Harrod and Domar
on Dynamic Economics

1. Introduction

Harrod and Domar developed their theories of growth indepen-
dently even though Harrod's first publication of the main elements of
his theory was in the *Economic Journal* in 1939, while Domar's was in
Econometrica in 1946. Domar notes that he first became aware of
Harrod's article after he had sent off to the printer a companion article
to the 1946 paper that was published in 1947.[1] The particular circum-
stances of the time — the intervening world war and its disruptions of
normal academic exchanges — explain why Harrod's pioneering article
was overlooked by Domar (and others). Most economists became aware
of the Harrod and Domar theories — Harrod lectured on his approach
to dynamic economics at the University of London in 1947, and these
lectures were first published in 1948 — at roughly the same time. Both
authors broke out of the static framework of Keynes's *General Theory*
by recognizing the productive-capacity creating, as well as the
aggregate-demand creating, effects of investment. Their equilibrium
rates of growth — involving propensities to save and coefficients that
could be taken to represent some normal relation between the change in
productive capacity and net investment — appeared to be the same, and
the two approaches were joined in the textbooks and in numerous
articles as the Harrod-Domar model.[2]

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[1] The two key papers in which Domar's theory was first presented are DOMAR (1946) and
DOMAR (1947). In addition there was a third paper, DOMAR (1948), written after Domar had read
Harrod's 1939 article, that introduced a growth rate that is similar to the basic growth rate in
Harrod's dynamic economics. All three papers; with a few minor modifications, were reprinted in
DOMAR (1957), and all references here will be to that book.
[2] See, for example, the well-known survey article by HAHN and MATTHEWS (1964), where
they write: "Our procedure will now be to take as our point of departure one well-known model,

Domar's published comments on Harrod's theory noted that the ideas it contains "are similar to those presented here" (Domar 1957, 92n), but he did not attempt any detailed comparison of the two. He contented himself with noting in a 1948 paper that "[A] good theoretical exposition [of the problem of capital accumulation] belongs to R.F. Harrod in a paper published in 1939" (*ibid.*, 121), and then in a 1952 paper he referred "to Harrod's now famous creation of 1939, which... had to wait for almost a decade and to be repeated in his book to receive its deserved recognition" (*ibid.*, 18). Harrod, on the other hand, wrote an article "Domar and Dynamic Economics" published in 1959 that noted some differences between the theories but stressed their similarities. In doing this, Harrod wrote that "I by no means wish to follow a precedent that may occur to the minds of some readers and to stress the difference between us" (Harrod 1949, 455). The reference here is probably to the confusion and controversy surrounding the almost simultaneous publication of E. H. Chamberlin's *The Theory of Monopolistic Competition*, and Joan Robinson's *The Economics of Imperfect Competition*, but this example does not justify the avoidance of a critical comparison of the two approaches. In spite of some similarities — the use of downward-sloping demand curves for individual firms and the determination of equilibrium by the equating of marginal revenue and marginal cost — there were significant differences to the approaches found in these two books as Chamberlin (1948, 191-218) kept insisting.[3] There are also important differences between the Harrod and Domar approaches to the theory of growth, and a critical comparison of these theories will help to better understand the strengths and weaknesses of each.

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the Harrod-Domar model from which much subsequent work has stemmed" (783). In constrast, ASIMAKOPULOS and WELDON (1965) in their presentation of some simple models of growth write: "we maintain that it is a mistake to link Harrod's model with that of E.D. Domar in the often encountered Harrod-Domar model..." (52).

[3] Robinson, although recognizing similarities, admitted important differences, "... I have never been able to grasp the nature of the distinction between *imperfect* and *monopolistic* competition to which Professor Chamberlin attaches so much importance ... It appears to me that where we dealt with the same question, in our respective books, and made the same assumptions we reached the same results (errors and omissions excepted). When we dealt with different questions we naturally made different assumptions. In many respects Professor Chamberlin's assumptions were more interesting than mine, in particular in connection with oligopoly and with product differentiation as a dynamic process" (ROBINSON, 1960, 222n, italics in original).

2. Equilibrium rates of growth

Both Harrod and Domar emphasize that once the productive-capacity creating effects of investment are introduced into a model, the crucial unknown variable is expressed in terms of the rate of growth. This is in contrast to Keynes's *General Theory*, where productive capacity was assumed to be constant while the demand effects of investment were examined, and the dependent variable, income (or employment), is expressed in absolute terms. Harrod made this distinction the essential feature of his definitions of statics and dynamics in economics.

> "In economic Statics we take certain fundamental conditions to be given and known, the size and ability of the population, the amount of land, tastes, etc., and these are deemed to determine the values of certain unknowns, the rates of output per annum of each of the various goods and services, the prices of the factors and of the goods and services. In Dynamics, on the other hand, the fundamental conditions will themselves be changing, and the unknowns in the equations to be solved will not be rates of output per annum but increases or decreases in the rates of output per annum" (Harrod 1948, 4).

The equilibrium concept around which the analysis turns differs for these two authors. For Harrod the fundamental growth equation is that which defines the warranted rate of growth, the entrepreneurial equilibrium rate of growth, where *entrepreneurs* feel that their investment in the period has been justified by the increase in output over the period. Achievement of a warranted rate of growth is not inconsistent with continuing involuntary unemployment. Domar's fundamental equation, in contrast, is concerned with the equilibrium rate of growth that will maintain full employment of labour. Harrod also introduced such a growth rate — his natural rate of growth — which could differ from the warranted rate, while Domar added (Domar, 1948) some consideration of the rate of growth that is "needed for a full utilization of *capital*" (Domar 1957, 114n, italics in original). This latter rate of growth is related to Harrod's warranted rate of growth. The derivation of these rates and the relative emphases given to them show important differences.

3. Harrod's fundamental equation[4]

Harrod's fundamental equation is the equation that defines a warranted rate of growth. It is derived by manipulating the identity between saving and investment in situations where *entrepreneurs* consider that the investment in the period has been justified by the increase in output over the period. Harrod's "analysis relates to a single point of time" (Harrod 1939, 24), where this "point" is Keynes's or Marshall's short period. Marshall (1920, 379) referred to "a few months or a year" as the possible length for such a period, and Harrod's explicit references to the length of the period over which the warranted rate of growth is measured fall in this range.[5] Harrod changes Keynes's identity between gross investment and gross saving, to an identity between net investment and net saving, since the relation between net investment and the increase in output is a crucial element in his theory. The amount subtracted for depreciation in any period in order to arrive at "net" estimates is, presumably, based on some accounting convention. This amount will not measure actual depreciation unless the period is one of a sequence of periods in which long-period equilibrium conditions hold and the depreciation accounting convention is an appropriate one for that situation. Using I to represent net investment and S net saving, we have

$$(1) \qquad I = S$$

If both sides of equation (1) are multiplied by equivalent forms of $1/Y$ (the left-hand side by $(\Delta Y/Y)(1/\Delta Y)$, and the right-hand side by $1/Y$, where Y is national income or output), this equation is transformed into an equation that incorporates a rate of growth, and thus according to Harrod becomes dynamic.[6] It is written as

$$(2) \qquad GC = s \quad \text{or} \quad G = s/C$$

where $G (=\Delta Y/Y)$ is the rate of growth of income; $C (=I/\Delta Y)$ is

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[4] This is how HARROD (1939, 15-16) refers to the equation for the warranted rate of growth.
[5] There is, for example, the reference to six months in his first paper (HARROD 1939, 26), and a later reference to three months (HARROD 1951, 269).
[6] "I prefer to define dynamic as referring to propositions in which a rate of growth appears as an unknown variable" (HARROD 1939, 17).

the ratio of net investment to the increase in income over the period, and s $(=S/Y)$ is the fraction of income saved.[7] The increase in output (ΔY) is the increase in this period's output as compared with last period's output, and the value for output (Y) used in these expressions is the current period's output.[8] The value of the product GC is independent of the length of the period used for the "point of time", with G being expressed as a pure number per unit of time, and C being expressed in terms of this unit of time.

Equation (2) — the transformation of the familiar identity between investment and saving into a "dynamic" equation — is itself transformed into Harrod's fundamental equation that defines the warranted rate of growth, if it so happens that the *entrepreneurs* judge the investment they have undertaken to be justified by the increase in output over the period. In this case C is equal to C_r, where C_r is "the required capital coefficient ... [it] is of course a marginal notion ... [and] may not be equal to the capital coefficient in the economy as a whole" (Harrod 1948, 82-84). Its value "depends on the state of technology and the nature of the goods constituting the increment of output. It may be expected to vary as income grows and in different phases of the trade cycle; it may be somewhat dependent on the rate of interest" (Harrod 1939, 17). In this fundamental equation the fraction of income saved is equal to the average propensity to save in the economy, which is denoted by s_d. It is "the fraction of income which individuals and corporate bodies choose to save" (*ibid.*, p. 16). The equation for the warranted rate of growth can thus be written as

$$(3) \qquad G_w C_r = s_d \quad \text{or} \quad G_w = s_d/C_r$$

where G_w is the warranted rate of growth.

Although, as the derivation of equation (3) makes clear, this fundamental equation refers only to conditions at a point in time, Harrod wants it to generate a trend line — with the trade cycle being

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[7] There have been some changes in the notation used by Harrod in his writings on this topic. The notation used here is that employed in HARROD (1973).
[8] Harrod had initially used the previous period's income in the denominator in calculating the rate of growth of income, although he had noted that the current period's income could also have been used for this purpose (HARROD 1939, 16). After ALEXANDER (1950) pointed out that the identification of s as the proportion of income saved required that the income referred to be the current period's income, HARROD (1951, 268-9) adopted the convention used here.

regarded as oscillations around this line of steady growth.[9] He thus not only states that the warranted rate of growth "will leave all parties satisfied that they have produced neither more nor less than the right amount" (Harrod 1939, 16), but he continues, "it will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth" (*ibid.*). The second of these statements does not follow from the first. It is one thing to feel satisfied with production and investment in a period, it is quite another matter to try to bring about the same rate of growth in subsequent periods.[10] Decisions about investment are taken by a very large number of *entrepreneurs* in Harrod's model, and he recognizes that not all may be satisfied with economic conditions — "what applies to the system in general may not apply to each individual separately" (Harrod 1939, 22). But Harrod assumes that when a warranted rate of growth is achieved, the effects of very favourable and unfavourable experiences for producers cancel out, so that "if one feels he has over-produced or over-ordered, this will be counterbalanced by an opposite experience of an equal importance in some other part of the field" (*ibid.*).

With his concentration on a line of steady advance, Harrod abstracted from the time lags between changes in income and the resulting changes in consumption expenditures and from the time lags between investment activity and the use of the resulting plant and equipment to produce goods. On a warranted growth path saving is in the desired relation to income, and the full multiplier effects of higher investment in the period are completed within the period. Some of this investment activity would be the result of the perception of future needs, and its justification could not be determined on the basis of

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[9] Harrod, in a historical note on the development of his views on dynamic economics, wrote: "From early days I have held the view that we are not likely to make a correct analysis if we proceed by regarding the phenomena of boom and slump as deviations from a static equilibrium. Instead, I have proposed that they should be regarded as oscillations around a line of steady growth" (HARROD 1951, 261). The position taken in this paper is that Harrod tries to use the "normal" warranted rate of growth to define an equilibrium growth path, "a moving equilibrium" (Harrod 1939, 22), that provides the required "line of steady growth" around which to build his trade cycle analysis. A different view is expressed by KREGEL (1980, 102) who writes that with the 1939 "Essay" "[T]he analysis is no longer primarily concerned with explanation of fluctuations around the trend rate of growth, but with the explanations of the possibility of fluctuations *in the trend rate itself*" (italics in original). Harrod's analysis "relates to a single point of time" (HARROD 1939, 24), but it is also concerned with the steady rate of growth implied by the values for the parameters, as Harrod made clear in his paper on Domar. "My [fundamental] equation purported to specify what the steady rate of growth at a given point in time would be, given the values of the dynamic determinants" (HARROD 1959, 454).

[10] This was pointed out by ALEXANDER (1950, 728).

current changes in output. Harrod showed that his method for the derivation of the warranted rate of growth can be readily adapted to allow for such long-range investment. The investment that falls in this category, which Harrod denotes by K, is subtracted from both sides of equation (1). We have

$$(4) \qquad I-K = S-K$$

Multiplying both sides of equation (4) by different forms of $1/Y$, we again obtain an equation involving the rate of growth of output. In place of equation (2) we have

$$(5) \qquad GC = s-k$$

where the capital coefficient, C, is now equal to $(I-K)/\Delta Y$, the ratio of net investment whose appropriateness is to be judged by short-term changes in output, to this period's increase in output. k is the fraction of the period's output that is devoted to long-range investment projects and, if it is positive, both the right-hand side of equation (5) and the value for C are smaller than the comparable values in equation (2). If the investment, adjusted for its long-range portion, is considered to be justified, given the period's increase in output, then C is equal to C_r and the equation that includes the warranted rate of growth is

$$(6) \qquad G_w C_r = s_d - k$$

Harrod recognized that even for investment geared to short-term needs "there must be some lag between the increased provision of equipment (and stocks?) and the increased flow of output which they are designed to support" (Harrod 1939, 20). He argued that the neglect of this lag "is necessary in order to get the clearest possible view of the forces determining the trend and its influence as such" (*ibid.*). Further, for a steady advance — the focus of Harrod's attention — "it matters not whether we regard the increment of capital as required to support the increment of total output in the same period or in the one immediately succeeding it" (*ibid.*), since the differences in the increases in output between two pairs of adjacent periods is "of the second order of small quantities" (*ibid.*). For these reasons he believed that equation (3) (or equation (6)) showed the relations that hold on a trend line of steady growth.

The transformation of equations (2) and (5) to arrive at Harrod's warranted rate of growth makes clear that it is not based on some *ex ante* investment function. Whether a rate of growth of output is warranted or not depends on an *ex post* judgement of the appropriate relation between the investment (suitably adjusted) in the period and the increase in output that occurred in that period. Harrod's fundamental equation thus refers to "justified investment" (Harrod 1951, 270) rather than to *ex ante* investment.[11] The distinction between these two concepts was emphasized in a book based on Harrod's lectures. " ... C_r refers to the amount of capital that entrepreneurs would like to find themselves with ... C_r is emphatically not an *ex-ante* concept. Entrepreneurs may have planned to have something quite different from what they *now* find it convenient to have, since when they made their plans it could not be foreseen what the demand for their products would be" (Harrod 1969, 165n, italics in original).

Harrod's initial presentation of his dynamic theory emphasized the "uniqueness" of his warranted line of growth — the line of steady advance. This uniqueness was due to the assumption of neutral technical progress and the implicit assumption that only one distribution of income between wages and profit was consistent with dynamic equilibrium over time. With neutral technical progress, and a constant rate of interest, C_r will be constant over time. There is a unique value for s_d in equilibrium — even though the propensities to save out of wages and profits differ — if there is only one distribution of income that is consistent with Harrod's steady growth equilibrium. Associated with this single distribution of income is a particular rate of interest and, given that rate and technology, there is an appropriate value for C_r. It is this value, and the single value for equilibrium s_d, that then determine the unique steady-growth value for G_w from equation (3). Harrod noted in a later comment that "if there is more than one possible equilibrium profit share in a dynamic equilibrium, consistent with other dynamic determinants, there must be more than one equilibrium growth rate" (Harrod 1970, 738). He recognized that in his analysis "I did not go deeply into the question of income distribution" (*ibid.*), but he inclined

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[11] In his 1939 "Essay", Harrod tried to relate the capital coefficients in his growth equations to the *ex ante-ex post* terminology. There was no problem in recognizing C as an *ex post* quantity, but he was "not clear [that C_r] should be regarded as its corresponding *ex ante*" (HARROD 1939, 19). At that time he decided to employ the term "*ex ante*" to stand for what he later labelled as "justified" investment.

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to the view that it was "unlikely" that there would be many possible equilibrium profit shares.[12]

The warranted rate of growth in general — as opposed to that on a line of steady growth for the economy — could have one of a large range of possible values, since both C_r and s_d are not restricted, outside that special case, to unique values. The value for C_r depends, among other things, on initial conditions and, if the economy is not initially on a warranted growth path, with plant and equipment in the desired relation to output, it could be higher or much lower than its "normal" value. Similarly, s_d depends critically on the distribution of income that happens to obtain at the point in time which is being considered. Harrod thus distinguished "between the 'normal' warranted growth that applies when the system is in equilibrium... and special warranted growth rates, that apply only in the course of runaways, when the system is not in equilibrium at all" (Harrod 1973, 101).

Harrod introduced a second equilibrium rate of growth in his basic writings, which he called the "natural rate of growth". "This is the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing that there is always full employment in some sense" (Harrod 1939, 30). This rate is conceptually distinct from the warranted rate of growth, since the desired ratio of saving to income on a warranted line of growth may differ from that which is consistent with a full employment rate of growth.[13] The relation of the natural to the warranted rate of growth plays a role in Harrod's explanation of cyclical movements, but it is the latter that is the centrepiece in Harrod's initial vision of dynamic economics.

4. Domar's fundamental equation

Domar's analysis of capital accumulation and employment revolves around the determination of the rate of growth of investment that will allow the economy "to remain in a continuous state of full employment"

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[12] It is suggested in ASIMAKOPULOS (1985) that Harrod's comments on the distribution of income are consistent with a Marshallian theory of distribution.

[13] In his "Second Essay on Dynamic Theory" (HARROD, 1960), which Harrod saw as a "companion piece" to the 1939 "Essay", he concentrated on the saving requirement for a natural rate of growth.

(Domar 1957, 71). For the purposes of this determination it is assumed that the productive capacity of an economy can be specified quantitatively and that it is equal to the value of "the economy's total output when its labor force is fully employed in some conventional sense" (*ibid.*). The analysis assumes a constant general price level and abstracts from time lags so that, among other things, consumption at each point in time is in the desired relation to income, with the full multiplier effects of any changes in investment being reflected in output. Investment also affects productive capacity and this must be explicitly allowed for in an analysis concerned with the conditions under which full employment will be maintained over time. Saving and investment are represented net of depreciation, with depreciation reflecting the cost of replacing the productive capacity that has been worn out or that has become obsolescent in the normal course of events.

With the measure of productive capacity being tied to the full employment of labour, rather than to the full utilization of capital equipment, there is a distinction between the potential addition to productive capacity of net investment in a year — Domar uses a year as the length of his basic, or short, period — and what is added to productive capacity. The former is indicated by Is and the latter by $I\sigma$. The latter would be less than the former if the utilization of existing equipment must be reduced and/or the new equipment cannot be used at designed capacity because of insufficient labour. In this case there is premature "junking"[14] of existing equipment that affects the measure of capital in the economy.

Both s and σ are expressed as pure numbers per year, so that a given amount of net investment (measured in dollars) in the year results in the indicated additional amount of potential productive capacity that is measured in dollars per year. The value for s depends on the operating characteristics of the new capital equipment, but the value for σ, which is called the "potential social average investment productivity", is also affected by technological developments in the economy and the size of the labour force. "σ is concerned with the increase in productive capacity of the whole society... its magnitude depends to a very great extent on technological progress. It would be more correct to say that σ refers to an increase in capacity accompanying rather than

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[14] This is the term used by Domar to indicate the writing off, over and above normal depreciation, of capital equipment.

caused by investment" (*ibid.*, 74). The value for σ is thus based on *ex post* calculations made after the period is over and it is not predetermined by the nature of the capital investment taking place. Domar asserts that "s is the maximum that σ can attain" (*ibid.*), but this does not follow from his definitions of these terms. For example, if changes in technical knowledge allow some existing plants to produce more even while utilizing less labour, so that there is sufficient labour to fully man existing plants, the value of σ will exceed that of s. The former includes in its numerator the increase in potential output from the existing plants as well as the full potential output of the new plants, while the latter's numerator has only the potential output of the new plants. Domar recognizes in a footnote that it "is also possible that... σ>s". He then adds that his assertion about the relative values of these parameters is based on the belief that the marginal propensity to save "in our society is sufficiently high to make σ>s in a continuous state of full employment more an exception than a rule" (*ibid.*, 76n). That is, the rate of investment needed to absorb full employment saving is so high that the full utilization of new equipment requires the premature scrapping of existing plant.

Domar is interested in determining the time path of investment that will maintain full employment in the economy and, for purposes of this calculation, he assumes that the economy is initially in full employment and that both s and σ, are constant over time. The marginal propensity to save is denoted by α, and it is also assumed to be constant. Domar abstracts from time lags and thus output in each period reflects the full multiplier effects of investment in that period. Output can thus be represented by equation (7)

$$(7) \qquad Y = I/\alpha$$

If both sides of equation (7) are differentiated with respect to "time", we obtain for the increase in output over the period,

$$(8) \qquad dY/dt = (dI/dt)/\alpha$$

The maintenance over time of the initial full employment situation, which is characterized by output being equal to Domar's definition of productive capacity, requires that output grow at the same rate as

productive capacity. The increase in the latter in each period is equal, by definition, to $I\sigma$, while the increase in output is equal to the product of the increase in investment and the multiplier (the reciprocal of the marginal propensity to save). We thus obtain what Domar calls "our fundamental equation" (*ibid.*, 75).

(9) $I\sigma = (dI/dt)/\alpha$

the solution of which is

(10) $I = I_o e^{\alpha\sigma t}$

The rate of growth in investment that will maintain full employment of labour is thus equal to $\alpha\sigma$, the product of the economy's marginal propensity to save and the potential social average investment productivity.

Given the possible difference between σ and s, there is implicit in Domar's model another equilibrium growth rate, one that will maintain the full utilization of capital equipment. This rate (αs), which can be derived by substituting s for σ in equation (9), was only made explicit in Domar (1948), written after Domar had read Harrod's 1939 paper. There is some similarity between this equilibrium rate of growth of investment and the rate of growth of output on Harrod's warranted path, since with both investment turns out to be justified by the accompanying growth in output over time. For Harrod though, this growth rate is the centrepiece of his analysis and the starting point for consideration of the stability of equilibrium, while Domar's primary concern is with the rate of growth of investment that will maintain the full employment of labour. Harrod, as we saw, also made reference to the rate of growth of output that would be consistent with the full employment of labour (the natural rate of growth), but in his basic model it only serves to indicate a ceiling for the actual rate of growth over time. Producers' responses to unutilized plant and equipment are also considered by Domar, but since this consideration does not follow from a focus on entrepreneurial equilibrium, it does not have the same scope or degree of integration into the main analysis as does Harrod's treatment of deviations from his equilibrium rate of growth.

5. The instability principle

Harrod considered that one of the most important aspects of his economic dynamics is the conclusion that his equilibrium (the warranted rate of growth) is unstable. This instability, a key element in his explanation of the trade cycle, is a consequence of his definition of the warranted rate of growth and an implicit assumption about producers' reactions to investment that turns out not to be what is required by the increase in output that has just occurred. It is assumed that producers respond to an apparent shortfall of investment in any period by increasing investment demand in the succeeding period and they respond to what appears to be overinvestment by decreasing investment demand. For example, if G exceeds G_w (with s being approximately equal to s_d), then the investment in the period is judged to be insufficient in relation to the increase in output "and the system will be stimulated to further expansion. G, instead of returning to G_w, will move farther from it in an upward direction, and the farther it diverges, the greater the stimulus to expansion will be"[15] (Harrod 1939, 22). The situation is reversed if G falls below G_w, since the investment that occurred is considered to be excessive in relation to the actual increase in output "and a depressing influence will be exerted; this will cause a further divergence and a still stronger depressing influence; and so on" (*ibid.*). Harrod thus concludes that "G_w represents a moving equilibrium, but a highly unstable one. Of interest this for trade-cycle analysis!" (*ibid.*).[16]

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[15] In HARROD (1973, 34), the case where $s > s_d$ when $G > G_w$, is considered but the conclusion is the same as that in the text above. The higher than normal ratio of saving to income in one period will tend to increase purchases in the subsequent period, thus giving a further boost to G, as required by the instability principle.

[16] HARROD (1939, 24) observes that his "demonstration of the inherent instability of the moving equilibrium, or warranted line of advance", depends on the assumption that s_d and C_r are independent of the value of G at the point in time at which G deviates from G_w. He believes that only s_d could change in the direction required to bring G_w towards G, but the required change is so extreme that he concludes that the instability principle is very robust. To illustrate his argument, he assumes that G_w has an annual rate of 2-1/2 per cent, and if six months is taken as the length of the basic period, G_w is 1-1/4 per cent per six months. He demonstrates that the instability principle will hold in this case unless the fraction of the output above the initial warranted level saved is more than 80 times the desired average saving ratio. Let x_e be the extent to which the experimental increase in output is above the warranted level, then $s_m x_e$ is the extent to which saving (also investment) is above the warranted line, with s_m being the fraction of this higher output that is saved. This investment will appear to be insufficient, in light of this increase in output, if $s_m x_e < C_r x_e$, and thus according to Harrod's implicit assumption it will lead to higher investment in a subsequent period. The condition for the validation of the instability principle can

Experience of non-equilibrium positions — situations where G is not equal to G_w — affects the value of the warranted rate of growth, since the values of both C_r and s_d are affected by conditions ruling at the point in time being considered. This is why Harrod, although inclining to the view that there is a "unique warranted line of growth" (*ibid.*, 23), emphasizes that "there is no unique warranted rate; the value of warranted rate depends upon the phase of the trade cycle and the level of activity" (*ibid.*, 30). He hypothesized that the lower value for s_d in a depression, where the profit share is lower, outweighs the lower value for C_r when there is unutilized productive capacity so that the "warranted rate is dragged down by depression" (*ibid.*). The higher value for the profit share in the expansionary phase of the cycle means that the warranted rate "may be twisted upwards by an inflation of prices and profits" (*ibid.*). These changes in the value for the warranted rate of growth may help reverse the direction of cyclical forces.

The natural rate of growth (G_n) enters into Harrod's explanation of the trade cycle because it sets a limit to the possible value of G over an interval of time. If the value of G_w, under conditions of full employment,[17] is greater than G_n, then G will often be constrained to a value lower than G_w and "there will be a chronic tendency to depression; the depressions drag down the warranted rate below its proper level, and so keep its average value over a term of years down to the natural rate. But this reduction of the warranted rate is only achieved by having chronic unemployment" (*ibid.*).

Domar does not have any comparable discussion of the stability of his equilibrium, although there is the hint that failure of investment to grow at the equilibrium rate might lead to depression, even when σ is equal to s, because of unused capacity and employment. More concern is expressed over the inability to maintain the equilibrium rate of growth when σ<s, as the depressing effects on investment of otherwise viable plant that cannot be utilized, permeate the economy. Even though Domar's analysis does not have as its centrepiece an entrepreneurial equilibrium rate of growth, it is Keynesian in nature. The

independent variable is investment, with desired saving then being equated to this investment through the operation of the multiplier.[18] After deriving the rate of growth of investment required to maintain full employment, he then examines what would happen if investment grows at some constant percentage rate r, which could be lower than the required rate ασ. Results are presented for the special case where the average and marginal propensities to save are equal and "the ratio of productive capacity to capital for the whole economy is equal to that of the new investment projects" (Domar 1957, 76). These show that, in the limit, the ratio of output to productive capacity is equal to the ratio of r to ασ. If σ is equal to s, any gap between ασ and r represents both unemployed labour and unused productive capacity. In what Domar takes to be the more general case, where σ is less than s, equilibrium may not be maintained, even if r is initially equal to ασ, because of the depressing effects on investment of unutilized productive capacity. He refers to a "junking process" to reflect the writing off of otherwise viable capital equipment that cannot be operated due to insufficient labour.

This junking process is seen to represent a potential danger to the maintenance of full employment growth "because the owners of capital assets headed for the junk pile will try to avoid the losses" (*ibid.*, 79). This could lead to downward pressure on real-wage rates, and the increase in reserves for possible losses, all of which act to increase the propensity to save and thus to increase the equilibrium rate of growth. In addition, there is a tendency for the actual rate of growth to fall as the experience of capital losses weakens the resolve to invest. Domar thus concludes that "*unemployed capital is extremely important, because its presence inhibits new investment.* It presents a grave danger to a full employment equilibrium in a capitalist society" (*ibid.*, italics in original).

There is no explicit statement by Domar on the stability of his equilibrium rate of growth. It is implied that an actual growth rate that is less than the equilibrium rate, since it results in unused capacity, tends to inhibit new investment and thus equilibrium could be unstable

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be written as $s_m < C_r$, or $s_m < s_d / G_w$, and with G_w equal to .0125, as $s_m < 80 s_d$. The introduction of long-range investment outlays changes the above condition to $s_m < (s_d - k) G_w$. If these outlays absorb a large proportion of desired saving, then the marginal saving requirement for the voiding of the instability principle is much less extreme. Harrod concludes that in this case "the attainment of a neutral or stable equilibrium of advance may not be altogether improbable in certain phases of the cycle" (*ibid.*, p. 26).

[17] HARROD (1939, 30) refers to the G_w value when in full employment as the "proper" warranted rate.

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[18] A reader may be misled about the Keynesian nature of his model by a statement that identifies the ratio of I/Y with the average propensity to save (DOMAR 1957, 76), implying that it is saving that determines investment. It is clear, however, from the general nature of Domar's model that it is investment that determines saving and not *vice versa*. This is made explicit in a 1952 paper in which Domar outlines his general approach to the theoretical analysis of growth. "Of the two ... variables — investment and consumption — we shall follow the Keynesian custom of treating investment as the active (independent) one and of tying consumption to its tail" (*ibid.*, 20).

downward. There is not even an implicit treatment of the case where r exceeds $\alpha\sigma$, since such a situation is ruled out by his initial assumptions of full employment, and that r is a constant rate. In contrast, it is Harrod's concentration on the warranted rate of growth, an entrepreneurial equilibrium, and the assumed producer responses to such a rate not being achieved — responses that are themselves related to those that will tend to maintain an equilibrium once achieved — that make the instability principle an integral feature of Harrod's analysis.

6. Harrod on Domar

A consideration of Harrod's comments on Domar's model is of interest, even though it will be argued that Harrod made a crucial error in his interpretation of Domar's fundamental equation, because it can serve to point out some of the technical differences in the two models.

Harrod's statement that "subject to two relatively minor reservations ... Domar's equation is identical to mine" (Harrod 1959, 452) ignores the nature of Domar's fundamental equation and its concern with the full employment of labour rather than with entrepreneurial equilibrium. Harrod's statement is based on an incorrect description of Domar's coefficient σ. "If we carefully note the fact that Domar's σ is valued on the basis that the new investment is properly utilised — it is in Domar's words the measure of a 'potential' — and the fact that my C_r is valued on the basis that the new investment is no more nor less than that required to produce a growth of output, *i.e.* that it is properly utilised, it is evident that $\sigma = 1/C_r$." (*ibid.*). But, as we have seen, Domar's "σ refers to an increase in capacity accompanying rather than caused by investment" (Domar 1957, 74). It is Domar's s that is used to indicate the increase in output from the proper utilization of equipment resulting from new investment. Domar characterized the situation where σ is equal to s as a "special simple case" — in contrast to "the more general case when $\sigma < s$" (*ibid.*, 76). Harrod's attempt to identify Domar's equilibrium rate of growth of investment, $\alpha\sigma$, with his warranted rate of growth of output, is thus mistaken.[19]

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[19] This attempt to treat the two growth rates as identical is surprising, given Harrod's references later in his paper to Domar's distinction between s and σ. Harrod then identifies $\alpha\sigma$ with his natural rate of growth and αs with his warranted rate of growth (HARROD 1959, 156).

The first of the two minor reservations referred to by Harrod in identifying his fundamental equation with Domar's is that the latter is derived using the assumption that the rate of growth of output is equal to the rate of growth of investment.[20] Harrod deals only with the rate of growth of output and it is not necessarily equal to the rate of growth of investment. This difference is a reflection of the ways in which these writers go about establishing their equilibrium rates and the basic characteristics of these rates. Domar's equilibrium deals with the rate of growth of investment *over time* and it necessitates assumptions about the values of the parameters over time. Domar assumes that s and σ, and the marginal propensity to save α, are all constant. Further, in considering what would happen if the rate of growth of investment (r) differs from the equilibrium rate, he only deals with the case where r is constant over an indefinitely long interval of time. The average and marginal propensities to save are also assumed to be equal, as are the ratios of productive capacity to capital, when fully utilized, for the economy and for new investment projects. Harrod, with his concentration on conditions at a point in time, requires none of these assumptions for the derivation of his basic equation. The marginal propensity to save does not even appear in his model. The operation of the multiplier is reflected in the appearance of the desired saving ratio (the economy's average propensity to save) in the equation for the warranted rate of growth. This means that the full multiplier effects of any change in investment have been completed within the period. Both Harrod and Domar neglect time lags in the multiplier process.[21] Harrod also notes that his equation and its derivation, unlike Domar's examination of r and $\alpha\sigma$, make no reference to total capital (Harrod 1959, 453), since his required capital coefficient is "a marginal notion" (Harrod 1948, 83). Harrod's approach is more general than Domar's, with the derivation of the equilibrium rate of growth not being tied to the assumption of a steady rate of growth. Such an outcome is possible under Harrod's schema, and as we saw he paid considerable attention to a warranted line of output with a constant rate of growth, but it can also accommodate changing values for the warranted rate of growth.

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[20] This equality is implied by equations (7) and (8) above.

[21] In his affirmation of the existence of the multiplier effects in his equation, Harrod mistakenly asserts that the desired saving ratio appearing in his equation "is the reciprocal of the multiplier" (HARROD 1959, 454). This would only be the case, of course, if his average propensity to save was always equal to the marginal propensity to save, a condition that he does not impose on his basic equation.

The second of Harrod's reservations on the identity of his and Domar's fundamental equations, revolves around the belated recognition that the counterpart to his warranted rate is really αs, with Domar's $\alpha\sigma$ sharing the characteristics of Harrod's natural rate. This difference is not, of course, a minor one! Harrod also takes issue with Domar's treatment, as two separate phenomena, of the possible difference between s and σ and the failure of the actual rate of growth of investment to keep up to $\alpha\sigma$. For Harrod, with his instability principle, a greater value for the warranted rate of growth (when there is full employment) than for the natural rate, tends "to cause chronic depression" (Harrod 1959, 458).[22] He reaffirms his faith in the instability principle, with the addition of a disclaimer about the balance of forces in situations when over time the actual and warranted rates diverge substantially.

> "While I hold that the instability theorem is safe, in the sense that the warranted rate of growth is surrounded by centrifugal forces and that a chance divergence from the warranted rate will be accentuated, I do not claim to have made any thorough-going analysis of the regions lying farther afield from the warranted rate" (*ibid.*, 460).

To the two reasons previously given for the possible termination of a boom before the full employment ceiling is reached — a sufficient rise in the value of s_d over time, allowing G_w to catch up with G, and the slowdown in G due to the appearance of bottlenecks in the supply of specific types of capital equipment — he adds a third one. *Entrepreneurs'* scepticism about the economy's ability to continue to grow at a rate in excess of the natural rate "could serve to bring the boom to an end before full employment is reached" (*ibid.*, 463).

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[22] Harrod parenthetically adds here: "I judge that Domar broadly endorses the instability principle" (*ibid.*). This judgement has no basis in Domar's analysis, for the reasons noted in the preceding section. With his focus on the full employment rate of growth of investment, and the assumption of a constant rate of growth of investment, Domar never considered the case where the latter exceeded the former. At most there is only one part of the instability principle — that dealing with the failure of investment to keep up with the required equilibrium rate — that is relevant to Domar's analysis.

7. Conditions of production

The differences in the formulation of the key equilibrium rates of growth in the Harrod and Domar analyses are reflected in their statements about the conditions of production. Harrod's analysis focusses on entrepreneurial decisions and judgements. The relation between the value of investment and the productive capacity it creates depends on the choices *entrepreneurs* make from the available techniques of production, choices that could be affected by the terms at which firms can borrow funds. Harrod's required capital coefficient, assuming normal rates of utilization of productive capacity, depends not only "on the state of technology and the nature of the goods constituting the increment of output", but "it may be somewhat dependent on the rate of interest" (Harrod 1939, 17). There is thus no basis in Harrod's analysis for the view that the "fundamental opposition of warranted and natural rates turns out in the end to flow from the crucial assumption that production takes place under conditions of *fixed proportions*. There is no possibility of substituting labor for capital in production". (Solow 1956, 65, italics in original).

Harrod allowed for the possibility of substitution, but he did not think that the changes in the rate of interest likely to occur, and the responses to them, would be such as to change substantially the choice of techniques. He concluded that "it is doubtful if productive methods in mature economies are much affected by the rate of interest at all. The effect that it should have in principle is swamped by uncertainty factors". (Harrod 1953, 557). The short period, or "point in time" in which Harrod's analysis is based, has all the characteristics of Keynes' analysis, with the degree of utilization of existing productive capacity being determined by effective demand. Output per man will be inversely related to the level of employment because less efficient plant is utilized the higher the level of effective demand (cf. Keynes 1936, 17). This is also true of Domar's "year", but his analysis goes well beyond that year.

Domar calculates the rate of growth of investment that will maintain full employment over time, given the values of both the marginal propensity to consume and the productive capacity added per dollar of investment. This calculation is a technical matter — *entrepreneurs* are given no role in the determination of the fundamental equilibrium rate in the model — and Domar simply assumes that there is a constant value for the ratio of added productive capacity to

investment. Thus when he turns to growth paths with constant rates of growth of investment over long periods of time, it is assumed that "the ratio of productive capacity to capital for the whole economy is equal to that of the new investment projects" (Domar 1957, 76). The value of capital enters into his analysis, while Harrod avoided all reference to it. In response to Robinson (1970), Harrod (1970, 739) writes: "Professor Robinson introduces the total stock of capital (K). In my book [Harrod 1948] to which she refers I deliberately avoided the use of this concept. It does not occur in my fundamental equations". He then goes on to take issue with Robinson's identification of the equilibrium rate of growth in output with the ratio of current net investment to the value of capital. "... this implies that the incremental capital-output ratio, viz., that pertaining to I in [a] year, is equal to the average capital-output ratio in the economy. I would say that this assumption is highly unlikely to be true in most cases — it would be true only by what Professor Robinson would call a 'fluke' " (ibid.). Harrod makes use here of the flexibility of his analysis that defines an entrepreneurial equilibrium rate of growth at a point in time. This analysis then tries to infer a steady rate of growth, in the absence of external barriers, but it does not actually trace this growing output over time. There is no necessary relation, at the point in time of Harrod's analysis, between the values of the incremental and average capital-output ratios.

8. Harrod on Harrod

Harrod kept returning to the main themes in his dynamic economics in a series of articles that appeared after the publication of *Towards a Dynamic Economics* in 1948 and that culminated in his 1973 book *Economic Dynamics*. The net effect of these writings — especially the later ones — is to blur the sharp outlines of the theory as first presented.[23] Harrod was working in the tradition of Keynes — knowledge of future conditions is uncertain — and he accepted that

"it is the essence of the theory that the market is very largely uncertain as to what is to happen in the future. In Dynamics we must not, any more than in Statics, think away uncertainty. Even if we postulate that the fundamental conditions are changing steadily, so as to determine, if all could be assessed accurately, a steady rate of advance and therewith a steady fall of interest, we must not postulate that it is known that these conditions will be such" (Harrod 1948, 65).

[23] On this evolution of Harrod's view of his theory, see ASIMAKOPULOS (1985).

This tradition set a serious problem for Harrod in his attempt to provide an analytical basis for his view that "the phenomena of boom and slump ... should be regarded as oscillations around a line of steady growth" (Harrod 1951, 261), as his response to Alexander (1950) makes clear. Harrod tried to give substance to this line of steady growth by identifying it with the "line of output traced by the warranted rate of growth [that] is a moving equilibrium, in the sense that it represents the one level of output at which producers will feel in the upshot that they have done the right thing, and which will induce them to continue in the same line of advance" (Harrod 1939, 22). The definition given for a warranted line of advance *assumes* that *entrepreneurs*, if the economy happens to grow at the warranted rate corresponding to the full utilization of equipment,[24] will act to maintain that rate of growth. This experience of a warranted advance is an average, "what applies to the system in general may not apply to each individual separately" (*ibid.*), and it is difficult to find any reason why, in a world of uncertainty, the actions of *entrepreneurs* in such a circumstance will result in the continuance of the same average rate of growth. Harrod tried to meet Alexander's criticism by postulating a representative *entrepreneur* whose "state of mind ... may be applied to the macro-economy" (Harrod 1951, 273). There is, of course, no such *entrepreneur* and although Harrod returns to this formulation in his 1973 book: "The idea that G_w is an equilibrium rate of expansion implies a certain behavioural parameter in the representative entrepreneur" (Harrod 1973, 19); he no longer commits himself to the view that this *entrepreneur* will act to maintain the same rate of advance. The only "proposition" he stands firm on is one that is true by definition, *viz.*, that the equation $G_w = s_d/C_r$ defines the warranted rate of growth (*ibid.*, 20).

There is a similar drawing back from the stark version of the instability principle presented initially. There we find: "Departure from the warranted line sets up an inducement to depart farther from it. The moving equilibrium of advance is thus a highly unstable one" (Harrod

[24] At each point of time along Harrod's warranted line of output, firms may be said to be, on average, in long-period equilibrium. This is consistent with Davidson's report on a 1969 conversation with Harrod, in which the latter indicated that on a warranted path "at any point of time the existing stock of capital is optimal for this period's demand" (DAVIDSON 1978, 45n). In another place Harrod also expressed concern that if producers were in long-period equilibrium at any point in time "they might lapse into stationary conditions" (HARROD 1948, 86), and perhaps a condition for continued growth is that productive capacity be slightly over-utilized at each point of time.

1939, 23); and: "If the aggregated result of trial and error by numerous producers gives a value for G which is different from G_w, there will not be any tendency to adapt production towards G_w, but, on the contrary, a tendency to adapt production still farther away from it, whether on the higher or lower side" (Harrod 1948, 87). But in 1973:

> "It would be almost a miracle if the aggregate of decisions resulted in an actual growth rate equal to the 'warranted' growth rate. There are likely to be some deviations all the time. But if they are of moderate dimensions, I would not suppose that they would bring the instability principle into operation. That is why I so much object to the knife-edge idea. It requires a fairly large deviation, such as might be caused by a revision of estimates across the board in some important industry, like the motor car industry, to produce a deviation sufficient to bring the instability principle into play" (Harrod 1973, 33).

The potential responses of *entrepreneurs* in a Keynesian world of uncertainty are too varied to allow them to be fitted into the rather rigid patterns that produce Harrod's warranted line of advance and his instability principle. Domar's calculation of the growth rate of investment that will maintain full employment, with all the special conditions under which it is calculated, is less susceptible to this type of criticism, since it is only a calculation, with no presumption that entrepreneurial decisions will guide the economy along such a growth path.

9. Conclusion

The dynamic analyses of both Harrod and Domar are Keynesian, with investment as the independent variable and saving being brought into the desired relation to income as a result of changes in the level and distribution of income. Harrod's is the more general and ambitious model, with its focus on the conditions for entrepreneurial equilibrium at a point of time allowing for consideration of conditions consistent with a steady advance, as well as for the many different equilibrium rates that would be applicable in different phases of the trade cycle. The consideration of the stability of this equilibrium follows naturally from its specification with its reference to entrepreneurial responses. In Harrod's model, producers pass judgement on the appropriateness of

the investment that has occurred in the short period with reference to the change in output that has taken place over that period. If investment is judged to be insufficient, investment orders are increased and output is given an upward boost, with this process being reversed if investment is judged to be too great. Domar focusses on a different equilibrium rate, the full employment rate, which is similar to Harrod's natural rate of growth. Domar's derivation of this rate assumes constancy of the values of the parameters over time and it does not have the flexibility of Harrod's derivation of the conditions for equilibrium at a point of time. There is also not the same scope for consideration of the stability of this equilibrium, since it is not an entrepreneurial equilibrium rate.

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