Italian Treasury Credit Certificates (CCTs): Theory, Practice and Quirks

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1. Introduction

CCTs were first introduced in 1977 to meet the need for more effective management of the public debt. They are a classic example of financial innovation implemented on an "experimental" basis in advance of theory.[1]

The CCT was born, and accepted in the market, as a *centaur*:[2] a short-term security as regards its yield and risk features, but with a long maturity. Investors are guaranteed a long-term contract that nonetheless offers a rate of return in line with short-term yields in the primary market for Italian Treasury bills. This has the advantage of providing automatic management of the capital invested (virtually a rollover of the reference Treasury bill).

When the bond is valued for trading purposes, its dual nature emerges again with reference to the components of income: the coupons, the value of which is determined in the primary market, and the price, which is fixed in the secondary market.

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[1] For a discussion of the role played by CCTs in Italy's debt management policy, see Salvemini (1992a).

[2] The metaphor was coined by Spaventa (1988).

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From the technical point of view, the indexation of the coupon has a stabilizing effect on the price of the security; the rationale (and the efficiency) of the stabilization mechanism is based on the uniqueness of the term structure of interest rates used in determining both the coupons and the price. The degree of stabilization is not always easy to measure owing to the complexity of the indexation mechanisms adopted. The market pricing of CCTs is affected not only by their technical features but also by market imperfections and uncertainty about factors other than interest rates (such as the credibility of the issuer, the exchange rate of the lira and a rather nebulous notion of illiquidity).

Initially it appeared possible to define variable rate securities as "banknotes carrying a coupon".[3] The monetary authorities themselves had indicated that CCTs would be "easily convertible into money" since indexation would reduce price fluctuations and thus limit the risk of capital losses. More than a technical diagnosis, the observation concerning price stability needs to be seen as a hypothesis, or hope, about the market. It was based on the assumption that operators would correctly assess the quantitative and qualitative features of CCTs, that there would not be significant misalignments between the primary and secondary markets and that illiquidity would not be a serious problem in the secondary market.

In practice, however, the behaviour of prices shows that the indexation mechanism was a poor immunizer: it resulted in CCTs appearing "subject at times to a risk of capital losses comparable to that on fixed rate securities".[4] Even recently it was noted that "in the secondary market the indexation of CCT coupons has not prevented the average price of these securities from varying in the same way as that of fixed rate securities".[5]

The "serious difficulties" CCTs have encountered cannot be used, however, to discredit the technical value of the indexation project; rather, they point to a failure on the part of operators to appreciate the complex financial significance of such securities and in general of the authorities' failure to manage the markets efficiently, both directly through issue policies and indirectly through their regulatory activity and the promotion of transparency.

[3] Banca Commerciale Italiana (1984), p. 4.
[4] Banca Commerciale Italiana (1988), p. 10.
[5] Banca d'Italia (1991), p. 237.

The Treasury Ministry has frequently appeared in two minds about the role to be played by variable rate securities in the management of the public debt. Ideas have changed with respect to the technical features that influence the quality of the indexation, either directly (the rules for calculating coupons, tax treatment) or indirectly (the auction techniques adopted for the reference Treasury bills), with the result that the market has had to cope with a great many different securities going under the name of Treasury credit certificates.[6]

In terms of techniques of financial analysis and portfolio management, the methodological approach to CCTs appears to be generally confused. Although stochastic models of the term structure of interest rates (falling within the so-called *new term-structure theory*) allow formulas to be developed for valuing variable rate securities that take account of their interest rate risk features, the recourse made to CCTs has still not been set within an officially recognized theoretical reference framework.

2. Market Models and Practice

The spread in the seventies of inherently risky instruments (variable rate securities and options, for which the cash flow that will be generated is not known) posed critical questions for traditional theory, based on the hypothesis of certainty, and gave rise to a golden decade of revolution in the methods of financial analysis on the basis of probability theory.[7]

[6] The Advisory Committee on the Public Debt judged the lack of uniformity of CCT issues to be one of the causes of their "crisis" (Ministero del Tesoro, 1989).

[7] According to Merton, the revolution that occurred in the seventies followed "the ... wave of financial innovations in capital markets". The sign of the break with tradition was "the mathematical tool of continuous probability *à la* Norbert Wiener and *à la* Kiyoshi Itô. Suddenly, what had been complex approximation became beautifully simple truth" (Samuelson). The effect of theory on markets has been truly notable in some cases (*e.g.* options pricing), as a result of a sort of natural link between formal techniques and economic significance, and of the ability (or lucky chance) of not confusing "rigor in the mathematical sense with rigor in the economic sense" (Merton). For the citations and further reading, see Merton (1990), Duffie (1992) and Samuelson's preface to Merton.

The new theory of the term structure of interest rates is one of the key products of this golden decade; it exploits all the key aspects of the new approach, ranging from stochastic methods of calculation to the underlying economic logic.

The new theory develops a formal analysis of the relationships between interest rates and states of the economy; it is built around the three basic constraints on asset prices: absence of arbitrage, single-agent optimality, and market equilibrium. Under this approach, CCTs can be classified as interest rate sensitive securities (irs) in the sense that their price depends "essentially" on the term structure of interest rates; they can also be classified as derivative securities since the flow of coupons they give rise to is a function of the interest rates observed or, from another point of view, because they can be replicated with portfolios of other securities. The valuation of CCTs therefore requires reference to be made to a market model: the prices of individual securities have to be determined within the overall system of prices, as a function of the state variables of the economy, and subject to consistency constraints (the arbitrage argument) to ensure that their replication does not permit riskless profits.[8]

For Italian Treasury bills (BOTs), bonds (BTPs), credit certificates (CCTs) and optable certificates (CTOs),[9] it is possible to derive valuation methods based on stochastic models of the market, in which the short-term interest rate is the only state variable. In line with theory, the prices of BTPs provide the empirical basis for the estimation of the typical parameters of the term structure. Once this has been obtained, the valuation formulas permit the analysis of CCTs and CTOs (as well as other derivatives such as options on BOTs, BTPs and CCTs and instruments derived from CCTs using stripping techniques).[10]

The model is thus not aimed at identifying the term structure, it is not just "a complicated method of curve fitting",[11] but is required

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[8] Even in the case of indexation based on parameters that are not comparable with the price (yield) of market securities, valuation still requires a specific model of the state variable and the price is obtained by exploiting the arbitrage argument; see Cox, Ingersoll and Ross (1980), pp. 394-5 for example. Similar considerations apply to the pricing of options, see Ingersoll (1987), pp. 381-3.

[9] BOTs, BTPs and CTOs are irs securities like CCTs, and CTOs can also be classified as derivate securities since they incorporate an option on a bond.

[10] See De Felice and Moriconi (1991b).

[11] Brown and Schaefer (1991), p. 14.

to serve as the (theoretical and empirical) basis for the comparative assessment of all the irs securities that are or could be traded.

For the purposes of empirical analysis, the single-factor Cox, Ingersoll and Ross (CIR) model appears to strike a good balance between the accuracy of the description of the market, mathematical tractability and ease of interpreting the results. Concepts such the structure of expected interest rates, the structure of liquidity premiums, and the measurement of interest rate risk, which are ambiguous in the language of traditional term-structure theory, take on a precise formal meaning and operational significance.[12]

In addition to the empirical implications, the definition of a model of the market plays an important heuristic role in the correct "identification" of the technical properties of contracts and hence in the choice of the appropriate management methods.[13]

Even without making specific hypotheses regarding changes in the state variable over time, the logical structure of the model shows that irs securities are subject to two types of interest rate risk, related to the uncertainty surrounding the value of the discount factor and the coupons. Consequently, the aim of the indexation mechanism is to achieve a negative correlation (autoimmunization) between the two components of risk. In practice, this cannot be perfect, so that the problem of measuring the two components of risk remains important.

On the other hand, the fact that the cash flow mechanism of irs contracts (maturities and/or coupons) is generally known, results in interest rate risk having an "inherently intertemporal" dimension ("an unexpected change in the interest rate now affects all future returns, so interest rate risk 'compounds' over time")[14] that reflects investors' preferences with regard to the allocation of resources over time and is difficult to determine in the case of stocks (since stock "risk is the contemporaneous resolution of returns").[15]

It follows that the management of portfolios consisting entirely of irs securities cannot use the methods typical of the stock market, based on diversification and the offsetting of risks. Under the tra-

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[12] See Cox, Ingersoll and Ross (1985).

[13] Identification of these properties is a major problem for all financial contracts that are "born without a theoretical father" and therefore (generally) seen in a distorted light by the market. The problem is recognized to be important and complex and affects the empirical response of products and the quality of the market.

[14] Ingersoll (1987), p. 404.

[15] Ingersoll (1987), p. 404.

ditional approach to portfolio selection theory (*à la* Markowitz), the risk associated with the uncertain value of stocks can be controlled by selecting a "sufficiently large" number of securities with yields that are not highly correlated (obviously, the effectiveness of the diversification is inversely related to the correlation between the yields). In the case of portfolios consisting of irs financial claims, since the price of the securities depends on the (uncertain) rate of interest, the risks are highly correlated. Consequently, the riskiness of a pure investment portfolio cannot be reduced through diversification. The riskiness of intermediation portfolios, consisting of both assets and liabilities, can be controlled by selecting contracts that match as closely as possible (perfect matching is the limiting case in which there is perfect negative correlation).

It is important (also for the purpose of assessing the overall operational value of stochastic models of the market) that the one-factor model permits the development of portfolio selection methods based on the theory of financial immunization and therefore suited to irs securities.[16] Following this approach, it is possible to develop correct forms of measurement and control of the interest rate risk in securities business.[17] There may also be useful indications for the management of the public debt; the widespread adoption of asset-liability management in the management of portfolios and financial institutions could improve the efficiency and completeness of markets.[18]

In practice, however, it does not appear to have been possible to overcome the difficulties encountered at the start, when the need to value securities "born without a theory" justified the use of the only analytical instruments available (based on the traditional hypothesis of certainty), despite their inability to cope with the innovation of uncertain cash flows. In fact, the tools provided by the new theory have not acquired the status of operating standards, nor does there

[16] See De Felice and Moriconi (1991a).

[17] "Regulators and financial folk are increasingly concerned that the risks in financial derivatives are neither measured nor priced correctly"; subtitle to the article, "Taming the derivatives beast", published in *The Economist*, 23-29 May 1992.

[18] See Merton (1990), pp. 451 and 467-71. The reference to the public debt sinking fund is important in several respects. The methods based on stochastic models of the market could meet the need for the fund to be able to *read* the secondary market in order to manage selective policies and make corrective interventions in segments of the market that are marked by assessment errors on the part of operators or negatively affected by the distribution of maturities (Salvemini, 1992b).

appear to be agreement on identifying the quality of innovative financial instruments. Paradoxically, the urgent need (imposed by the growth of the market) to apply probability theory has resulted in a sort of "uncertainty malaise", which, despite the hope that it would be overcome,[19] is clearly reflected in portfolio management practice.

The valuation of CCTs is still (generally) based on "fixed-rate practices", with the result that "more or less arbitrary conceptual generalizations and approximations are often found in the analysis of the yields of variable rate securities".[20] As regards the measurement of interest rate risk, the information available to the public is still contradictory: in the case of operators, risk is measured by the "liquidity horizon" (defined as "the time until the next coupon revision date"),[21] which was recently adopted in regulations governing the Italian securities market,[22] but it is also measured by the Macaulay duration computed on a stream of known cash flows obtained by assuming a deterministic coupon equal to the latest coupon on all the subsequent payment dates, thereby "sterilizing" the indexation mechanism. Accordingly, on 26 November 1991 a CCT 1.3.91/98 (ABI code 13096) had a risk of two months on the basis of its liquidity horizon and of four and a half years when considered as a sterilized security.[23]

3. Preliminary Diagnosis of the Effects of Indexation

The typical amortization features of CCTs provide for the lump sum repayment of principal at maturity coupled with the payment of coupons at regular intervals.

In the analysis of the indexation mechanism important roles are played by the so-called indexation lag (the interval between the fixing of the reference index and the payment of the indexed coupon) and

[19] In its report on "Asset and Liability Management by Banks" the OECD draws attention to the urgent need to "managing risk... and continually monitoring the bank's position with respect to risks..." (OECD, 1987, p. 16).

[20] Monti and Onado (1989), p. 298.

[21] Banca Commerciale Italiana (1984), p. 4.

[22] See Article 32(2) of the Regulations issued by the Bank of Italy on 2.7.1991 implementing Law 1/1991: "variable rate securities are attributed to the residual maturity bracket corresponding to the date of the next interest rate revision".

[23] Figure published in *Il Sole 24 Ore* on 27 November 1991.

by the relationship between the coupon interval (*i.e.* the interval between coupon payments) and the maturity of the reference security.

If (as in the case of CCTs) the indexation parameter is the rate of return on a zero-coupon bond, the index maturity corresponds to the maturity of the zero-coupon bond, which in this sense is the underlying security of the indexation, and the CCT the derivative security.

In the case in which each fixing of the reference index coincides with the start of coupon entitlement and the coupon interval is equal to the maturity of the reference security ("synchronous" indexation), the indexed security perfectly replicates, over a horizon equal to its maturity, a rollover of the nominal capital in the underlying zero-coupon bond.

Consider a market for irs securities that satisfies the usual perfect market conditions and with an equilibrium term structure of interest rates; let t be the moment of observation and $t_1, t_2, \ldots, t_m$ the coupon dates of the indexed security ($t \leq t_1 \leq t_2 \leq \ldots \leq t_m$), which perfectly replicates a rollover in short term bonds traded on the market. It can be shown, as a direct consequence of the arbitrage argument, that the flow of indexed coupons, referred to a unit amount of nominal capital, is equivalent, in terms both of value and of sensitivity to changes in the interest rate, to a pair of deterministic unit zero-coupon bonds: an asset with maturity t_1 and a liability with maturity t_m (see Appendix 1).

This result is important because it allows the risk associated with the (uncertain) indexed payment stream to be measured by calculating the Macaulay duration of the equivalent deterministic stream, which is usually found to be negative.

If, in addition to the indexed amounts, account is taken of the redemption of the principal at maturity (a positive unit zero-coupon bond), the resulting cash flows reduce to a deterministic unit zero-coupon bond redeemable at t_1, so that the duration of the stream of uncertain coupons combined with the principal at maturity is equal to $t_1 - t$.

Adding the first coupon (which is paid at t_1 and known at t), increases the value of the payment stream, but does not alter the duration.

At each coupon payment date, the price of the security is equal to its par value; in the period between two coupons, the ex-coupon price remains "very close to the nominal value", but only if in this interval the market rate of return remains "very close" to the coupon rate of return. In general, therefore, there are capital risks in the period between two successive coupons. In this sense the duration coincides with the liquidity horizon or, in other words, measures the time that has to pass before the next theoretical parity is reached; from this point of view as well it can be considered as a measure of risk. On the other hand, the fact that the duration of the indexed cash flow on its own is negative, so that it diminishes the deterministic components in the calculation of the overall duration of the security, indicates that a risk-reduction (autoimmunization) mechanism is implicit in indexed securities.

It is also clear that the spread adopted as part of the CCT indexation mechanism is spurious to the rationale of indexation: the addition of a flow of deterministic amounts increases the overall duration of the security, by lengthening the liquidity horizon, and alters the theoretical par value to the advantage of the holder through the addition of an amount that, under the hypothesis of a stable term structure of interest rates, is a decreasing function of the maturity of the security.

The typical indexation mechanisms adopted for CCTs do not differ in qualitative terms from synchronous indexation, but their quantitative evaluation requires a more detailed analytical framework and a model of the reference market.

4. A Theory for CCTs: Measurement of the Price and Risk Components

Stochastic models of the market make it possible to define a comprehensive method for valuing CCTs that takes account of their special contractual features (indexation lags that differ from the coupon interval, the dependence of the coupon on the results of more than one auction, the rules governing the calculation of the reference index and the size of the spread).

In addition to the determination of the price consistent with the equilibrium term structure of the market (and with the expectations implicit in this structure), it is possible to measure the risk, in the form of the stochastic duration (the semi-elasticity of the price with respect to changes in the "short-term" rate of interest, measured on a time scale), distinguishing between the component due to the uncertainty of the coupons (coupon risk) and that due to the variability of the discount factor (discount risk).

The model also makes it possible to define operational criteria of equivalence between certain and uncertain pay-offs. In particular, it is interesting to calculate the so-called *ex ante* value of each uncertain coupon given by the model, defined as the certain amount that, when substituted for the coupon, has the same equilibrium price, calculated on the basis of the model. (The formal structure of the approach is developed in Appendix 2.)

With reference to the situation observed in the screen-based market (*Mercato Telematico*) on 26 November 1991, we have analyzed a hypothetical five-year CCT with a semi-annual coupon linked to the auction prices of six-month Treasury bills in the two months ending one month before the day on which coupon entitlement begins. We assume that the security is issued on the valuation date and ignore taxation and the spread (adding a spread of 0.50 points would result in this hypothetical security being very similar to some actual CCTs, *e.g.* that with ABI code 13208).

The main features of the security are summarized in Table 1. The cash-flow schedule is analyzed by showing for each coupon payment date the model present-value (P), the *ex ante* value (z), the overall risk (Ω), the discount risk (Φ) and the coupon risk (Γ). The corresponding figures are also shown for the redemption value and the security as a whole. The security is found to have a stochastic duration (D) of about 5 months, which coincides with the conventional liquidity horizon. The reduction in risk (autoimmunization) achieved with the indexation mechanism is revealed by the negative sign of the coupon risk of the uncertain coupons (the coupon risk of the first coupon is nil because its value is known at the valuation date).

In Table 2 the CCT is compared with securities having fixed (deterministic) coupons and the same payment dates. Four different deterministic streams are considered having various forms of equivalence with the indexed security. In addition to *ex ante* coupons and flat coupons, which are all equal to the first known coupon, we consider forward coupons (each of which is obtained using the forward rate implicit in the estimated term structure relative to the period of coupon entitlement) and par coupons (calculated on the basis of the par yield, defined as the nominal rate of a bullet bond

TABLE 1

COUPONS					
payment date	P	z	Ω	Φ	Γ
26/05/92	5.1920	5.4763	0.4602	0.4602	0.0000
26/11/92	4.9291	5.4843	–7.0054	0.8490	–7.8543
26/05/93	4.6777	5.4905	–5.4433	1.1769	–6.6203
26/11/93	4.4358	5.4928	–4.1257	1.4535	–5.5791
26/05/94	4.2045	5.4925	–3.0140	1.6864	–4.7004
26/11/94	3.9841	5.4905	–2.0762	1.8825	–3.9588
26/05/95	3.7750	5.4880	–1.2852	2.0476	–3.3328
26/11/95	3.5768	5.4852	–0.6183	2.1864	–2.8047
26/05/96	3.3888	5.4821	–0.0565	2.3031	–2.3596
26/11/96	3.2109	5.4791	0.4168	2.4012	–1.9844
REDEMPTION VALUE					
payment date	P	z	Ω	Φ	Γ
26/11/96	58.6034	100.000	2.4012	2.4012	0.0000
SECURITY					
$P(X) = 99.9781$			$D(X) = 0.4061$		
$\Omega(X) = 0.3796$		$\Phi(X) = 2.0495$		$\Gamma(X) = -1.6698$	

with an equilibrium price, calculated on the basis of the reference term structure, equal to par). The *ex ante* coupons differ from the forward coupons because the indexation lag of around seven months does not coincide with the coupon interval (or with the maturity of the underlying security). The *ex ante* coupons are systematically lower than the corresponding forward coupons and the model price of the *ex ante* security is below par. The model price of the flat-coupon security does not diverge from the equilibrium price only because the slope of the relevant yield curve is not very large. The notable difference between the risks (and the durations) of the deterministic cash flow with *ex ante* coupons compared with those of the indexed security is a measure of the risk-reduction effect of the indexation mechanism for securities that (by construction) have the same time to maturity and the same model price.

TABLE 2

payment date	forward rate	Fixed rate: forward coupon	Fixed rate: par coupon	Fixed rate: *ex ante* coupon	Fixed rate: flat coupon	Variable rate: coupon
26/05/92	0.0548	5.4763	5.4891	5.4763	5.4763	5.4763
26/11/92	0.0549	5.4880	5.4891	5.4843	5.4763	I_2
26/05/93	0.0549	5.4939	5.4891	5.4905	5.4763	I_3
26/11/93	0.0550	5.4960	5.4891	5.4928	5.4763	I_4
26/05/94	0.0550	5.4955	5.4891	5.4925	5.4763	I_5
26/11/94	0.0549	5.4937	5.4891	5.4905	5.4763	I_6
26/05/95	0.0549	5.4914	5.4891	5.4880	5.4763	I_7
26/11/95	0.0549	5.4886	5.4891	5.4852	5.4763	I_8
26/05/96	0.0549	5.4855	5.4891	5.4821	5.4763	I_9
26/11/96	0.0548	5.4825	5.4891	5.4791	5.4763	I_{10}
	Price	100.0000	100.0000	99.9781	99.9036	99.9781
	Risk	2.0494	2.0493	2.0495	2.0498	0.3796
	Duration	3.5060	3.5057	3.5063	3.5073	0.4061

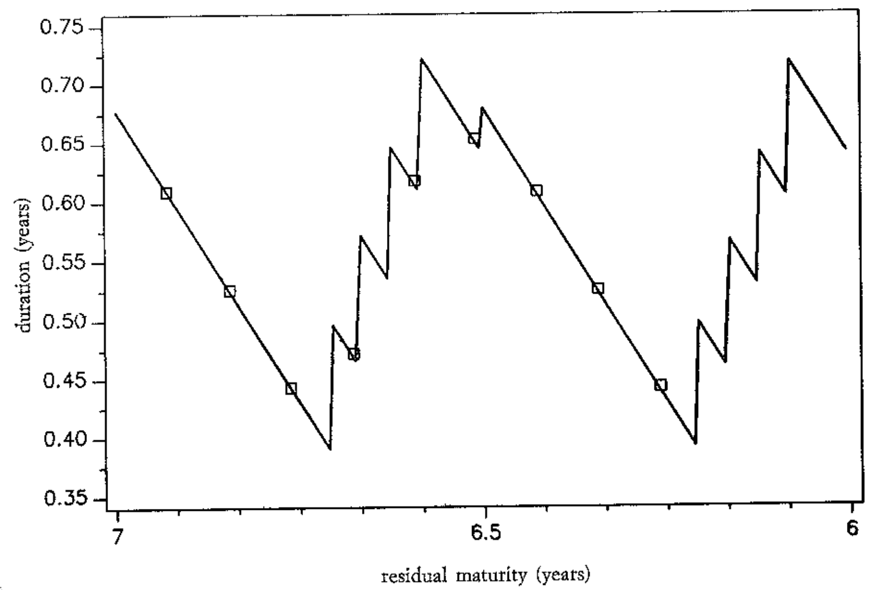
Figure 1 shows the changes in the stochastic duration of a CCT (ABI code 13208) calculated with a given term structure of interest rates (estimated on 26 November 1991). It is worth noting the effect of the increase in information occasioned by Treasury bill auctions, which result in the coupon to which they refer becoming progressively less uncertain, and hence less indexed (autoimmunized); the auctions thus produce brusque increases in overall risk that are larger than those due to the payment of the coupons (in the figure the latter is shown in correspondence with the residual maturity of six and a half years).

The figure can also be seen as a description of the market risk associated, at a given time, with a set of securities having the same indexation mechanism and ordered by residual maturity. The squares in the figure correspond to the levels of risk that could be measured on 26 November 1991 (for securities actually traded in the screen-based market on 26 November 1991 that differed from ABI 13208 only in their date of issue).

FIGURE 1

CHANGES IN DURATION

26 November 1991



It is commonly held that, by reducing uncertainty, increases in information reduce risk; however, CCTs demonstrate that this view has no sense unless it is properly specified. The apparent paradox can be overcome by recalling that it is always necessary to measure risk in terms of the uncertainty of achieving the underlying objective. In our case, we are not concerned with the uncertainty surrounding the individual elements (coupons) of the contract but with the uncertainty as to whether the contract will achieve the objective for which it was designed (producing the market rate of return).

The quantitative analysis of indexation mechanisms makes it possible to test the technical effects of the contractual features that can be considered of strategic importance for issue policy. Changes we simulated in various market situations to the method of calculating the indexed part of the coupon compared with that in use did not produce significant changes in the quality of the securities.[24]

5. Quirks in the Empirical Analysis of Prices

The empirical analysis was carried out on a daily basis with reference to the prices observed in the screen-based government securities market from 1 January 1990 to 30 June 1992. These prices can be taken as a reliable index of market conditions in the period,[25] and more representative than those recorded on the stock exchange.

Figures 2A and 2B show, in relation to the observation date, the ex-coupon prices of BTPs and CCTs (for a nominal value of 100 lire), which, together with the prices of Treasury bills, constitute the reference data base for the analysis (with 7.951 BTP prices and 13.538 CCT prices).

The term structure of interest rates, in the form defined by the single-factor CIR model, was estimated on a daily basis using the prices of Treasury bills and BTPs by a non-linear regression procedure of the kind proposed by Brown and Dybvig.[26] Since the payments relative to each security have been calculated net of withholding tax, the estimate gives the term structure of net yields.

[24] In this simulation, CCTs were hypothesized with three, six and twelve-month coupon intervals, related to the auction prices of three, six and twelve-month Treasury bills (as the price of a single auction and as the average of the prices of four successive auctions), with the indexation lag set equal to the coupon interval. The theoretical prices of these securities, calculated for the observation period using the CIR model, did not diverge significantly from the theoretical prices of the CCTs traded in the market. This, of course, does not guarantee that the market prices of such securities would have behaved in the same way if they had actually been issued, since it cannot be excluded that the market uses different valuation criteria from those of the test model.

[25] "Only in 1991 did the screen-based government securities market become comparable with the Anglo-Saxon markets in terms of efficiency and scale of trading (number of operators, number of listed securities, turnover and ability to reflect market conditions and operators' expectations)", Bianchi, 1992, p. 58.

[26] The method consists basically in estimating the "risk-adjusted" parameters of the model that produce the best fit, in the least squares sense, between the theoretical and actual prices. The underlying theoretical idea is that an intertemporal arbitrage model of

FIGURE 2A

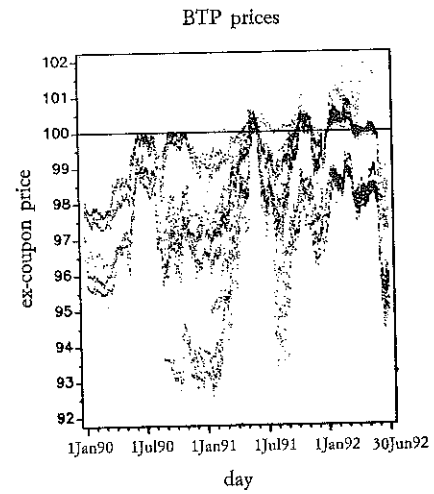
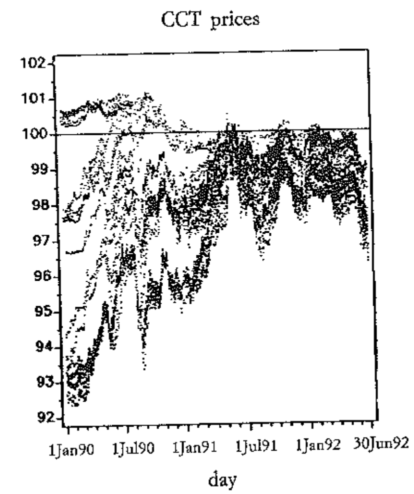


FIGURE 2B



In the 626 observation days the instantaneous interest rate *r(t)* fluctuated between a minimum of 0.0935 and a maximum of 0.1244, with a mean value of 0.1045 (standard deviation 0.0042). The mean value of the long-run asymptotic rate, r_L, was 0.1202, with a range of about 0.05 (a standard deviation of 0.9143). The mean value over the

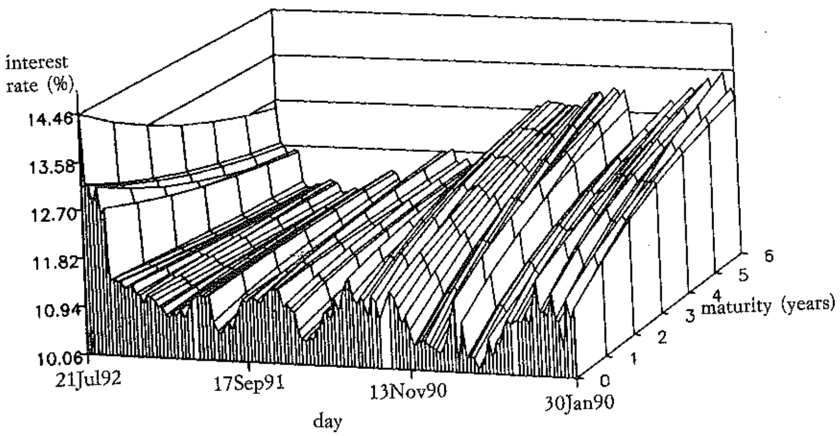
the yield curve implies restrictions on the structure of the prices of the bonds traded on a given date. Even if the complete specification of the model can only be obtained by estimates based on time series of prices, the information contained in individual cross sections is sufficient for the purpose of valuing irs securities consistently with the exclusion of arbitrage opportunities.

If the model were to be used for forecasting, it would have to be estimated on time series. However, as was to be expected, all the empirical tests of single-factor models of the CIR type have indicated that the parameters estimated on time series are highly unstable, over different subperiods. This conflicts with the basic hypotheses, which require that the values of the parameters be constant over time, and shows the inadequacy of such models for forecasting purposes. This result may seem obvious, in view of the fact that the model is extremely simple while the reality is extremely complex. On the other hand, the estimation based on successive cross sections, which implies a continuous recalibration of the risk-adjusted parameters has shown, also when applied to the Italian market (see Barone, Cuoco and Zautzik, 1989), that the stability of some parameters (in particular implicit volatility) is comparable with that obtained on time series. This suggests that the combined use of the information provided by time series and that obtained from the observation of successive cross sections of prices (as in the "three-step" estimation method proposed by De Felice and Moriconi, 1991b) could whole observation period of the volatility coefficient of the instantaneous rate, $\rho\sqrt{r(t)}$ was 0.024 (dev. st. 0.002).[27]

Figure 3 shows the term structures estimated on each Tuesday in the observation period for maturities up to six years.

FIGURE 3

THE TERM STRUCTURE OF INTEREST RATES
(on Tuesday, from 9 January 1990 to 21 July 1992)



Initially rising, the slope of the yield curve decreased between the early months of 1991 and January 1992 as a result of the fall in longer-term yields (attributable to expectations of convergence towards the average level of European rates). Subsequently, the curve flexed sporadically before acquiring the downward slope that marked the final part of the period.

The slope of the yield curves and the twist of the surface they form are also revealed by the estimated path of the instantaneous rate

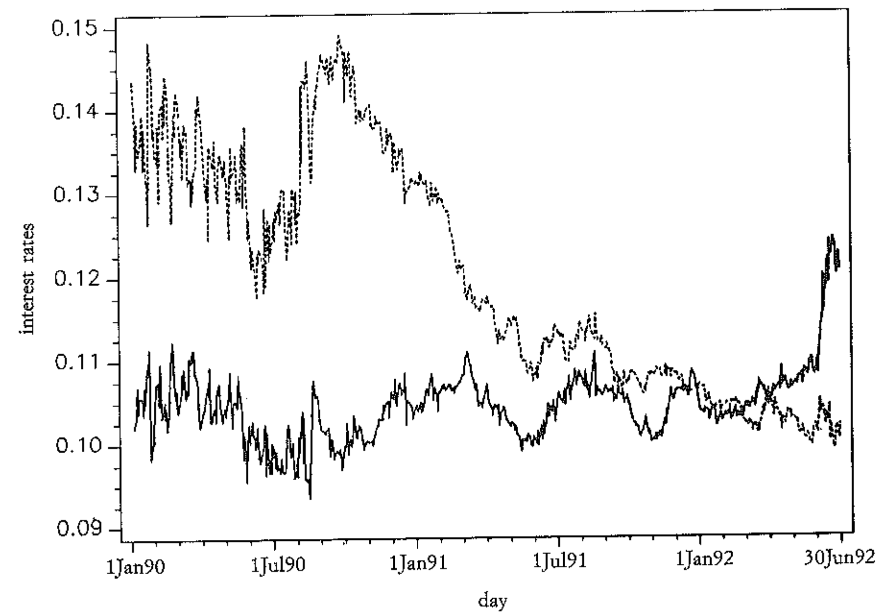
represent an acceptable compromise in the practical use of such models. "This process of routine reparameterization is theoretically inconsistent ... but seems to some degree unavoidable", see Duffie (1992, p. 216).

[27] The definitions of $r(t)$, r_L and ρ are given in Appendix 2.

$r(t)$ (the continuous line in Figure 4) and that of the long-term rate r_L (the broken line). Excluding the rare non-monotonic curves, the yields show a rising trend when the long-term rate is greater than the short-term rate and a twist when the dominance is reversed.

FIGURE 4

INSTANTANEOUS AND LONG-TERM RATES



Analysis of the Price of CCTs

In order to enhance the homogeneity of the data, the observation period is divided into two subperiods at 1 March 1991. In fact, this month saw a significant change in the set of securities traded in the screen-based market since the number of CCTs having technical features that can be summarized as C6/A/2[28] came to exceed that of all the other types of CCTs. In the second subperiod C6/A/2 se-

[28] The code stands for "semi-annual coupon, indexation on annual Treasury bills and taxation at the second rate (12.5%)". These securities have the type of indexation mechanism classified as ψ_9 in Barone and Folonari (1992) and a spread of 50 basis points. Since May 1988 only this type of CCT has been issued.

curities make a fundamental contribution to broadening the range of maturities available in the market (roughly from 3 to 7 years), while in the first subperiod other types of security covered the extremes of the range. From the point of view of transactions, the tendency for the prices of all traded CCTs to increase systematically was confirmed, with a significant reduction in the difference between face values and ex-coupon prices, and a decrease in the dispersion within each cross section (the set of prices at a given date). The pattern can be seen in Figure 2B and March 1991 can conveniently be taken as the moment when the prices of CCTs entered a narrow fluctuation band close to the par value and about 3.5 lire wide. In fact, the average price rose from 97.24 lire in the first period to 98.84 in the second, while the standard deviation in the first period was nearly three times that in the second (2.17 lire, as against 0.77 lire).

For all the CCTs listed in the observation period we have calculated the theoretical price on each quotation day (using the term structure of interest rates estimated on the prices of fixed-rate securities on the same date), taking account of all the specific features of the indexation mechanism (including the spread), so that the values obtained are comparable with the observed prices.

The analysis reveals a general tendency for the model to overvalue CCTs in comparison with their actual prices. Over the whole observation period the daily overvaluation averaged 4.5 lire and it was never less than 3 lire.

This systematic overpricing can be interpreted both as a symptom of an inherent weakness of the model attributable to poor information (the inadequacy of the single-factor stochastic process in describing the state of the market in government securities) and as a result of the extreme idealization implicit in the perfect market hypotheses (no taxation or transactions costs and the possibility of selling short, etc.).

The results of the test can nonetheless be used heuristically to identify the typical market conditions that diverge from the basic hypotheses of the model. Several factors appear likely to reduce the explanatory power of the model; they are also complex (taxation), difficult to quantify ("supply pressure") and their effect can be cumulative (*e.g.* in the case of an incorrect perception of the quality of the securities). Along these lines, it is possible to use the model to construct and test possible causes of the mispricing and thereby enhance the explanatory power of the model.

The Effects of Taxation

Under the provisions of Italian tax law, there are both taxed investors, who receive interest income net of withholding tax, and others whose large interest payments result in their being *de facto* exempt and therefore interested in the gross amount of interest.[29]

The existence of tax-exempt investors results in a price structure that is higher on average than would be found in an ideal market consisting entirely of taxed investors. Consequently, the existence of exempt investors should result in the term structure of net interest rates estimated on the prices observed in the market being lower than the ideal net structure. The converse applies for the structure of gross interest rates with respect to a market consisting exclusively of tax-exempt investors.

The structure of the market, and hence the definition of the system of prices, are further complicated by the existence of securities taxed at different rates, which can give rise to substantial tax-clientele effects.[30]

The distortions produced in the estimation of the term structure of interest rates by differences in taxation can alter the logic of pricing models, with effects that are more complex for variable-rate securities than for fixed-rate securities. In the case of CCTs the disturbance can be shown by looking at the reinvestment security (without having to refer to the stochastic valuation model). If the cash flows have been considered net of withholding tax, it can be seen that the factor $1/v(T,s) - 1$, obtained from the estimated term structure and used to represent the reference index, may not coincide with the reinvestment rate best suited to tax-exempt investors, who faces a higher specific term structure. Accordingly, there is no longer equivalence between holding the indexed security and a rollover commitment at market

[29] In practice there exists a whole range of intermediate positions, depending on the type of income earned and the structure of the investor's balance sheet, with important implications for prices; see Bernareggi (1986). For the sake of clarity, we consider only the two extreme cases of taxed and exempt investors.

[30] The problem of identifying a specific term structure for a given tax category of investors is closely linked to that of determining the ideal securities portfolio that such investors should rationally hold. As has been shown in the literature, from the model of Hodges and Schaefer (1977) to the work of Dermody and Rockafellar (1991), there is a relationship of duality (in the linear programming sense) between the two problems. For a discussion of the possible tax-clientele effects as a result of the changes in the tax provisions regarding the Italian market, see Rovelli (1991), especially pp. 21-22.

rates.[31] If the market were to perceive tax asymmetries of this type as offering an arbitrage opportunity, it might react by lowering the market prices of CCTs as if the indexed coupons were valued on the basis of the (estimated) structure of gross interest rates.[32]

This type of retuning can easily be incorporated in single-factor stochastic models and has given satisfactory results in a certain period.[33] The gradual increase in the prices of CCTs that began in the first half of 1991 led to systematic underpricing by the model corrected for the effect of taxation (with a mean error for each cross section of between approximately 0.5 lire and 1 lira). Since no significant changes in the relevant tax provisions occurred in this period, the credibility of the mechanism adopted to correct for tax segmentation is diminished.

The Significance of the Reference Index

Another lack of correspondence between the theoretical model and reality is the fact that CCTs are indexed to prices observed in the primary market, and hence determined on the basis of equilibria and mechanisms that may differ from those that determine prices in the secondary market.

The hypothesis of an undervaluation of CCTs as a result of a misalignment of the index compared with the yields typical of the

[31] Tax segmentation also produces more direct effects. For example, with reference to the reinvestment security (considered on the after-tax hypothesis), a tax-exempt CCT indexed on the net Treasury bill yield allows a tax-exempt investor to reinvest only at the net rate. Another factor that alters the equivalence between a strategy of holding CCTs and a Treasury bill rollover strategy is the difference between the ways in which tax is paid on Treasury bills and securities with coupons.

[32] The possibility of valuing CCTs by "crossing" net and gross term structures is also considered by Drudi and Scalia (1992, p. 23).

[33] Until the early months of 1991 the analysis carried out using the single-factor CIR model gave theoretical price cross sections that were not very different from the prices observed on the Milan Stock Exchange. For instance, on 24 July 1989, for a sample consisting of the twelve C6/A/2 CCTs listed, the error (difference between market price and model price) ranged from -1.15 lire to 0.04 lire, with a mean of 0.65 lire (standard deviation 0.38); on 8 January 1991, for the 27 CCTs traded the error ranged from -0.62 lire to 1.56 lire, with a mean of 0.96 lire (standard deviation 0.57); see Castellani, De Felice and Moriconi (1989) and De Felice and Moriconi (1991b). Ristuccia (1991) provides an analysis of the reliability of the model in the period May 1988-December 1989, distinguishing between the various types of indexation.

secondary market does not appear to be confirmed by *ex post* analysis if the index is considered inclusive of the spread.

Figure 5 shows the difference between the coupons paid (net of withholding tax) on C6/A/2 CCTs and the net return estimated on the market for a maturity of six months.

With two exceptions, the coupons were always higher than the market return. Net of the spread (*i.e.* measuring the distance of the asterisks from the horizontal line at 4.375 lire, which corresponds to the after-tax value of the spread), the coupons were lower than the market return on 19 occasions out of 30. It can therefore be said that the rollover guaranteed by CCTs (at least in the case of the C6/A/2 type) was more profitable than the market rollover, since the spread offset the difference between the annual return on the primary market and the annualized semi-annual return estimated in the secondary market.[34]

"Supply Pressure"

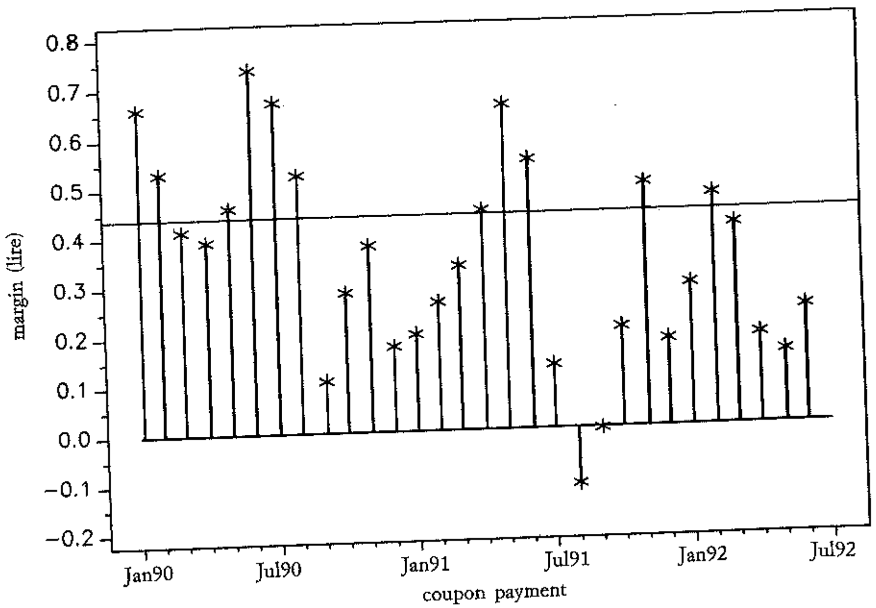
Ignoring the spread, an issue of CCTs is equivalent, for debt management purposes, to replacing a series of Treasury bill issues made at each maturity of the CCT coupons to finance an amount equal to the face value of the CCTs (naturally, the maturity of the Treasury bills is equal to the coupon interval of the CCTs).

If the Treasury were to finance the borrowing requirement with issues of Treasury bills, it would run the risk of having to pay high returns as a consequence of "supply pressure". From this point of view an

[34] The misalignment between the returns observed in the primary market and those estimated in the secondary market does not appear significant for three and six-month maturities. This excludes the possibility of the misalignments at twelve months being caused by excess demand in connection with the uniform-price auction techniques, which encourage multiple bids; on this point, see Buttiglione and Prati (1991). The explanation lies in the interaction between the segmentation of demand and the very active role played by the central bank in the primary market. In the case of twelve-month Treasury bills, banks are led to make auction purchases at unaligned prices in order to supply customers whose requirements cannot be met with securities bought in the secondary market and who do not generally indicate exact prices since they are afraid of being excluded from the allotment as a result of the purchases made by the central bank (with the aim of excluding the demand at prices it judges to be too low). It is worth noting that in the period in question purchases by the central bank were less frequent for three and six-month bills.

FIGURE 5

C6/A/2 TYPE CCTs: MARGIN ON THE INDEXED COUPON



issue of CCTs reduces the cost borne by the Treasury compared with a series of Treasury bill issues.

However, considerations of this type undermine one of the basic hypotheses of the theoretical model, *i.e.* the assumption that operators cannot influence the level of prices. On the one hand we have a large number of operators, who, taken individually, are price takers; on the other there is just one very large operator whose supply tends to depress prices and who therefore has an interest in diversifying issues as widely as possible over the whole range of maturities.

The analysis of the equilibrium of a market with such a structure is theoretically complicated and it is not clear how far this type of asymmetry influences prices in the secondary market.

The Lengthening of the Liquidity Horizon

The explanation of the mispricing based on the illiquidity of CCTs appears to be of a circular nature that exploits the centaur-like ambivalence of these securities: a CCT is underpriced in comparison with the model price because of its limited liquidity (in other words it is considered as a long-term contract), if it were quoted at the ideal price, it would be highly liquid (because it would be equivalent to the underlying Treasury bill). On the other hand, the relationship between price and liquidity shows how the price can influence the quality of the security, with the risk of destabilizing effects: vicious circles (a low price implies low quality and therefore a lower price) or virtuous circles (a high price has an announcement effect of high quality and therefore a higher price).

The relationship between liquidity and price provides a more interesting interpretative key if it is coupled with a tradeoff approach: the mispricing of CCTs with respect to the equilibrium line can be measured in terms of underpricing, but also in terms of the lengthening of the liquidity horizon, *i.e.* as additional risk (equivalent to the loss of liquidity) measured on the time axis. This approach has a direct operational significance by representing the market in the (*Duration, Normalized price*) plane.[35]

Figure 6 shows the situation in the market on 26 May 1992.

The continuous line represents the equilibrium line $P=v(D)$: if the market prices coincided with those of the model, the normalized prices would lie on this curve. The normalized price is plotted against the stochastic duration for the securities quoted in the screen-based market during the day: using ("Zs") for Treasury bills, ("Fs") for

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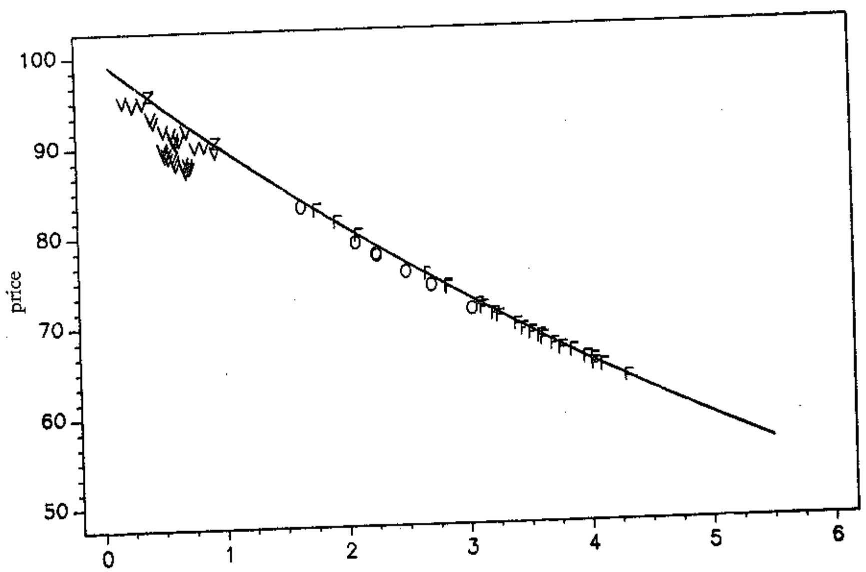
[35] In terms of the theory of stochastic immunization, an irs security with contractual features c can be considered equivalent at time t to a zero-coupon bond having the same value and stochastic duration (the same basis risk). At time t let $v(t,s)$ be the price structure estimated in the market. Consider a security c with value P, stochastic duration D and price Q. The value $v_D = v(t, t+D)$ is the equilibrium value at t of a unit zero-coupon bond with maturity D, so that the quantity $z = P/v_D$ is the face value of a zero-coupon bond that, at time t, has the same value and basis risk as c. Defining the "normalized price" of c as the ratio $\Pi_n = 100Q/z = 1000v_D/P$, it follows that Π_n is the price paid at t in the market for a security c, equivalent, in terms of price and duration, to a zero-coupon bond with a face value of 100 lire and maturity at $t+D$. In the (*Duration, Price*) plane the curve $P = v(D)$ can be interpreted as the equilibrium frontier of the market at time t. For more details, see Castellani, De Felice and Moriconi (1992).

BTPs, ("Vs") for CCTs and ("Os") for CTOs. The mispricing of the CCTs is revealed by the fact that the "Vs" are below the equilibrium curve. The deviation from the ideal position can be measured as the distance from the curve on the x-axis, which gives increases in the liquidity horizon on the day in question ranging from 0.07 to 0.55 years.

FIGURE 6

EQUILIBRIUM LINE AND NORMALIZED PRICES
26 May 1992



The Influence of the Residual Maturity

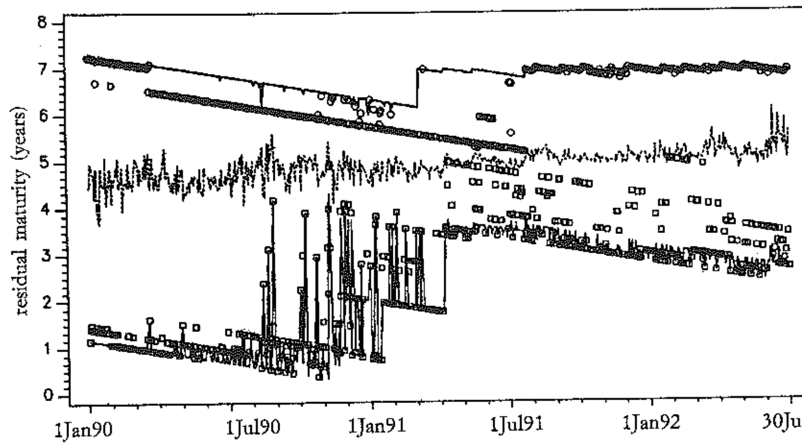
The analysis of the market prices of individual CCTs in relation to their residual maturities suggests that the market has an incorrect perception of these securities (or that fixed-interest valuation rules have been improperly applied).

The relationship between the residual maturities of all the listed CCTs and their market prices is summarized in Figure 7, which shows the mean value (broken line) and the minimum and maximum values

FIGURE 7

RESIDUAL MATURITY OF LISTED CCTs

market values



(continuous lines) of the time to maturity of the CCTs listed on each day in the observation period.

The securities with the lowest price for the day (whose maturity is shown with a circle) tend to be those with the longest residual maturity and those with the highest price (whose maturity is shown with a square) those with the shortest residual maturity. The line corresponding to the mean values divides the securities with maximum and minimum prices almost perfectly.

These results are inconsistent with the condition embodied in the theoretical model of the indexation mechanism that the price of a security is basically independent of its residual maturity.

Price Paths

The comparison of the empirical relationship between market price and maturity and the corresponding theoretical relationship can be taken further by looking at the changes in the prices of CCTs over their lives.

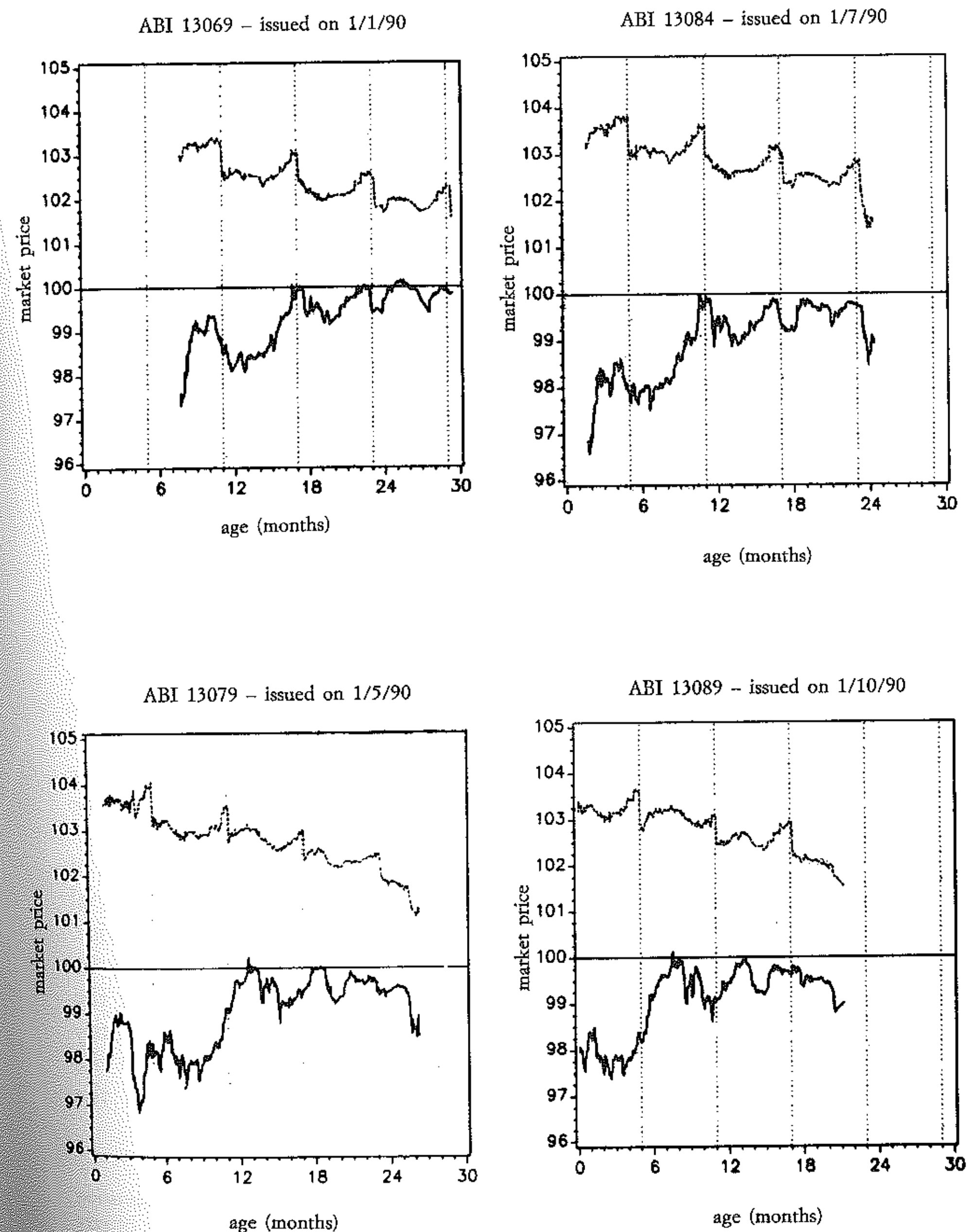
CCTs of the C6/A/2 type have been examined. The theoretical price follows a falling path as the residual maturity shortens owing to the price component corresponding to the spread. This also explains why the theoretical price is (generally) higher than the face value. Net of the spread component, the theoretical price of securities with asynchronous indexation mechanisms follows a path similar to that of securities with synchronous indexation (in particular, the value is close to the theoretical parity around the time at which coupons are fixed). By contrast, prices in the screen-based market are well below par. For some securities the theoretical price and the observed price follow similar paths, albeit with mispricing. The reaction of the theoretical prices to the results of the auctions used to fix the coupons is clear. However, for other securities of the same type but issued in different periods the two price paths are dissimilar. The hypothesis that the market has a correct perception of the indexation mechanism does not appear to be confirmed; rather, such a perception appears to be a random effect produced as market prices draw closer to the par value following a cyclical trend in the second subperiod (as Figure 2B shows). Figure 8 shows, for four CCTs of the C6/A/2 type, the paths followed by the model and market prices net of the accrued interest (respectively the broken line and the continuous line). The CCTs with ABI codes 13069 and 13084 can be considered as representative of "in-phase" paths and those with ABI codes 13079 and 13089 of "out-of-phase" paths.

The Cost of the "Uncertainty Malaise"

In the context of a market that has an incorrect perception of the indexation mechanism, instead of using the valuation model to explain mispricing on rational grounds, it is possible to provide an acritical measure of the effects of management customs and practices typical of fixed-rate securities.

The difference between the model and market prices can be attributed to the "uncertainty malaise" associated with CCTs as a result of the uncertainty surrounding coupons. In this sense the misalignment can be seen as the price paid for the extra risk arising from indexation.

FIGURE 8



A CCT is equivalent to a portfolio with fixed, deterministic components (the redemption value, the spread, the known coupons – the first and sometimes the second) and indexed components (the unknown coupons). Accordingly, the price P of the security can be defined, in view of its linearity, as $P = F + V$, *i.e.* as the sum of the price of the fixed components, F, and that of the variable components, V.

At any time the prices F and V can be valued separately using the CIR model, estimated on the situation prevailing in the market (*e.g.* F is the present value of the fixed amounts calculated using the discount factors obtained from the estimated term structure of interest rates).

The extra-risk component can be defined as the factor, k, with a value between 0 and 1, that reduces the price of the indexed payment stream that, summed with the price of the fixed component, gives the market price, Q. For each cross section of prices $\{Q_i\}$, the reduction can be measured in a natural way using the model $Q_i = F_i + kV_i + \varepsilon_i$. With the usual assumptions on the error term, ε_i, the parameter k can be estimated by means of a linear regression of V on the differences $Q - F$, observed on the date the price refers to.

The values of k estimated daily for C6/A/2-type CCTs over the observation period reveal two different patterns: until March 1991 they are highly erratic in connection with the high volatility of the market prices and the small number of observations in each cross section; they are nonetheless "centered" around the level equivalent to a tax rate of 12.5%, thus confirming the possibility of explaining the observed prices on the basis of the tax-effect hypothesis.[36] In the second subperiod k appears to be at a constantly higher level.

In the terminology of fixed-rate securities, the estimated level of the reduction parameter k can be measured in terms of the additional return the market requires (at the estimation date) to buy the security. Assuming, in conformity with an actual valuation method, that all the indexed coupons are equal to the last known coupon c (net of the withholding tax and the spread, which is included in F), the quantity $R = 2c(1-k)$ provides a direct measure, in percentage points, of the increase in the return, on an annual basis, required to offset the uncertainty surrounding the value of the coupons.

[36] The compensation mechanism associated with the existence of tax segmentation would result in a reduction in the model price of the variable component, which can be very roughly approximated by the reduction in the price corresponding to the tax rate.

The continuous line in Figure 9 represents the values of R, while the broken line represents the corresponding standard error of the residuals (stdr).

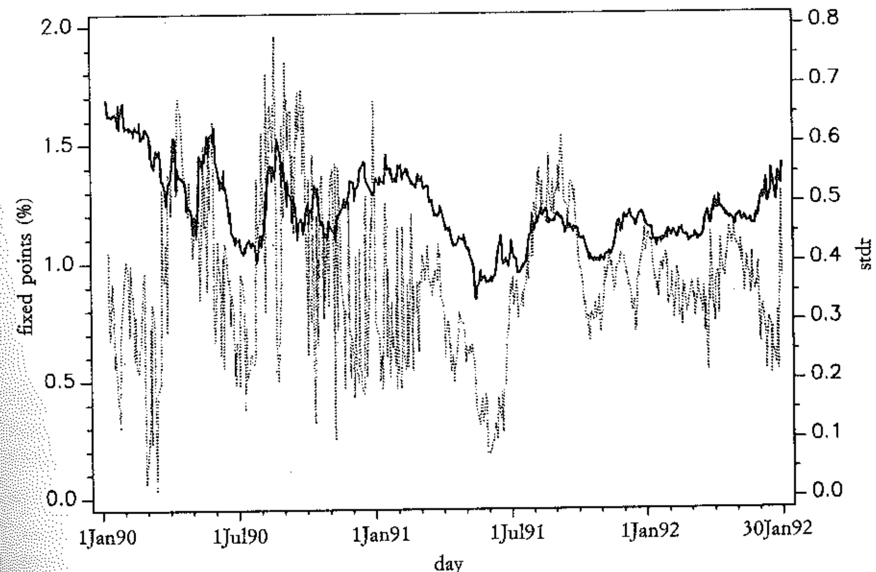
The average premium required on each day of the whole observation period ranged from 0.85 percentage points (in the middle of May 1991) to 1.7 points (at the beginning of the period). The pronounced variability of the standard error of the residuals in the first period (which implies a highly unstable quality of fit) can probably be attributed to the small number of C6/A/2-type CCTs listed in the screen-based market until the beginning of 1991.

The residuals reveal a negative correlation of the difference between the observed and estimated prices with the residual maturity of the securities in question. In other words, the model tends to undervalue the CCTs with shorter residual maturities.

FIGURE 9

C6/A/2-TYPE CCTs: EXCESS RETURN REQUIRED

single-parameter linear model



In order to improve the interpretative power of the model, it appears plausible to assume that each indexed coupon is valued by reducing its ideal (equilibrium) value by a constant coefficient k and by a second coefficient, $k_2 = -\alpha m_i$, that is proportionally related, by means of a constant α, to the time to maturity of the coupon, m_i. Taking S to denote the sum of the ideal values of the coupons, c_i, multiplied by m_i, the value of an indexed security is given by $P = F + kV - \alpha S$. Consequently, the parameters k and α can still be estimated by means of linear regression on each cross section of prices $\{Q_i\}$ on the basis of the model: $Q_i = F_i + kV_i - \alpha S_i + \varepsilon_i$.

As in the case of the single-parameter model, the estimated coefficients can be expressed as an additional return. The mispricing of each CCT is thus measured as a number of fixed percentage points (determined by k) and a variable number for each year of residual maturity of the security.

We have estimated the two-parameter linear model on a daily basis for C6/A/2-type CCTs with reference to the 335 working days of the second subperiod (from 1 March 1991 to 30 June 1992). The estimation errors are sufficiently small and stable (the standard error of the residuals oscillates around 0.19 lire with a standard deviation of 0.05). In addition, they are not significantly correlated with the residual maturity of the securities.

The results show that during this subperiod CCTs were valued, on average, by adding a fixed premium of 70 basis points and a premium of 8 basis points for each year of residual maturity to the return required on fixed-rate securities with a coupon equal to the known coupon.[37]

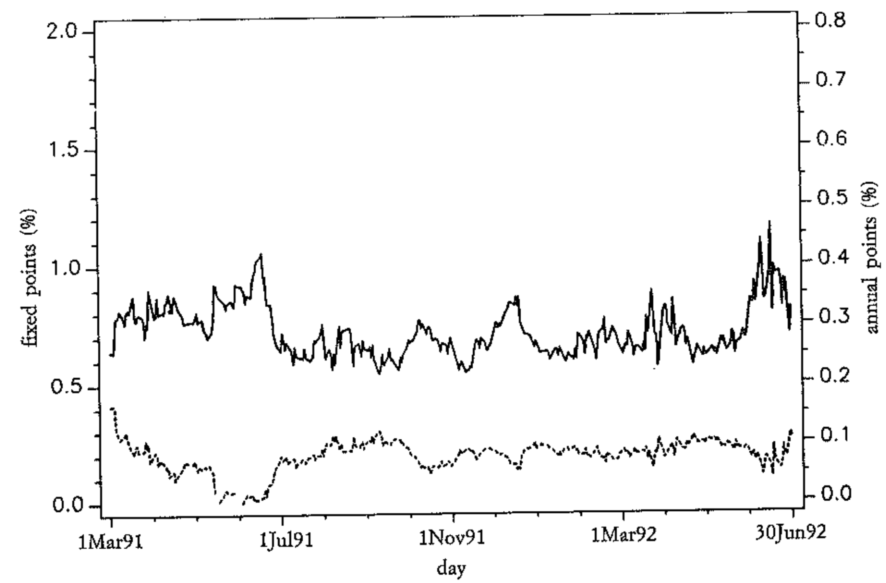
[37] It would, of course, be possible to consider the single parameter model $Q_i = F_i - \alpha S_i + \varepsilon_i$, which measures the mispricing in terms of a premium that increases linearly with the maturity (without a fixed component). In this case the estimation carried out for the whole observation period produced residuals that were positively correlated with the residual maturity.

These linear models, whether based on one or two parameters, have only a descriptive value. However, they represent a more correct methodological approach than attempts to explain the mispricing by reparameterizing stochastic models of the term structure. In the end, reparameterization attributes meanings to the parameters that are not consistent with the original theoretical reference framework and their estimation may be biased (since estimations on cross sections only permit the determination of the risk-adjusted parameters, it is not possible to interpolate the CCT prices while constraining the values of the parameters to those obtained by interpolating the prices of fixed-rate securities).

FIGURE 10

C6/A/2-TYPE CCTs: EXCESS RETURN REQUIRED

two-parameter linear model



6. Conclusions

The CCT is a security that was born without a theory, but which, analyzed using the methods of new term-structure theory, has revealed a complex theoretical nature.

The technical price and risk features show that the indexation mechanism provides protection against interest-rate risk and, in this respect, confirm the original judgement concerning the quality of CCTs, which continue to be "securities of interest both for the public and for the issuer",[38] in view of their autoimmunization properties.

The empirical analysis of the secondary market based on the single-variable CIR model reveals quirks in the behaviour of market prices compared with that which theory indicates would be correct. Not only are there misalignments between the theoretical and ob-

[38] Ministry of the Treasury (1986), Ch. 6.

served prices, but price paths are found that generally do not appear to be influenced by the indexation mechanism, in the sense of their correspondence with the theoretical parity. We have found that prices are affected by the residual maturity of securities (thereby flouting the independence hypothesized at the theoretical level) and that they can be explained in terms of a perception of the security as involving extra risk, in the sense of a lengthening of the liquidity horizon.

The empirical results thus appear to corroborate the hypothesis that the critical factor in the pricing of CCTs is operators' incorrect perception of this type of security. The mispricing can be interpreted as the effect of market practices, whereby indexed securities are valued using methods customarily applied to fixed-rate securities, and measured as the cost of the "uncertainty malaise" caused by the uncertainty surrounding the value of coupons.

Other factors naturally play a role in the complex phenomenon of mispricing and can be identified by examining the disparities between the hypotheses underlying the model of the market and the conditions in which it actually functions. It is plausible to imagine that the tax situation and the pressure of supply make a significant contribution to the mispricing found. The analysis only hints at the effects of the deterioration in the standing of CCTs as a result of historical factors such liquidity crises and doubts about the credibility of the reference index.[39] However, it is worth noting that in the observation period the spread incorporated in the indexed coupon offset all the misalignment between the annual returns in the primary and secondary markets, in the sense that the rollover guaranteed by CCTs was more profitable than the ideal rollover observed in the secondary market.

Technically speaking, it does not appear possible to prevent CCTs from being underpriced by means of changes in the indexation mechanism, such as a switch to synchronously indexed securities with a short coupon interval (*e.g.* a security with a three-month coupon, a three-month indexation lag and based on three-month Treasury bills), since this has been found to produce only a small change in the theoretical parity. Rather, the greater simplicity of synchronous indexation could be used to guide the market by making identification of the technical properties of indexed securities easier and hence their recognition by the public.

[39] See, for instance, Alesina, Prati and Tabellini (1990).

It nonetheless appears more important to try and overcome the misalignment between the returns at twelve months in the primary and secondary markets by reducing the interaction between the segmentation of demand and the intervention of the central bank at auction. The guarantee of alignment between the returns in the two markets and a reduction in the spread would improve the standing of CCTs.

Turning to an assessment of the methods used, the limited empirical correspondence of the model is not in itself evidence against the single-variable version of the CIR model or, more generally, of diffusion stochastic models. Rather, it can serve as the basis for a critique of single-variable arbitrage models. But such a critical approach would also undermine the (traditionally recognized) significance of the relationship between spot rates and implicit rates.

The proposal to improve the performance of the model by increasing the number of state variables is theoretically valid, but brings to mind the debate on the accuracy of share-market models: the generalness of a model does not always guarantee its explanatory power, and the additional effort required to estimate the extra parameters is not always matched by an increase in the model's accuracy.[40] Moreover, in the special case of the market for government securities the additional sources of risk appear difficult to incorporate satisfactorily in a model that remains logically consistent without weakening its structure by adopting simplistic *ad hoc* hypotheses.

The single-variable approach based on the no-arbitrage hypothesis retains a certain validity as a reference model. In any case, the synchronous model already permits the key properties of indexation to be identified, thereby providing an example of how a simple model built on robust foundations can "be useful in establishing logical limits to rhetoric".[41]

[40] Jarrow and Rudd (1983), p. 228.
[41] Bray (1985), p. 187.

APPENDIX 1

Consider a bond market that satisfies the usual perfect-market assumptions (frictionless, complete and with no arbitrage opportunities).

Denoting the valuation time (the current date) by t, $v(t, s)$ denotes the price at time t of a unit zero-coupon bond with maturity s ($t \leq s$).

In equilibrium conditions the function $v(t, s)$ represents the discount factor of the market in the interval from t to s (naturally, $v(t, s) \geq 0$ for every s, $v(s, s) = 1$ and $v(t, s)$ is a decreasing function of s). The annual interest rate relative to the period from t to s is given by $i(t, s) = v(t, s)^{-1/(s-t)} - 1$; as a function of s, $i(t, s)$ gives the term structure of interest rates at time t.

A payment stream x consisting of the non-negative amounts $x_1, x_2, ..., x_m$ payable at $t_1, t_2, ..., t_m$ (with $t \leq t_1 \leq t_2 \leq ... \leq t_m$) will have a value (price) at time t given by $P(t,\mathrm{x}) = \Sigma_{k=1}^{m} x_k\, v(t, t_k)$ and a (Macaulay) duration given by $D(t, \mathrm{x}) = \Sigma_{k=1}^{m} (t - t_k)\, x_k\, v(t, t_k) \,/\, P(t, \mathrm{x})$.

Consider the following:

Definition – *Given the dates t, T and s with $t \leq T \leq s$, we define as (unit) reinvestment security from T to s the zero-coupon bond, acquired at time t, that pays the uncertain amount $X_{T,s} = 1 / v(T, s)$ at time s.*

This is clearly a security that guarantees, at time s, the value of one lira invested from T to s at the market rate prevailing at T (not known at time t).

We therefore have the following:

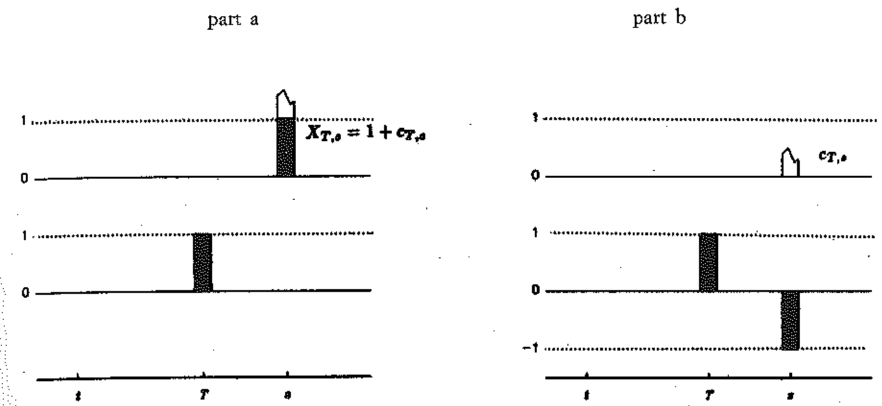
Theorem – *In order to prevent arbitrage opportunities at time t, the price $P(t, X_{T,s})$ of a reinvestment security from time T to time s must be equal to the price of a deterministic zero-coupon bond with maturity T; i.e. $P(t, X_{T,s})$ must be equal to $v(t, T)$.*

The proof of this theorem is a direct consequence of the arbitrage argument (Moriconi, 1992). It is easy to show, moreover, that the property continues to hold even after uncertain perturbations of the term structure of interest rates.

It follows from the theorem that, at time t, the uncertain amount "$X_{T,s}$ lire" payable at time s, is equivalent in terms of both price and sensitivity to changes in the interest rate, to a certain "1 lira" payable at time T.

In general, since $v(t, s) \leq 1$, the amount $X_{T,s}$ can be broken down into a certain, unit component (conventionally taken as corresponding to the principal) and into an uncertain interest component (the coupon). The arbitrage argument also results in the coupon, $c_{T,s} = [1 / v(T, s)] - 1$ being equivalent (in terms of its value) to the portfolio consisting of the uncertain positive amount $X_{T,s}$ payable at time s and a deterministic negative unit zero-coupon bond with maturity s, and hence can be replicated by compounding two deterministic zero-coupon bonds with maturities T (asset) and s (liability). The equivalence is shown schematically in Figure A1: part a shows the result of the application of the theorem to the uncertain amount $X_{T,s} = 1 + c_{T,s}$; part b shows the consequent equivalence between the uncertain coupon and the two deterministic zero-coupon bonds.

FIGURE A1



We refer to the maturity schedule (based on a coupon interval of one year), $t_1, t_1 + 1, t_1 + 2, ..., t_1 + m - 1$, with $t \leq t_1$, and to a security that pays $m - 1$ coupons starting from the maturity $t_2 = t_1 + 1$. More specifically, we consider the security with unit face value that at time $t_k = t_1 + k - 1$ ($k = 2, 3, ..., m$) pays the indexed coupon $c_k = i\,(t_{k-1}, t_k)$, amounting to the interest earned by investing one lira in the market from t_{k-1} to t_k (through the purchase of the zero-coupon bond with a unit duration, the underlying security of the indexation).

Since the indexed component, c_k, of each coupon can be replicated by means of a pair of zero-coupon bonds maturing at t_{k-1} and t_k (respectively positive and negative), the stream x made up of the $m - 1$ indexed coupons will be equivalent to a pair of deterministic unit zero-coupon bonds (respectively positive and negative) maturing at t_1 and t_m (since each component of the net payment stream generated by the portfolios replicating the indexed components is zero at the intermediate maturities), as can be seen in part a of Figure A2 in the case of four maturities.

It follows immediately that the cash flow **X** of the indexed components, with reference to a principal amount with face value C, has the following value (price) at time t:

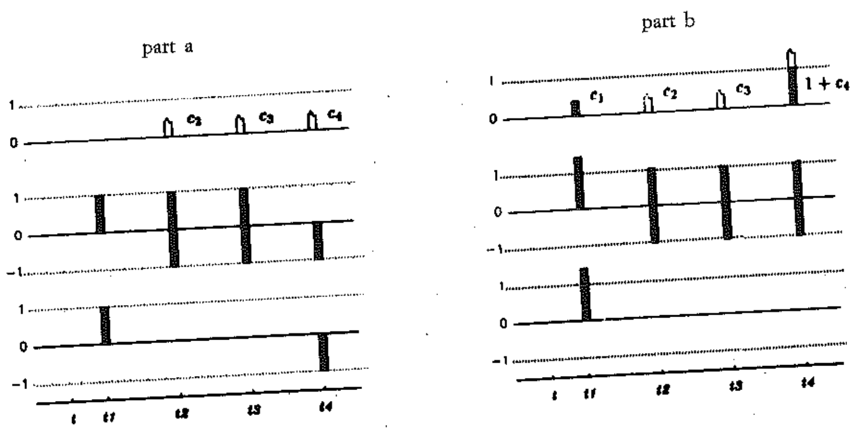
$$P(t, \mathrm{X}) = C\,[v(t, t_1) - v(t, t_m)]$$

and a (Macaulay) duration given by:

$$D(t, \mathrm{X}) = \frac{(t_1 - t)\, v(t, t_1) - (t_m - t)\, v(t, t_m)}{v(t, t_1) - v(t, t_m)} .$$

When $v(t, t_1) / v(t, t_m) < (t_m - t) / (t_1 - t)$ (as in the cases of practical interest), the duration takes on negative values.

FIGURE A2



If the (certain) amount C of the principal redeemed at maturity (t_m) is added to the indexed amounts, the resulting cash flow **X** will be equivalent to the deterministic zero-coupon bond with a value of C, payable at t_1 (as can be immediately shown by continuing the physical analogy of part b of Figure A2); we therefore have: $P(t, \mathbf{X}) = C\, v(t, t_1)$ and $D(t, \mathbf{X}) = t_1 - t$.

If account is also taken of the first (known) coupon (amounting to c_1 in t_1), we obviously have: $P(t, \mathbf{X}) = (C + c_1)\, v(t, t_1)$ and $D(t, \mathbf{X}) = t_1 - t$, since in this case the payment stream **X** that describes the indexed security is equivalent to a zero-coupon bond with a redemption value of $C + c_1$ and maturity t_1.

APPENDIX 2

Consider a diffusion model of the financial market in which the state variable (the only source of uncertainty) is the instantaneous interest rate, or the spot rate, $r(t)$, which follows Ito's stochastic differential equation:

$$dr = f(r, t)\, dt + g(r, t)\, dZ,$$

where f and g^2 are the infinitesimal parameters of the diffusion process $\{r(t)\}$, and $\{Z(t)\}$ is a standard Brownian motion.[1] Under the standard hypotheses of the new term-structure theory, the price at time t, $P(t)$, of a generic irs security is a function of the state variable $r(t)$, time and the vector **c** of the contractual parameters:

$$P(t) = P(r, t; \mathbf{c});$$

the general valuation equation, obtained using the arbitrage argument, is given by the differential equation:

$$P_t + (f + qg)\, P_r + \frac{1}{2} g^2\, P_{rr} - r\, P = 0, \qquad (1)$$

where $q = q(r, t)$ is a function (indipendent of **c**) that expresses the increase in expected instantaneous rate of return per additional unit of risk and defines the "market price of risk".

From (1) it is possible to derive the price at time t, $v(r, t; s)$, of an ordinary zero-coupon bond with maturity s ($t \leq s$), by imposing the boundary condition: $v(r, s; s) = 1$. The price, $v(r, t; s)$, as a function of s, describes the price structure of the market at time t; the term structure of interest rates is given by: $h(r, t; s) = - \log v(r, t; s) / (s - t)$; the long-term (asymptotic) rate of return r_L is defined as: $r_L = lim_{s \to \infty}\, h(r, t; s)$.

The reference framework for the valuation of CCTs can be set up by assuming that the coupons I_k ($k = 1, 2, ..., m$) are paid at the maturities $t_k = t + k\theta$, with $\theta > 0$, and determined in accordance with indexation rule:

$$I_k = \begin{cases} C\,[\frac{1}{v(T_k, T_k + \mu)} - 1 + \sigma], & \text{se } T_k > t, \\ C\,[\frac{1}{v(t, t + \mu)} - 1 + \sigma], & \text{se } T_k \leq t, \end{cases}$$

where:

$$T_k = t_k - \lambda, \qquad k = 1, 2, ..., m,$$

and $\mu > 0$, $\lambda > 0$, $\sigma \geq 0$ (the coupons are stochastic zero-coupon bonds).

[1] More exactly, the local rate is a force of interest (hence measured in years^{-1}); the corresponding value of the interest rate, on an annual basis and in percentage points, is given by: $i(t) = 100\, (e^{r(t)} - 1)$.

Under this rule each coupon depends on the issue price of an Italian Treasury bill maturity μ, observed λ units of time before each payment; the amount of the coupon I_k is given by the rate of return $i(T_k, \mu)$ plus a spread σ.

The indexed security is thus represented by the vector of random cash flows:

$$X = (I_1, I_2, ..., I_m + C),$$

paid at the times $t_1, t_2, ..., t_m$, and can be considered (and treated) as a portfolio of stochastic zero-coupon bonds.

The price function is given by:

$$P(r, t; X) = \sum_{k=1}^{m} P(r, t; T_k, t_k, I_k) + C\, v(r, t; t_m) \tag{2}$$

$$= C \left\{ \sum_{k=1}^{m} [\beta(r, t; T_k, t_k, \mu) - (1 - \sigma)\, v(r, t; t_k)] + v(r, t; t_m) \right\},$$

being:

$$\beta(r, t; T, s, \mu) = E_t \left[\frac{v(r, T; T + \lambda)}{v(r, T; T + \mu)} e^{-S(t, T)} \right], \tag{3}$$

where $S(t, s)$ is the stochastic integral:

$$S(t, s) = \int_t^s r(u)\, du + \frac{1}{2} \int_t^s q^2(r(u), u)\, du - \int_t^s q(r(u), u)\, dZ(u),$$

and E_t represents the conditional expectation taken with respect to the probability distribution that describes the process $\{r(t)\}$, specified by the value of the spot rate observed at time t.

Using the *ex ante* amount of the stochastic coupon I_k:

$$z(r, t; T_k, t_k, I_k) = \frac{P(r, t; T_k, t_k, I_k)}{v(r, t; t_k)}, \qquad k = 1, 2, ..., m,$$

the price of the indexed cash flow can be expressed in the form:

$$P(r, t; X) = \sum_{k=1}^{m} v(r, t; t_k)\, z(r, t; T_k, t_k, I_k) + C\, v(r, t; t_m).$$

The basic risk is given (using a simplified notation) by:

$$\Omega(r, t; X) = - \frac{P_r(r, t; X)}{P(r, t; X)}$$

$$= - \frac{1}{P} \left\{ \sum_{k=1}^{m} v_r(t_k)\, z(t_k) + C\, v_r(t_m) + \sum_{T_k > t} v(t_k)\, z_r(t_k) \right\},$$

since if $T_k \leq t$ the *ex ante* coupon is independent of r. In particular, for a zero-coupon bond with price $v(r, t; s)$ the basic risk is given by:

$$\varphi(r, t; s) = - v_r(r, t; s) \;/\; v(r, t; s).$$

The (overall) basis risk $\Omega(r, t; X)$ can be represented, by distinguishing the discount risk from the coupon risk, in the form:

$$\Omega(r, t; X) = \Phi(r, t; X) + \Gamma(r, t; X),$$

where the discount risk is given by:

$$\Phi(r, t; X) = - \frac{1}{W} \left[\sum_{k=1}^{m} \varphi(t_k)\, v(t_k)\, z(t_k) + \varphi(t_m)\, v(t_m)\, C \right]$$

and the coupon risk by:

$$\Gamma(r, t; X) = - \frac{1}{W} \sum_{T_k > t} v(t_k)\, z_r(t_k).$$

If $\lambda = \mu = \theta$ and $\sigma = 0$, we have the same results as with the synchronous indexation (Appendix 1). In fact, for each k:

$$\beta(r, t; T_k, t_k, \lambda) = v(r, t; T_k) = v(r, t; t_{k-1}),$$

where $t_0 = t$. Accordingly, the *ex ante* coupon expressed in the form:

$$z(r, t; T_k, t_k, I_k) = C \left[\frac{v(r, t; t_{k-1})}{v(r, t; t_k)} - 1 \right] = C\, j(t; t_{k-1}, t_k),$$

where $j(t; T, s)$ represents the forward rate at time t, for the period from T to s.

From (2), we have:

$$P(r, t; X) = C,$$

so that the indexed security is priced at par, as if it were a fixed-rate security with "forward" coupons.

For the basis risk, we have:

$$\Omega(r, t; X) = \varphi(r, t; t_1),$$

which corresponds to the stochastic duration.

$$D = t_1 - t = \theta.$$

In the one-factor model of the term structure of interest rates proposed by Cox, Ingersoll and Ross, it is assumed that the market is characterized by a diffusion process for the spot rate, with the following parameters:

$$f(r, t) = \alpha (\gamma - r), \qquad \alpha, \gamma > 0,$$

$$g(r, t) = \rho \sqrt{r}, \qquad \rho > 0;$$

and by a function for the market price of risk:

$$q(r, t) = \frac{\pi \sqrt{r}}{\rho}, \qquad \pi \text{ constant.}$$

Under these assumptions, the fundamental valuation equation (1) is specified in the form:

$$P_t + [\alpha \gamma - (\alpha - \pi)\, r]\, P_r + \frac{1}{2} \rho^2\, r\, P_{rr} - r\, P = 0. \tag{4}$$

The discount factor (obtained from equation (4) under the boundary condition $v(r, s; s) = 1$) is given by:

$$v(r, t; s) = A(t, s)\, e^{-r(t)\, B(t, s)},$$

where

$$A(t, s) = \left\{ \frac{2d e^{(\alpha - \pi + d)(s - t)/2)}}{(\alpha - \pi + d)\, e^{d(s-t)} - 1 + 2d} \right\}^{2\alpha\gamma/\rho^2},$$

$$B(t, s) = \frac{2\,[e^{d(s-t)} - 1]}{(\alpha - \pi + d)\,[e^{d(s-t)} - 1] + 2d}$$

and

$$d = \{(\alpha - \pi)^2 + 2\rho^2\}^{1/2}.$$

The long-term interest rate is constant: $r_L = 2\,\alpha\gamma\, /\, (d + \alpha - \pi)$.

The explicit form of the function β can be obtained from equation (4), with the boundary condition:

$$\beta(r, T; T, s, \mu) = \frac{v(r, T; s)}{v(r, T; T + \mu)}$$

(which is obtained from equation (3) by putting $t = T$).[2] The solution, given in Castellani (1988), is:

$$\beta(r, t; T, s, \mu) = M(t; T, s, \mu)\, e^{-r(t)\, N(t; T, s, \mu)},$$

where

$$M(t; s) = \frac{A(T, s)}{A(T, T + \mu)}$$

$$\left[\frac{2\, d e^{(\alpha - \pi + d)(T - t)/2}}{(\alpha - \pi + d)\,(e^{d(T-t)} - 1) + 2d + \rho^2\, \Delta B\, [e^{d(T-t)} - 1]} \right]^{2\alpha\gamma/\rho^2},$$

$$N(t; s) = \frac{2\,(e^{(T-t)} - 1) - \Delta B\, [(\alpha - \pi - d)\,(e^{d(T-t)} - 1) - 2d]}{(\alpha - \pi + d)\,(e^{d(T-t)} - 1) + 2d + \rho^2\, \Delta B\, [e^{d(T-t)} - 1]}$$

and

$$\Delta B = B(T, s) - B(T, T + \mu).$$

M.D.F. – F.M. – M.T.S.

[2] For further details, see De Felice and Moriconi (1991a).

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