

Methods to Determine Capital Requirements for Options

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1. Introduction

Measuring the risks associated with options is a complex business, not only because option prices depend in a non-linear fashion on a number of variables, but also because these risks need to be seen in connection with other positions. Uncertainty is a major factor that complicates matters here. In order to monitor these risks, it should preferably be possible to calculate the risks in a relatively simple manner, while the principles must also be compatible with the regulations emanating from Brussels (European Commission 1993) and Basle (BIS 1994).

The purpose of the present paper is to clarify the nature of the risks involved in options so as to be able to assess whether the various capital requirements being proposed are reasonable.¹ For this purpose we shall, in Section 2, approximate the risks involved in options by means of a second order Taylor expansion after the manner of the proposed capital requirements. In Section 3 we shall consider four different methods, namely the BIS's simple method for purchased options, the delta method proposed by the European Commission, the delta-plus method proposed by the BIS and the scenario analysis preferred by the BIS, as well as a few possible alternatives. We shall end with a summary of the main conclusions.

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¹ For a comparison of the Brussels and Basle capital requirements for other instruments, see Hall (1995).

2. The risks involved in an option

The price of European options (which can only be exercised on the expiry date) can be calculated by means of the Black-Scholes formula (1973). This calculates the value of the option on a share without dividend payments from the current spot price of the share, the strike price and the time to expiry of the option, the risk-free nominal interest rate and the volatility of the share price, quantified by means of its standard deviation. With a few minimal adjustments the formula can also be used for shares with dividend payments or other financial or non-financial underlying assets. The strike price and the time remaining to expiry of the option are known and so are not subject to risk.² The risk involved in options lies exclusively in unexpected changes in the price and volatility of the underlying and the interest rate.

Figures 1, 2 and 3 show the sensitivity of the value of a call and put option to changes in, respectively, the price of the underlying, the interest rate and volatility in accordance with the above-mentioned Black-Scholes formula. The option price seems particularly sensitive to changes in the price of the underlying, where a non-linear relationship is clearly evident. On the other hand, as far as the interest rate and volatility are concerned, a linear approximation appears to be adequate. The negligible effect on the option price of an interest-rate change is particularly striking here. In the case of options with a long period to expiry the interest-rate effect is somewhat greater. However, most options traded on the stock exchange have a time to expiry of at most six months. Volatility has a much greater influence on the option price, especially in the case of out-of-the-money options.

The risks involved in an option can also be illustrated by means of the second order Taylor approximation in the three uncertain factors. In the case of a call option it takes the following form:

$$C = C|_0 + C_{s|_0}(s-s|_0) + C_{r|_0}(r-r|_0) + C_{\sigma|_0}(\sigma-\sigma|_0) + 0.5 [C_{ss|_0}(s-s|_0)^2 + C_{rr|_0}(r-r|_0)^2 + C_{\sigma\sigma|_0}(\sigma-\sigma|_0)^2 + 2C_{sr|_0}(s-s|_0)(r-r|_0) + 2C_{s\sigma|_0}(s-s|_0)(\sigma-\sigma|_0) + 2C_{r\sigma|_0}(r-r|_0)(\sigma-\sigma|_0)].$$

² *Ceteris paribus*, the value of the option does diminish during the option period, since the uncertainty concerning the price of the underlying on the expiry date decreases. In the literature this decrease in value (in the case of a purchased option) is referred to as the theta risk. However, since this decrease in value is known in advance, there is no risk present here.

FIGURE 1

OPTION PRICE FOR DIFFERENT PRICES OF THE UNDERLYING
x = 100; r = 6%; volatility = 20%; t = 0.5 year

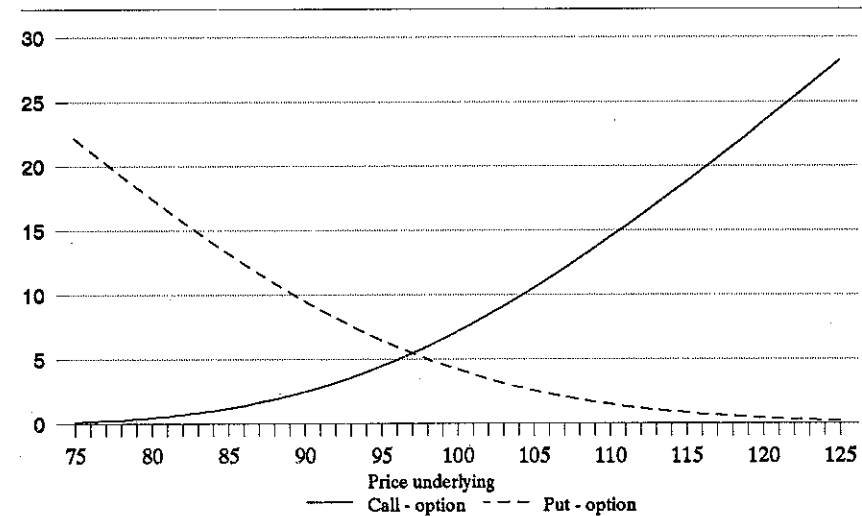


FIGURE 2

OPTION PRICE FOR DIFFERENT INTEREST RATES
x = 100; volatility = 20%; t = 0.5 year

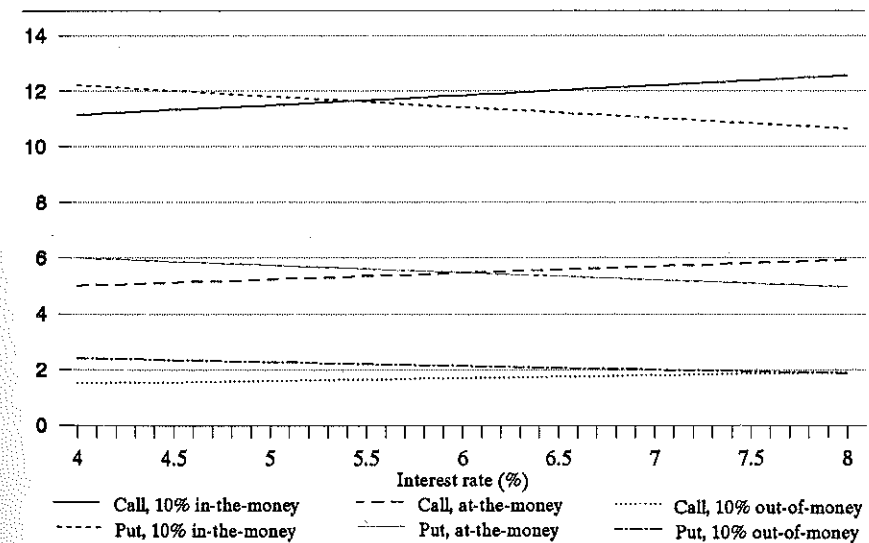
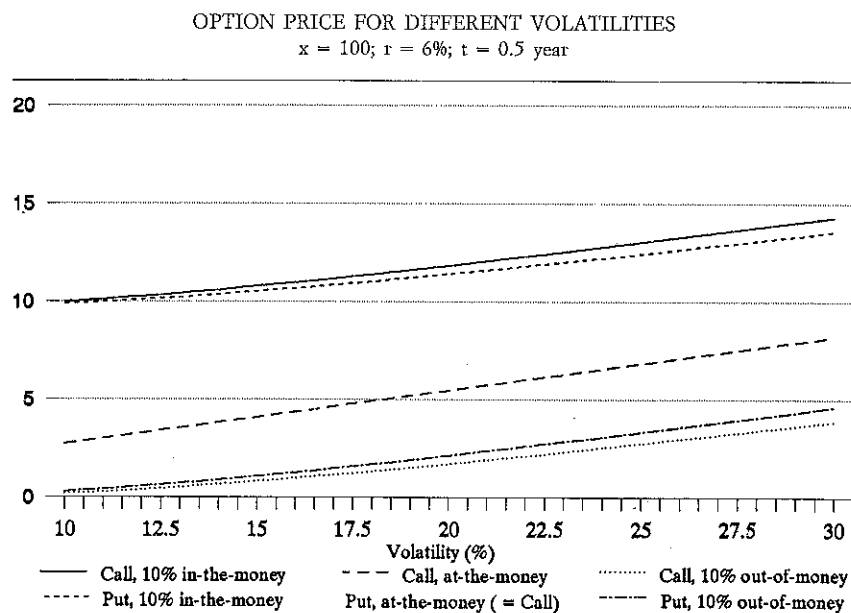


FIGURE 3



Here C , s , r and σ are respectively the call option price, the price of the underlying, the risk-free interest rate and volatility. C_i is the partial derivative from C to i and the $|_0$ subscript denotes the current value of a variable. Given the almost linear relationship between the option price, on the one hand, and the interest rate or volatility, on the other, the quadratic terms in these variables can be omitted. In the literature on options it is also assumed that the correlation between the various explanatory variables is insignificant, so that the cross terms can also be ignored. In the case of the cross terms with the interest rate this is quite justified, since the option price is hardly sensitive to the interest rate and consequently the second order effects are negligible. Volatility changes, on the other hand, have greater influence, and a simultaneous change in the price and the volatility of the underlying is not improbable. We shall here refer to the risk associated with this cross term ($c_{s\sigma|_0}$) as 'beta'. What remains are the first four terms of the approximation, which in the literature are referred to respectively as the delta ($C_{s|_0}$), rho ($C_{r|_0}$), vega ($C_{\sigma|_0}$) and gamma ($C_{ss|_0}$) risk.

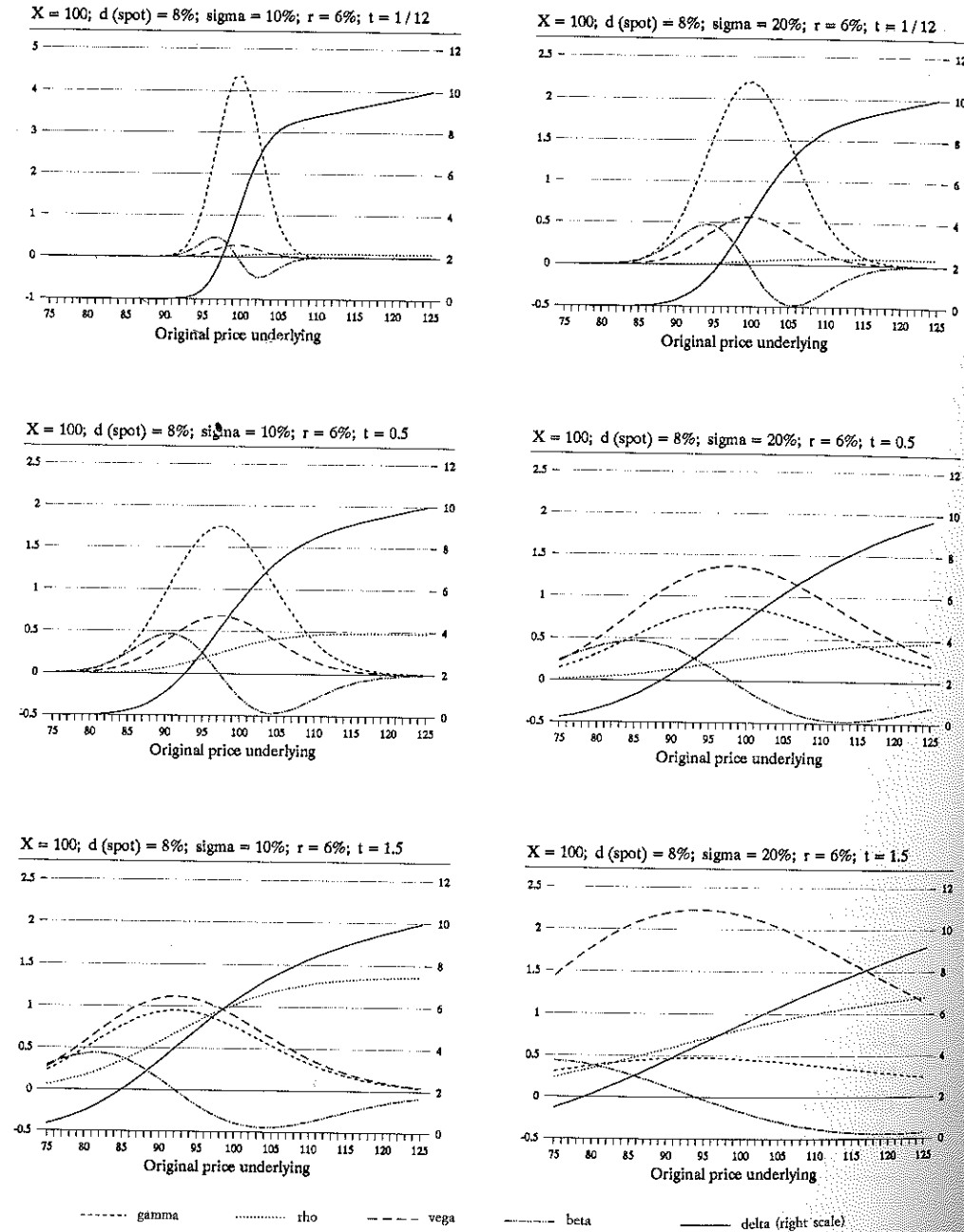
In Figure 4 these risk factors for a call option have been set against the price of the underlying.³ For the purposes of calculating the risk, a price rise in the underlying of 8%, an increase in volatility of 25% and an interest-rate rise of 1 percentage point have been assumed in each case. These percentages correspond to the assumed maximum changes in a share index in the BIS's and European Commission's proposals for determining capital requirements. The assumed strike price of the option is 100 ($X = 100$). The original volatility level (sigma) has been set at 10% (first column) and 20% (second column). This has been done in order to help compare the risks in the case of a low and high level of volatility. The assumed interest-rate is 6%, but another rate gives practically identical results. The risks have been calculated for a time remaining to expiry of respectively 1 month ($t = 1/12$; top pair of graphs), 6 months ($t = 0.5$; middle pair of graphs) and eighteen months ($t = 1.5$; bottom pair of graphs).

The delta risk seems to be the most significant risk factor for almost all types of options. The gamma and vega risks are also very important. The gamma risk is especially great if the assumed volatility is low and the time remaining to expiry short, since then there is less chance that the price change will be offset in the period to expiry. The vega risk is greater, the higher the volatility and the longer the time remaining to expiry. This is because the assumed volatility change over the period to expiry increases with the time remaining to expiry and volatility (measured on a yearly basis). The rho risk is only significant for options with a long period to expiry. For options with a time to expiry of up to six months the rho risk is even smaller than the beta risk. However, over-the-counter (OTC) options can run for much longer periods. Therefore, for such options a capital requirement in respect of the rho risk seems essential. The maximum loss in respect of the beta risk does not appear to be sensitive to the time remaining to expiry or to volatility. The level of this risk depends on both the price risks (gamma) and volatility (vega). As regards the beta risk, we see that an increase in volatility that is accompanied by a price rise in the underlying results in a relative increase in the value of the option if the option was out-of-the-money (and is thus moving in the direction of the at-the-money point), but results in a fall in value if the option was already in-the-money (and is thus moving away from the at-the-money point).

³ See Annex 1 for calculation of the risk parameters.

FIGURE 4

RISK FACTORS FOR A WRITTEN CALL-OPTION



3. Possible methods for determining capital requirements

This paper will consider four proposed methods, ranging from the simple to the sophisticated. As a general rule, it can be said that simple methods offer fewer opportunities for hedging and make less of a distinction between different types of option. For this reason capital requirements based on such methods will be relatively conservative. A balance therefore has to be struck between the computational burden and precision.⁴

3.1. A simple method for purchased options

The BIS's proposals for capital requirements for options make use of a simple method which can be applied to banks that only buy options (and so do not have any written options in their portfolios). Both the European Commission and the Nederlandsche Bank have adopted this proposal. The proposal consists of two parts. For a purchased option the capital requirement is simply the lowest of the market value of the option and the sum of the capital requirements in respect of the specific and market risk on the underlying. Only in the case of options which are very deeply in-the-money will these requirements be lower than the value of the option.

In the case of a purchased option combined with an offsetting position in the underlying, *i.e.* a short position in the case of a call option and a long position in the case of a put option, the capital requirement will be the sum of the capital requirements in respect of the specific and general market risks relating to the underlying less the amount that the option is in-the-money, with a minimum of zero. A number of objections can be made concerning these requirements. They are linked to a possible change in price, whereas the possible loss resulting from a price change is relatively small in such a combination if the option is not too deeply out-of-the-money. A change in value in the underlying is partially offset by a change in the value of the option if delta is not equal to zero. The discount given for in-the-money options is also questionable. For example, no capital requirement will be imposed for a combination involving an option

⁴ See Annex 2 for the calculation of the capital requirements.

which is deeper in-the-money than the sum of the capital requirements for the underlying. However, if volatility decreases and the price of the underlying remains the same, the value of the option, and hence the value of the portfolio, will decline (see Figure 5). In fact, the most important source of risk for this combination appears to be volatility, since if volatility decreases the option loses part of its premium, defined as the additional value of the option with respect to a future which derives from the right not to sell or buy the underlying.

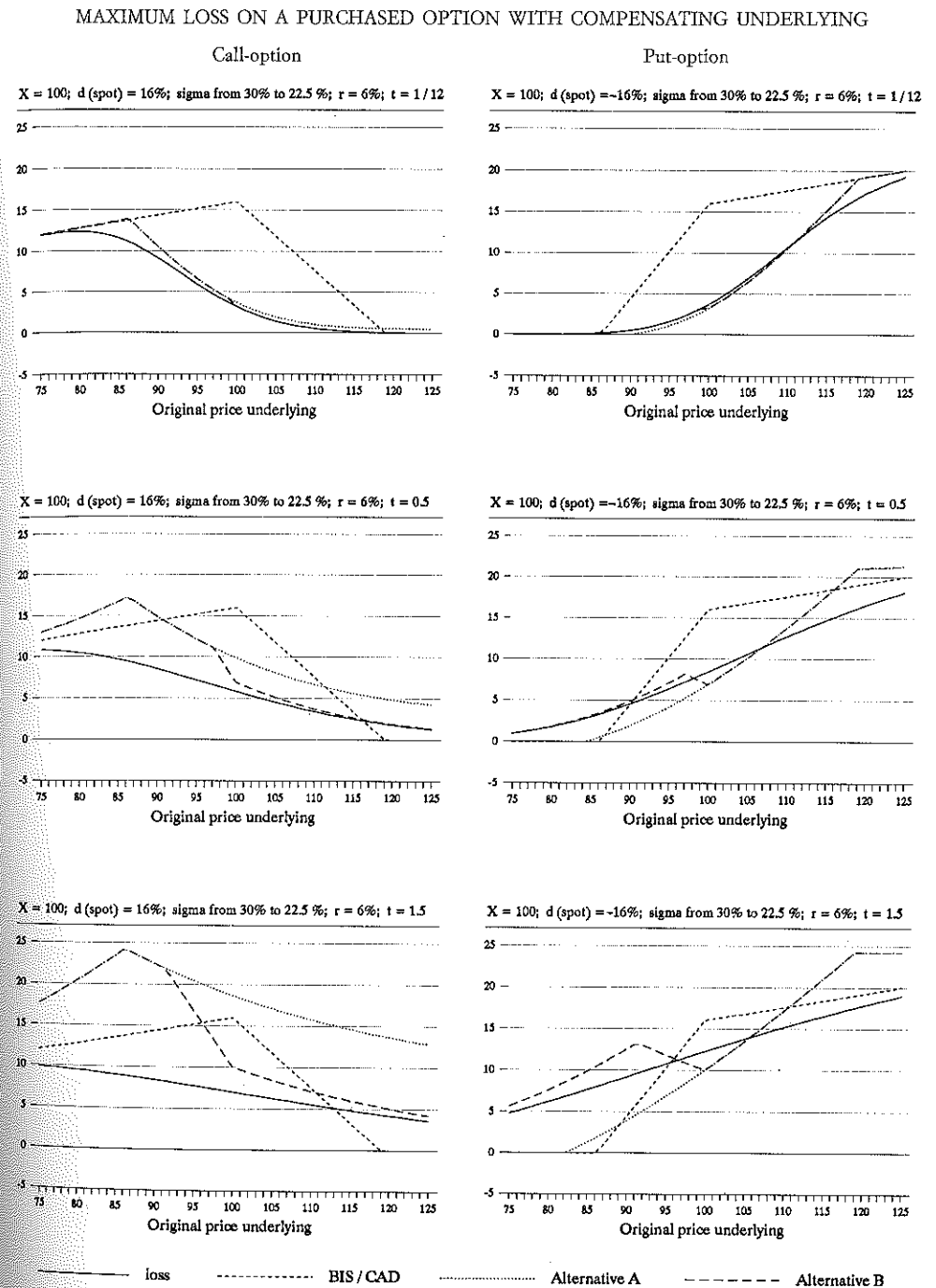
For this reason it would be more natural to take as the capital requirement the value of this premium plus the minimum of the amount that the option is out-of-the-money (in relation to the current spot price) or the sum of the capital requirements for the underlying, with a minimum of zero.

The results of these requirements can be seen in Figure 5. Here the at-the-money point has been taken as equal to the current spot price, exactly as in the examples given in the BIS proposal. In alternative A the premium has been determined on the basis of the spot price, in alternative B on the basis of the forward price. The graphs show that the BIS/CAD requirement practically never properly reflects the actual risks and is particularly unsatisfactory in the case of a short time remaining to expiry. The alternative is much better, especially if interest costs are taken into account when calculating the premium (alternative B).

3.2. The delta method

The delta method forms the basis of the Brussels proposals. In contrast to the simple method already discussed, this method can also be applied to written options. The method hinges on the linear approximation (the delta, see Annex 1) of the change in the price of the option resulting from a price change in the underlying. The option position is then taken for its delta equivalence value, *i.e.* delta times the underlying, together with other positions in the underlying in order to determine the capital requirement. In Figure 1 this means that the option price is approximated, in the region of the current price of the underlying, by the tangent. As Figure 1 makes clear, this linear approximation is only suitable when the assumed price change is small. In practice, however, we want to take account of price changes of more

FIGURE 5



than 10%, in which case a linear approximation can produce very misleading results. Moreover, the delta method does not take account of the consequences of possible changes in the volatility of the underlying value and possible interest-rate changes. Therefore, a minimum adequate mark-up will need to be found in order to take account of such risks. In the Brussels proposals the various supervisors can at their own discretion define the capital requirements for these risks.

When determining the minimum necessary mark-up a distinction needs to be made depending on whether one is dealing with written or purchased options. As can be seen from Figure 1, the risk incurred in respect of price changes in the underlying is always smaller in the case of the buyer of an option and greater in the case of the writer than is shown by the delta method. This is because the linear approximation always overestimates a price fall in an option resulting from a price change in the underlying and underestimates a price increase.

3.2.1. Written options on share indexes or on foreign currency

Figure 6 shows the maximum loss in addition to delta losses which the writer of respectively a call and put option can incur if the option relates to a share index or a foreign currency. The assumed maximum price change of 8% for these underlyings is the same as that assumed in the BIS guidelines. The assumed interest and volatility changes are likewise the same as those assumed in the BIS proposals. The volatility level for which the calculations have been carried out is 20%, which historically speaking is on the high side.⁵ This has been done in order to obtain a conservative estimate. Although the additional loss resulting from the non-linear relation between the option price and the price of the underlying decreases with the volatility, this is more than offset by higher possible losses resulting from a change in volatility, at least if the time to expiry of the option is not extremely short (see Figure 4). For the purposes of an upward or downward movement in interest rates, an interest rate of 6% has again been assumed, but a different rate produced practically identical results. Finally, the calculations have been carried out for options

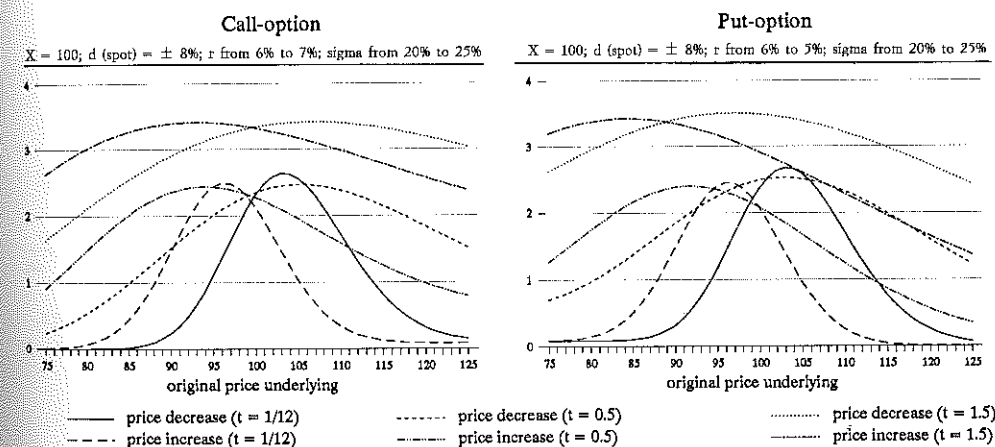
⁵ The average volatility level during the last few years of, e.g., the Amsterdam EOE Index and the guilder/dollar exchange rate has been about 15% per annum.

with three difference periods to expiry, namely 1, 6 and 18 months. The possible costs resulting from interest-rate or volatility changes increase and those resulting from a price change in the underlying decrease with the time to expiry.

The differences in risk between a call and a put option seem to be minimal. For both types of option the maximum risk in addition to delta losses is around 3.8% of the underlying. However, this percentage is only found with options with a long period to expiry (1.5 years). The majority of options traded on the stock exchange have a time to expiry of at most six months. For such options an additional capital requirement of 2.7% should suffice (for written options). The advantage of a capital requirement based on the underlying (instead of, e.g., the delta equivalence value) is that the highest risks, which determine the level of the capital requirements for all options, are incurred with options that are around the at-the-money point. Since this is the case for the vast majority of options, the capital requirements will be unnecessarily strict only for a few options.

FIGURE 6

MAXIMUM LOSS ON A WRITTEN OPTION IN A 'DELTA-NEUTRAL' PORTFOLIO

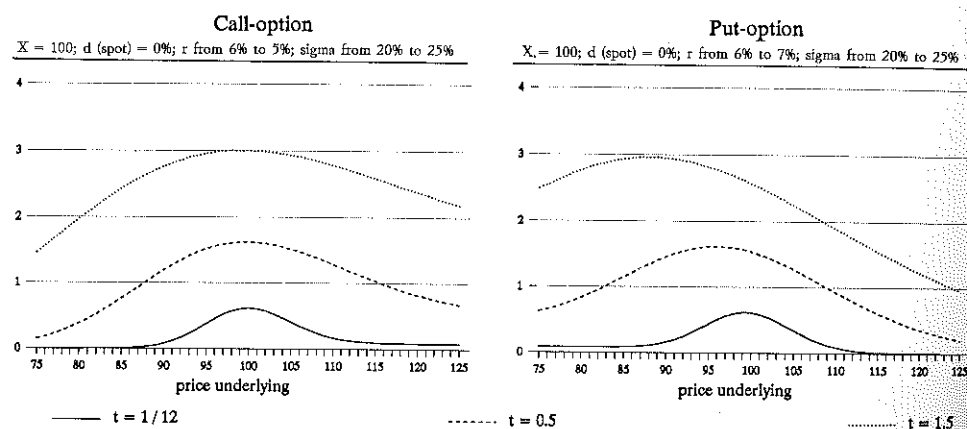


3.2.2. Purchased options on a share index or on foreign currency

Figure 7 shows the maximum risks in addition to the delta risks on respectively a purchased call and put option. Since the delta method systematically overestimates the risks for the purchaser of an option from price changes in the underlying, this price change has now been assumed to be zero. The maximum risks on a call and put option are again of the same order of magnitude. The risks clearly increase with the option's time to expiry, since both the interest-rate and volatility risks increase with the time to expiry, and these are no longer offset by the risks in respect of price movements. A simple mark-up on the capital requirement for purchased options on foreign currency or share indexes in order to cover the higher order risks (in addition to the delta risk) might be 1.7% of the underlying in the case of purchased options with a time to expiry of not more than six months and 3.4% in the case of options with a longer period to expiry.

FIGURE 7

MAXIMUM LOSS ON A PURCHASED OPTION IN A 'DELTA-NEUTRAL' PORTFOLIO



3.2.3. Other options

Comparable charts have also been produced for raw materials, individual shares and bonds. For raw materials and shares, the calculations assume a maximum price change of 15% and 12% respectively, in line with the BIS proposals. The assumptions for the maximum price changes for bonds, which depend heavily on the remaining times to expiry, also correspond to the BIS and European Commission proposals. Volatility is once again equal to two-and-a-half

times the maximum price change assumption (as it is for the other underlyings). Because of the close similarity with earlier charts the results for raw materials, shares and bonds are not reproduced in chart form here. But the results are summarised in Table 1. If the differences in capital requirements for options or bonds according to maturity is likely to present difficulties, it would of course always be possible to apply the strictest requirement to several categories of options. In the last resort, the current value of the option could serve as an alternative capital requirement as far as purchased options are concerned.

TABLE 1

CAPITAL REQUIREMENT FOR HIGHER ORDER RISKS ON OPTIONS

	Maximum price change (%)	Capital requirement as percentage of the underlying			
		Written option		Purchased option	
		t > 0.5 yr	t ≤ 0.5 yr	t > 0.5 yr	t ≤ 0.5 yr
Raw materials	15	6.8	4.8	5.1	2.9
Shares	12	5.5	3.9	4.3	2.3
Currency/share index	8	3.8	2.6	3.4	1.7
<i>Bonds remaining time to expiry</i>					
> 20 years	6	3.1	2.0	2.6	1.3
15 to 20 years	5.25	2.8	1.8	2.4	1.2
10 to 15 years	4.5	2.5	1.6	2.1	1.0
7 to 10 years	3.77	2.2	1.4	2.0	0.9
5 to 7 years	3.26	2.1	1.2	1.8	0.8
3 to 5 years	2.74	1.9	1.1	1.7	0.7
to 3 years	1.76	1.6	0.8	1.5	0.6

Note: The volatility assumption is equal to two-and-a-half times the maximum price change assumption.

3.3. The 'delta-plus' method

The 'delta method' described above merely divides losses into delta losses and other losses. The delta-plus method proposed by the BIS – which may also be applied to written options – makes a further sub-division of these other losses into gamma, vega and interest-rate losses, which means that a greater distinction can be made between various sorts of options and better use can be made of hedging opportunities. The method resembles a second order Taylor approximation, as described in Section 2. There are, however, two differences from the approximation considered there. Firstly, the BIS proposal

concerning interest-rate risk (which is, though, mentioned specifically in the BIS proposals only in respect of options on interest rate or debt instruments) is based on the capital requirements for an interest-rate instrument with a maturity corresponding to the time to expiry of the option and to an amount equal to the delta equivalence value (the costs of the hedge). These risks are thus not determined directly by the rho. Secondly, no account is taken of simultaneous changes in the volatility and the price of the underlying (the beta). The success of this method depends in part on the extent to which the Taylor expansion approximates the actual option price changes. It has been successively tested for the separate price-determining factors and for simultaneous changes. Figures 8, 9 and 10 show the Taylor approximation fault for a change in price, interest rate and volatility respectively.

The effects of a *price change* are approximated reasonably well by the delta and the gamma.⁶ It is only for options with a very short time to expiry and a low volatility that substantial differences may emerge between the actual change in the option price and the second order approximation. And these are the options on which the gamma risk is high (see Figure 4).

The rho provides a good approximation of the *interest-rate risk*, which is not very surprising, bearing in mind the virtually linear relationship between the interest rate and the option price (see Figure 2). The delta-based approximation is somewhat less satisfactory, especially for options which are deep in-the-money. The reason for this is that no account is taken of the return from the option for the writer of the hedge. The capital requirement calculated on the basis of the delta equivalence minus the value of the option appears to correspond exactly to the requirement calculated on the basis of the rho (see Annex 1). Adjusting for the value of the option is of some considerable significance, since the value of the option increases with the period to expiry while the interest-rate risk is significant only for long periods to expiry. One advantage of the delta over the rho in this context is that it is simpler to incorporate into the system for computing other interest-rate risks, making it easy to identify hedging opportunities.

⁶ A Taylor approximation ceases to perform as well for larger price changes; see Estrella (1995). However, the biggest problems in percentage terms occur with options which are far out-of-the-money. Given the small number of options for which this is the case and the low value of these options, this is not much of a problem in practice. See also Fase *et al.* (1990).

FIGURE 8

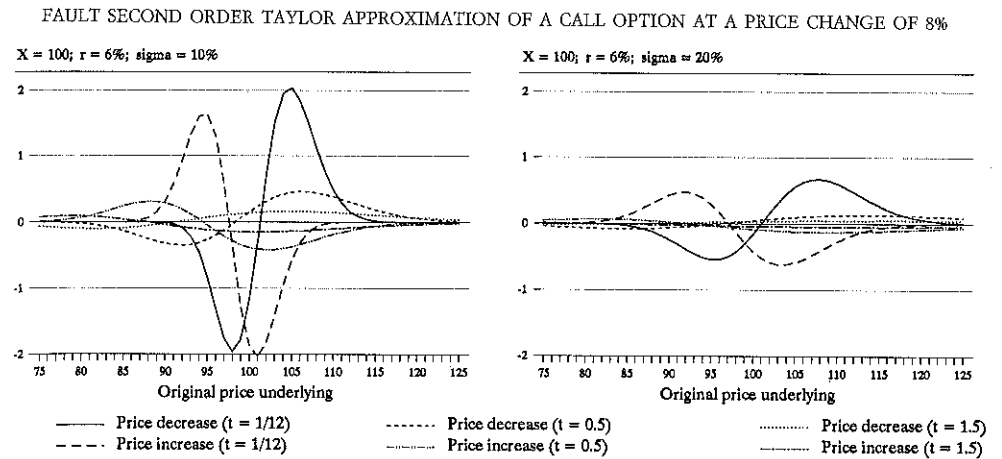


FIGURE 9

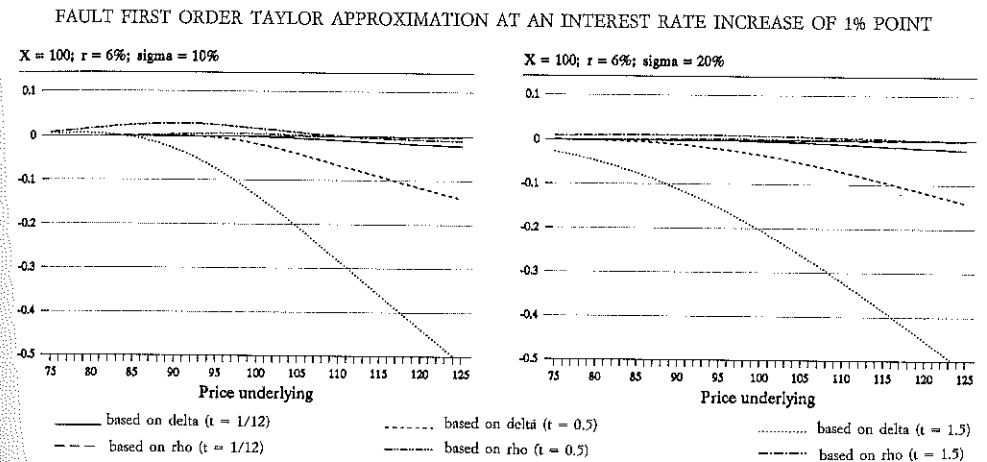
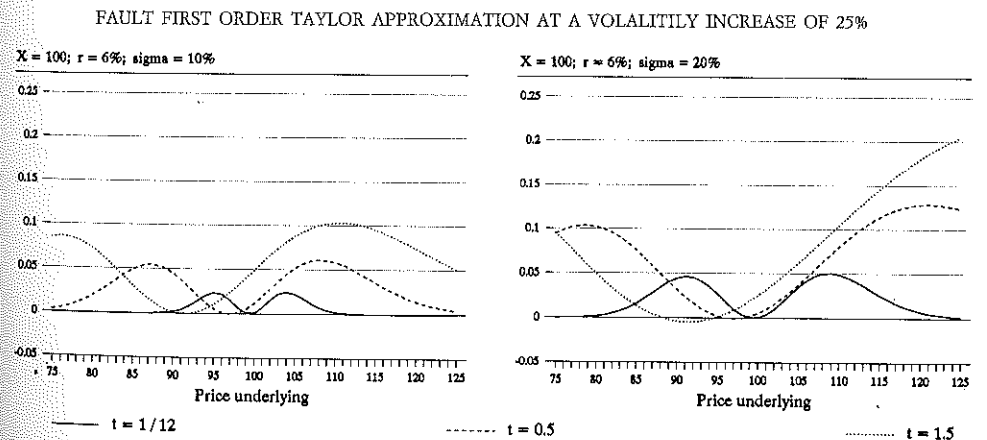


FIGURE 10



The vega provides a good approximation of the *volatility risks*. In virtually all cases the risk for the writer of the option is somewhat greater than would appear to be the case on the basis of the vega; but the differences are minimal.

Figure 11 shows the simultaneous effects of a change in price and volatility in relation to the capital requirements in terms of the delta, the gamma, the vega and (in some cases) the beta. The interest-rate risk is not involved in this simultaneous change because the rho provides a good indication of the interest-rate risk. Second order interest-rate effects do not appear to be significant. There is a high degree of overlap between the simultaneous effects and the effects of a price change alone. Once again the greatest deviations occur when the remaining time to expiry is short and the volatility is low. If no account is taken of the beta, as is the case in the standard delta-plus method, the simultaneous effects are greater, however.

Table 2 shows the maximum losses on top of the capital requirements based on the delta-plus method for a written call-option. For purchased options the maximum risk is always smaller than the capital requirements based on the delta, vega and rho (see Figures 9 and 10). It is only for options with a low volatility and a short time to expiry that the capital requirements based on the delta-plus method may prove substantially inadequate, especially if no account is taken of the beta risk. For far and away the majority of options the chances of a greater loss than accounted for by the delta-plus method including the beta appear negligible, however.

In both Table 2 and Figure 11 the absolute value of the beta is taken into account for the purposes of calculating the capital requirement. However, if several option positions are held in the same underlying, any positive beta position (computed on a positive change in price and volatility) may be offset by a negative position. The total capital requirement taking into account the beta for the underlying is in that case equal to the absolute value of the net balance of the beta positions. A similar offsetting effect can also occur with the vegas and the gammas, on the understanding that if the net balance of the gammas is positive (this is the case if more purchased options than written options are held in the portfolio), no capital requirement based on the gamma needs to be imposed in respect of the underlying in question.

FIGURE 11

LOSSES MINUS 'DELTA-PLUS' CAPITAL REQUIREMENTS ON A WRITTEN CALL OPTION AT A PRICE CHANGE OF 8% AND A VOLATILITY RISE OF 25%

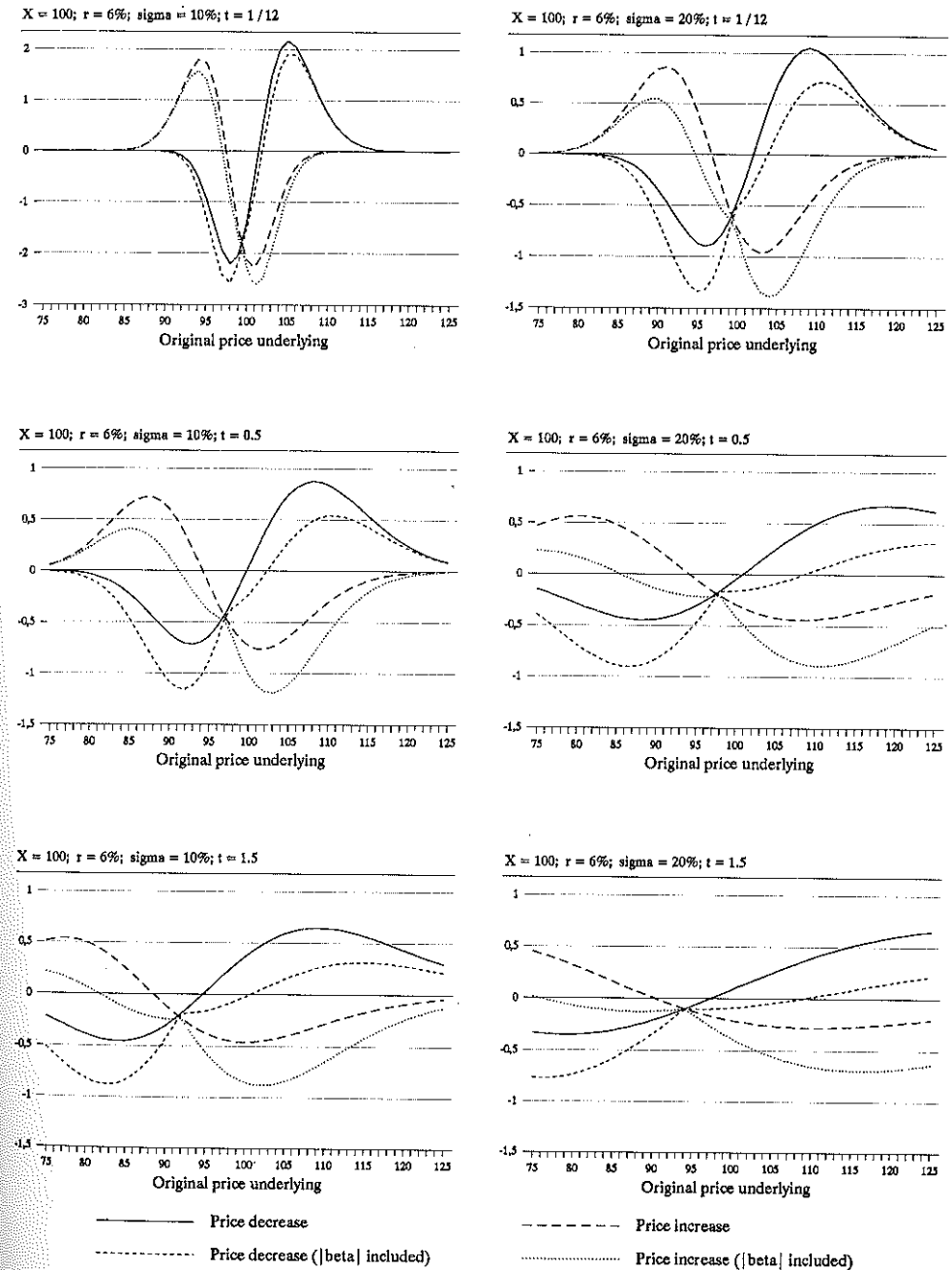


TABLE 2

MAXIMUM LOSS IN ADDITION TO CAPITAL REQUIREMENTS
UNDER DELTA-PLUS METHOD

	Max price change (%)	Maximum loss as percentage of underlying					
		Low volatility			High volatility		
		t=1/12	t=0.5	t=1.5	t=1/12	t=0.5	t=1.5
Raw materials	15	3.2 (2.4)	1.6 (1.1)	1.1 (0.7)	1.8 (1.3)	1.2 (0.2)	0.7 (0.1)
Shares	12	2.5 (1.8)	1.2 (0.8)	0.9 (0.5)	1.5 (1.0)	1.1 (0.2)	0.7 (0.2)
Currency/share index	8	1.6 (1.2)	0.8 (0.5)	0.6 (0.3)	1.0 (0.7)	0.7 (0.3)	0.6 (0.2)
Bonds remaining time to expiry							
>20 years	6	1.2 (0.9)	0.6 (0.4)	0.4 (0.2)	0.7 (0.5)	0.5 (0.2)	0.5 (0.2)
15 to 20 years	5.25	1.0 (0.8)	0.5 (0.3)	0.4 (0.2)	0.6 (0.4)	0.4 (0.2)	0.5 (0.2)
10 to 15 years	4.5	0.9 (0.6)	0.5 (0.3)	0.3 (0.2)	0.5 (0.4)	0.4 (0.2)	0.4 (0.2)
7 to 10 years	3.77	0.7 (0.5)	0.4 (0.2)	0.2 (0.1)	0.5 (0.3)	0.3 (0.1)	0.3 (0.1)
5 to 7 years	3.26	0.6 (0.5)	0.3 (0.2)	0.2 (0.1)	0.4 (0.3)	0.3 (0.1)	0.3 (0.1)
3 to 5 years	2.74	0.5 (0.4)	0.3 (0.2)	0.2 (0.1)	0.3 (0.2)	0.2 (0.1)	0.2 (0.1)
to 3 years	1.76	0.3 (0.2)	0.2 (0.1)	0.1 (0.1)	0.2 (0.1)	0.1 (0.1)	0.2 (0.1)

Note: The original volatility level is equal to one-and-a-quarter and two-and-a-half times, respectively, the maximum price change assumption. The volatility increase assumption is 25. The figures in brackets show the maximum losses if the beta risk is also taken into account.

All in all, the risks on options appear to be approximated reasonably well by the delta-plus method. The administrative burden of the method – five parameters need to be computed – could, however, be seen as a drawback.

3.4. Scenario analysis

From the previous sub-section it would appear that the rho or the delta provide a simple and good approximation of the interest-rate risk on an option. Furthermore, this risk can also be easily compared with other interest-rate risks, with the result that hedged positions can be held without coming under the capital requirement.

Hedging an option on the underlying against a price change in the underlying is possible under the delta-plus method only in respect of the linear approximation (the delta). Furthermore, it can be seen from Figure 11 that the price and volatility risks are not always approximated correctly by the second order Taylor expansion. In order to overcome these difficulties, when it comes to determining the price and volatility risks on options and other investments, it should be possible to calculate the value of the whole portfolio for the complete range of possible price and volatility changes. In fact, this amounts to a scenario analysis. This method is preferred by the BIS to the delta-plus method, but as yet the European Commission has sanctioned the method only for internal use within banks.

The most important advantages of the method are that it allows all possible hedge positions in the portfolio to be taken into account and that the risks are calculated exactly. The additional computational burden of the method, as opposed to the delta-plus method, depends primarily on the model that is used to determine option prices. If the option pricing model is not too complicated, this method is preferable to the delta-plus method.

4. Conclusions

This paper has looked into the risks attaching to options and compared these risks with the capital adequacy requirements proposed by the BIS and the European Commission as well as some alternatives. It covers risks arising from unexpected changes in the price or volatility of the underlying security or changes in interest rates. The non-linear relationship between the option price and the price of the underlying security, in particular, makes quantifying the risks a complex matter.

The following conclusions can be drawn regarding the various methods.

1) The simple method for purchased options, as proposed by the BIS and adopted by the European Commission and the Nederlandsche Bank, performs poorly in terms of approximation of the potential losses. A simple alternative is available, but this might not fit in with the EU Capital Adequacy Directive, which comes into binding force in January 1996.

2) For the delta method, on which the CAD is based, a simple mark-up can be calculated for higher order risks. A disadvantage of this method, though, is that interest-rate risk and volatility risk cannot be hedged at all and the price risk can be hedged only in respect of the linear portion. It is partly for that reason that the capital requirements are relatively high.

3) The delta-plus method proposed by the BIS approximates the risks well – with the possible exception of options with a very short time to expiry and a low volatility – provided it is adjusted in two ways. Firstly, a simultaneous change in the price and the volatility of the underlying (the beta) needs to be taken into account. Secondly, in determining the interest-rate risk the option price needs to be deducted from the delta equivalence value. This method allows both interest-rate and volatility risks to be hedged. Partly because of these adjustments, the total computational burden of this method is relatively high.

4) The advantage of scenario analysis (preferred by the BIS) is that optimal use can be made of all hedging opportunities and that the risks are calculated exactly instead of being just approximated. If a sophisticated model is used to determine option prices, the computational burden can be high, though.

All in all, when it comes to choosing the method, the precision that is thought desirable will have to be weighed against the benefits of simplicity. Simple methods make less distinction between different types of option and take less account of hedging opportunities, leading to relatively high capital requirements. Sophisticated methods, on the other hand, give rise to a higher computational burden. The outcome of these deliberations will depend primarily on the importance of options in the total investment portfolio.

ANNEX 1

The Black-Scholes formula and its derivatives

According to the Black-Scholes formula the call option price (C) is given by $C = sN(d_1) - xe^{-r(T-t)}N(d_2)$

$$\text{with } d_1 = \frac{\ln(s/x) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \text{ and } d_2 = d_1 - \sigma\sqrt{T-t}$$

where s , x , r , $T-t$ and σ are, respectively, the price of the underlying security, the strike price, the interest rate, the remaining time to expiry and the volatility of the underlying security, and where $N(d_1)$ is the value of the standard normal distribution function for the value d_1 .

The relevant partial derivatives are as follows:

$$\text{delta} = \frac{\partial C}{\partial s} = N(d_1)$$

$$\text{gamma} = \frac{\partial^2 C}{\partial s^2} = P(d_1) \frac{1}{s\sigma\sqrt{T-t}}$$

where $P(d_1)$ is the value of the standard norm density function in d_1 .

$$\text{rho} = \frac{\partial C}{\partial r} = x(T-t)e^{-r(T-t)}N(d_2)$$

$$\text{vega} = \frac{\partial C}{\partial \sigma} = sP(d_1)\sqrt{T-t}$$

$$\text{beta} = \frac{\partial^2 C}{\partial s \partial \sigma} = P(d_1) \left[\frac{\sqrt{T-t}}{2} - \frac{\ln(s/x) + r(T-t)}{\sigma^2\sqrt{T-t}} \right]$$

ANNEX 2

Algebraic expressions for the capital requirements

The maximum expected price change of the underlying (denoted Δs_{\max}) is given in Tables 1 and 2. The requirements are computed for a call-option (C). The future price is denoted f .

1. Simple method for purchased options

- single purchased option: $REQ_{\text{purchased}} = \max(\Delta s_{\max}, C)$
- combination purchased option + offsetting position in underlying
 - BIS/CAD: $REQ_{\text{combination}} = \min[0, \Delta s_{\max} - \max(0, s - x)]$
 - alternative A: $REQ_{\text{alternative A}} = C + x - s + \min[\Delta s_{\max}, \max(0, x - s)]$
 - alternative B: $REQ_{\text{alternative B}} = C + x - f + \min[\Delta s_{\max}, \max(0, x - s)]$

2. The delta method: $REQ_{\text{delta}} = \text{abs}(\text{delta}) \Delta s_{\max} + \text{mark-up}_{\text{table 1}} s$

where the delta-position can be hedged by other positions in the same underlying.

3. The delta-plus method: $REQ_{\text{delta-plus}} = \text{abs}(\text{delta}) \Delta s_{\max} - 0.5 \min(0, \text{gamma}) \Delta s_{\max}^2 + 0.01 \text{rho} + 0.25 \text{abs}(\text{vega}) \sigma + 0.25 \text{abs}(\text{beta}) \Delta s_{\max} \sigma + \text{mark-up}_{\text{table 2}} s$

where the delta-, gamma-, vega- and beta-positions can be hedged by other positions in the same underlying, and the rho-risk can be hedged by other interest rate instruments with equal maturity.

4. Scenario analysis: $REQ_{\text{scenario}} = \text{value}_{\text{portf.}} - \min_{s, \sigma, r} \text{value}_{\text{portf.}}$

The minimum value of the entire portfolio is computed over the ranges:

$$\begin{aligned} s - \Delta s_{\max} &< s < s + \Delta s_{\max} \\ 0.75 \sigma &< \sigma < 1.25 \sigma \\ r - 1\% &< r < r + 1\% \end{aligned}$$

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