Money, Prices and Fiscal Lags: A Note on the Dynamics of Inflation [1]

The monetarist-structuralist controversy, which was in full swing in Latin America at the turn of the decade, appears to be now well past its climax. Few Latin American economists would doubt to-day that sectoral disequilibria can have inflationary repercussions; and still fewer would deny the importance of errors in monetary policy as a possible factor of inflation. The eclectic view tends to prevail that both real and monetary causes, combined in different degrees, are responsible for the graver instances of continuing inflation within the area. The absolute monetarist, no less than the structuralist *pur sang*, have become rare species. And the same latitude of mind seems to preside over some current stabilization endeavours.

But there is one limitation which both strands of thought have in common; one which, in consequence, cannot be removed by simply twining them together. For both are (to borrow the phraseology of economic dynamics) exogenous theories, similar in nature to those models of the trade cycle that depend on the recurrence of autonomous shocks. Chronic inflation, in the pure monetarist view, results from the persistence of active inflationary policies; in the structuralist view, it reflects the continuance of underlying real pressures. Both conceptions tend to underrate, if not to disregard entirely, the part of mere inertia in the process. But the elements of inertia may be strong from the outset; and, at all events, they are certain to become stronger and more refractory as inflation moves on.

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The most familiar source of inflationary inertia is the anticipation of new price rises, whenever it derives from the experience of previous increases in the price level. This subjective fact tends to keep up inflation in two ways. On the side of demand, it carries with it a change both in the terms of choice between present and future consumption and in the terms of choice between money and other assets. On the side of supply, it induces an upthrust of wage claims. Yet, generally speaking, expectations can preserve the impetus of price rises only in a limited measure. Their efficacy from this viewpoint will burn out if unsupported by other inflationary factors.

More dangerous, in the last resort, is the possibility of inertia arising from objective circumstances: built-in inertia, so to speak. It is not difficult to imagine dynamic situations in which this will take place. Let us suppose, for example, that the envisaged economy tends to oscillate finitely about the equilibrium values. Relative prices are subject accordingly to a cobweb-type regular fluctuation. Hence, if money prices are inflexible downwards, any departure from equilibrium is bound to entail inflation at a sustained pace. An outside shock will be necessary to set the process going; but, once started, it will be kept in motion by its own internal drive (2).

It is not proposed here, nevertheless, to consider any such general-inertia model. Instead we shall spotlight attention on a specific element of inertia, derived from the fiscal system. It is often found, in practice, that Government expenditure follows the rise of the price level more rapidly than does government income. This fact is traceable to various circumstances, namely: (a) Taxes collected in any given period are partly based, to a greater or lesser degree, on the level of private income at an earlier period. (b) Some sources of revenue depend on the exchange rate, which as a rule lags temporarily behind the internal price changes. (c) Prices paid by the Government tend to be, on the whole, more responsive to inflation than prices of public utilities. Thus a general price expansion (no matter whether it is unleashed by

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(2) This type of sequence represents a possible form of structural inflation, as described in our previous paper: " On Structural Inflation and Latin-American Structuralism ", *Oxford Economic Papers*, 16, No. 3 (November 1964), pp. 321 sqq.

monetary or by real causes) is liable to originate a certain amount of "passive" fiscal deficit (3).

The inflationary spiral that may obviously develop from this fact has some interesting and relevant features, which seem however to have escaped scrutiny. A little mathematical formulation will bring them into focus. Let it be assumed, to begin with, that the fiscal deficit determines an identical increase of the money supply:

$$[1] \qquad D_t = \Delta M_t \,,$$

where D denotes the fiscal deficit and M the stock of money, whilst the suffix identifies the time interval. Let us suppose further that the total amount of real cash balances tends to remain fixed. It will be realistic to allow for some delay in the operation of this tendency. Accordingly,

$$[2] \qquad \hat{P}_t = \frac{\Delta M_{t-1}}{M_{t-1}} = \frac{D_{t-1}}{M_{t-1}} = \frac{D_{t-1}}{Y_{t-1}} V \,,$$

where $\hat{P}$ is the proportional rate of price change, Y the national income, and V the income-velocity of money. Thus the inflationary effect of any given fiscal deficit is larger, other things equal, the lower the national income and the higher the velocity of circulation. We now insert the alluded fiscal lag,

$$[3] \qquad D_t \equiv G_t - I_t = hY_t - kY_{t-1} \,,$$

in which G represents Government expenditure, I Government income, h and k institutional coefficients. Substituting [3] into equation [2] yields

$$\hat{P}_t = \left(h - k \frac{Y_{t-2}}{Y_{t-1}} \right) V \,.$$

Moreover, with real income unchanged at the full-employment level,

$$Y_{t-1} = Y_{t-2} \, (1 + \hat{P}_{t-2}) \,,$$

(3) Concerning the importance of this fact in the Latin American economies, see, especially: N. Macrae, "These Inflationary Recessions", *The Economist*, 25 September - 1 October 1965, pp. XXIV sqq.; A. Ferrer, *The Argentine Economy* (transl. M. M. Urquidi), Berkeley and Los Angeles, 1967, ch. 17, pp. 185 sqq.; R. F. Mikesell, "Inflation and Growth: Observations from Latin America", in P. L. Kleinsorge (Ed.), *Public Finance and Welfare Essays in Honor of C. Ward Macy*, Eugene (Oregon), 1966, pp. 255 sqq.

and so

$$[4] \qquad \hat{P}_t = \left(h - k \frac{1}{1 + \hat{P}_{t-2}} \right) V \,.$$

Now, in order to isolate the working of the fiscal lag, suppose that the Budget would keep in balance if prices were unaltered. This implies $h = k$, so that the preceding formula resolves into

$$[5] \qquad \hat{P}_t = hV \left(1 - \frac{1}{1 + \hat{P}_{t-2}} \right) ,$$

which summarises the dynamic nexus that concerns us here.

We are now in a position to consider its properties. It is easy to find out that equation [5] possesses two "equilibrium" solutions, under which

$$\hat{P}_t = \hat{P}_{t-2} = \text{constant} \,,$$

for all periods. One of them is the price-stability situation:

$$[6] \qquad \hat{P} = 0 \,,$$

the other equilibrium being

$$[7] \qquad \hat{P} = hV - 1 \,,$$

that coincides with the former solely in the fluke case of $V = 1/h$; otherwise it involves a distinct value. But prices, for the most part, are downward inflexible. We must therefore distinguish two states of things: if $V < 1/h$ there is only one attainable equilibrium, which corresponds to price stability; if $V > 1/h$ there are two attainable equilibrium points, one corresponding to price stability, the other to steady inflation.

Let us turn next to the stability of equilibrium. We observe that

$$\frac{d\hat{P}_t}{d\hat{P}_{t-2}} = \frac{hV}{(1 + \hat{P}_{t-2})^2}$$

is necessarily positive, whilst

$$\frac{d^2\hat{P}_t}{d\hat{P}^2_{t-2}} = - \frac{2hV}{(1 + \hat{P}_{t-2})^3}$$

is negative. The curve defined by [5] is thus monotone increasing and concave to the $\hat{P}_{t-2}$ axis. This is all we need to know for our quaesitum. Graph 1 a presents the movement of prices under the hypothesis $V < 1/h$. It indicates clearly that the solution $\hat{P}=0$ is stable. Graph 1 b, on the other hand depicts the case $V > 1/h$. As indicated by the arrows, the solution $\hat{P}=0$ turns out to be *unstable* in this case, whilst the steady-inflation point is stable. The system tends to settle down along a path of self-sustained inflation.

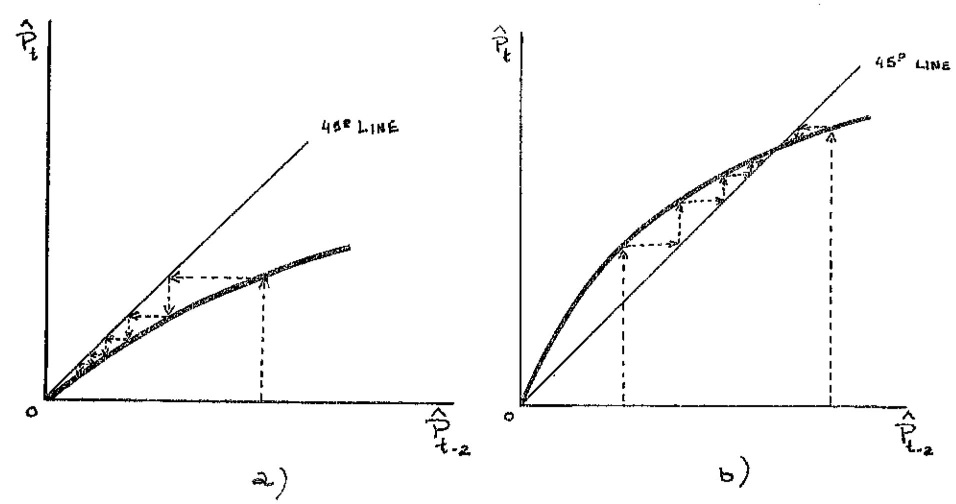

Fig. 1

We have thus reached the crux of the matter. Suppose an initial non-inflationary state ($\hat{P}=0$) and then a supervening shock, either from the monetary or from the real sector, which induces a rise of the price level. Whenever $V < 1/h$, the system will converge back to the non-inflationary equilibrium. But if $V > 1/h$, the system will shift away from price-stability on to steady inflation. That is to say, inflation will become a lasting self-contained process. Any further inflationary shock will add for some time an extra layer of price increases on top of the steady-inflation rate. And any possible stabilization effort, if not strong enough to achieve the $\hat{P}=0$ target, will reduce the rate of inflation only in a precarious and transitory form.

It may be observed, parenthetically, that these results are not conditional upon the lag assumption made in [2]. The element of

tardiness in price responses can be either accentuated or dropped out, to suit the actual experience, without effect on our conclusions. We may verify this by the introduction of the generalised distributed lag:

$$\hat{P}_t = a_0 \frac{\Delta M_t}{M_t} + a_1 \frac{\Delta M_{t-1}}{M_{t-1}} + a_2 \frac{\Delta M_{t-2}}{M_{t-2}} + \dots ,$$

where the a's are nonnegative coefficients that add up to unity. Instead of [5] we obtain

$$\hat{P}_t = Vh \left\{ a_0 \left(1 - \frac{1}{1 + \hat{P}_{t-1}} \right) + a_1 \left(1 - \frac{1}{1 + \hat{P}_{t-2}} \right) + \right.$$
$$\left. + a_2 \left(1 - \frac{1}{1 + \hat{P}_{t-3}} \right) + \dots \right\}$$

which has obviously the same stationary solutions ($\hat{P}=$constant) as [5]. The stability analysis also carries over to the generalised equation, since the

$$\frac{\partial \hat{P}_t}{\partial \hat{P}_{t-i}} , \; (i = 1, 2, 3 \dots)$$

are positive, and the

$$\frac{\partial^2 \hat{P}_t}{\partial \hat{P}^2_{t-i}} , \; (i = 1, 2, 3 \dots)$$

are all negative.

Likewise, it can be easily shown that our results are largely independent of the particular lag structure assumed by [3], in so far as there exists some relative delay of fiscal income as compared with fiscal outgoings. But, in contrast to the lag pattern, the length of the unit period has a key role throughout. This is so because V is a flow-stock ratio whereas h is a flow-flow ratio. To avoid arbitrariness, we may choose our time unit in such a way that the fiscal lag equals one period. Thus, if the actual duration of the interval is θ, we can put $\theta = 1$ and switch all magnitudes to the time scale so determined.

Up to this point we have proceeded on the hypothesis of "static" expectations, each new price level being regarded by everybody as destined to rule *ad infinitum*. This assumption permitted us to isolate the working of the fiscal lag, in abstraction from other sources of inflationary inertia. But having accomplished this aim, we may now render the above scheme of analysis more realistic by taking account of "dynamic" expectations. Income-velocity will therefore be made to depend on the expected rate of price increase. A well-attested hypothesis (4) in this connexion is

$$1/V_t = \varphi_0 \, e^{-\alpha E_t}$$

where E_t designates the rate of price increase ($\hat{P}_t$) expected to come about in t and subsequent periods, whilst φ_0 and α represent constants. Whenever $E_t = 0$ then $1/V_t = \varphi_0 = 1/V$, and the analysis reduces to the foregoing model. The anticipated rate of inflation, in its turn, will somehow reflect the experience of past price-movements. It may be assumed that this factor is subject to a geometric lag:

$$E_t = (1 - r)\,(\hat{P}_{t-1} + r\hat{P}_{t-2} + r^2\hat{P}_{t-3} + \ldots) ,$$

where r $(0 < r < 1)$ is a given ratio. It will simplify the argument if we approximate these functions linearly:

[8] $$1/V_t = \varphi_0\,(1 - \alpha E_t) ,$$

and, for small r,

[9] $$E_t = \hat{P}_{t-1} .$$

Now, replacing V by V_t in [5] we get

[10] $$\hat{P}_t = \frac{h}{\varphi_0\,(1 - \alpha\hat{P}_{t-2})}\left(1 - \frac{1}{1 + \hat{P}_{t-2}}\right)$$

which sums up the sequential process in the case of adaptive expectations. This obviously admits the stationary root $\hat{P} = 0$. There are, moreover, two additional equilibria corresponding to the roots of the quadratic equation

[11] $$(\hat{P})^2 + \frac{\alpha - 1}{\alpha}\,\hat{P} + \frac{h - \varphi_0}{\alpha\varphi_0} = 0 ,$$

obtained from [10] on setting $\hat{P}_t = \hat{P}_{t-2} =$ constant. We have now three situations to distinguish: first, $\varphi_0 > h$, where the equation

(4) See Ph. Cagan, "The Monetary Dynamics of Hyperinflation", in M. Friedman (ed.) *Studies in the Quantity Theory of Money*, Chicago, 1956, pp. 25 sqq.

has one positive and one negative root; second, $\varphi_0 < h$, $\alpha > 1$, where the equation has two negative roots; and finally, $\varphi_0 < h$, $\alpha < 1$, in which it has two positive roots. Putting aside the possibility of complex roots (5), this evidently implies that there exists one steady-inflation point in the first case, none in the second, and two in the last case.

Let us next take up the problem of dynamic stability. We must search a little more into the characteristics of the recursive formula [10]. It is at once apparent, upon differentiation, that

$$\frac{d\hat{P}_t}{d\hat{P}_{t-2}} \rightarrow \frac{h}{\varphi_0} \text{ as } \hat{P}_{t-2} \rightarrow 0 .$$

Therefore, if $\varphi_0 > h$, the slope of the curve in the neighbourhood of price stability ($\hat{P} = 0$) is less than 1, whilst it is greater than 1 if $\varphi_0 < h$. This also indicates the stability (or lack of it) for each of the other solutions. The resulting phase diagrams are shown in Fig. 2.

As we should expect, the main consequence of introducing dynamic expectations into the previous schema is the possibility of hyperinflation. In all three cases whereinto the model subdivides the upper equilibrium point turns out to be unstable. If the rate of price-increase is pushed beyond this point by some exogenous disturbance, then (whatever the character of the impulse factor) the system becomes explosive and hyperinflation sets in. Thus the extreme equilibrium point represents at the same time the hyperinflation barrier.

From a realistic viewpoint, the case described by Fig. 2b is rather a freak possibility. Whatever the magnitude of the fiscal lag, and even if all the other conditions of the model are fulfilled, it is hard to think of real cases where the slightest deviation from price-stability should give rise to hyperinflation. As a matter of fact, from what is known about the relevant parameters (6), α should be expected to be less than 1. It is, accordingly, the model represented in Fig. *2c* which comes forth as typical when $\varphi_0 < h$.

(5) The necessary and sufficient condition for the roots to be real is

$$\frac{(1-\alpha)^2\,\varphi_0}{4\,\alpha} \geqq h - \varphi_0$$

(6) Cagan, *op. cit.*, pp. 69 sqq. In our treatment of price expectations Cagan's β equals 1, and so his condition $\alpha\beta < 1$ means that our α is less than unity.

Let us confine ourselves, therefore, to the first and third cases. It will be noticed that there is a significant difference between them. In the comparatively low-velocity system ($\varphi_0 > h$) the zero-inflation point is dynamically stable. In the alternative, high-velocity system

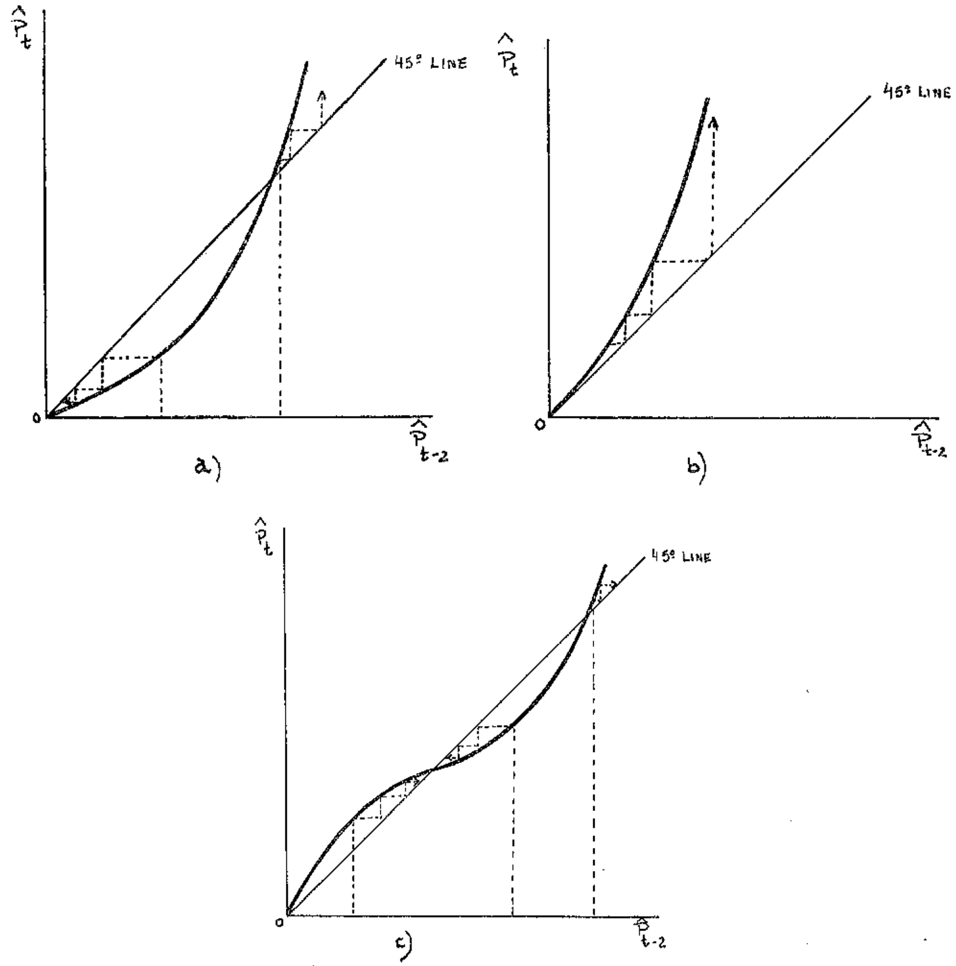

Fig. 2

($\varphi_0 < h$) the dynamically stable solution implies continually rising prices. This stands out clearly from the direction of the adjustment path, as shown by the diagram. The chief result deduced in the first part of this paper is thus confirmed upon our present, more general assumptions.

We shall not undertake here to apply the above model to any actual case of inflation, since it is not designed to convey a full-scale picture of the inflation process in its real complexity. It is not intended to supersede either the monetary or the structural interpretation of the ultimate causes, or any hybrid between the two. But it may be of some service in identifying certain factors that need to be watched. A true stabilisation policy, if it aims at having more than ephemeral success, should go to the root of the inertial tendencies whenever these are important. Otherwise it may find itself vainly rolling uphill the stone of Sisyphus.

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