

An Economy of Industries and its Aggregate Representation *

1. Purpose

It is customary in many macroeconomics textbooks to represent the functioning of the economic system as a whole by the fiction of the one good economy. In contrast, the present paper demonstrates that Keynes, in *The General Theory*,¹ theorized in more complex terms, referring to an economy made up of industries in the Marshallian tradition. In his approach Keynes anticipated a number of problems that were to be debated later, in relation to microfoundations. In the ensuing discussion I would like to trace these problems, in order to show how, rather than ignored they are hidden within the Keynesian aggregate analysis.²

In chapter 4 of the G.T., "The choice of units", Keynes discusses the possibility of making employment homogeneous by taking an hour of ordinary labour as a unit and weighing an hour's employment of special labour in proportion to its remuneration. In a footnote he points out that "interesting complications obviously arise when we are dealing with particular supply curves since their shape will depend on the demand for suitable labour in other directions. To ignore these complications would, as I have said, be unrealistic. But we need not consider them when we are dealing with employment as a whole, provided we assume that a given volume of effective

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¹ In the following discussion the *General Theory* is referred to as G.T.

² Some years ago I have already written a note on this subject (D'ADDA, 1981). In the present paper I take these ideas further, trying to express them in a more satisfactory way.

demand has a particular distribution of this demand associated with it". My contention here, is that, "if a given volume of effective demand has a particular distribution of this demand associated with it", a Marshallian model of an economy made up of industries may be adequately represented through the Keynesian aggregate demand and supply functions.

2. Individual industry output demand and supply price

The economy described here is characterized as having as many markets as there are industrial outputs. With regard to labour we assume a given hourly money wage and an infinite elasticity of supply until full employment. This is a standard Keynesian assumption.

The final demand function for output of industry r , qf_r , can be represented as

$$qf_r = qf_r(wN, k, s_r, pd_r)$$

where the symbols have the following meaning: w money wage, N aggregate employment (so that wN is the wage bill), k autonomous expenditure in money terms, s_r is the r th component of a vector (s) of coefficients or functions linking the volume of effective demand with a particular distribution of final demands in money terms for the outputs of the various industries, and pd_r is the demand price of output of industry r . A practical example is the qf_r function as belonging to the linear expenditure system originally introduced by Stone (1954), that could lead to the simple specification

$$qf_r = \frac{1}{pd_r} s_r (cwN + k)$$

where the sum of the components of s amounts to one, c represents the Keynesian propensity to consume and the expression $(cwN + k)$ indicates the money value of the effective demand.

This representation of the final demand function for the output of an individual industry is completely within the framework of

the Keynesian view. In comparison with the demand functions ordinarily obtained by utility maximization in modern general equilibrium analysis the present representation is not too restrictive, if we take into account: first, that the lack of a labour market renders labour endowments uninfluential and that therefore some version of the *dual decision hypothesis* according to the tradition of Keynes and Clower is in order;³ second, that at the level of an industrial analysis making the final demand for output dependent only on its own price, rather than on the whole set of output prices, it may be seen as a reasonable first approximation.⁴

In order to describe the total demand function for output of the r th industry, qd_r , an intermediate demand component must be added to the final demand.⁵ For the sake of simplicity we may assume that such a component can be described by the expression

$${}_rA f(n).$$

where the symbol ${}_rA$ represents the r th row of the A input-output matrix and $f(n)$ represents a vector having the industrial production function $f_h(N_h)$ for h th component.⁶ The total demand function for output of industry r may therefore be expressed by

$$qd_r = qd_r(wN, k, s_r, pd_r) + {}_rA f(n).$$

In an explicit specification

$$qd_r = \frac{1}{pd_r} s_r (cwN + k) + {}_rA f(n).$$

Alternatively the demand function for the output of industry r

³ The expression *dual decision hypothesis* was firstly introduced by CLOWER (1965) to connect individual demand with income generated by other's decisions, rather than with the notional value of individual resources; but such a concept clearly goes back to Keynes, G.T., chap. 3, sect. I, "The principle of effective demand".

⁴ The second assumption is equivalent to hypothesizing a world where all goods are complementary goods. Notice in addition that there is no need to conceive the final demand function for the output of industry r as the allocation of an aggregate variable. We can conceive individual agents as having all the same c and the same s_r . This seemingly naïf assumption may be made without loss of generality for the following argument.

⁵ In my 1981 paper I neglected to stress this point.

⁶ Throughout this paper fixed rather than variable coefficients of intermediate inputs may be assumed without loss of generality.

may be expressed by carrying the (demand) price on the left hand side

$$pd_r = \frac{s_r (cwN + k)}{qd_r - A f(n)}$$

The individual industry supply price may be represented by

$$ps_r = \frac{w}{f_r'(N_r)} + u_r'(p^e)$$

where the symbol $f_r'(N_r)$ means marginal product of labour in industry r , and $u_r'(p^e)$ indicates the marginal user cost in industry r as a function of the vector of the expected prices (p^e). In other words the individual industry supply price is given by the marginal cost of labour per unit of output, plus the marginal user cost per unit of output. This is exactly the assumption of Keynes in chapter 4, section IV of the G.T. As is usual the short run industrial production function [$f_r(N_r)$] is characterized by diminishing marginal product of labour. Within the pricing procedure labour is imputed at its marginal cost per unit of output, whereas it is remunerated at its average cost, the difference representing the source of the industrial profit.

In what follows for the sake of simplicity I shall assume that

$$u_r'(p^e) = p^e A_r$$

where A_r represents the r th column of the A input-output matrix accounting for current material "ingredients" and fixed capital replacement. Consequently we shall assume

$$ps_r = \frac{w}{f_r'(N_r)} + p^e A_r$$

We are in this way confronted with the demand and supply functions for the output of an individual industry. Provided that both industrial employment and expected prices are given, these two functions may be represented in the ordinary Marshallian diagram. Every market for the output of any industry may consequently be described fully.

3. The working of an economy made of industries

Our description begins with the decision, made by individual industries, about the daily (or weekly, according to the length of the production process) rate of output. Within a Marshallian context such a decision requires reference to short run expectations.⁷ These expectations are essentially expectations as to the prices that will prevail in the course of the production period: prices at which inputs may be bought and outputs may be sold. Following a Keynes' suggestion,⁸ we will assume that individual industries price expectations are normally based on the prices prevailing at the end of the previous period

$$p^e = p_{-1}$$

where the symbol p_{-1} represents the vector of such prices. Subsequently industries will utilize these expectations to make decisions about their individual rates of output. Marshallian perfectly competitive industries will decide employment and output levels by equating their supply prices (which are functions of employment levels) to the expected prices

$$\left[w \frac{1}{f_r'(N_r)} \right] + p_{-1} A = p_{-1}$$

where the expression in square brackets represents the vector of the unit labour costs of the individual industries. It must be noticed that the previous system is one of independent equations, each one solvable, so to speak, by the corresponding industry. It determines, equation by equation, every industrial level of employment (N_r) as a function of $[(1/w) p_{-1}]$. Let us represent the vector of such industrial levels of employment by the symbol n . Since the industrial production functions $f_r(N_r)$ are given, the vector of the industrial outputs, call it (q), may also be calculated. In addition the total employment N may be obtained by the total sum of the industrial records

$$N = ni$$

⁷ G.T., chap. 5, sect. I.

⁸ G.T., chap. 5, sect. II.

where the symbol i represents a vector which has all components equal to one.

Until expected and realized variables coincide, n and q represent "tentative" solutions within a sequence; therefore we shall indicate them, for the time being, by the symbol n_1 and q_1 . At the end of the production period the industrial outputs flow on to their individual markets and output prices fluctuate so as to make each output demand equal to the corresponding supply, as it appears in the system

$$qd(wN, k, pd, Aq_1) = q_1$$

where the symbol $qd(\dots)$ represents the vector of total demand functions for outputs of the various industries and pd represents the vector of the demand prices for industrial outputs. In explicit form the system may be expressed as

$$\hat{p}d^{-1} s(cwN + k) = (I - A) q_1$$

where the script $\hat{p}d^{-1}$ indicates the diagonal matrix having the inverses of the components of the vector pd as non zero elements. Such a system is again made of independent equations, each one corresponding to the two sides of a particular market.

General equilibrium systems usually do not include independent but rather interdependent market clearing equations. It must be emphasized that the reason for having independent market clearing equations such as ours is for simplicity and perhaps in the very short run it offers greater realism, but in no way is such independence a necessary condition, either for the solution of the whole system or for the representation of the Keynesian system at the industrial level.

A system of market clearing equations, where supplies are given and demands for the outputs of the various industries are functions only of their own prices, lends itself to represent the coordination process between supply and demand on the various markets in a very simple way. There is in fact no need of a Walrasian auctioneer to explore in a timeless way which price vector to select within an infinite variety in order to clear at one go the whole set of interdependent markets. We may in fact think of a more modest middleman in every market, exploring the demand function for the

corresponding output and permitting perfect balance between demand and supply.⁹

Coordination within our economy made up of industries consequently is easily achieved and once market equilibrium prices have been established, a new round of production may take place starting from new (revised) price expectations and leading to new industrial outputs q_2 . Provided that the autonomous expenditure in money terms (k) flows regularly over time and provided the price revision process converges to a stable price vector $p(k)$ within a sufficiently short time, a situation of short run equilibrium employment $n(k)$ and industrial production $q(k)$ is reached, which may be considered to last as long as the autonomous expenditure in money terms remains unchanged.

4. Perfect foresight and aggregation

If the sequence of daily outputs and prices converges quickly to short run equilibrium and/or agents are quick to learn how the system works, backward looking expectations may be replaced by *perfect foresight* or its modern equivalent, *rational expectations*. One symbol (p) may be used to represent the equilibrium price vector and one symbol [$f(n)$] to represent the equilibrium output vector, every output being obviously a function of employment in the corresponding industry. Recourse to a time sequence is no longer necessary. The supply side of the system may be expressed by

$$p = w \left[\frac{1}{f_r, (N_r)} \right] + pA$$

and the demand side, in explicit form, by

$$f(n) = \hat{p}_1 s(cwN + k) + A f(n).$$

⁹ Alternatively, since income distribution is irrelevant to the industrial distribution of demand, transactions out of equilibrium might be allowed for, provided the average transaction price times the quantity of output on sale in every market were equal to the share of effective demand assigned to that particular market.

Solving for the price vector, setting in column form and equating to the previous expression for the supply side gives

$$(\mathbf{I} - \mathbf{A}^T)^{-1} w \left[\frac{1}{f_r' (N_r)} \right] = \text{diag} [(\mathbf{I} - \mathbf{A}) f(\mathbf{n})]^{-1} s (cwN + k)$$

where the script $(\mathbf{I} - \mathbf{A}^T)^{-1}$ indicates the inverse of $(\mathbf{I} - \mathbf{A}^T)$, \mathbf{A}^T represents the transpose of \mathbf{A} and $\text{diag} [\dots]$ indicates the diagonal matrix having the components of the vector in square brackets as non zero elements. The former is obviously an interdependent equation system, but far-looking agents now expect that the price vector will be realized, and therefore the help of middlemen is no longer needed. The present simultaneous system admits a vector of industrial levels of employment (\mathbf{n}) as a solution. We expect that for any k (or better k/w) the solution is unique. When \mathbf{n} is known, be it $\mathbf{n}(k)$, also a vector of industrial outputs and a vector of industrial prices may be calculated, which must coincide with the vectors $\mathbf{q}(k)$ and $\mathbf{p}(k)$ obtained as limiting elements of the converging sequences mentioned in the previous section.

According to the suggestion of Keynes the "industrial" system may be synthesized into the aggregate demand function and aggregate supply function. In order to perform the aggregation process let us first concentrate on the supply side. By using the equilibrium equation of the whole system that connects the solution vector $\mathbf{n}(k)$, as well as the vectors $\mathbf{p}(k)$, $\mathbf{q}(k)$ and the scalar N to any value of k , it is possible to define

$$Z = \mathbf{p}(k) (\mathbf{I} - \mathbf{A}) \mathbf{q}(k) = \bar{Z}(k)$$

as the *aggregate supply function* or value of the whole industrial output net of user cost (i.e. in modern terminology *value added*¹⁰). Such a function is generated in a continuous way as long as we keep increasing the value of k between zero and some maximum level implying the full use of the available labour force. Since a value of N may be connected to every value of k , the *aggregate supply function* may also be defined as

$$Z = Z(N).$$

¹⁰ Notice that Z may also be expressed as

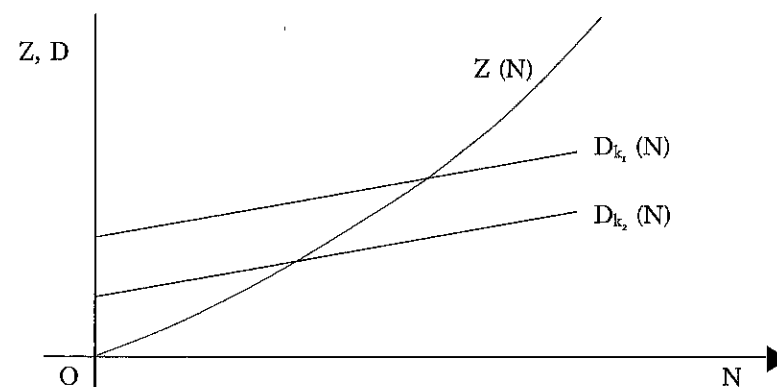
$$Z = \left[\frac{w}{f_r' (N_r)} \right] \mathbf{q}(k).$$

In order to justify why the function $Z(N)$ has been denominated aggregate supply function, even though every solution of the complete industrial system belongs simultaneously to the supply and to the demand side, we have to complete the description of the aggregate system. To this end let us consider the autonomous expenditure (k) as given and take N as the independent variable. Looking at the demand side of the system, it is then possible to define the *aggregate demand function* as the value of the final demand for the whole industrial output,¹¹ which would become effective if N workers were employed. In its explicit form

$$D_k = cwN + k = D_k(N).$$

Such a function will obviously have only one point in common with the aggregate supply function.

The following chart depicts Z and D as functions of N . The second one is represented with respect to two levels of the autonomous expenditure k_1 and k_2 .



It is apparent that a whole family of aggregate demand curves may be generated in connection with a variable level of autonomous expenditure. Along every such curve only one point may become effective; in contrast, the whole succession of effective points depicts the aggregate supply curve. Qualitatively such a curve is similar to a Marshallian supply price curve of an industry.

¹¹ i.e. demand for the whole industrial output net of user cost.

As a matter of fact the aggregate supply function may be interpreted as a weighted average of industrial supply prices (net of user costs) with weights given by the outputs of the various industries.

5. Concluding argument

The third chapter of *The General Theory* has been criticized a number of times even by followers of Keynes. This was mainly due to Keynes' lack of attention to imperfect competition — the principal concern of his disciples — when Keynes was writing the G.T. My impression is that Keynes' recourse to Marshallian categories cannot be considered as a disregard for the economics of imperfect competition. It rather reflects his aim to explore the consequences of "injecting" his intuitions, like the principle of effective demand, the idea of ordinary ineffectiveness of the supply of labour and the distinction between short and long run expectations, into a comprehensive framework, such as the one depicted by Marshall in book V of the *Principles*. Yet the implications of general interdependence of that scheme were not explored and it was Keynes, with his interest in the problem,¹² who made the implications clear.

The present paper may be considered as an attempt to interpret chapter 3 of *The General Theory* in the light of Keynes' theoretical views. Of course within the literature a Marshallian foundation of that chapter has already been noted by, among others E.R. Weintraub (1979),¹³ Casarosa (1981) and D'Adda (1981). What has been added here is to show how the Keynesian system can be viewed as the synthetic representation of a complete industrial system in the short run equilibrium, one that does not require recourse to expectations about aggregate variables or to macroeconomic behavioural functions for its solution. In contrast with the opinions of several authors, including Patinkin (1979), who allege the obscurity of the third chapter of *The General Theory*, this paper shows that there is no apparent inconsistency in Keynes' construction.

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¹² See especially chap. 5 of the G.T., on "expectations as determining output and employment".

¹³ See chap. 3.

BIBLIOGRAPHY

- CASAROSA, C. (1981), "The Microfoundations of Keynes's Aggregate Supply and Expected Demand Analysis", *The Economic Journal*, vol. 91 (March)
- CLOWER, R. (1965), "The Keynesian Counterrevolution: A Theoretical Appraisal", in Hahn and Brechling (1965).
- D'ADDA, C. (1981), "La determinazione delle funzioni aggregate di offerta e di domanda: una nota di 'ermeneutica' keynesiana", *Giornale degli economisti e Annali di economia*, Anno XL (Nuova Serie), n.1-2.
- HAHN, F.H. and BRECHLING, F.P.R. eds. (1965), *The Theory of Interest Rates*, London, Macmillan.
- KEYNES, J.M. (1936), *The General Theory of Employment, Interest and Money*, London, Macmillan.
- MARSHALL, A. (1920), *Principles of Economics*, eighth ed., London, Macmillan.
- PATINKIN, D. (1979), "A Study of Keynes Theory of Effective Demand", *Economic Inquiry*, vol. XVII (April).
- STONE, R. (1954), "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand", *The Economic Journal*, vol. 64 (September).
- WEINTRAUB, E.R. (1979), *Microfoundations*, Cambridge, Cambridge University Press.