 0.5	S	0.0	0.0	0.7	0.7
	UK	0.5	0.0	0.0	0.5

Source: calculations based on equation (A.19) and (A.21) in the Appendix.

Required fiscal effort to meet deficit criterion.

Additional effort to 'diminish sufficiently'.

Additional effort to 'approach at a satisfactory pace'. - = not necessary to determine.

In addition, a disinflation effort is necessary.

order to meet R2 the debt ratio has to diminish by σ % points ($\sigma = -0.5$). Also in Austria the necessary effort is relatively high (1.5%) points) due to the combined result of a rising debt ratio in 1997 and the, on average, relatively low real GDP growth.

In Table 5, the results are presented if EU countries were to implement the necessary fiscal measures as pointed out in Table 4. It is assumed that in 1997 half of the fiscal measures relating to E_{R} and E_{B} are implemented in a non-recurrent way. The structural level would be attained from 1998 onwards. In the left panel of Table 5 the planned deficit in 1997 (d '97), the necessary deficit under NRI in 1997 (d-nri '97) and the structurally needed deficit under NRI (d-nri > '97) are shown. In the right panel of Table 5 the debt ratio in 1996 and 1997 is shown, and the debt ratio in 1997 which would prevail if each country implemented the necessary fiscal measures.

The EMU Debt Criterion: An Interpretation 525 From the values of the structural deficit needed, as shown in Table 5, it is clear that for most countries the deficit should be well under 3% of GDP. For five countries the structural deficit is close or equal to 3% due to the fact that either the debt ratio is (still) under 60% of GDP or nominal GDP growth is exceeding 5% per annum (while not exceeding the reference value for inflation). The above mentioned fiscal consolidation should be taken as the minimum since no buffer has been included for the effects of a cyclical downturn or other financial setbacks. These measures can therefore be seen as a first step towards the medium-term goal of deficits close to balance or surplus as set out in the Stability and Growth Pact.

TABLE 5

THE IMPACT OF THE NRI ON DEFICIT AND DEBT in % of GDP

	d '97	d-nri '97	d-nri* > '97	s '96	s '97	s-nri '97
В	2.9	2.4	1.9	130.6	127.0	126.0
DK	0.3	0.3	0.3	70.2	67.8	67.8
D	2.9	2.1	1.3	60.8	61.9	60.3
GR	6.5	3.0	3.0	110.6	109.3	105.8
Е	3.0	2.9	2.9	67.8	67.1	67.0
F	3.0	3.0	3.0	56.4	58.1	58.1
IRL	0.9	0.9	0.9	74.7	70.0	70.0
I	3.3	2.5	2.0	123.4	122.3	121.0
L	-0.5	-0.5	-0.5	7.8	8.8	8.8
NL	2.5	2.4	2.3	78.7	76.8	76.6
Α	3.0	2.2	1.5	71.7	72.2	70.7
Р	2.9	2.9	2.9	71.1	69.0	69.0
SF	2.2	1.9	1.5	61.3	61.5	60.8
S	2.9	2.5	2.2	78.1	77.6	76.9
UK	3.5	3.0	3.0	56.2	57.0	56.5

^a Reported value is equal to the minimum of the structural deficit needed and the planned deficit in 1997.

5. Conclusions

In this paper a procedure has been described that can be used to assess whether EU member states satisfy the EMU debt criterion. The theoretical properties of that procedure (called a non-redundant interpretation) can be summarized as follows:

- the procedure takes into account the extent to which the other convergence criteria are fulfilled;

- the procedure is parametrized with respect to the "sufficiently diminishing" requirement and with respect to "approaching the reference value at a satisfactory pace". In addition to a few assumptions, the two corresponding parameters quantify to a large extent the amount of judgement needed to assess whether the EMU debt criterion is fulfilled;

- the procedure is country-independent and can be applied before and after the start of the third stage of EMU.

On the practical side, it is shown that the NRI-procedure is able to mechanically reproduce to a large extent the results of the excessive deficit procedure in the years 1994 through 1996. Application of the NRI-procedure to the latest available Commission data for the crucial year 1997 reveals that five countries unconditionally satisfy the EMU deficit and debt criteria in 1997 under the suggested non-redundant interpretation. Most other countries will need to implement fiscal measures of about 1% of GDP on average.

MATHEMATICAL APPENDIX

This Appendix contains the mathematical background to the concepts in the main text regarding the decomposition of the change in government debt over one or more periods (Sections 1 and 2) and the behaviour of variables in first-order difference equations (Section 3).

1. Decomposition of the annual change in government debt

The following equations are accounting identities:

I

$$S_t \equiv S_{t-1} + D_t + SF_t \tag{A.1}$$

$$D_t \equiv PD_t + IP_t \tag{A.2}$$

where:

D : government deficit;

g : annual growth rate of nominal GDP (= $GDP_t/GDP_{t-1} - 1$)

i : effective interest rate on the stock of government debt (%);

PD: primary government deficit;

py : annual growth rate GDP-deflator;

IP : actual interest payments on government debt;

S : stock of government debt (nominal value at the end of the year);

SF : stock-flow adjustment (and other statistical items);

y : annual growth rate of real GDP;

Subscripts t refer to time.

This representation is independent of the delimitation of the government sector. However, the definition of the government sector used in the main text refers to "general government" as defined in Article 2 of the Protocol on the excessive deficit procedure annexed to the Treaty.

Combination of (A.1) and (A.2) gives:

$$S_t = S_{t-1} + IP_t + PD_t + SF_t$$
(A.3)

Dividing (A.3) by nominal GDP gives an expression for the debt ratio (variables in lower case are expressed as a ratio to GDP):

$$s_t = \frac{1}{1 + g_t} s_{t-1} + ip_t + pd_t + sf_t$$
 (A.4)

The change in the debt ratio can be written as:

$$\Delta s_{t} = - \frac{g_{t}}{1 + g_{t}} s_{t-1} + ip_{t} + pd_{t} + sf_{t}$$
 (A.5)

The first term on the right-hand side represents the denominator effect and indicates the change in the debt ratio attributable to the growth of nominal GDP. This can be decomposed into a real component and an inflation component assuming the second-order term in $1 + g_t = 1 + y_t + py_t + y_t \cdot py_t$ is approximately zero $(y_t .py_t \approx 0)$:

$$\Delta s_{t} = -\frac{y_{t}}{1+g_{t}} s_{t-1} - \frac{py_{t}}{1+g_{t}} s_{t-1} + ip_{t} + pd_{t} + sf_{t}$$
(A.6)

The interest payments IP, depend on the stock of debt at the end of the previous year and the average rate of interest on that stock of debt:

$$IP_t = i_t S_{t-1}$$

or, expressed as a ratio to GDP:

$$ip_t = \frac{i_t}{1+g_t} s_{t-1}$$
 (A.7)

Substitution of (A.7) into (A.6) yields equation (1) in the main text:

$$\Delta s_{t} = -\frac{y_{t}}{1+g_{t}}s_{t-1} - \frac{py_{t}}{1+g_{t}}s_{t-1} + \frac{\dot{i}_{t}}{1+g_{t}}s_{t-1} + pd_{t} + sf_{t}$$
(A.8)

2. Debt decomposition over n periods

The decomposition shown above can be generalised to n periods (n > 0; with index running from t - n to t). Simplify (A.4) by letting $f_t \equiv r_t + pd_t + sf_t$ and $a_t \equiv 1/(1+g_t)$ such that:

$$\mathbf{s}_{t} = \mathbf{a}_{t} \, \mathbf{s}_{t-1} + \mathbf{f}_{t} \tag{A.9}$$

Subsequently, from (A.9) the change in government debt over two periods, denoted by $\Delta_2 s_t (\Delta_n s_t \equiv s_t - s_{t-n})$, it follows that:

$$\Delta_2 s_t = a_t s_{t-1} + f_t - s_{t-2} \tag{A.10}$$

Substitution of s_{t-1} into (A.10) by the one-period lagged version of (A.9) yields:

$$\Delta_2 s_t = a_t \left(a_{t-1} s_{t-2} + f_{t-1} \right) + f_t - s_{t-2} = \left(a_t a_{t-1} - 1 \right) s_{t-2} + a_t f_{t-1} + f_t \qquad (A.11)$$

Analogous for $\Delta_3 s_t$:

$$\begin{array}{l} \Delta_3 \ s_t = a_t \ s_{t-1} + \ f_t - s_{t-3} \\ = \ a_t \ (a_{t-1} \ s_{t-2} + \ f_{t-1}) + \ f_t - s_{t-3} \\ = \ a_t \ (a_{t-1} \ (a_{t-2} \ s_{t-3} + \ f_{t-2}) + \ f_{t-1}) + \ f_t - s_{t-3} \\ = \ (a_t \ a_{t-1} \ a_{t-2} - 1) \ s_{t-3} + \ a_t \ a_{t-1} \ f_{t-2} + \ a_t \ f_{t-1} + \ f_t \end{array}$$

The general case ($\Delta_n s_t$; n > 0) can be derived by repeated substitution of lagged versions of (A.9):

$$\Delta_{n}s_{t} = \left(\prod_{i=0}^{n-1} a_{t-i} - 1\right)s_{t-n} + \sum_{i=1}^{n-1} \left(\prod_{j=0}^{n-i-1} a_{t-j}\right)f_{t-n+i} + f_{t}$$
(A.12)

3. First-order difference equations

The behaviour over time of government debt can be described using equation (A.9) which is a first-order difference equation. Under the assumption that for $t = 1, 2, ..., sf_t = 0$, it follows that:

$$s_{t+1} = \frac{1}{1 + g_{t+1}} s_t + d_{t+1}$$
 (A.13)

Solving (A.13) for s_t

In the following, it is shown how (A.13) can be solved for s_{t-1} . Following a standard textbook on difference equations (e.g. Mickens 1987, pp. 46-48), let x_i denote a variable in period t and let p_i and q_i be time-varying coefficients; then a first-order difference equation consists of the following general form:

$$x_{k+1} - p_k x_k = q_k$$
 (A.14)

and its general solution equals

$$\mathbf{x}_{k} = \mathbf{A} \prod_{i=1}^{k-1} \mathbf{p}i + \left(\prod_{i=1}^{k-1} \mathbf{p}i\right) \sum_{i=1}^{k-1} \left(\frac{\mathbf{q}_{i}}{\prod_{r=1} \mathbf{p}r}\right)$$
(A.15)

where A is an arbitrary constant. In general, (A.15) can be used to solve for x_t . Our case, equation (A.13), is a simple instance of (A.14) with x_{k+1} equivalent to s_{t+1} , p_k equivalent to $(1+g_{t+1})$ and q_k equivalent to d_{t+1} . Furthermore, in the main text the simulations are based on constant values for deficit d and growth g, so that the general solution (A.15) in our case reduces to:

$$S_t = A(1+g)^{-t+1} + (1+g)^{-t+1} \sum_{i=1}^{t-1} \left(\frac{d}{(1+g)^{-1}} \right)$$

Furthermore, using the fact that for any r, the equation for the finite

sum
$$\sum_{i=1}^{t-1} r^{i}$$
 equals $\frac{r-r^{t}}{1-r}$, it follows that:

$$s_{k} = A(1+g)^{-t+1} + (1+g)^{-t+1} d \frac{1-(1+g)^{t-1}}{(1+g)^{-1}-1}$$
Since $\frac{1}{(1+g)^{-1}-1} = -\frac{(1+g)}{g}$,

$$s_{k} = A(1+g)^{-t+1} - \frac{(1+g)d}{g}(1+g)^{-t+1} + \frac{(1+g)d}{g}$$
Let $C = \frac{(1+g)d}{g}$ then the solution for s_{t} is as follows:

 $s_t = (A - C)(1 + g)^{-t+1} + C$ (A.16)

From (A.16) it follows that if $t \to \infty$, $s \to C$. C can be named the convergence value of s_n , i.e. s_i approaches C asymptotically under the earlier stated assumptions of zero stock-flow adjustment, constant deficit d and constant growth g. Another observation from (A.16) is that if we choose d = 3% GDP and C = 60% GDP (the reference values for the EMU-debt and deficit criteria), it follows that the compatible value for the annual nominal GDP growth is $g = 1/19 \approx 5\%$ GDP. The consistency of the budgetary EMU-

criteria with an annual nominal GDP of *exactly* 5% holds if the analysis is carried out in continuous time rather than in discrete time. In principle the latter is in this case more appropriate since debt is measured at non-infinitesimal intervals.

Solving (A.16) for t-1: the number of years between s_1 and s_2

Equation (A.16) can also be used to solve for t-1 (the number of years between s_1 and s_1). First solve (A.16) for A when t = 1:

$$s_1 = (A - C)(1 + g)^{-1+1} + C$$
 hence $A = s_1$ (A.17)

Substitute (A.17) into (A.16) so,

$$s_t = (s_1 - C) (1 + g)^{-t+1} + C$$
 (A.18)

hence, $(1 + g)^{-t+1} = \frac{s_t - C}{s_t - C}$ or,

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$$t-1 = \frac{1}{\ln(1+g)} \ln(\frac{s_1-C}{s_t-C})$$
 (A.19)

Equation (A.19) solves for t-1 which can be used to answer questions when the debt will reach a certain level. Suppose the government debt equals at present 80% GDP ($s_i = 80$) and we want to know when the EMU-debt reference value will be attained ($s_i = 60$). In addition, assume d = 2 and g = 5. Then it follows that the convergence value C = (1.05)2/0.05 = 42% and t-1 = $[\ln(80 - 42) - \ln(60 - 42)]/\ln(1.05) = 15.3$ years. Hence under the stated assumptions, it takes a little more than 15 years to diminish the debt ratio from 80 to 60% GDP. Note that t-1 can only be determined if in (A.19) ($s_1 - C$)/($s_1 - C$) > 0. This reflects the fact that, because the convergence value C cannot be 'crossed', s can only be reached starting from s_1 if they are both located on the same side of C.

Solving (A.18) for d

In the following, it is shown how (A.18) can be used to solve for d. Let $d_{t-1} = d$ denote the constant deficit such that it is compatible with the change in the debt ratio from s_t to s_t in t-1 periods. From (A.18) we have

$$C = \frac{s_t - s_1 (1 + g)^{-t+1}}{1 - (1 + g)^{-t+1}}$$

Using the definition $C = \frac{(1+g)d}{g}$, we can write:

$$d = -\frac{g}{(1+g)} - \frac{s_i - s_1 (1+g)^{-i+1}}{1 - (1+g)^{-i+1}}$$

(A.20)

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For example, suppose the present debt ratio is 100% GDP and, assuming g = 5%, the goal is to reach the EMU-reference value in 40 years. Then from (A.20) it follows that the constant deficit compatible with that goal, d_{40} , equals:

$$d_{40} = \frac{0.05}{1.05} - \frac{60 - 100 (1.05)^{-40}}{1 - (1.05)^{-40}} = 2.54\% \text{ GDP}$$

Derivation of the required total fiscal effort

In the following, derivation of the required total fiscal effort (E_T) is shown. E_T is the minimal required increase in the fiscal balance necessary both to meet the reference value of the deficit criterion (E_{ref}) and approaching the reference value at a satisfactory pace (E_{sp}) . E_{ref} is obviously equal to the excess deficit over 3% GDP and hence can be calculated as MAX(d - 3, 0) where d is the actual value of the deficit. E_{sp} is the fiscal effort needed to reduce the debt to 60% GDP within a given time-span τ and hence equals MAX(d - d_t, 0). τ can be determined from (A.19) using the values of the 'reference curve' parameters $g_{ref} = 5\%$, $C_{ref} = 52,5$, $s_{ref} = 60\%$ GDP. Given actual values for g, d, s_1 and τ , E_{sp} can be calculated on the basis of (A.20):

$$d - d_{r} = d - \frac{g}{(1 + g)} \cdot \frac{s_{ref} - s_{1} (1 + g)^{-r}}{1 - (1 + g)^{-r}}$$
(A.21)

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