The Role of Commercial Banks in Foreign Exchange Speculation*

Commercial banks normally strive to balance their positions in foreign currency at the close of each day, and thus avoid bearing any exchange risk of a speculative kind. This attitude may be prompted by the, perhaps unconscious, association of speculation with instability and profiteering. Here I suggest that, on the contrary, banks can play a useful role in foreign exchange speculation if they are given a chance to draw a permanent profit from it and are convinced of the ability of the monetary authorities to maintain the exchange rate within certain pre-established boundaries.

Definitions and Assumptions

I first offer a certain number of definitions:

By exchange rate I mean the deviations of the absolute level of the exchange from the exchange parity, this being referred to as the X axis (1). These deviations are expressed in units of local currency. The exchange rate may, therefore, be represented either by a positive or by a negative quantity. Expected rates are expected for a definite date in the future.

A speculative stock of foreign exchange is defined as the difference between the total stock of foreign exchange at the disposal of a certain individual and the stock that the same individual would hold if he did not enter any speculative commitment. This excess

* This note expands section 1.3 of my article "La speculazione su cambi esteri ed i trasferimenti internazionali di fondi privati", Parte I, in Banca Nazionale del Lavoro, *Moneta e Credito*, No. 60, December 1962. Thanks are due to this Review for permission to include some previously published material.

(1) Unless specified to the contrary, in this paper "exchange rate" means, therefore, deviations around parity.

can be either positive or negative. For an individual whose normal non-speculative stock of foreign exchange is zero, a negative speculative stock means a (temporary) accumulation of local currency to be later converted into foreign exchange. For an individual who normally runs a positive non-speculative stock of foreign exchange, a negative speculative stock may mean a (temporarily) lower total stock of foreign exchange.

Speculative profits are defined as the difference between the sums received through speculative sales and the sums paid out for speculative purchases of foreign currency. To be represented by a "real" magnitude, these profits have to be reckoned for a period of time such that speculative stocks are the same at the beginning as at the end. Profits are measured in units of local currency.

Non-speculators are traders engaged in trading operations with foreign countries. Their profit depends mainly upon the structure of relative prices at home and only to a secondary extent upon the behaviour of the exchange rate (2).

Banks' speculative activities are designed to obtain a permanent flow of profits from non-speculators. The latter, therefore, share with banks part of their non-speculative profits. When occasional speculators intervene (see below), banks may gain at their expense too (provided certain conditions are fulfilled).

Occasional speculators enter the foreign exchange market whenever they see the opportunity to make a quick profit and quit. These opportunities mostly present themselves at the time when the monetary authorities are in danger of being forced to change the exchange parity. Occasional speculators may even think that they will be able to force the authorities to change the parity itself.

The following assumptions are adopted:

There is no forward market in foreign exchange.

All groups are homogeneous; there is no particular aggregation problem to solve in order to obtain the excess-demand functions of the entire group from the excess-demand functions of the individual members of each group. All groups behave competitively, within and between them. No strategy or information process is at work.

(2) If non-speculators were allowed to hedge through a forward exchange market, their profit would be entirely independent of the behaviour of the exchange rate. In this model, however, dealings in forward exchange are assumed away.

Expectations about the future level of the exchange rate are held with certainty.

The difference between two stocks (actual or desired) can be treated as a flow. There is, therefore, a regularity assumption about the rhythm of accretion or diminution of stocks.

The monetary authorities are engaged in maintaining the exchange parity, and to this end they are willing to let the exchange rate oscillate between certain pre-assigned limits.

The Excess-Demand Functions

Non-speculators' excess demand at time t is given by

$$E_{n_t} = a_1\ SR_{n_t} + A\ \cos\omega t \qquad a_1 < 0 \text{ and } A > 0 \qquad [I]$$

where SR_{n_t} is the exchange rate at time t. The oscillating element in [I] represents external factors, such as seasonal influences, acting both on the demand and on the supply side. By putting $E_{n_t} = 0$, one obtains the solution $SR_{n_t} = A_1\ \cos\omega t$, where $A_1 = -\dfrac{A}{a_1} > 0$. Non-speculators therefore determine a path for the exchange rate showing oscillations of constant amplitude around the parity level.

Banks' speculative excess demand for foreign exchange at time t is given by

$$E^B_{s_t} = k\ [ER_t - SR_t] \qquad k > 0 \qquad [II]$$

where ER_t is the expected exchange rate and SR_t the actual exchange rate (3). The intensity of banks' intervention is represented by k. k is a parameter reflecting the factors which enter into the description of a speculative equilibrium position (4). Broadly speaking, k will be the higher, the higher the level of speculative profit obtained in the past compared with the profit obtainable from non-

(3) SR_t is different from SR_{n_t} because, while non-speculators may operate independently of the existence of any other group, banks' speculative activity requires that there be non-speculators from whom to derive a profit. SR_t therefore reflects the outcome of the combined activity of banks and non-speculators. Of course, when banks enter the exchange market, SR_t becomes the exchange rate relevant for the non-speculators too.

(4) For an analysis of these factors see, beside Keynes' and Kaldor's well-known work on the subject, S. C. Tsiang, "A Theory of foreign Exchange Speculation under a Floating Exchange System", *The Journal of Political Economy*, Vol. LXVI, No. 5, October 1958, pp. 399-418 and, idem, "The Theory of Forward Exchange Market", *Staff Papers, International Monetary Fund*, Vol. VII, No. 1, April 1959, pp. 75-106.

speculative operations of the same degree of risk and liquidity. On the other hand, k will be the lower, the higher the costs (including the risk coefficient) of the operation.

The expected rate of exchange is represented by

$$ER_t = n\ SR_t + m\ \frac{dSR}{dt},\quad n < 0,\ m < 0$$

Is it plausible for banks to hold these expectations about the future behaviour of the exchange rate? I think the answer is yes. Banks take for granted that the exchange rate will oscillate within certain limits imposed by the intervention of the monetary authorities (and have in fact observed this behaviour happening in the past). Therefore they judge that, whenever the rate is falling, it is subsequently going to rise and viceversa ($m\dfrac{dSR}{dt}$), and, furthermore, the greater the difference (positive or negative) between the exchange rate and the exchange parity, the sooner the change in direction is going to take place ($n\ SR_t$).

Substituting the expression for the expected rate into [II] above, one obtains

$$E^B_{s_t} = k\ l\ SR_t + k\ m\ \frac{dSR}{dt},\quad l = n - 1 \qquad [IIa]$$

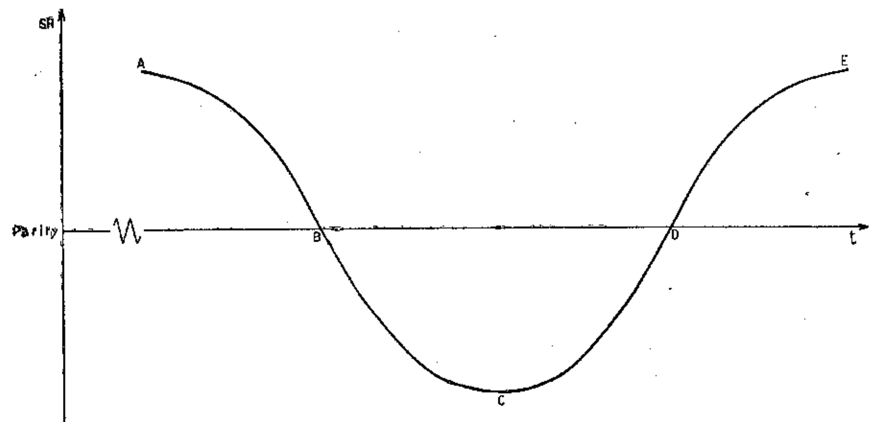

FIG. 1

The speculative buying and selling (5) by banks in the foreign exchange market can be graphically shown in fig. 1 above.

(5) Speculative buying corresponds to a positive excess demand and speculative selling to a negative excess demand.

Banks concentrate their purchases in segment BC and their sales in segment DE. Segments AB and CD are uncertain. More specifically, in segment AB, banks buy if $|m|$ is sufficiently greater than $|l|$, and sell if $|l|$ is sufficiently greater than $|m|$. The contrary is true for segment CD. For convenience' sake, I shall assume $|l|$ sufficiently greater than $|m|$.

Occasional speculators' excess demand is given by (6)

$$E^0_{s_t} = b\,\frac{dSR}{dt} + c\,\frac{d^2SR}{dt^2},\; b>0,\; c<0 \qquad [III]$$

Selling and buying by occasional speculators are concentrated respectively in segments BC and DE (see fig. 1 above), reflecting the fact that occasional speculators choose for their intervention those moments which they think most favourable to push the exchange rate pattern off gear and force the monetary authorities into changing the parity.

The Working of the Model

The model will be put to work in three stages. I will first show how banks will reduce the amplitude of fluctuations in the exchange rate due to the activity of non-speculators. Then I will show how occasional speculators, if unimpeded by the banks' (or the monetary authorities') intervention, will destabilize the exchange pattern. I will then consider the simultaneous activities of all three groups to show how banks will hamper the occasional speculators' actions, the final outcome depending upon the amount of financial means the two groups are willing to pour in to the battle. Implications for the policy of the monetary authorities will be briefly indicated at each stage.

(6) The occasional speculators' excess demand is given directly in its explicit form without first showing the expected rate and then substituting into the general form. The formal derivation of such an excess demand is very simple. The occasional speculators' expectations, however, partaking of the nature of targets, are different from those of the banks. This seems to justify the treatment in the text.

(a) *Banks and Non-Speculators*

To find the exchange rate resulting from the combined activity of banks and non-speculators, put $E_{n_t} + E^B_{s_t} = 0$ and solve. This gives (7)

$$SR_t = Be^{pt} + C\cos(\omega t + \alpha) \qquad [IV]$$

where $p<0$, $C<A_1$ and B is an arbitrary constant. Neglecting the recessive term (8) (Be^{pt}), the exchange rate shows regular fluctuations of amplitude $C<A_1$ around the parity level. Banks' speculative activity is therefore seen to be stabilizing.

To show that banks' speculative activity is also profitable, it is first necessary to find an interval of time where banks' speculative stocks of foreign exchange are the same at the beginning as at the end (see above p. 217). The extremes of such interval are found to be tied by the relationship $s_2 = s_1 + \frac{2r\pi}{\omega}$ ($r = 1, 2, 3, \ldots$) where s_1 is chosen arbitrarily. Banks' profits P per cycle of fluctuation of the exchange rate ($r=1$) are given by (9)

$$P = -k\, l\, C^2\,\frac{\pi}{\omega}$$

a positive amount (10). Banks, however, are more interested in the rate of profit than in its absolute amount. Calling S the sums spent by banks, this rate is found to be

$$\frac{P}{S} = \frac{2\pi C}{4R - \pi C}$$

where R is the parity level of the exchange rate (11). The rate of profit shows the convincing property of being a decreasing function

(7) Solutions of the various differential equations, stability conditions and amount of profit obtained are all mathematically discussed in the appendix.

(8) It can be shown that, under plausible assumptions, this term represents from the very beginning a negligible quantity (see the appendix).

(9) Always neglecting the recessive term Be^{pt}.

(10) *Prima facie*, it would seem that the larger is C, the larger are banks' speculative profits, which would run counter to the result previously arrived at of a stabilizing behaviour on the part of banks. C, however, depends *inter alia* on k. A larger k means a smaller C, and it can be shown that, within certain values for k, a smaller C means larger profits. The expression of the absolute profit conceals also a problem of scale. Both problems are dealt with in the appendix.

(11) To arrive at a figure for the sums spent, it is necessary to take explicit account of the exchange parity (see the appendix).

of the amount of speculative stocks employed by banks, which ensures the existence of an equilibrium in banks' assets distribution.

As far as the intervention of the monetary authorities is concerned, it all depends upon the pre-assigned limits of fluctuation of the exchange rate. If these limits are kept reasonably apart (12), no official intervention will be needed, and banks can be expected to take care of stabilizing the foreign exchange market.

(b) *Occasional Speculators and Non-Speculators*

The result of an attack on the exchange parity by occasional speculators is found by putting $E_{n_t} + E^0_{s_t} = 0$ and solving. This gives

$$SR_t = D_1 \, e^{q_1 t} + D_2 \, e^{q_2 t} + E \, cos \, (\omega t + \beta) \qquad [V]$$

where q_1 and q_2 are of unstable type, D_1, D_2 are arbitrary constants and $E < A_1$. Occasional speculators therefore destabilize the exchange rate. While it is impossible to calculate profits according to the afore-mentioned criterion (13), it can easily be seen that, if successful (i.e., if the monetary authorities are forced to change the parity), speculative activity will be profitable.

If they want to maintain the exchange parity, monetary authorities have no choice but to indulge in a tug of war with occasional speculators. Success will depend upon their relative strength (and nerve?).

(c) *Banks, Occasional Speculators and Non-Speculators*

To see the consequences of the simultaneous operations of all the groups, put $E_{n_t} + E^B_{s_t} + E^0_{s_t} = 0$ and solve. This gives

$$SR_t = G_1 \, e^{w_1 t} + G_2 \, e^{w_2 t} + H \, cos \, (\omega t + \gamma) \qquad [VI]$$

where G_1 and G_2 are arbitrary constants and $H < A_1$; w_1 and w_2 are of unstable type if $|km| < b$ (i.e., occasional speculators win over banks) and of stable type if $|km| > b$ (i.e., banks prove stronger than occasional speculators).

(12) For instance, a fluctuation of 5% on each side of the exchange parity twice a year generates a profit of roughly 16% on the speculative funds of banks. This figure seems to be high enough to ensure that sufficient speculative activity will in fact be forthcoming.

(13) While occasional speculators exert their pressure, speculative stocks are always varying.

The two cases must be discussed separately. Case one is similar to that under *b*) above. Banks do not succeed in overcoming the pressure from occasional speculators, and the exchange pattern is destabilized. w_1 and w_2 are, however, less unstable than q_1 and q_2 (see the appendix) and so the intensity of the monetary authorities' intervention to maintain the exchange parity is, *pro tanto*, reduced. Case two shows that banks would eventually succeed in repelling the occasional speculators' attack, and establish, in the course of

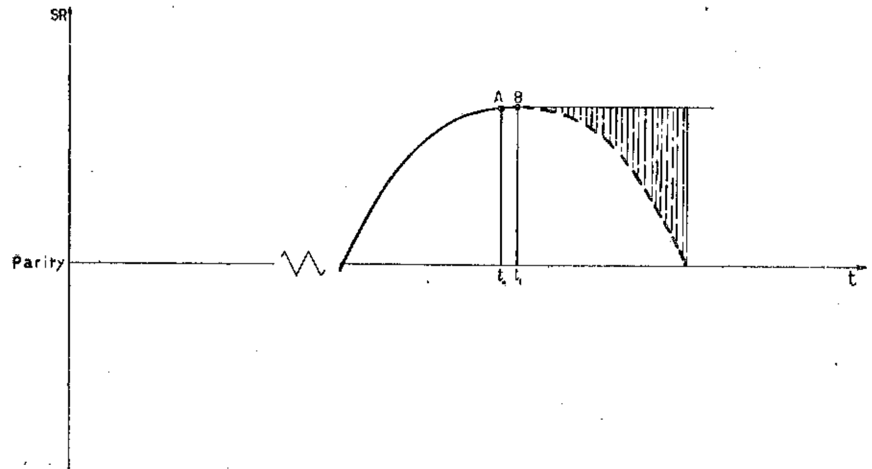

FIG. 2

time, a path for the exchange rate showing regular fluctuations of costant amplitude ($H < A_1$) around the parity level, provided occasional speculators remain in the market (14). This proviso points out to the hypothetical nature of [VI], which also implies that the monetary authorities do not intervene in the struggle. This condition is very unlikely to be fulfilled, since the values for the exchange rate given by solution [VI] will normally (15) exceed the pre-established limits of oscillation, and force the monetary authorities to intervene in order to maintain the exchange at one of its boundaries. To repeat, solution [VI] is only hypothetical since it shows

(14) This contradicts the definition for occasional speculators previously offered (see p. 217, above). The essence of the occasional speculators' behaviour lies in its temporary character.

(15) This will only be true temporarily; but temporariness is all that matters in the case of occasional speculators.

what would happen to the exchange rate if monetary authorities did not intervene and occasional speculators remained in the market. The importance of solution [VI] lies, however, in its demonstration that banks have a built-in tendency of fighting against occasional speculators even when, if left alone, they will not be successful. This tendency is emphasized by the behaviour of banks' profits during an attack by occasional speculators. In figure 2, above, if occasional speculators intervene at point A, the exchange rate will move to B and remain at its upper limit held constant by the monetary authorities' intervention. After time t_1, banks will reap an additional profit on their speculative stocks at the expense of occasional speculators (16). Banks may even decide to utilize additional funds for speculative purposes to continue selling in the face of an extremely favourable rate, when it becomes apparent that "abnormal" forces hold the exchange rate against its upper limit. Be that as it may, the first impact of an attack by occasional speculators finds the banks side by side with the monetary authorities in repelling it.

Concluding Remarks

Banks' speculative activity is, therefore, beneficial both in stabilizing the exchange rate in the face of non-speculators' operation and in repelling a sudden attack by occasional speculators. To carry on this activity, banks require: (a) the exchange rate to oscillate with wide enough swings to permit them to gain a sufficient profit; (b) the certainty that the monetary authorities will prevent the exchange rate from moving outside its pre-assigned limits.

Rome BRUNO CUTILLI

(16) In the absence of occasional speculators, the exchange rate would have followed the dotted line in figure 2. Now banks obtain an additional profit by selling all their remaining speculative stock at the most favourable price to them.

APPENDIX

The purpose of this appendix is to show how the results stated in the text have been derived analytically.

I) *Banks and non-speculators*

The condition $E_{n_t}+E^B_{s_t}=0$ (see text page 221) yields the equation:

[1] $$km\frac{dSR}{dt}+(kl+a_1)\,SR_t+A\cos\omega t=0.$$

The general solution of the reduced (homogeneous) equation is:

[2] $$SR_t=B\,e^{pt}\ (B \text{ arbitray constant, } p=-\frac{kl+a_1}{km}).$$

Calling SR_o the value of the exchange rate at the moment when banks begin their speculative activity (i.e., for $t=0$), the solution becomes:

[2bis] $$SR_t=SR_o e^{pt}$$

It seems plausible to assume that banks, having observed the past behaviour of the exchange rate and having decided to set up as speculators, begin their activity when the (effective) exchange rate is just falling below the parity. This means that SR_o is very small in absolute value.

A particular solution of [1] is:

[3] $$SR_t=B_1\cos\omega t+B_2\sin\omega t$$

where: $$B_1=\frac{-A(kl+a_1)}{(kl+a_1)^2+\omega^2k^2m^2},\qquad B_2=\frac{-A\omega km}{(kl+a_1)^2+\omega^2k^2m^2}.$$

The solution [3] can be written as:

[3bis] $$SR_t=C\cos(\omega t+\alpha)$$

where:

$$[4] \qquad C = \frac{A}{+\sqrt{(kl+a_1)^2 + \omega^2 k^2 m^2}}, \quad sin\alpha = -\frac{B_2}{C}, \quad cos\alpha = \frac{B_1}{C}.$$

The general solution of [1] is then:

$$[5] \qquad SR_t = SR_o e^{pt} + C \cos(\omega t + \alpha).$$

The denominator of the fraction appearing in [4] is greater than $-a_1$ and so $C < A_1$.

It must now be shown that banks' stabilizing behaviour is also profitable (*).

Profits, for any period $s_1 - s_2$, are $P = -\int_{s_1}^{s_2} E^B_{s_t} SR_t dt$, where s_1 and s_2 must be such that $\int_{s_1}^{s_2} E^B_{s_t} dt = 0$ (**).

The value of $\int_{s_1}^{s_2} E^B_{s_t} dt$ is:

$$[6] \qquad klC \frac{1}{\omega} \Big[sin(\omega t + \alpha) \Big]_{s_1}^{s_2} + kmC \Big[cos(\omega t + \alpha) \Big]_{s_1}^{s_2}.$$

Having chosen s_1 arbitrarily, s_2 must be such that $s_2 = \frac{2r\pi}{\omega} + s_1$ ($r = 1, 2, 3, \ldots$) in order that both expressions between square brackets in [6] vanish. The condition is then identically satisfied.

The value of $-\int_{s_1}^{s_2} E^B_{s_t} SR_t dt$ is:

$$[7] \qquad -klC^2 \frac{1}{2} \Big[t \Big]_{s_1}^{s_2} - klC^2 \frac{1}{4\omega} \Big[sin2(\omega t + \alpha) \Big]_{s_1}^{s_2} - kmC^2 \frac{1}{4} \Big[cos2(\omega t + \alpha) \Big]_{s_1}^{s_2}.$$

Since $s_2 = s_1 + \frac{2r\pi}{\omega}$, the last two terms in [7] vanish, and so:

$$[8] \qquad P = -klC^2 \frac{r\pi}{\omega}$$

(*) SR_o being very small in absolute value and $p < 0$, the first term in the solution [5] will be neglected in all that follows.

(**) For this condition, see L. G. Telser, "A Theory of Speculation Relating Profitability and Stability", *The Review of Economics and Statistics*, Vol. XLI, No. 3, August 1959, pp. 295-296.

Considering the behaviour of P for different values of the intervention parameter k, it can be seen that P increases with k if $0 < k < k_1$, is stationary if $k = k_1$ and decreases if $k > k_1$, where $k_1 = \frac{-a_1}{+\sqrt{l^2 + \omega^2 m^2}}$.

Since a greater k means a smaller C, it follows that, between k and k_1, the smaller C, the greater the profits (i.e., the increase in k more than offsets the consequent decrease in C) and *vice versa* (the greater C, the smaller the profits). Obviously, the opposite holds if $k > k_1$.

So far, only absolute profits have been considered. But banks are mainly interested in the rate of profit.

The latter is given by $\frac{P}{S}$, where S is the amount of local currency employed by banks to obtain that P.

There is no loss of generality if only one cycle is considered and if s_1 is set at a point where the (effective) exchange rate is beginning to fall below parity. Since banks buy below parity, S is given by:

$$[9] \qquad S = \int_{s_1}^{s_1+\frac{\pi}{\omega}} E^B_{s_t} (SR_t + R) dt \ (*) \qquad (R = \text{parity})$$

where

$$s_1 = \frac{\pi - 2\alpha}{2\omega} + s \frac{2\pi}{\omega} \qquad (s = 0, 1, 2, \ldots).$$

The value of this integral is:

$$[10] \qquad S = klC^2 \frac{1}{2} \Big[t \Big]_{s_1}^{s_1+\frac{\pi}{\omega}} + klC^2 \frac{1}{4\omega} \Big[sin2(\omega t + \alpha) \Big]_{s_1}^{s_1+\frac{\pi}{\omega}} +$$
$$+ kmC^2 \frac{1}{4} \Big[cos2(\omega t + \alpha) \Big]_{s_1}^{s_1+\frac{\pi}{\omega}} + RklC \frac{1}{\omega} \Big[sin(\omega t + \alpha) \Big]_{s_1}^{s_1+\frac{\pi}{\omega}} +$$
$$+ RkmC \Big[cos(\omega t + \alpha) \Big]_{s_1}^{s_1+\frac{\pi}{\omega}}$$

and the result is:

$$[11] \qquad S = \frac{-klC(4R - \pi C)}{2\omega}$$

(*) When we reckon S, the effective exchange rate must be used (instead of deviations), whereas there is no need for it in reckoning P, since:

$$P = -\int_{s_1}^{s_2} E^B_{s_t} (SR_t + R)\, dt = -\int_{s_1}^{s_2} E^B_{s_t} SR_t dt, \text{ being zero the integral } -R \int_{s_1}^{s_2} E^B_{s_t} dt.$$

consequently, the rate of profit is:

[12] $$\frac{P}{S} = \frac{2\pi C}{4R - \pi C}$$

Considering the behaviour of $\frac{P}{S}$ for different values of the parameter k, it can be seen that the rate of profit decreases if k increases (this holds for any $k > 0$).

Something will now be said about the scale problem mentioned in the text (see page 221, note 10). To introduce it, it is necessary to start from non speculators' excess demand, $E_{n_t} = a_1 SR_{n_t} + A \cos\omega t$.

a_1 can be assumed to be about -1, SR_{n_t} cannot be more than parity in absolute value, $\cos\omega t$ is at most ± 1, and so the burden of giving the actual magnitude of E_{n_t} falls entirely on A. If we take it that one unit of A corresponds to one unit of currency, then A (and, consequently, A_1) must be about the same order of magnitude as E_{n_t}. The figure giving A_1 would then be preposterous and actually impossible. The way out is to assume that, when setting $E_{n_t} = 0$, some reasonable reduction of scale in A must be made. How and how much cannot be said on *a priori* grounds, but only on the basis of empirical research.

Being A so reduced, k too must be correspondingly reduced (it can be assumed that l and m are about the same order of magnitude as a_1, and so they do not need any reduction — only k does, being the intervention parameter), so that C will be about the same order of magnitude as A_1 (*). With these assumptions, there is no need to make the scale explicit, since it is enough — as far as C is concerned — to say that k is expressed in the same scale as A and A_1. But when reckoning $P = -\boxed{k}\, lC^2 \frac{\pi}{\omega}$, the true value must be given to the encircled $\boxed{k}$ (not to the k appearing in C, for the reasons just stated) and the same holds for $S = -\frac{\boxed{k}\, lC\,(4R - \pi C)}{2\omega}$.

Not knowing the scale, no value can be given *a priori* to P and S.

When the rate of profit is reckoned, the encircled k disappears, and so theoretical values for $\frac{P}{S}$ can be found, giving to C any (*a priori*) percentual value of R.

(*) For instance, applying this reasoning to the lira/dollar parity of 625 liras per U.S. dollar, if A_1 is, say, 50 lire, $C < A_1$ may then be, say, 30 lire.

II) *Occasional speculators and non-speculators.*

The condition $E_{n_t} + E^{o}_{st} = 0$ (see text page 222) yields the equation:

[1] $$c\frac{d^2SR}{dt^2} + b\frac{dSR}{dt} + a_1 SR_t + A\cos\omega t = 0.$$

A particular solution is:

[2] $$SR_t = E\cos(\omega t + \beta)$$

where:

[3] $$E = \frac{A}{+\sqrt{b^2\omega^2 + (a_1 - \omega^2 c)^2}} \quad (*)$$

$$\sin\beta = \frac{Ab\omega}{b^2\omega^2 + (a_1 - \omega^2 c)^2} E^{-1}$$

$$\cos\beta = \frac{-A(a_1 - \omega^2 c)}{b^2\omega^2 + (a_1 - \omega^2 c)^2} E^{-1}.$$

The denominator of E is greater than $-a_1$ and so $E < A_1$.

The general solution of the reduced equation is:

[4] $$SR_t = D_1 e^{q_1 t} + D_2 e^{q_2 t}$$

$$\left(D_1, D_2 \text{ arbitrary constants;} \quad q_1, q_2 = \frac{-b \pm\sqrt{b^2 - 4a_1 c}}{2c}\right)$$

q_1, q_2 are both positive (if real) or have positive real part (if complex).

The general solution of [1] is then either:

[5] $$SR_t = D_1 e^{q_1 t} + D_2 e^{q_2 t} + E\cos(\omega t + \beta)$$

or:

[6] $$SR_t = De^{\eta t}\cos(\zeta t + \varepsilon) + E\cos(\omega t + \beta)$$

$$\left(D, \varepsilon \text{ arbitrary constants;} \quad \eta = -\frac{b}{2c}, \quad \zeta = \frac{+\sqrt{|b^2 - 4a_1 c|}}{-2c}\right)$$

(*) Here too there is the same scale problem as in I) and, as before, it might be assumed that the intervention parameters b and c are in the same scale as A.

III) *Banks, occasional speculators and non-speculators.*

The condition $E_{n_t} + E^O_{s_t} + E^B_{s_t} = 0$ (see text page 222) yields the equation:

$$[1] \qquad c\,\frac{d^2SR}{dt^2} + (b+km)\,\frac{dSR}{dt} + (kl+a_1)SR_t + A\ cos\omega t = 0.$$

The general solution of the reduced equation is:

$$[2] \qquad SR_t = G_1e^{w_1t} + G_2e^{w_2t}$$

$(G_1,\ G_2$ arbitrary constants; $w_1,\ w_2 = \dfrac{-(b+km) \pm\sqrt{(b+km)^2 - 4c(kl+a_1)}}{2c})$.

The stability condition is $|km| > b$ (*). In this case, w_1 and w_2 will be both negative or with negative real part. On the contrary, if $b > |km|$, w_1 and w_2 will be both positive or with positive real part. However, in this second case, it can be seen that $w_1 < q_1$ and $w_2 < q_2$ (or real part w_1, $w_2 <$ real part q_1, q_2), so that the solution is less unstable than II, [4].

A particular solution of [1] is:

$$[3] \qquad SR_t = H\ cos\ (\omega t + \gamma)$$

where:

$$[4] \qquad H = \frac{A}{+\sqrt{(a_1+kl-c\omega^2)^2 + \omega^2(b+km)^2}}\ (^{**})$$

$$sin\gamma = \frac{A\omega(b+km)}{(a_1+kl-c\omega^2)^2 + \omega^2(b+km)^2}\ H^{-1}$$

$$cos\gamma = \frac{-A(a_1+kl-c\omega^2)}{(a_1+kl-c\omega^2)^2 + \omega^2(b+km)^2}\ H^{-1}$$

The general solution of [1] is then either:

$$[5] \qquad SR_t = G_1e^{w_1t} + G_2e^{w_2t} + H\ cos\ (\omega t + \gamma)$$

(*) The comparison between $|km|$ and b requires that both be in the same scale (see above, scale problem).

(**) Same scale problem as before. k, b and c must be in the same scale as A.

or:

$$[6] \qquad SR_t = Ge^{\lambda t}cos\ (\mu t + \delta) + H\ cos\ (\omega t + \gamma)$$

$(G,\ \delta$ arbitrary constants; $\lambda = -\dfrac{b+km}{2c},\ \mu = \dfrac{+\sqrt{|(b+km)^2 - 4c(kl+a_1)|}}{-2c})$.

The denominator of the fraction appearing in [4] is greater than $-a_1$ and so $H < A_1$.

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