Harrod and Robinson on the Equilibrium Rate of Growth

1. Introduction

R.F. Harrod was the pioneer in using the concepts found in Keynes's *The General Theory of Employment, Interest and Money* to examine the rate of growth of a capitalist economy. His attempt to develop an analysis that made use of a steady rate of growth — with the trade cycle being seen as oscillations around a line of steady advance — was, however, part of a research program that pre-dated *The General Theory* (Harrod 1951, 261). Harrod's book, *The Trade Cycle: an Essay*, which was published in 1936, was largely written independently of *The General Theory*. The main analytical tools used in *The Trade Cycle* were J.M. Clark's acceleration principle and R.F. Kahn's multiplier. From Harrod's perspective, an important limitation of *The General Theory* was its sole concern with the levels of output and employment at a point in time. This was sufficient for him to categorize Keynes's theory as "static", in spite of its inclusion of positive net saving, and its emphasis on uncertainty and other characteristics of historical time. He concluded his summary of Keynes's theory, in a paper presented to the Econometric Society in September 1936, by envisaging:

> "...in the future two departments of economic principles. The first, the static theory, will be elaborated on the assumption that there is no growth and no saving... In the second department, dynamic theory, growth and saving will be taken into account. Equilibrium theory will be concerned not merely with what size, but also with what rate of growth of certain magnitudes is consistent with the surrounding circumstances" (Harrod 1937, 86).

The first fruits of Harrod's approach to dynamic theory using Keynes's concepts were to be found in an article published in the *Economic Journal* in March 1939. The centerpiece of his analysis was the entrepreneurial equilibrium growth rate, which he referred to as the warranted rate of growth.

Joan Robinson was involved as a sympathetic reader and critic in the various stages of the writing of *The General Theory*. Even before it was published, she wrote a paper (Robinson 1936) that tried to deal with long-period aspects of this theory. This work was severely limited because it considered only situations of stationary equilibrium in which net investment (saving) was zero. Twenty years were to pass before Robinson was able to present her "extension of Keynes's short-period analysis to long-run development" (Robinson 1956, vi), in her *magnum opus, The Accumulation of Capital.* She acknowledged the importance of the pioneering work of Harrod — "Our analysis of accumulation in the long run is largely an elaboration of R.F. Harrod's model..." (404) — but there are important differences, both in the manner of presentation and of content.

Robinson's model was also centered around an entrepreneurial equilibrium growth rate, the "desired rate of accumulation", which Robinson (1962, 49n) noted "is very similar to Harrod's *warranted rate of growth* and has a similar role in the analysis". The reason she gave for not using Harrod's term is that Harrod "has never removed the ambiguity as to whether the firms are supposed to be content with the stocks of productive capital that they are operating or with the rate at which it is growing". Her statement about their similarity ignores an important difference in the role of the propensity to save in the determination of these two equilibrium growth rates.

In Harrod's model the enterprise of producers has no explicit, or direct, role in the determination of the warranted rate of growth. Its value "is determined by certain 'fundamental conditions' — namely, the propensity to save and the state of technology, etc." (Harrod 1939, 17), while the actual rate of growth that is determined by investment decisions could differ from this rate. A higher value for the economy's propensity to save, other things given, results in a higher value for his warranted rate of growth. Robinson's desired rate of accumulation, on the other hand, is derived from two functions. One indicates the responsiveness of investment decisions to the expected rate of profits, while the other shows the realized rate of profits to be negatively related to the propensity to save. With the

expected rate of profits dependent on the realized rate of profits in her model, a higher propensity to save results in a lower desired rate of accumulation. This is Robinson's dynamic version of Keynes's paradox of saving. Harrod can only find room in his theory for this paradox in the interplay between the warranted, natural, and actual rates of growth.

In Robinson's theory, there is not necessarily the same opposition between the entrepreneurial equilibrium rate of accumulation and the possible rate of accumulation as there is in Harrod's model between the warranted and natural rates of growth. The animal spirits of entrepreneurs, by affecting the rate of technical progress, often bring the two rates in close proximity. There is also a tendency in the central core of Robinson's theory to identify the actual rate with the equilibrium rate, because of the special assumptions she makes about entrepreneurial expectations. An examination of the different ways in which the propensity to save affects these two theories can thus provide a useful way of comparing some of their aspects.

2. Harrod's warranted rate of growth and the propensity to save

Harrod calls his entrepreneurial equilibrium rate of growth of output the warranted rate of growth. A rate of growth of output is said to be a warranted rate if entrepreneurs consider the investment they have undertaken in the period to have been "justified by the circumstances" (Harrod 1939, 18), that is, by the growth in output that has occurred over the period. The equation for this warranted rate of growth can be derived by manipulating the *ex post* identity between net investment and net saving for any short period,[1] if certain conditions are met. The saving must be in the desired, or equilibrium, relation to income, so that it is equal to the product of the average propensity to save in the economy and the level of income. Harrod recognized that the value for the economy's propensity to save depended on the distribution of income (see, *e.g. ibid.,* 21), but given this distribution and the propensities to save out of wages and profits, we take as given the desired saving ratio (s_d) for the economy. In this special case of short-period equilibrium, the

necessary equality between net investment (I) and net saving (S) can be written as

$$(1) \qquad\qquad I = s_d\ Y$$

where Y is the level of income (output) in this period.

Equation (1) goes beyond the *ex post* identity between saving and investment, because with saving in the equilibrium relation to income, it assumes that the full multiplier effects of any changes in investment that have occurred have worked themselves out. If both sides of equation (1) are divided by Y, and the ratio $\Delta Y/\Delta Y$ is introduced and re-arranged on the left-hand side (where ΔY is the change in output in this period as compared to last period's output), we have

$$(2) \qquad\qquad \frac{\Delta Y}{Y} \cdot \frac{I}{\Delta Y} = s_d.$$

If the period's investment is considered to be appropriate in the light of the increase in output,[2] that is, if $I/\Delta Y$ is equal to the required capital coefficient, C_r, then $\Delta Y/Y$ is said to be equal to the warranted rate of growth G_w, and equation (2) can be written as

$$(3) \qquad\qquad G_w C_r = s_d.$$

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[1] In the initial presentation of his theory, HARROD (1939, 26) used six months to illustrate the length of the short period, or point of time, on which the theory is based. This was assumed to be roughly the time required by producers to react in a meaningful way to investment that was *not* justified by the circumstances.

[2] Harrod recognizes that part of investment activity in any period is undertaken in line with long-range objectives, and its appropriateness could not be judged in the light of the increase in output over a single period. His equation for the warranted rate of growth can be easily adapted to allow for this by starting with a modified form of equation (1), where the amount of investment activity undertaken for long-term objectives is subtracted from both sides of the equation (HARROD 1948, 79). The investment used to calculate the capital coefficient is then only that investment undertaken to meet expected near-term increases in output. Harrod recognizes that even for this investment there is some time lag before the appropriateness of the resulting equipment in relation to output can be judged, but he felt that the neglect of this lag is not unreasonable when attention is focussed on the trend growth of output, where the difference in increments of output in successive periods is small (HARROD 1939, 20).

It is clear from the derivation of Harrod's warranted rate of growth that its value depends on conditions in the period, in particular, on the distribution of income (which affects s_d) and the degree of utilization of productive capacity (which affects C_r). This is why Harrod wrote "there is no unique warranted rate; the value of warranted rate depends upon the phase of the trade cycle and the level of activity" (*ibid.*, 30). But there is one value for the warranted rate that played a central role in his dynamic analysis, and that is the rate corresponding to normal utilization of productive capacity with output being sold at prices that allow for a normal rate of profit. It is this rate of growth that "represents a moving equilibrium" (*ibid.*, 22), and gives rise to what Harrod referred to as a "unique warranted line of growth"(*ibid.*, 23).The uniqueness of this rate depends on the uniqueness of the value for the economy's propensity to save, and on a single value for the required capital coefficient along the warranted line of growth. Harrod recognized that the propensities to save out of wages and profits differ, so that the former depends on the existence of only one possible equilibrium distribution of income. He subsequently noted that "if there is more than one possible equilibrium profit share in a dynamic equilibrium, consistent with other dynamic determinants, there must be more than one equilibrium growth rate" (Harrod 1970, 738), but he inclined to the view that it was "unlikely" that there would be many possible equilibrium profit shares.[3] The single value for the required capital coefficient is dependent on a constant rate of profit and neutral technical progress over time.

If we consider this "normal" warranted rate of growth of output for two economies which have access to the same technology, the one with the higher propensity to save will have the higher warranted rate of growth. The higher propensity to save thus appears to have a beneficial effect — it makes the entrepreneurial equilibrium growth rate higher — but because this equilibrium is not stable, the net effect might be *lower* values for the actual rate of growth (G). The higher the value for G_w, the more likely it is that G will fall short of G_w, thus precipitating a slump. Keynes's paradox of saving is thus reflected in Harrod's Keynesian dynamic theory, not in the value for

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[3] See ASIMAKOPULOS (1985) for a brief discussion of Harrod's tentative approach to the theory of distribution.

the entrepreneurial equilibrium rate of growth, but in the actual rate of growth. The relation between the natural and warranted rates of growth also serves to depress the actual rate, the higher the propensity to save. The higher this propensity, other things given, the greater the value for the warranted rate of growth, and the likelier it is to exceed the natural rate (G_n). The latter sets an upper limit to the possible values for G over time, and thus it tends to force G below G_w, when G_w exceeds G_n. This triggers the operation of the instability principle, which causes G to fall still further below G_w and results in a slump. It is in describing this feature of his model that Harrod refers to the paradox of saving.

> "It is the departures from G_w, not the value of G_w itself, which have paramount influence in producing boom and slump. If the value of G_w is too great (greater than that of G_n) there will be a prevailing tendency for departures to be in a downward direction. From that there is no escape. I believe that this paradox is very near the heart of the contrast between Keynesian economics and classical economics. Saving *is* a virtue and benefical so long as G_w is below G_n. While it is disastrous to have G_w above G_n, it is not good to have it too far below, for in that case, although we may have plenty of booms and a frequent tendency to approach full employment, the high employment will be of an inflationary and thereby unhealthy character. In these circumstances saving is a virtue since, by raising G_w, it enables us to have good employment without inflation. But if G_w is above G_n saving is a force making for depression" (Harrod 1948, 88-9).

This paradox of saving is reflected in the equilibrium rate of accumulation in Robinson's theory, to which we now turn.

3. The desired rate of accumulation and the propensity to save

Robinson uses the term the *desired* rate of accumulation to describe the entrepreneurial rate of growth in her model. This difference in terminology reflects in part the difference in the variable whose rate of growth appears in the entrepreneurial equilibrium. Robinson refers to a rate of accumulation, and thus requires a value of capital for her model,[4] while Harrod deals only with the rate

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[4] See ASIMAKOPULOS (1984) for critical comments on the way in which Robinson introduces a value for capital into her model.

of output, and the value of capital does not appear in his model. The required capital coefficient is, as Harrod (1948, 83) emphasized, "a marginal notion", and it is a ratio of current investment to the current period's increase in output. More fundamental is the difference in the derivation of their equilibrium rates. Harrod's warranted rate of growth is determined, as we have seen, by his "fundamental conditions", the propensity to save, and technology as it affects the required capital coefficient. Robinson's rate is derived from a double-sided relationship between the rate of profit and the rate of accumulation, a relationship that can be traced to Kalecki (1971, chapter 1). It is this difference that leads to a different prediction of the effects of a higher propensity to save on the entrepreneurial equilibrium rate of growth in these two models.

Robinson adopted Kalecki's explanation for the determination of profits, with their value depending on capitalists' expenditures. She usually assumes that the propensity to save of workers is zero, and if the time lag between profits and capitalists' consumption expenditures is ignored, a simple linear relation between profits and investment that incorporates the propensity to save out of profits can be obtained. With no workers' saving, and assuming a closed economy with no government economic intervention, the short-period equilibrium value for profits (P) is given by equation (4).

$$(4) \qquad\qquad P = I + \lambda P$$

where λ is the propensity to consume out of profits. It can be re-written as

$$(5) \qquad\qquad P = I/(1-\lambda).$$

From equation (5) we can deduce that the greater the propensity to save out of profits (the greater the propensity to save in the economy), the smaller the level of profits for any given value for investment. This linear relation between the level of profits and investment in a short-period equilibrium, can be transformed into a linear relation between the rate of profits and the rate of accumulation, if both profits and investment are divided by the value of capital. The latter relation is illustrated by line A in Figure 1, which is based on the diagram presented by Robinson (1962, 48). A higher

FIGURE 1
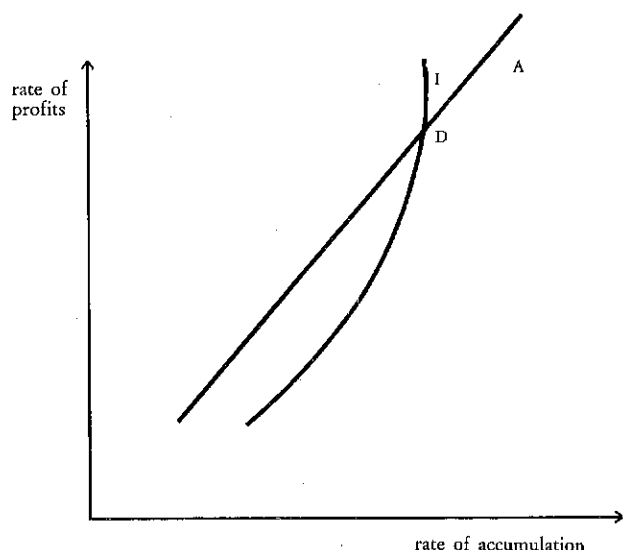
propensity to save would be reflected in a lower value for the slope of this line.

Robinson notes the importance of "animal spirits" and conditions of finance in determining the rate of accumulation, but given values for these, she postulates that the inducement to invest is positively related to the expected rate of return on investment. This relation is indicated by the I-curve in Figure 1; the state of finance and animal spirits help determine the position of this curve. Robinson's *desired* rate of accumulation is represented by point D, the intersection of the I and A curves.[5]

If the inducements to invest in two economies are roughly similar, as shown by their I-curves, then the economy with the higher propensity to save would have a lower value for the desired rate of growth. The entrepreneurial equilibrium rate of growth in Robinson's model is thus inversely related to the economy's propen-

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[5] ROBINSON (1962, 48) also showed a second, low-level, point of intersection of the two curves, but it is not relevant to the issues discussed here.

sity to save. She wrote that "[W]hen the actual rate of growth is limited only by the desired rate, therefore, greater thrift is associated with a lower rate of accumulation. This is the central paradox of the *General Theory* projected into long-period analysis" (*ibid.*, 60). Unlike Harrod's model, the paradox of saving is reflected in the entrepreneurial equilibrium growth rate. This difference in the two models is due to the inclusion of an independent inducement to invest function in Robinson's determination of the desired rate of accumulation, and its absence in the determination of Harrod's warranted rate of growth.

4. The inducement to invest and the paradox of saving

Harrod assumed that if producers experience a warranted rate of growth, then they will place investment orders that will continue that rate of growth. This is obviously a very special investment function. Why will achievement of a warranted rate of growth for the economy as a whole — with output of some producers increasing at a faster rate, and output for others increasing at a slower rate — result in the maintenance of the same rate of growth? Harrod simply asserted that "the ups and downs should balance out and, in the aggregate, progress in the current period should be equal to progress in the last preceding period" (Harrod 1948, 82). When challenged on this point by Alexander (1950), Harrod (1951, 273) tried to take refuge in the behaviour of a non-existent "representative entrepreneur". This was no more than a "fudge", and in his final book on dynamic economics, Harrod (1973, 19-20) implicitly recognized that the evocation of a mythical representative entrepreneur did not solve the problem of how to justify the assertion that producers will act to maintain the same rate of growth if that rate turns out to be warranted.

With the implicit assumption of an accommodating investment function that maintains the warranted rate of growth, once it is achieved, the possible adverse effects of a higher propensity to save are not relevant as long as external factors do not intervene. This means that a higher propensity to save, given the required capital coefficient, simply results in a higher value for the warranted rate of

growth, which once achieved will be maintained by the accommo-
dating investment function in the absence of external constraints. In
Robinson's model, as we saw above, there is an independent
inducement to invest function that has a role in determining the value
for her desired rate of accumulation. This function shows that the
rate of accumulation that firms try to undertake is positively related
to the expected rate of profits on new investment. With the actual
rate of profits (an important determinant of the expected rate) being
negatively related to the propensity to save, the higher the propensi-
ty to save the lower the desired rate of accumulation. It is this
independent investment function that is a critical element in having
the paradox of saving affect Robinson's equilibrium rate of accu-
mulation, and it is a key feature that distinguishes this equilibrium
rate from Harrod's warranted rate of growth.

The paradox of saving is reflected in Harrod's model when the
actual and warranted rates of growth differ. It is only in these cases
that the actual rate of growth is determined by an independent
investment function. This function is not fully specified by Harrod,
but two features are made clear. There is that portion of investment
determined by long-term prospects that are unaffected by current
changes in output, and then there is the instability principle. The
latter is brought into operation when the changes in current output
differ from the changes that would have turned out to be warranted.
Investment is changed in such a way as to increase the difference
between the actual and warranted rates. A *higher* propensity to save
would tend to result in *lower* rates of growth of output when this
independent investment function is brought into operation. This
could happen, for example, if the resulting warranted rate of growth
exceeds the natural rate, with this conjuncture eventually forcing the
actual rate below the warranted rate, and bringing the instability
principle into operation. Harrod also recognized situations where a
higher propensity to save would have beneficial effects. They occur
where a higher propensity to save raises the warranted rate towards a
higher natural rate, thus limiting the scope for upward deviations of
the actual rate that produce strong inflationary pressures. He
believed that in developed economies the warranted rate of growth
tended to exceed the natural rate, with the reverse tending to hold in
less-developed economies. Harrod's model thus allows for "clas-
sical" as well as "Keynesian" effects of saving.

Robinson's model does not have the same opposition between

the entrepreneurial equilibrium and the natural (or possible) rates of
growth, because there is a range of possible values for the desired rate
of accumulation, and technical progress is, in part, induced. The
value for the desired rate of accumulation varies with the state of
enterprise, even with given technical conditions of production,
because the distribution of income is a function of investment. The
greater the inducement to invest, the further to the right is the
I-curve in Figure 1, the higher is the equilibrium rate of profits and
the greater the value for the desired rate of accumulation. If this rate
tends to outrun the possible rate, then the pace of induced technical
progress is assumed to be increased. Inflationary pressures would
arise when this induced technical progress is not sufficient to close
the gap, and the rate of accumulation might have to be restrained by,
for example, tight credit controls. In such a situation higher thrifti-
ness could lessen, or even eliminate, the need for restraint, by
bringing about a lower equilibrium rate of profits, and thus a lower
desired rate of accumulation. This is a situation where a higher
propensity to save tends to eliminate inflationary pressures brought
on by a strong urge to accumulate, without harming the actual rate of
growth, which would be restrained by the possible rate in any case.
Saving could, in these circumstances, be accorded some "classical
virtue". This case differs from the situation where Harrod sees
"virtue" in a higher propensity to save, because that occurs where the
entrepreneurial equilibrium rate (G_w) lies *below* the possible rate
(G_n).

For the working out of her theory, Robinson uses an assump-
tion about expectations that tends, in the absence of constraints, to
make the actual and desired rates of accumulation coincide.

> "We shall therefore conduct our analysis (except when otherwise stated) on
> the assumption that at every moment entrepreneurs expect the future rate of
> profit obtainable on investment to continue indefinitely at the level ruling at
> that moment; that they expect the rate of technical progress (which may be
> nil) to be steady; and that they fix amortisation allowances for long-lived
> plant accordingly. When something occurs which causes a change, we assume
> that expectations are immediately adjusted, and that no further change is
> expected" (Robinson 1956, 67).

It was this assumption that made it possible for her to obtain a value
for capital that was used in the derivation of the desired rate of
accumulation. The trade cycle is then discussed in relation to a

desired rate of accumulation, and it can be set in motion by random shocks, which result in movements around the desired rate.[6] In this case, the cyclical movement appears to be more "controlled" than those initiated by Harrod's instability principle. Brief consideration is also given to situations where expectations are based on projections of recent changes. Inherent instability is observed in this latter case, with the model incapable of arriving at a steady rate of accumulation. "Uncertainty, through the volatility of expectations to which it gives rise, is continually leading the firms into self-contradictory policies" (Robinson 1962, 67). The paradox of saving would hold in all the cases where the economy is experiencing cyclical movements, because the mechanism behind these movements is Keynesian.[7]

5. Conclusion

This examination of the entrepreneurial equilibrium rates of growth in the Keynesian models of Harrod and Robinson shows important differences between them. In particular, they respond in different ways to the value for the economy's propensity to save. In Robinson's model this equilibrium rate is determined from the combination of two relations, one showing the effect on the rate of profits of the rate of accumulation, and the other showing the relationship between the expected rate of profits and the planned rate of accumulation. In equilibrium the actual rate of profits is equal to the expected rate, and the planned and actual rates of accumulation are the same. A higher propensity to save results in lower rates of profits being associated with any specified values for the rate of accumulation, and it thus leads (assuming that it is not related to the

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[6] ROBINSON (1962, 64n) notes that the interplay between the current (and expected) rate of profits and changes in productive capacity, which she uses in discussing the cycle "...resembles Kalecki's trade-cycle model... It differs from his in that the central point around which the cycles cycle is a rate of accumulation, not a stock of capital".

[7] This mechanism could, alternately, be labelled Kaleckian, because it and the paradox of saving were clearly specified in Kalecki's 1933 outline of the business cycle (KALECKI 1971, chapter 1).

inducement to invest) to a lower value for the desired rate of accumulation.

In Harrod's model there is no explicit reference to the inducement to invest in the derivation of the warranted rate of growth, nor is there any explicit way in which the propensity to save can affect the distribution of income. As a consequence the warranted rate of growth is seen to be positively related (given technology) to the propensity to save, since the growth in output must be large enough to justify the higher investment that is the counterpart of the higher saving. The paradox of saving is reflected in Harrod's model only when a higher propensity to save makes the warranted rate of growth larger than the natural rate of growth. This tends to result in actual rates of growth that are lower than the warranted rate, with the instability principle then causing even greater deviations between these two rates.

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