Least-Squares Construction of the Yield Curves for Italian Government Securities, 1957-1967

Part II - TECHNICAL NOTES ON THE CONSTRUCTION OF THE YIELD CURVES FOR THE ITALIAN TREASURY SECURITIES (B.T.P.) (*)

A. 1) Some General Considerations

The actual construction of yield curves relating yields to term to maturity involves two logically different problems. First of all observed rates should be adjusted so as to make them alike in every respect, except term to maturity. This, in turn, implies a thorough understanding of the relationships between yields from securities of the *same* maturity and all other differentiating characteristics: credit rating of borrower, marketability, call provisions, coupon rates, etc. (1). However, only recently accurate (and quantitative) studies of

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(1) Some of these determinants have a clear *a priori* effect on bond-yield differentials. Thus, e.g., it is immediately evident that, for a given term to maturity, an increase in the risk of default in promised payments will determine an increase in the redemption yield.

The overall effect of other determinants may, however, be indeterminate on the basis of purely theoretical arguments. Thus, e.g., the direction of the coupon effect on yields should be the resultant of at least two forces acting in opposite directions. On the one hand, one might in fact argue, on "duration» lines (see pp. 348-49 of Part I), that a bond with a higher coupon returns the loan, in an effective sense, sooner than a bond with a lower coupon. Liquidity considerations might therefore be thought to cause, for a given term to maturity, a negative relation between the conventionally calculated internal rate of return and coupon rates. On the other hand, however, the presence of preferential capital-gains tax rates in actual financial markets implies that, *ceteris paribus*, investors tend to prefer deep-discount (low-coupon) bonds, which would imply a positive relation between yields to maturity and coupon rates. The existing evidence seems to point out that tax considerations more than offset "duration" considerations, as high-coupon bonds tend in reality to have higher yields than low-coupon ones. See, for the U.S. experience, D. Durand, *Basic Yields of Corporate Bonds*, Technical Paper No. 3, N.B.E.R., New York, 1942, pp. 20-21, and, for the British experience, D. Fisher, "The Structure of Interest Rates: A Comment", *Economica*, November 1964, p. 413, n. 4 (in this "Comment» Fisher, however, stressed the relevance of "duration" considerations, which is clearly not consistent with his positive correlations between deviations in yields and deviations in coupon rates. In his subsequent paper, "Expectations, the Term Structure of Interest Rates, and Recent British Experience", *Economica*, August 1966, p. 323, n. 2, Fisher does in fact explain the observed evidence in terms of differential tax rates).

some of these relationships have been pursued (2), thus the methods for washing out yield differentials due to factors other than term to maturity have normally been, as will be seen, rather tentative. We have therefore to take into account the possibility of what I shall call *errors of the first type*: we can attribute to term to maturity bond-yield differentials which are in fact due to other determinants.

But, even if we succeed in sorting out a group of securities perfectly homogeneous, apart from term to maturity, a second problem must be faced. What we observe in the market is but a limited number of yields from securities of different maturities. However, what we are normally interested in is a continuous yield curve, the by-product of which are yields at desired (definite) maturities, in order to have the possibility of precise quantitative comparisons through time (or space); the obvious difficulty consisting precisely in how to obtain the continuous curve from the finite number of observable points. Hence we shall always face the possibility of *errors of the second type*: the *estimated* yields to maturity, determined on the basis of our continuous curve, may be different from the *true* ones, even if we did not incur any error of the first type (3).

Two points should be investigated in relation to what I called errors of the second type. First of all it might be argued that *true* yields to maturity defined by a continuous curve in fact do not exist, as long as the market will anyhow record but a finite number of yields. Hence true should be understood in the sense of *hypothetically true*. While this argument is in principle relevant, I would not attach to it a great importance: in fairly developed financial markets one can actually observe, at any given moment of time, a very wide number of different securities with different maturities, for any of which a "pure" redemption yield is in principle defined. So if we split time in reasonable intervals, say months, we must admit the logical existence of true (though generally not obtainable) term to maturity yields for the whole maturity range (within a reasonable limit on the long side). Hence, we may be allowed to refer

(2) See, for example, L. Fisher, "Determinants of Risk Premiums on Corporate Bonds", *Journal of Political Economy*, June 1959; P. Sloane, "Determinants of Bond Yield Differentials", *Yale Economic Essays*, Spring 1963; W. Baumol, B. Malkiel and R. Quandt, "The Valuation of Convertible Securities", *Quarterly Journal of Economics*, February 1966; F. Jen and J. Wert, "Imputed Yields of a Sinking Fund Bond and the Term Structure of Interest Rates", *Journal of Finance*, December 1966; R. Johnson, "Term Structure of Corporate Bond Yields as a Function of Risk of Default", *Journal of Finance*, May 1967 (Paper and Proceedings); F. Jen and J. Wert, "The Effect of Call Risk on Corporate Bond Yields", *Journal of Finance*, December 1967; R. Weil, Jr., J. Segall and D. Green, Jr., "Premiums on Convertible Securities", *Journal of Finance*, June 1968.

It should be pointed out that, unfortunately, studies of this kind are not available, as far as I know, in relation to the Italian experience.

(3) Alternatively, these two types of error might be described as "first-stage" and "second-stage" errors, respectively. This would immediately stress the step procedure involved.

to true term to maturity yields at a given moment of time. The second point, strictly related, is concerned with how to consider the finite data (yields) that have been selected for the construction of the yield curve. Even if we have successfully taken into account all the other most important factors, so as to leave term to maturity as the only " responsible " for observed yield differentials, we may want to leave room for some stochastic error term affecting our observations, following the normal econometric procedure of tackling with series of observed data (4).

A sharp distinction has been drawn between the two main problems which must be taken into account when constructing yield curves in order to point out the logical differences involved and the step procedure which in principle should be followed. What we are really interested in, is clearly the minimization of the probability of a global function: E_2 of second type errors; $P(E_2)$, however, is itself dependent on the probability of a global function of errors of the first type, say $P(E_1)$. In practice the two problems are closely connected. If we want to consider a very homogeneous group of securities, thus minimizing errors of the first type, we shall consider a relatively limited number of points, thus facing a high probability of errors of the second type; if, on the contrary, we select a large number of securities and proceed in some way to wash out yield differentials not due to term to maturity, we shall increase the probability of errors of the first type, but *eventually* decrease the probability of errors of the second type. An example may perhaps be helpful: suppose we are interested

(4) The introduction of a disturbance term in this second stage may be rationalized on the basis of two general arguments. Firstly, however homogeneous the group of securities considered (or however precise our adjusting procedures) may be, in practice many factors of minor importance will anyhow influence the theoretical yield curve (these extraneous and spurious random disturbances seem *a priori* particularly important in relatively " thin " markets). Secondly, we should always allow for the possibility of errors of observation in the basic yield data. Following these lines it seems therefore necessary to allow for the presence of a disturbance term; however, in this case it seems plausible to assume that the error term has a probability distribution centered at zero and a relatively small and approximately constant variance (as we shall see, this approach is at the basis of the model devised to obtain yield curves for the Italian B.T.P.).

It would not be meaningful, in my opinion, to introduce an error term from the beginning of the first stage (i.e. when considering unadjusted yields from non-homogeneous securities) so as to account for the influence of *all* the excluded variables affecting the theoretical yield curve. This would in fact certainly increase the overall variance of the disturbance term, but, in particular, it would make it impossible to formulate plausible *a priori* assumptions on its average value and on the constancy of its variance. All this, in turn, would imply a practical impossibility of deriving reliable curves from the observed data. The reason for these difficulties lies essentially in the fact that, in general, we are not able to formulate *a priori* economic assumptions on the exact shape of the theoretical yield curve: in other words, the construction of yield curves is, or should be reduced to, a problem of *interpolation* analysis. Empirically constructed yield curves must necessarily be an approximation of the underlying " true " theoretical curves, but if we pool together all the extra factors, in addition to term to maturity, that have an influence on observed yields, it becomes impossible to separate the two in any meaningful way.

in obtaining a yield curve spanning the maturity range from one to thirty years, and suppose we can either rely on data from a perfectly homogeneous, but very limited, group of securities free from any risk of default, with, say, only three different maturities: one year, four years and six years, or on data from a more numerous group of non-homogeneous securities, spanning the whole maturity range we are interested in, and we have a fair knowledge of yield differentials due to explanatory variables other than term to maturity. If our confidence in the adjustment procedure is relatively high, we may well decide to construct our continuous yield curve on the basis of the larger group of securities, even if this will imply probable errors of the first type, which were avoided considering the homogeneous group of securities.

Given the above set-up, an optimal method of yield-curve construction might in principle be obtained by minimization of $P(E_2)$ (5). In the light of these considerations we shall now turn to examine the actual methods of yield-curve construction most commonly used, and finally set out the method developed in this study.

A. 2) Durand's «Basic Yields», and U. S. Treasury Securities Curves: Free-Hand Interpolation

A pioneering work on yield-curve construction was done by D. Durand (6), who in 1942 constructed yield curves for U.S. best-grade corporate bonds on annual basis, since 1900 and for the whole maturity range from zero to ninety years (7). Subsequent annual and quarterly estimates, using his original procedure, have since appeared, thus providing a consistent series of yield curves for well over half a century (8). Durand's technique of construction may be viewed as an attempt to solve simultaneously the two problems previously pointed

(5) Clearly this optimal procedure would imply some *a priori* knowledge on the shape of the " true " yield curve, a determined degree of confidence in the adjusting procedure of raw observed redemption yields and, if we allow for a stochastic disturbance term, precise assumptions on its distribution (in other words it necessarily implies a Bayesian approach).

(6) See D. Durand, *Basic Yields of..., op. cit.*

(7) Yields to maturity, however, were read off the curves at one-year intervals only for the maturity range from one to ten years, then at two-year intervals, five-year and finally ten-year intervals up to sixty years (see D. Durand, *Basic Yields of..., op. cit.*, pp. 5-6). The curves were plotted up to ninety years, except in a few years, when no yields of very long maturities were available (*Ibid.*, Basic Charts, pp. 25 ff.).

(8) See D. Durand and W. Winn, *Basic Yields of Bonds, 1926-1947: Their Management and Pattern*, Technical Paper No. 6, N.B.E.R., New York, 1947; and *Idd.*, in *The Economic Almanac*, 1953-1954, New York, T. Cromwell Company, 1953; D. Durand, " A Quarterly Series of Corporate Basic Yields, 1952-1957, and Some Attendant Reservations ", *Journal of Finance*, September 1958; and S. Homer, *A History of Interest Rates, op. cit.* Durand's " basic yield " curves were used in Meiselman's study on the term structure. See D. Meiselman, *The Term Structure..., op. cit.*

out. High and low sale prices (not including transactions costs) for a relatively homogeneous (9) group of the highest grade corporate bonds traded on the New York market during each of the months of the first quarter of the year (10) were converted to yields to maturity by means of standard bond tables. Average yields were thus obtained (11). These average yields were then plotted on a scatter diagram. A free-hand trend line was subsequently drawn as an envelope of the lowest yields plotted (12), care being taken to check that the lowest yield for each maturity was not influenced by spurious elements.

The obvious difficulty of free-hand interpolation lies in the actual method employed to fit the continuous curve, and Durand's curves are no exception. He decided *a priori* that all humps in "basic yields" curves would be spurious and hence restricted himself to the three theoretical curves usually drawn for illustrative purposes: "1) a horizontal straight line, 2) a smooth curve falling at a decreasing rate until it approaches a horizontal straight line at the long-term end, 3) a smooth curve rising at a decreasing rate until it approaches a horizontal straight line" (13). The only extra possibility he allowed for being an ascending line rising at a constant rate through the short-dated region and then taking the usual ascending shape.

This, however, was just the first step. As is well known, a major disadvantage of free-hand interpolation is given by the fact that, when the curve is translated into numbers, many irregularities are detected, even if the curve is, at first sight, sufficiently regular. Since these irregularities are essentially artificial some extra smoothing procedure is usually needed. Durand did in fact proceed to smooth the preliminary curves obtained, until the successive differences between maturities became sufficiently regular.

Even without going into further details, Durand's yield curves appear to be rather artificial and of dubious validity as a starting point for sophisticated

(9) Nearly three thousand different outstanding bonds have been considered, but only between fifty and a hundred issues were actually used each year to construct the curve. Care was taken to eliminate all issues clearly characterized by spurious elements affecting the term to maturity yield, such as low quality ratings, uncommon call provisions or sinking funds, scarse marketability, etc. (see D. DURAND, *Basic Yields of...*, *op. cit.*, pp. 6-8).

(10) Yields after 1941 were calculated using January and February prices, and since 1951 the yields were based on only February prices. The quarterly curves are based on prices for the central month of each quarter.

(11) The yields were rounded "to the nearest twentieth of a per cent below the true yield », i.e. down to the nearest 0.05%, hence "the yields in the basic charts are located on the average 0.025% below their true positions". See D. DURAND, *Basic Yields of...*, *op. cit.*, p. 8.

(12) In drawing the curve as an envelope of low yields, Durand intended to wash out risk of default presumably affecting average yields. In other words, he wanted to select "best" and not average corporate bonds, so that the line drawn would give good estimates of riskless, or "basic" yields.

(13) See D. DURAND, *Basic Yields of...*, *op. cit.*, p. 7.

manipulations (14). And, as a matter of fact, Durand himself, in pointing out the purposes of his curves, frankly admitted that "basic yield curves are designed to create a quick and crude impression of the term structure of high-grade bond yields at a moment of time; and for this they are adequate", however they "may err badly for any particular maturity", and therefore a "type of refined analysis for which the basic yields are not entirely appropriate is the calculation of implied forecasts of future short-term bond yields" (15).

The whole procedure followed is in fact suspect. By pooling together yields from a wide, but not homogeneous, group of securities any possibility of drawing a distinction between errors of the first and second type is precluded. Hence there is no criterion by which one might judge whether humps and irregularities are random or not. The over-smoothed curves finally obtained provide therefore, so to speak, biased evidence in favour of a pure expectations model, in that any humps, which might have been produced by segmentations in the market, were *a priori* washed out. The difficulties are enhanced by the fact that the number of low yields from which the curve is actually obtained is rather limited, despite the initial number of obligations considered, and there are often gaps in the maturity continuum; additionally the method itself of choosing these low, but not too low, yields is highly dubious. Another serious deficiency of Durand's curves is to be found in the free-hand fitting procedure. The essential drawback of free-hand interpolation being its subjectivity. The curves obtained are in a sense "unique"; even the same person trying to fit a curve to the same data will normally obtain different results at each attempt (16). While this may not be very relevant when we know *a priori* that the "true" curve does have a very smooth shape, it becomes extremely serious when we would like to obtain this information precisely from the scatter of observable points.

All this is not to deny the validity of Durand's curves for the purpose they were constructed, i.e. obtaining reliable and consistent "eyeball" estimates, but simply to point out their inadequacies for highly sophisticated mathematical and economic studies. The forward rates obtained from them are obviously affected by serious errors. If one uses forward rates as the independent variable of an econometric model (as in Meiselman's study), such errors, apart from any other difficulty, will invalidate the assumption of independence between the disturbance term and the explanatory variables. Hence, as is well known, the resulting ordinary least-squares estimates are biased and inconsistent.

On these grounds the same sort of criticism can be made on another series of commonly used free-hand yield curves: the U.S. Government securities yield

(14) The first person to seriously question the validity of the Durand curves for econometric investigations such as Meiselman's was Grant. See J. GRANT, "Meiselman on the Structure of Interest Rates: A British Test", *Economica*, February 1964, pp. 58-62.

(15) See D. DURAND, "A Quarterly Series...", *op. cit.*, pp. 348, 351, 353.

(16) This is particularly true when smoothing of the preliminary curves obtained takes place.

curves (17), which are published monthly in the *Treasury Bulletin* (18). There are, however, some differences of construction between the two series which are perhaps worth mentioning. First of all, yields used for the construction of U.S. Government curves are net of transactions charges to the buyer. And this is in fact what one should be looking for, as long as investors are concerned with realized yields. But, in my opinion, the most important difference is to be found in the fact that the shapes of U.S. Government yield curves are not so severely constrained on *a priori* grounds as was the case in the Durand ones. So even if they are free-hand fitted and smoothed, they are often humped in the early maturities, which tends to point out that humps are not only attributable to random factors.

The general conclusion which seems to be drawn from this analysis of free-hand yield curves is that the method of "fitting by eye" is not very reliable. It has certain advantages, an obvious one being speed of execution, and admittedly it can be quite useful to give "a quick and crude impression of the term structure". However, numerical data obtained from free-hand fitted curves are too subjected to errors to be used for mathematical and econometric manipulation. This is especially true when one has no well-founded *a priori* knowledge to decide whether humps and irregularities are to be smoothed out as purely random, or should be left in as significantly important. It may therefore be interesting to examine the alternative methods of constructing yield curves which have been recently adopted.

A. 3) Grant's British Government Securities Yield Curves: Linear Interpolation

In a recent paper Grant (19) tried to replicate Meiselman's test on British data. Since there is not any consistent time series of yield curves for British securities the first step was precisely the construction of these curves. "Normal" Government securities were considered, i.e. securities with peculiar coupon rates and redemption features were excluded (20). From a quarterly series of prices

(17) The reasons why Durand did not use U.S. Government securities as a starting point of his studies were essentially the following: (a) large gaps in the maturity range, (b) changing value of tax exemption, and (c) changing features and special privileges. It was therefore impossible to construct a long and consistent time series of yield curves. It should, however, be pointed out that comparisons of the two estimated yield curves for recent years show that Durand's curves generally lie above Government curves, which makes one suspect that Durand was not completely successful in purging for risk of default. Government securities curves appear on the whole to be more reliable for recent years, and in fact have been used in many recent econometric studies.

(18) The curves appearing in the *Treasury Bulletin* are not directly tabulated, hence there is the additional difficulty that different observers may obtain different results when compiling numerical series.

(19) See J. Grant, "Meiselman on the Structure...", *op. cit.*

(20) The procedure of dropping "abnormal" securities is clearly open to the criticism of subjectivity, unless it is made on the basis of accurate studies of the effects of differentiating features on yields at a given maturity.

for these securities, covering the period 1924-1962, redemption yields were calculated (21) and plotted on graphs; when no security of less than a year to maturity was available the average allotment rate on 91-day Treasury bills at the nearest tender was introduced as a proxy for a three-month yield. The maturity range considered was from one year to infinity, as 2½ consols were included; while the number of observations ranged from six to fourteen.

Grant's method of fitting the yield curve was very different from the ones already examined. He avoided any smoothing procedure and calculated yields at various maturities by linear interpolation between the two adjacent observed values including the maturity considered. Once again, however, this method of fitting seems highly dubious. Admittedly, it meets the criticism of subjectivity and over-smoothing of free-hand estimates, but it goes too far away in the opposite direction. Firstly, as long as coupon differences and other differentiating features are still present among different securities, it may well be that observed yields are affected by errors of the first type. Moreover, as was pointed out, even if we were prepared to admit that the differentiating factors still present have a negligible importance, it would seem plausible to allow for a stochastic error term influencing our observed values. And finally, if indeed we want to assume that there is no error term and that our estimates are in fact free from first-type errors, there is no reason to assume that true yields are obtained by linear interpolation, while there are, on the contrary, strong presumptions in favour of a continuous and relatively smooth theoretical yield curve (22). The damage of linear interpolation being, as is well known, very serious especially when the scatter of observed points is limited (this being clearly Grant's case).

So, in conclusion, even if we want to rule out errors of the first type, estimates obtained by linear interpolation are probably affected by serious errors of the second type. A related factor should be explicitly pointed out. Linear interpolation is quite dangerous, but it may be completely misleading when one has to extrapolate outside the range of actual observations. This presumably was the reason why Grant, when without observations on the shortest maturities, introduced the rate of 91-day Treasury bills as a proxy for a three-month yield. However, I gather that Treasury bills have a rather different market from the other securities considered and one might expect that this proxy implies serious errors of the first type. It seems therefore that also Grant's approach does not lead to very reliable results (23). A different and more promising method will now be examined.

(21) To deal with securities with optional call date Grant adopted the usual convention of assuming that when price was above parity the earliest (instead of the final) redemption date should be considered.

(22) This is especially true if expectations play a relevant role in explaining yield differentials.

(23) In principle the above criticisms might be applied also to the method of linear interpolation used by La Malfa and Savona to construct their yield curves (based on the

A. 4) Least-Squares Interpolation Yield Curves

If we want to avoid free-hand interpolation and adopt a general form of analytic interpolation the problem may be set out in the following terms: given the n observed points $(x_1, y_1; ...; x_n, y_n)$ we want to determine an analytic function $\hat{y}=\hat{y}(x)$ which will enable us to obtain for any given value of the independent variable the corresponding value of $\hat{y}$. The first decision to be taken is whether we have grounds to assume that the observed points are "true" values, not affected by any error term. If such an assumption is made, we will tend to choose a function $\hat{y}(x)$ such as $\hat{y}_i=y_i$ for $i=1, ..., n$. The crudest procedure is that of linear interpolation between successive points which, as we have seen, was adopted by Grant. In general, the choice among interpolating functions is restricted to rational whole functions of the general form (function of degree m) $\hat{y}=\sum_{i=0}^{m} a_i x^i$. As is well known, if the x values of the n observed points are different, a unique interpolating polynomial of degree n—1 passing through the n given points can be obtained. If, however, we have reasons to believe that the observed points are affected by some disturbance term, it will not be required that observed and estimated points be equal, but, after having determined the type of analytic function $\hat{y}$ which seems to give a good fit to the observed scatter of points, and defined a global deviation function $\delta\hat{y}=\delta\left[\sum_{i=1}^{n} \hat{y}_i - y_i\right]$, the actual form (i.e., the parameters) of the $\hat{y}$ function will be obtained by minimization of $\delta\hat{y}$; the most common choice for $\delta\hat{y}$ being the square deviation function, which leads to the ordinary least-squares method of interpolation, normally used in regression analysis.

Regression analysis has in fact been used very recently to construct yield curves, both in the U.S. and in Great Britain. D. Fisher, who wanted to replicate Meiselman's test for British Government securities, using data from a set of yield curves constructed in a different way from that followed by Grant, presented the following regression model (24)

$$Y_t = \alpha_t + \beta_1 M + \beta_2 M^2 + \beta_3 M^3 + \beta_4 \log M + \beta_5 C + \beta_6 C^2 + \beta_7 \log C$$

were Y stands for yield, M for term to maturity and C for the coupon rate.

B.T.P. data) in relation to the Italian experience (see G. LA MALFA and P. SAVONA, "Le relazioni tra saggi di rendimento su titoli di diversa scadenza in Italia dal 1958 al 1968", *Moneta e Credito*, March 1967). The two authors, however, limited their curves to the maturity intervals for which yield-data were available, and, above all, they did not present any series of data on estimated redemption yields. Their paper is essentially aimed at providing a graphical or "eyeball" analysis of the term structure, and for this purpose the method of linear interpolation seems perfectly acceptable.

(24) See D. FISHER, "Expectations, the Term Structure of...", *op. cit.*, p. 323, n. 1. The model was applied to quarterly observations for the period 1951 to 1963 (twenty observations for each quarter, the R^2 ranging from 0.66 to 0.99). It should perhaps be pointed out that while Meiselman's model performed rather poorly with reference to Grant's data (see J. GRANT, "Meiselman on the...", *op. cit.*, Table 2, p. 62), it gave very good results when applied to Fisher's data (see D. FISHER, "Expectations, the Term Structure...", *op. cit.*, Table 1, p. 324), which underlines the importance of having reliable estimates of the yield curves.

The variables were introduced in a step-wise fashion in the order of their significance, and accepted if they reached the 0.10 level of significance in a two-tailed test. By introducing the coupon rate Fisher tried to identify and hence remove coupon effects from the representative yields. The procedure followed seems, however, difficult to rationalize: Fisher reports that in 10 of the 52 estimated yield curves the coupon effect does not reach the 0.10 level of significance and hence is dropped from the equation. Now, obviously, coupon effects may vary through time; however, it does not seem plausible to assume that they are *not* present in 10 curves (quarters), while present and presumably important in the other 42. It would therefore appear that in these cases, due to misspecification of the model, the estimated residuals are eating up coupon effects, and hence the estimated yield to maturity curves are not reliable. In other words if one has firm *a priori* knowledge that coupon effects are present (25), there is no reason to drop coupon as an explanatory variable even if it does not reach the 0.10 level of significance (26).

A regression approach has also been adopted by Cohen, Kramer and Waugh to estimate yield curves for U.S. Government securities (27), in alternative to the free-hand curves reported in the *Treasury Bulletin*. After various experiments the model proposed is the following:

$$Y = a M + b (\log M)^2 + d$$

where Y stands for yield and M for term to maturity. The model has been tested on five different dates, with $\bar{R}^2$ ranging from 0.67 to 0.96. The weak points of this model are essentially two. Firstly, differentiating features among Government securities were accounted for only by introducing tax considerations, i.e. before-tax yields and after-tax yields, which seems to leave ample room for errors of the first type. More particularly, five experiments are not sufficient to judge the goodness of the model through time.

(25) As Fisher seems to have; see D. FISHER, "Expectations, the Term Structure...", *op. cit.*, p. 323, n. 2.

(26) For other comments and criticisms of Fisher's procedure see A. BUSE, "The Structure of Interest Rates and Recent British Experience: A Comment", *Economica*, August 1967. See, however, also Fisher's "Reply", *ibid.*

(27) See K. COHEN, R. KRAMER and W. WAUGH, "Regression Yield Curves for U.S. Government Securities", *Management Science*, December 1966.

A. 5) The Regression Model Used for Italian B. T. P. and the Resulting Estimated Yield Curves

As was pointed out, the bond market in Italy is rather peculiar in comparison with the U.K. and U.S. markets. This partly depends on the fact that there are no securities comparable to the Treasury bills (i.e. a free market for securities of very short maturities at the date of issue), partly on the existence of a very important sector of semi-public securities (I.R.I., E.N.I., ...), almost free of default risk, which overlaps both the public and private sectors, and partly on the portfolio investment behaviour of the commercial banks. Since no reliable study on the effect of yield differentiating factors other than term to maturity is available, it was felt that considering sets of different securities would have implied a high probability of serious first-type errors, which, in turn, might have badly affected the estimated redemption yield curves. The analysis was therefore restricted to a homogeneous group of securities: the B.T.P. (*Buoni Tesoro Poliennali*), with a maximum maturity of nine years, which, as we have seen, are very actively traded and form an important share of the whole market. This procedure should essentially avoid the difficulty of errors of the first type [on this point see, however, footnote (28)], but it faces the problem of a rather limited number of observations at any moment of time. Fortunately, the situation is not so bad for the B.T.P. The number of observations admittedly is not high; however, one has to bear in mind that the maturity range considered is restricted to nine years: for the period 1957-1967 the number of observations ranged in fact from 6 to 9, and these observations were well spread, i.e. there have not been large gaps in the maturity continuum (28).

(28) The average yields during each month for the existing maturities are calculated by the Bank of Italy and reported in the *Bollettino della Banca d'Italia*. The B.T.P. have a nominal yield of 5% and go in (for every series of Lit. 10 md.) for the following yearly prizes: 1 of 10 million liras, 4 of 5 million, 20 of 1 million, i.e. 25 yearly prizes for a total of 50 million liras. The prizes are taken into account in obtaining redemption yields. Some difficulties arise for issues very near the redemption date, in that they may be considered as tickets of a lottery, where the initial price is always refunded. This problem is taken into account by the compiler of the series reported in the Bulletin, who drops issues near redemption with possibly spurious yields. [For specific references on the procedures adopted, see the *Nota Introduttiva* (*Mercato Finanziario*) which is reported in every number of the Bulletin]. Inspection of the data led however to consider it safer anyhow to drop issues with seven or less months to the redemption date. (The number of observations reported clearly refers to those *effectively used*).

Some errors of the first type can also affect the yields very near the redemption date because of the opportunity which is always offered to owners of B.T.P. near redemption to buy an equal amount of new B.T.P., at the conditions of issue. This may be an advantage when the supply of new B.T.P. cannot satisfy the total demand. In these conditions the price for B.T.P. very near redemption can even go above par. It seems difficult to evaluate in quantitative terms the influence of these factors; and the very crude procedure adopted here, as we have seen, was simply to drop yield data with less than eight months to maturity. A necessary condition for a quantitative appraisal of the errors thus introduced appears to be the construction of yield curves for other sets of securities and a comparative analysis of the yields obtained in the near-redemption maturity range.

Finally, since the redemption date is always at the beginning of the month, in order to avoid half-months as unit of measure, the average yield during each month was considered as the yield at the beginning of the month, which implies a fifteen-day bias in calculated maturities.

The approach adopted in this study may be summarized in the following terms: since no *a priori* knowledge of the influence on redemption yields of factors other than term to maturity was available, the degree of confidence of possible adjustment procedures was rather low. Hence, given the fact that a reasonable number of observations on a set of homogeneous securities was available, it was assumed to be preferable to construct yield curves by means of a relatively limited number of observations, but essentially avoiding first-type errors, instead of constructing them by means of a larger set of observations, but having to cope simultaneously with errors of the first and second type.

It was then assumed that observed redemption yields were however affected by a random disturbance component. An interpolating procedure was therefore adopted, trying to smooth out the error term and simultaneously minimize the probability of errors of the second type. A least-squares interpolation was adopted and the form of the interpolating function was chosen on the basis of a large number of experiments on yield curves of different shapes, by means of the usual criteria of goodness of fit (R^2) and significance tests (29). It appeared however that in certain cases the R^2 remained steadily low, even when all sorts of synthetic interpolating functions were applied. To elucidate this point, which apparently has never happened in previous attempts of least-squares yield curve construction, one should bear in mind the meaning of the R^2 statistic when used in interpolation analysis. As is well known, the square of the coefficient of multiple correlation may be generally defined:

$$R^2_{1,2\ldots k} = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

where $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$, and all the other symbols have already been introduced. It follows therefore that if $R^2 = 0$, $\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2$, which in interpolation analysis simply means that no gain is obtained in passing from the least-squares interpolation polynomial of degree 0 (i.e. $\bar{y}$) to the least-squares interpolation function $\hat{y}$ (30).

(29) This tentative approach reflects the already mentioned difficulty that we do not have any exact information on the shape of the " true " yield curve.

(30) A simple example which, as will be seen, is relevant to the interpolation model used in this study, may be interesting. We can generally define the global deviation function

One would therefore expect that in all cases where the "true" yield curve is flat the R^2 statistic should remain steadily low if the fitted function does, in fact, succeed in washing out the error term which accounts for random deviations from the true yield curve of the actually observed points. However, for this analysis to be satisfactory, one should also want in these cases the actual variance of the observed points to remain rather low, so that it may be inferred that the error term has a small variance and that the whole interpolating procedure is to be relied upon.

This was in fact the case for the constructed yield curves for the B.T.P. The model employed did generally give remarkably good fits; when it failed to do so the observed yields suggested the idea of a flat yield curve, and the variance of the yields was low relative to its average value. The interpolating equations fitted were, in conclusion, of the following form (y = redemption yield, m = term to maturity):

[1] $y = \alpha_1 + \beta_1 m + u$

[2] $y = \alpha_2 + \beta_2 m + \gamma_2 (\log m)^2 + u$

[3] $y = \alpha_3 + \beta_3 m + \gamma_3 (\log m)^2 + \delta_3 m^2 + u.$

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for interpolating functions restricted to polynomials of degree k: $[p_k(x)]$, as $\delta^2_{p_k} = E[y - p_k(x)]^2$. By minimization of $\delta^2_{p_k}$ for any value of k we shall obtain the parameters of $p_k(x)$, i.e. $\bar{p}_k(x)$, the least-squares polynomial of degree k, so that:

$$\bar{\delta}^2_{p_k} = \min E\,[y - p_k(x)]^2 = E\,[y - \bar{p}_k(x)]^2$$

In can easily be seen that $\bar{\delta}^2_{p_{k-1}}(x) - \bar{\delta}^2_{p_k}(x) \geqslant 0$. We can therefore construct a general class of coefficients of correlation of degree k:

$${}_k r^2_{yx} = \frac{\bar{\delta}^2_{p_{k-1}} - \bar{\delta}^2_{p_k}}{\bar{\delta}^2_{p_{k-1}}} \quad \text{where } 0 \leqslant {}_k r^2_{yx} \leqslant 1.$$

Considering the first coefficient of this class (k = 1) we have:

$${}_1 r^2_{yx} = \frac{\bar{\delta}^2_{p_0} - \bar{\delta}^2_{p_1}}{\bar{\delta}^2_{p_0}}$$

which measures the gain in goodness of fit which is obtained by passing from the best interpolating polynomial of degree 0 to the best interpolating polynomial of degree 1 (if ${}_1 r^2_{yx} = 0$, $\bar{\delta}^2_{p_0} = \bar{\delta}^2_{p_1}$, i.e. no gain is obtained; if ${}_1 r^2_{yx} = 1$, $\bar{\delta}^2_{p_1} = 0$, i.e. y is a linear function of x). It can, however, be easily verified that ${}_1 r^2_{yx}$ is in fact equal to $\frac{\sigma^2_{xy}}{\sigma^2_x\ \sigma^2_y}$: the square of the normal product-moment coefficient of correlation.

On every date during the period considered all the three models were fitted, the function chosen was that which gave the best fit (measured by $\bar{R}^2$), provided that all coefficients were significant at the 0.10 level. If none of the three functions had significant coefficients, the interpolating function was assumed to be the $\bar{y}$ value: a flat yield curve. The results: function fitted, $\bar{R}^2$, s_y, and number of observations are reported in Section A.6. The annual estimated redemption yields are reported in Section A.7, and the charts of the time-series of the B.T.P. yields for the maturity range from 1 to 9 years are given in Section A.8.

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A. 6) Statistical Data Relative to the Least-Squares Construction of Italian B. T. P. Yield Curves (1957-1967)

Date	Equation fitted (1)	$\overline{R}^2$ (2)	s_y (3)	$\overline{y}$ (4)	Number of observations (5)
1957 1	3	0.993	31.91	697	7
» 2	3	0.993	30.08	721	7
» 3	2	0.946	28.75	720	7
» 4	2	0.950	20.07	720	7
» 5	2	0.878	23.98	715	7
» 6	1	0.818	28.00	719	8
» 7	1	0.795	28.82	724	8
» 8	2	0.923	28.53	721	8
» 9	1	0.911	32.98	735	8
» 10	1	0.876	36.80	741	8
» 11	1	0.848	32.34	738	8
» 12	1	0.848	31.44	738	8
1958 1	2	0.964	19.44	699	8
» 2	2	0.612	10.75	654	8
» 3	3	0.999	56.10	633	8
» 4	3	0.997	58.54	626	8
» 5	3	0.996	60.01	616	8
» 6	3	0.990	64.24	593	8
» 7	3	0.991	87.91	568	8
» 8	3	0.955	127.52	529	8
» 9	2	0.941	15.25	575	7
» 10	3	0.945	11.06	592	8
» 11	3	0.960	8.03	582	8
» 12	2	0.900	6.38	568	8
1959 1	2	0.803	18.83	556	8
» 2	3	0.984	43.31	533	8
» 3	3	0.991	64.81	523	8
» 4	3	0.913	55.41	524	8
» 5	3	0.978	41.10	511	8
» 6	3	0.992	31.83	522	8
» 7	3	0.899	6.78	554	8
» 8	3	0.941	19.93	554	8
» 9	3	0.865	3.66	564	7
» 10	3	0.971	11.08	556	8
» 11	3	0.946	16.06	550	8
» 12	2	0.930	17.37	544	8
1960 1	2	0.982	19.71	535	8
» 2	2	0.967	24.83	528	8
» 3	3	0.982	25.77	527	8
» 4	3	0.987	38.13	513	8
» 5	3	0.990	57.97	491	8
» 6	2	0.969	21.85	513	8
» 7	2	0.984	23.85	508	8
» 8	2	0.971	30.66	499	8
» 9	2	0.996	24.01	508	8
» 10	3	0.962	13.34	528	8
» 11	2	0.991	15.38	528	8
» 12	2	0.980	22.31	523	8
1961 1	3	0.993	47.66	486	8
» 2	3	0.996	61.16	469	8
» 3	3	0.999	71.11	465	8
» 4	3	0.990	76.58	451	8
» 5	3	0.991	79.65	459	9
» 6	2	0.984	35.92	483	8
» 7	2	0.992	39.52	482	8
» 8	2	0.986	37.08	481	8
» 9	2	0.990	34.19	485	8
» 10	3	0.995	35.97	483	8
» 11	3	0.996	48.72	470	8
» 12	3	0.991	43.24	474	8
1962 1	3	0.995	82.75	443	8
» 2	3	0.991	86.74	428	8
» 3	3	0.994	98.17	422	8
» 4	3	0.995	124.16	406	8
» 5	3	0.990	101.17	432	8
» 6	0	(0.133)	6.98	542	8
» 7	1	0.395	7.02	532	8
» 8	2	0.977	13.39	519	8
» 9	2	0.458	4.26	532	8
» 10	2	0.587	5.56	535	8
» 11	2	0.988	25.12	491	8
» 12	1	0.937	21.00	479	8

(Ctd) A. 6) Statistical Data Relative to the Least-Squares Construction of Italian B. T. P. Yield Curves (1957-1967)

Date	Equation fitted (1)	$\overline{R}^2$ (2)	s_y (3)	$\overline{y}$ (4)	Number of observations (5)
1963 1	3	0.696	19.22	471	8
» 2	3	0.486	16.88	475	8
» 3	0	(0.313)	12.26	485	8
» 4	1	0.642	21.08	479	8
» 5	0	0	11.33	509	8
» 6	0	(0.028)	11.68	517	8
» 7	0	(0.245)	13.74	520	8
» 8	0	(0.316)	14.34	518	8
» 9	0	(0.028)	7.18	540	7
» 10	1	0.725	10.95	558	7
» 11	1	0.879	16.42	558	7
» 12	1	0.865	18.51	566	7
1964 1	1	0.880	23.00	563	7
» 2	1	0.865	20.32	562	7
» 3	1	0.877	38.41	613	7
» 4	1	0.824	31.32	609	7
» 5	1	0.737	44.44	618	7
» 6	1	0.697	65.76	636	7
» 7	1	0.818	43.77	617	7
» 8	1	0.723	32.92	592	7
» 9	2	0.884	9.39	571	7
» 10	1	0.743	6.54	574	7
» 11	0	(0.550)	6.22	569	7
» 12	0	(0.537)	5.95	559	7
1965 1	0	0	8.95	531	7
» 2	0	0	14.34	528	7
» 3	0	(0.343)	14.17	527	7
» 4	0	0	9.76	542	7
» 5	0	(0.288)	12.88	550	7
» 6	0	(0.009)	11.45	546	7
» 7	0	0	8.96	541	7
» 8	0	0	5.73	547	8
» 9	3	0.985	10.05	542	7
» 10	3	0.983	13.63	533	7
» 11	1	0.773	12.60	532	7
» 12	1	0.779	13.41	532	7
1966 1	1	0.748	17.18	513	7
» 2	1	0.872	26.13	496	7
» 3	2	0.981	12.45	525	6
» 4	1	0.560	1.25	552	6
» 5	2	0.473	13.74	551	6
» 6	2	0.909	4.45	558	6
» 7	2	0.774	4.63	552	6
» 8	2	0.884	5.96	557	6
» 9	2	0.883	7.44	566	7
» 10	2	0.920	9.10	568	7
» 11	3	0.995	11.05	569	7
» 12	0	0	5.39	562	7
1967 1	0	0	8.10	558	7
» 2	0	0	7.51	555	7
» 3	2	0.879	3.91	557	8
» 4	2	0.797	5.45	560	8
» 5	2	0.687	4.95	561	8
» 6	0	(0.002)	1.59	560	7
» 7	0	(0.372)	2.03	561	7
» 8	2	0.449	2.55	559	7
» 9	3	0.954	4.27	562	7
» 10	1	0.843	5.34	564	7
» 11	3	0.915	6.47	563	7
» 12	3	0.875	0.77	564	7

(1) The equation numbers reported refer to those given on p. 94. I indicated with of the case when the interpolating function was assumed to be the $\overline{y}$ value: a flat yield curve.

(2) When the $\overline{R}^2$ value is within brackets it reports the highest value obtained applying the interpolation equations for regressions with non significant coefficients. I indicated with of the case when all regressions yielded $\overline{R}^2$'s less than zero.

(3) s_y stands for the standard deviation of the observed yield values.

(4) $\overline{y}$ stands for the average value of the observed yields: units are in basis points.

(5) The number of observations refers to those actually used in the regressions.

A. 7) Yields to Maturity of Italian B. T. P. (Monthly Observations, 1957-1967)

Years to Maturity

Date		1	2	3	4	5	6	7	8	9
1957	1	7.05	7.41	7.36	7.18	6.96	6.77	6.63	6.56	6.58
»	2	7.45	7.63	7.56	7.39	7.21	7.03	6.90	6.80	6.77
»	3	7.34	7.49	7.48	7.40	7.27	7.11	6.92	6.72	6.50
»	4	6.93	7.33	7.45	7.43	7.31	7.15	6.93	6.69	6.41
»	5	7.01	7.29	7.37	7.33	7.23	7.08	6.90	6.69	6.46
»	6	7.76	7.55	7.44	7.33	7.22	7.10	6.99	6.88	6.77
»	7	7.71	7.59	7.48	7.37	7.26	7.14	7.03	6.92	6.80
»	8	7.34	7.48	7.48	7.41	7.30	7.16	7.00	6.82	6.63
»	9	7.89	7.76	7.62	7.48	7.35	7.21	7.07	6.93	6.80
»	10	8.00	7.85	7.70	7.55	7.40	7.25	7.10	6.95	6.80
»	11	7.88	7.75	7.62	7.49	7.36	7.23	7.10	6.97	6.84
»	12	7.86	7.73	7.60	7.48	7.35	7.22	7.10	6.97	6.84
1958	1	7.13	7.19	7.17	7.10	7.02	6.91	6.78	6.67	6.54
»	2	6.30	6.51	6.61	6.64	6.63	6.60	6.55	6.49	6.41
»	3	4.72	6.12	6.61	6.72	6.68	6.59	6.51	6.47	6.52
»	4	4.78	6.09	6.56	6.68	6.64	6.54	6.44	6.37	6.35
»	5	4.82	6.01	6.46	6.59	6.57	6.49	6.38	6.29	6.24
»	6	4.66	5.84	6.25	6.35	6.31	6.24	6.18	6.16	6.22
»	7	4.19	5.74	6.19	6.23	6.12	6.01	5.97	6.05	6.28
»	8	3.58	5.64	6.08	5.98	5.75	5.62	5.69	6.05	6.76
»	9	5.12	5.58	5.79	5.88	5.89	5.86	5.78	5.68	5.55
»	10	5.31	5.91	6.06	6.03	5.96	5.88	5.85	5.87	5.97
»	11	5.42	5.80	5.90	5.90	5.86	5.82	5.80	5.81	5.86
»	12	5.51	5.61	5.66	5.69	5.71	5.72	5.73	5.73	5.72
1959	1	5.06	5.39	5.56	5.65	5.70	5.70	5.68	5.65	5.59
»	2	4.07	5.05	5.44	5.58	5.60	5.58	5.56	5.56	5.61
»	3	3.46	4.99	5.52	5.65	5.62	5.55	5.51	5.53	5.65
»	4	3.90	5.16	5.54	5.58	5.51	5.43	5.40	5.47	5.67
»	5	4.25	4.99	5.25	5.33	5.33	5.32	5.33	5.39	5.50
»	6	4.62	5.15	5.34	5.40	5.40	5.39	5.39	5.42	5.49
»	7	5.44	5.57	5.59	5.58	5.56	5.54	5.54	5.56	5.60
»	8	5.31	5.65	5.71	5.67	5.61	5.57	5.55	5.59	5.68
»	9	5.53	5.68	5.70	5.67	5.64	5.61	5.60	5.61	5.64
»	10	5.14	5.51	5.63	5.64	5.62	5.59	5.58	5.60	5.66
»	11	4.98	5.43	5.57	5.60	5.59	5.56	5.55	5.57	5.62
»	12	5.00	5.31	5.46	5.54	5.56	5.56	5.54	5.49	5.43
1960	1	4.89	5.20	5.36	5.45	5.49	5.49	5.47	5.44	5.39
»	2	4.76	5.10	5.28	5.38	5.44	5.45	5.45	5.42	5.38
»	3	4.77	5.16	5.32	5.38	5.40	5.42	5.45	5.49	5.57
»	4	4.46	5.01	5.22	5.29	5.32	5.34	5.38	5.45	5.57
»	5	4.01	4.80	5.08	5.17	5.19	5.22	5.27	5.39	5.57
»	6	4.43	4.78	4.99	5.12	5.21	5.26	5.30	5.31	5.31
»	7	4.36	4.73	4.95	5.09	5.12	5.24	5.27	5.29	5.29
»	8	4.07	4.57	4.85	5.03	5.13	5.20	5.22	5.23	5.21
»	9	4.44	4.77	4.96	5.09	5.18	5.24	5.28	5.30	5.31
»	10	4.89	5.15	5.26	5.30	5.33	5.34	5.37	5.40	5.46
»	11	4.92	5.10	5.22	5.29	5.35	5.38	5.41	5.43	5.44
»	12	4.70	5.01	5.19	5.29	5.36	5.39	5.40	5.39	5.37
1961	1	3.71	4.57	4.92	5.05	5.10	5.12	5.15	5.20	5.30
»	2	3.34	4.37	4.78	4.94	5.00	5.02	5.05	5.11	5.23
»	3	3.24	4.35	4.78	4.96	5.02	5.04	5.09	5.17	5.32
»	4	3.14	4.21	4.65	4.84	4.91	4.95	4.99	5.07	5.20
»	5	3.21	4.25	4.68	4.86	4.94	4.99	5.04	5.12	5.26
»	6	3.60	4.23	4.59	4.82	4.97	5.05	5.10	5.12	5.11
»	7	3.53	4.20	4.59	4.83	4.98	5.08	5.12	5.14	5.12
»	8	3.64	4.27	4.63	4.85	4.98	5.06	5.09	5.09	5.06
»	9	3.87	4.39	4.69	4.88	5.00	5.08	5.12	5.14	5.14
»	10	3.74	4.43	4.75	4.90	4.98	5.03	5.08	5.13	5.21
»	11	3.32	4.22	4.63	4.84	4.94	4.99	5.03	5.07	5.13
»	12	3.58	4.36	4.72	4.89	4.96	5.00	5.01	5.03	5.06
1962	1	2.35	3.91	4.51	4.73	4.80	4.83	4.90	5.04	5.29
»	2	2.35	3.68	4.28	4.57	4.71	4.80	4.86	4.95	5.07
»	3	2.21	3.71	4.34	4.61	4.72	4.77	4.82	4.92	5.08
»	4	1.76	3.56	4.28	4.57	4.68	4.74	4.81	4.95	5.19
»	5	2.65	4.02	4.56	4.77	4.84	4.86	4.91	5.01	6.10
»	6	5.41	5.41	5.41	5.41	5.41	5.41	5.41	5.41	5.41

(Ctd) A, 7) Yields to Maturity of Italian B. T. P. (Monthly Observations, 1957-1967)

Years to Maturity

Date		1	2	3	4	5	6	7	8	9
»	7	5.23	5.25	5.27	5.29	5.31	5.34	5.36	5.38	5.40
»	8	4.74	4.97	5.10	5.19	5.24	5.28	5.30	5.31	5.31
»	9	5.18	5.28	5.33	5.35	5.36	5.35	5.33	5.31	5.28
»	10	5.16	5.29	5.36	5.39	5.39	5.38	5.36	5.34	5.30
»	11	4.18	4.57	4.80	4.94	5.03	5.09	5.12	5.13	5.13
»	12	4.46	4.55	4.64	4.73	4.82	4.92	5.01	5.10	5.19
1963	1	4.92	4.45	4.47	4.62	4.79	4.90	4.92	4.82	4.59
»	2	4.92	4.53	4.56	4.71	4.85	4.93	4.91	4.78	4.51
»	3	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85	4.85
»	4	4.52	4.60	4.68	4.76	4.84	4.92	5.00	5.08	5.16
»	5	5.09	5.09	5.09	5.09	5.09	5.09	5.09	5.09	5.09
»	6	5.17	5.17	5.17	5.17	5.17	5.17	5.17	5.17	5.17
»	7	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20
»	8	5.18	5.18	5.18	5.18	5.18	5.18	5.18	5.18	5.18
»	9	5.40	5.40	5.40	5.40	5.40	5.40	5.40	5.40	5.40
»	10	5.74	5.69	5.64	5.59	5.54	5.50	5.45	5.40	5.35
»	11	5.84	5.76	5.68	5.60	5.52	5.44	5.36	5.28	5.20
»	12	5.94	5.86	5.77	5.68	5.59	5.50	5.41	5.32	5.32
1964	1	5.97	5.86	5.75	5.63	5.52	5.41	5.30	5.19	5.08
»	2	5.91	5.81	5.71	5.61	5.52	5.42	5.32	5.22	5.12
»	3	6.67	6.48	6.30	6.11	5.92	5.74	5.55	5.36	5.18
»	4	6.51	6.36	6.21	6.07	5.92	5.77	5.62	5.47	5.33
»	5	6.73	6.53	6.33	6.13	5.93	5.73	5.53	5.33	5.13
»	6	7.13	6.84	6.55	6.26	5.97	5.68	5.39	5.10	4.81
»	7	6.70	6.50	6.29	6.09	5.88	5.67	5.47	5.26	5.05
»	8	6.29	6.14	6.00	5.85	5.70	5.55	5.41	5.26	5.11
»	9	5.99	5.82	5.72	5.67	5.64	5.63	5.63	5.64	5.65
»	10	5.82	5.80	5.77	5.75	5.72	5.70	5.67	5.65	5.62
»	11	5.69	5.69	5.69	5.69	5.69	5.69	5.69	5.69	5.69
»	12	5.59	5.59	5.59	5.59	5.59	5.59	5.59	5.59	5.59
1965	1	5.31	5.31	5.31	5.31	5.31	5.31	5.31	5.31	5.31
»	2	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28	5.28
»	3	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27
»	4	5.42	5.42	5.42	5.42	5.42	5.42	5.42	5.42	5.42
»	5	5.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50
»	6	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46
»	7	5.41	5.41	5.41	5.41	5.41	5.41	5.41	5.41	5.41
»	8	5.47	5.47	5.47	5.47	5.47	5.47	5.47	5.47	5.47
»	9	5.17	5.35	5.42	5.45	5.47	5.47	5.48	5.49	5.51
»	10	5.02	5.25	5.34	5.38	5.39	5.40	5.41	5.44	5.48
»	11	5.16	5.21	5.25	5.30	5.34	5.39	5.44	5.48	5.53
»	12	5.16	5.20	5.25	5.30	5.35	5.40	5.45	5.50	5.54
1966	1	4.93	4.99	5.05	5.11	5.17	5.23	5.29	5.36	5.42
»	2	4.64	4.74	4.84	4.94	5.03	5.13	5.23	5.33	5.43
»	3	4.85	5.06	5.18	5.26	5.31	5.35	5.37	5.39	5.40
»	4	5.50	5.50	5.51	5.51	5.52	5.52	5.53	5.53	5.54
»	5	5.56	5.52	5.51	5.50	5.50	5.51	5.51	5.52	5.54
»	6	5.74	5.64	5.58	5.56	5.54	5.54	5.55	5.57	5.58
»	7	5.67	5.55	5.49	5.47	5.47	5.49	5.52	5.56	5.60
»	8	5.76	5.61	5.55	5.52	5.51	5.52	5.55	5.58	5.62
»	9	5.87	5.74	5.67	5.63	5.61	5.60	5.60	5.61	5.62
»	10	5.92	5.78	5.70	5.66	5.63	5.62	5.61	5.61	5.62
»	11	6.01	5.76	5.67	5.63	5.63	5.63	5.63	5.62	5.60
»	12	5.62	5.62	5.62	5.62	5.62	5.62	5.62	5.62	5.62
1967	1	5.58	5.58	5.58	5.58	5.58	5.58	5.58	5.58	5.58
»	2	5.55	5.55	5.55	5.55	5.55	5.55	5.55	5.55	5.55
»	3	5.64	5.57	5.54	5.53	5.53	5.54	5.56	5.58	5.60
»	4	5.69	5.61	5.57	5.56	5.55	5.56	5.57	5.59	5.61
»	5	5.66	5.61	5.58	5.57	5.57	5.57	5.59	5.60	5.62
»	6	5.60	5.60	5.60	5.60	5.60	5.60	5.60	5.60	5.60
»	7	5.61	5.61	5.61	5.61	5.61	5.61	5.61	5.61	5.61
»	8	5.69	5.62	5.58	5.57	5.57	5.57	5.58	5.60	5.62
»	9	5.81	5.67	5.61	5.60	5.60	5.60	5.59	5.58	5.55
»	10	5.72	5.70	5.68	5.66	5.64	5.61	5.59	5.57	5.55
»	11	5.96	5.63	5.56	5.58	5.61	5.64	5.63	5.57	5.47
»	12	6.03	5.61	5.54	5.57	5.62	5.66	5.64	5.57	5.42

A. 8) Charts of the Time-Series (1957-1967) of the B. T. P. Redemption Yields for the Maturity Range from 1 to 9 Years

CHART A.1 - ONE-YEAR RATE (R_1) AND ITS MEAN VALUE ($\bar{R}_1$)

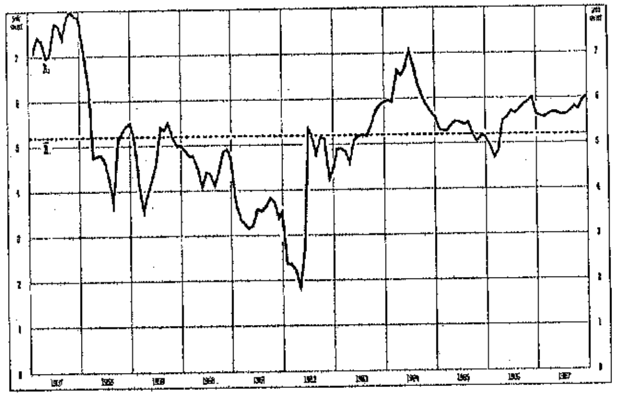

CHART A.2 - TWO-YEAR RATE (R_2) AND ITS MEAN VALUE ($\bar{R}_2$)

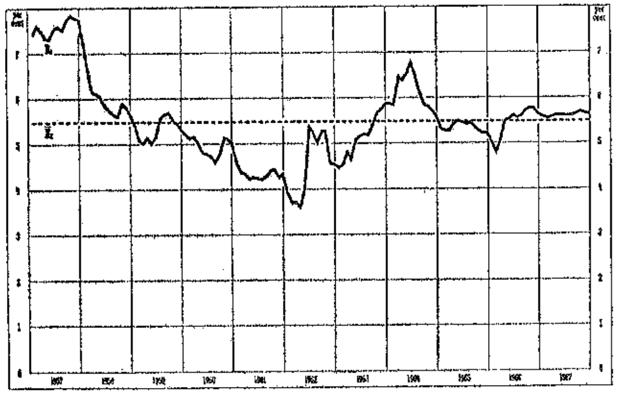

CHART A.3 - THREE-YEAR RATE (R_3) AND ITS MEAN VALUE ($\bar{R}_3$)

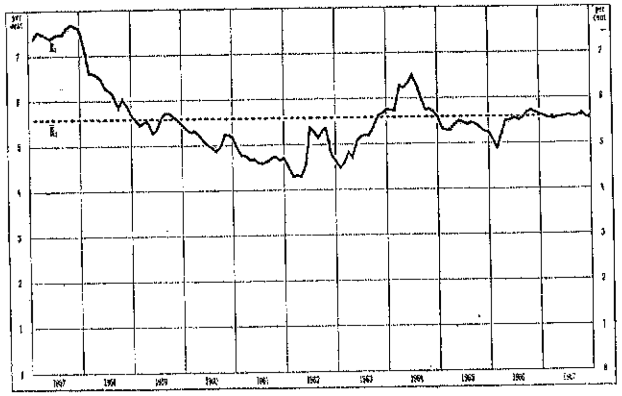

CHART A.4 - FOUR-YEAR RATE (R_4) AND ITS MEAN VALUE ($\bar{R}_4$)

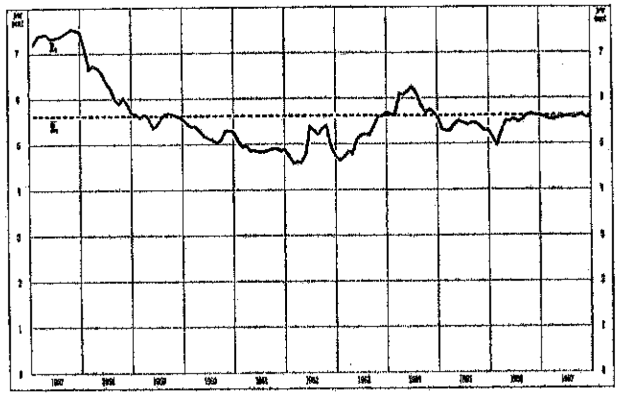

CHART A.5 - FIVE-YEAR RATE (R_5) AND ITS MEAN VALUE ($\bar{R}_5$)

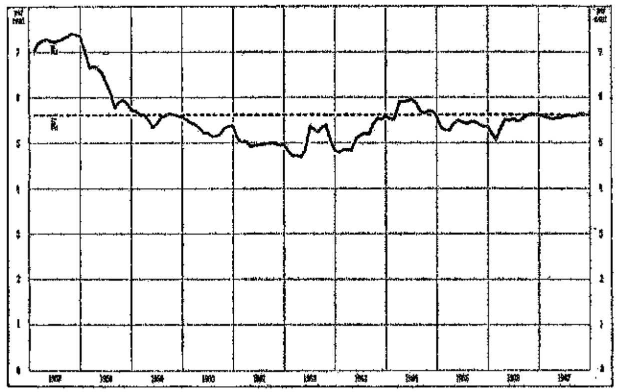

CHART A.6 - SIX-YEAR RATE (R_6) AND ITS MEAN VALUE ($\bar{R}_6$)

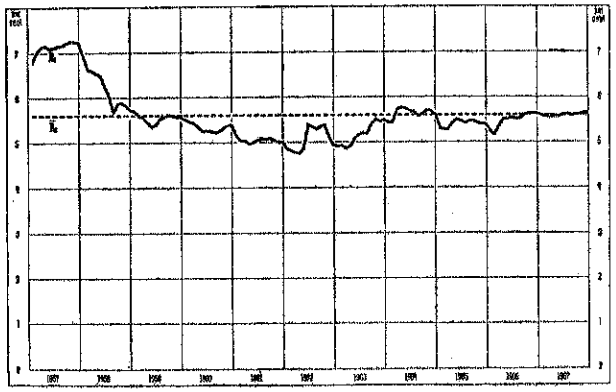

Chart A.7 - SEVEN-YEAR RATE (R_7) AND ITS MEAN VALUE ($\overline{R}_7$)

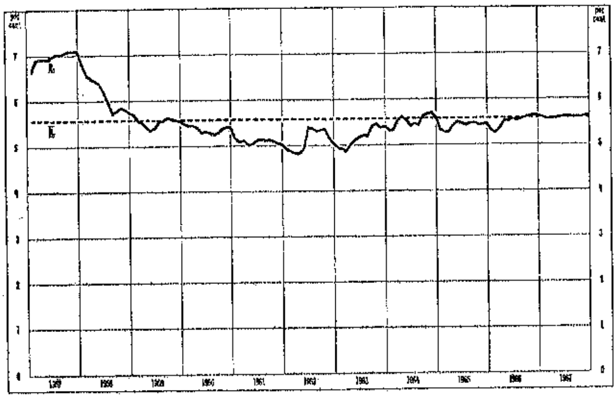

Chart A.8 - EIGHT-YEAR RATE (R_8) AND ITS MEAN VALUE ($\overline{R}_8$)

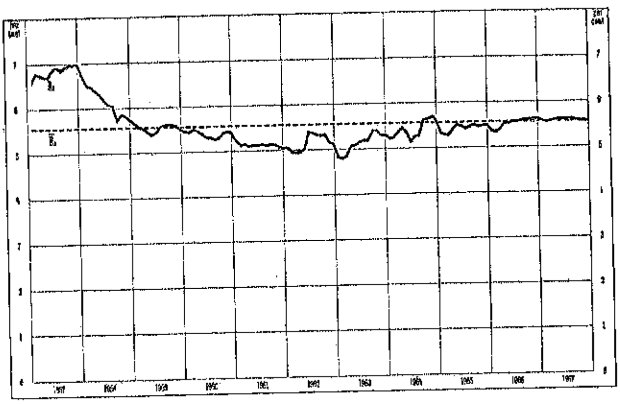

Chart A.9 - NINE-YEAR RATE (R_9) AND ITS MEAN VALUE ($\overline{R}_9$)