

## Outline of a Monetary Approach to the Theory of Tax Incidence

1. The relationship between taxation and prices raises two distinct preliminary questions. The first question is whether taxes of different technical nature have different effects on prices; the second, whether taxes can have any effect on prices when the quantity of money and its velocity of circulation are constant. The first question leads to analysis of the effects of various kinds of taxes in identical market conditions and on the assumption that the money supply adapts itself readily to the shifts in demand due to changes in government receipts and expenditure. The second question is concerned only with the monetary transfers and with price changes due to various kinds of taxes in conditions of a general monetary constraint.

The two types of analysis are strictly complementary, but are largely situated on different planes. The first is best conducted in terms of partial equilibria, at least so long as the enquiry is concerned solely with the distribution of the tax burden and leaves out of account the expenditure; the second must be in terms of general equilibrium and must always deal both with revenue and with expenditure. The latter type of analysis is a first step in providing the background and basic outline for further more detailed and more concrete research on the economic effects of taxation and on the distribution of the tax burden.

The relationship between taxes and prices also raises a third question which, methodologically, it is convenient to deal with after we have examined the other two. This is the question of the monetary consequences, and hence the effect on prices, of a public expenditure which may be greater or smaller than the diminution in private expenditure due to the tax levy. In this case the analysis is concerned with an additional problem: that of

the different effects of different kinds of taxes, at a given level of public expenditure, on private consumption, investment and saving. The view widens: it presents two types of analysis which are no longer distinct. One is still concerned with the distribution of the tax burden, and the other extends to the effects of government finance on aggregate national income and on its distribution, in dynamic conditions free of any constraint.

The present paper deals with the second question. An attempt will be made to discover whether it is possible for prices to change irrespective of changes in the quantity of money and in its velocity of circulation; and whether price changes are or are not the same in the case of a direct tax on income and in the case of an indirect tax on goods. The following analysis will lead to an examination of the validity of H. G. Brown's analysis (1), which was later developed by E. R. Rolph (2). Both authors, having implicitly assumed not only a constant quantity of money, but also a constant transactions velocity of circulation, reached the conclusion that direct taxes on income and indirect taxes on goods have the same effect: "Excise taxes, because they reduce private money incomes, operate in a fashion strictly comparable with so-called direct taxes" (3), both as regards prices, which remain unchanged in either case, and as regards incidence.

2. Brown had asserted (4) that "there is... no basis in monetary theory for supposing that a general tax on all goods will make average prices permanently higher. ... If a tax on the output of all goods neither decreases supply nor increases demand, on what basis is it to be argued that such a tax will raise prices?"

He had demonstrated this statement by means of a period analysis of expenditure: given constant values of the two monetary parameters, the tax does not cause effective expenditure to alter in the unit period, and hence the "wedge" of tax which is driven between cost and price, can only compress costs, or factor incomes.

(1) HARRY GUNNISON BROWN, "The Incidence of a General Output or a General Sales Tax", *The Journal of Political Economy*, April 1939, pp. 254-62.

(2) EARL R. ROLPH, "A Proposed Revision of Excise-Tax Theory", *The Journal of Political Economy*, April 1952, pp. 102-17.

(3) ROLPH, *op. cit.*, p. 117.

(4) *Op. cit.*, p. 255.

Other writers, before and at the same time as Brown, simply based their conclusions on the quantity theory of money, stating in general and always more or less similar terms that a tax on goods could raise the general price level only if the money supply increased or its velocity of circulation quickened, in other words if there were changes on the money side. It follows that if neither the volume of money nor its velocity of circulation alter (and there is indeed no obvious connection between taxation and the monetary variables), the general price level cannot change.

Rolph revived and reinforced Brown's period analysis and extended it to special excise taxes. His conclusion is that an excise tax introduced in an inflationary situation when public expenditure exceeds revenue (this is Rolph's initial situation) stops the rise in prices; and that one introduced when the budget is balanced paradoxically tends to reduce rather than to raise prices.

The adherents of the quantity theory, as well as Brown and Rolph and recently other authors (5) all make the same assumption, namely that the volume of money and its transactions velocity of circulation remain constant. The assumption is explicit with the former, with the latter often implicit. The period analysis of Brown and Rolph is, to use Rolph's own words "quantitative theory in the Fisherian tradition of treating money as an objective constraint on behavior" (6). Both groups of theorists reach the result of a constant price level of goods and services by wrongly deducing from a constant transactions velocity that circular (income) velocity must also remain constant; because the speed with which money changes hands between one payment and another does not quicken (and indeed there is no theoretical reason why the tax should alter the habits of the market), the authors conclude that money on the average still needs the same time to traverse its entire course from one income expenditure to the next.

(5) Among others VALERIO SELAN, "Intorno agli effetti delle imposte sul livello generale dei prezzi", *Bancaria*, April 1955, pp. 393-410; A. R. PREST, "Statistical Calculations of Tax Burdens", *Economica*, August 1955, pp. 234-45; WILLI ALBERS, "Preisliche Wirkungen der Besteuerung", *Finanzarchiv*, No. 2, 1955, pp. 227-57. For Selan, credit is an essential factor for a durable price rise; Prest considers that prices can rise only if the supply of money is sufficiently elastic to allow public expenditure to increase in step with the tax levy; Albers states that in a stationary economy the price level rises only if public requirements are covered by consumption taxes and the government is able to create money.

(6) ROLPH, *op. cit.*, p. 108, fn. 11.

3. The Brown-Rolph analysis and its results regarding equality of the effects of various kinds of taxes in conditions of a monetary constraint are open to both external and internal objections. We may question the authors' assumptions as well as the validity of their interpretation of the quantity theory of money, which they take to imply not only a constant volume of money, but also a constant velocity of circulation irrespective of changes on the other side of the equation of exchange; or else, accepting their assumptions, we may re-examine the internal logic of the model and query whether the results are consistent with the postulates. The principal objections of an external order have already been discussed in another paper of mine (7), where I compared the assumptions of the model with a more accurate interpretation of the quantity theory in Fisherian terms and stressed the circumstance that the exchange transactions include some which do not necessarily require any additional quantity of money. These are new transactions which are due to institutional and organizational market changes and which therefore act directly on the velocity of circulation. In this paper I propose to concentrate on internal criticism of the Brown-Rolph model and in general of the argument of those who reach the same or similar conclusions. The paper is a new presentation, along different lines and with new analytical developments, of another essay of mine (8); the conclusions are the same.

Just as in the Brown-Rolph model, the assumptions are: (a) a stationary economy; (b) free competition; (c) a closed economy; (d) rigid factor supply: any reduction in total expenditure is at once reflected in a fall in factor incomes; (e) government intervention in the distribution of national income without any government contribution thereto, such intervention taking the form either of direct purchases of goods and services or of the mere redistribution of tax revenue; (f) money has a constant transactions velocity of circulation, that is to say, its average resting period between successive payments is always the same irrespective of the nature of the transaction and of its being private or public; (g) an indirect

(7) "Imposte indirette su merci e livello generale dei prezzi", *Moneta e Credito*, 1954, No. 2, pp. 144-64 and No. 3, pp. 298-312.

(8) "Imposte, moneta e prezzi", *Rivista di diritto finanziario e scienza delle finanze*, June 1956, pp. 111-136.

tax on goods affects also the goods purchased with its own yield; and (h) both direct and indirect taxes are general.

The purpose of the analysis is to trace the effect on prices of the two principal forms of taxation, *i.e.* direct taxes on income and indirect taxes on goods, in order to ascertain whether their effect is or is not the same, and whether and in what direction prices change. By a tax on goods we mean any kind of indirect tax on the consumption, production, purchase, sale or turnover of goods or services.

We shall distinguish between two price levels: first, the price level of consumer goods and services (9), and secondly, the "general" price level of all goods and services: means of production, intermediate and final goods, capital goods and consumer goods, services to producers and consumers, as well as incomes. The more important price level in the context of this study is that of consumer goods. This is also the point of departure for the incidence theories of all the above-mentioned writers who, when discussing the relationship between taxes and prices, implicitly or sometimes explicitly mean prices to the consumer. In particular, the incidence theory of Brown and Rolph is in their model deduced from stability of the prices to consumers. The general price level of all goods is too general and hence too indeterminate to be relevant to a study of tax incidence, which is concerned with the relative changes in the prices of separate goods and categories of goods.

4. The entire money supply,  $M$ , is assumed to consist of "active" funds required for technical payments needs;  $V_i$  is average velocity of circulation with respect to income in the unit period, or circular velocity — that is the number of times in the unit period a money unit on the average passes from consumers to producers and back again to consumers through the expenditure, production and distribution of income;  $V_t$  is average Fisherian velocity with respect to all payments in the same unit period — called the transactions or exchange velocity; and  $D$  is the average number of stages or successive steps which make up the process of income production, distribution and spending, plus one — that is the number of payments stages through which a money unit on

(9) When, in the following, we speak of the price of goods, services will always be understood to be included.

the average passes in the course of the circular flow of income (10). On the assumption that all transactions occur in the production and spending of national income, we have the identity  $V_t = V_r D$ . Given  $M = K$  constant, changes in effective demand,  $E = MV_r = M \frac{V_t}{D}$ , and hence in prices to the consumer (assuming real income also to be constant), are determined by changes in the values of  $D$  and/or  $V_t$ . If  $V_t$  is also constant, changes in  $V_r$ , and hence in prices to the consumer, will be determined by changes in  $D$  due to the new payments to which the tax gives rise.

It follows that price stability implies new values for  $V_t$ . *Prices and transactions velocity cannot both remain unchanged, the conditions are mutually incompatible.*

The tax in fact modifies the number of times the same quantity of money has to change hands. In abstract terms, the introduction of a tax on income is equivalent to the insertion, into the existing circuit of formation, distribution and spending of income, of new public transactions serving the use of that portion of income which has been paid as tax. On the other hand, the introduction of a tax on goods amounts to the creation, alongside the monetary circuit of private income, of a smaller public monetary circuit consisting of two or more transactions: in the case of a sales tax, payment of the tax by the seller from whom it is levied and (direct or indirect) public expenditure on goods and services produced in the private sector; in the case of another type of indirect tax on goods, the same payment plus payments from the final seller to the producer or intermediate trader from whom the tax is levied.

5. As a result of a new tax there is a change in the number of times the existing money supply changes hands in the circular flow of income.

With a direct tax, consumption expenditure by income recipients is reduced by the amount of the tax. Part of the money flow coming into the hands of the income-receivers is no longer spent by

(10) With respect to these concepts and their use we recall two essays by JAMES W. ANGELL, "Money, Prices and Production: Some Fundamental Concepts", *Quarterly Journal of Economics*, November 1933, pp. 39-76, and "The Components of the Circular Velocity of Money", *ibid.*, February 1937, pp. 224-71.

these: the number of times which the part that is collected as tax is exchanged against goods and services becomes  $D - 1$  while the number of times which the remainder is exchanged is still  $D$ . On the average the number of payments-stages through which money passes in respect solely of private transactions falls to:  $a(D - 1) + (1 - a)D$  that is to  $D - a$ , where  $a$  is the tax rate. As against the diminution of private payments for consumption there are, however, new public transactions concerned with the collection of the tax and the spending of the tax revenue. In the case that the government spends the revenue on direct purchases in the market, there are added to the new number of private transactions,  $D - a$ , public transactions equivalent to  $2a$ . And if the tax yield is first distributed to others, say civil servants, who in turn spend it on goods and services produced in the private sector, the number of public transactions is  $3a$ . Hence the number of payments-stages traversed by the money stock during the entire income circuit becomes  $D + fa$ , where  $f$  stands for the number of purely financial transactions within the public monetary circuit, or that is for the total number of public transactions less one, since that which relates to expenditure on final goods is offset by the reduction of private transactions. Into the large money flow connected with the production, distribution and spending of private income is inserted a smaller money flow brought into existence by the public authority. The value of  $f$  varies between zero and 2; it is zero when the tax is levied at the source and spent by the tax authority on direct purchases in the market, and 2 when the tax is not collected at the source and when the tax authority distributes the yield to its own employees or to other persons.

Distinguishing post-tax values from pre-tax values (when they are not identical by hypothesis) by the superscript', we can write

$$MV_t = MV_r D = M V_r' (D + fa)$$

which gives

$$V_r' = V_r \frac{D}{D + fa} \quad (1)$$

$V_r'$  is always smaller than  $V_r$  except in the unlikely case that  $f = 0$ . This means that there is normally a reduction in

effective demand and a corresponding reduction in national money income. If  $P_c$  is the price level of consumers' goods,

$$P'_c = P_c \frac{D}{D + fa} \quad (2)$$

Prices to the consumer fall (11). Their reduction is a direct function of the tax rate and of the number of payments-stages through which the money collected as taxation has to pass before it is spent on goods, and an inverse function of the degree of market differentiation. The more numerous are the stages of private production and trade and the larger therefore is the number of payments-stages from which the money required for the public monetary flow may be drawn, the smaller will be the fall in prices to the consumer. In any case the absolute fall in the price level is always less than the absolute amount of tax. Given that  $D > f$ , as is in practice the case (12),

$$P_c \left( 1 - \frac{D}{D + fa} \right) < aP_c \frac{D}{D + fa}$$

6. A tax on goods, as we said above, creates a new "public" monetary circuit of its own. There are now two monetary flows, both ending in consumption spending: the old private circuit consisting of payments in the course of the production, distribution and spending of private income, and the new public circuit, comprising payments stages traversed by the tax. These include tax payment by the legally liable taxpayer to the government; direct or indirect payments by the government to retail sellers, according as the government itself makes purchases on the market or distributes the tax revenue to government employees or other individuals; and finally payments from the retail seller to the preceding seller from whom the tax is levied (the two coincide in the case of a tax on retail sales). The money supply is now channelled through two

(11) RICHARD A. MUSGRAVE reached the same conclusion in his paper "On Incidence", *The Journal of Political Economy*, August 1953, pp. 306-23. On the assumption of a constant money supply and constant velocity a direct tax on income depresses prices, with the single exception of a tax levied at the source.

(12) *A fortiori*, the price fall is less than the absolute amount of the tax calculated on the initial income. This follows from the inequality  $D > f(1 - a)$ .

separate monetary circuits and splits into two parts in proportion with the two parts of total sales receipts at market prices: one part which is and one which is not collected in tax. The latter continues, as before, to traverse an average of  $D$  payments stages in the circular flow of production, distribution and spending of private income. The remaining part traverses two or more stages depending on the type of tax.

That part of the money supply which is required for tax payment and public expenditure traverses an average number of payment stages  $h \geq 2$ . We can write

$$MV_t = MV_r D = V'_r [(1 - a)MD + aMh]$$

from which we get

$$V'_r = V_r \frac{1}{(1 - a) + \frac{ah}{D}} \quad (3)$$

When  $h = D$ , the denominator is equal to unity and hence  $V'_r = V_r$ ; when  $h < D$ , it is smaller than unity and  $V'_r > V_r$ ; and when  $h > D$  it is bigger than unity and  $V'_r < V_r$ . Since we have assumed real income to be constant, we can write

$$P'_c = P_c \frac{1}{(1 - a) + \frac{ah}{D}} \quad (4)$$

An indirect tax on goods does not change prices to the consumer when the average number of payments stages in the public monetary circuit equals the average number of payments stages traversed in the private circular flow of income; prices to the consumer rise when there are fewer payments stages in the public circuit than in the private one, and prices fall in the opposite case. The new price level of consumer goods is not only a direct function of the tax rate, as traditional theory says; it is also a direct function of the degree of differentiation of the economic process, that is of the number of transactions stages from which the additional money required by the public authorities is drawn, and over which, therefore, it is distributed. This new price level is besides

an inverse function of the number of payments stages traversed by the public monetary flow.

The equality  $h=D$  calls to mind two definite types of tax (or of their combination): an indirect tax levied at the earliest production stage, say on the sale of raw materials, when the government spends the tax revenue directly, or an indirect tax applied one stage later in case the tax revenue is paid out to government employees or others. The inequality  $h < D$  calls to mind the usual forms of indirect tax, either a turnover tax since this usually falls partly, or even exclusively, to stages close to consumption, or a tax on production which is normally levied at the last production stage, or, *a fortiori*, a sales tax. The inequality  $h > D$  calls to mind an indirect tax levied only on the first production stage, when the government does not spend the tax revenue directly. In this latter case  $h=D+1$ , that is to say,  $D=h-1$ . The tax enters into all prices as far as consumption; it does not enter into private incomes, but instead traverses two further stages which are not connected with any exchange of goods and services. These are from the taxpayer to the government, and from the government to its employees or other persons.

Of the three abstract cases discussed in the analysis the second is that which reflects the actual conditions of indirect taxation. We can consider it as the normal, or general, case that  $h < D$ . Normally a tax on goods raises prices to the consumer, and it raises them the more the longer is the private monetary circuit and the shorter the public one, and the higher is the tax rate. In the particular case of a sales tax (when  $h$  should be between 2 and 3 according as public expenditure is direct or indirect) a price rise can be avoided only if the private process of income production, distribution and spending comprises one stage, or two, respectively (13). In the particular case of a production tax levied at the stage immediately preceding sale to the consumer (when  $h$  would be 3 and 4 respectively), prices would not rise only if the private process of income production, distribution and spending comprises two stages, or three, respectively.

(13) The result of Musgrave's analysis, that prices remain unchanged if  $M=K$ , is due to the fact that he implicitly assumed not the general case, but the unrealistic one where  $h=D$  and both are equal to 2.

If prices to the consumer rise, the absolute price increase is always less than the absolute amount of the tax. For

$$P_c \left[ \frac{1}{(1-a) + \frac{ah}{D}} - 1 \right] < aP_c \frac{1}{(1-a) + \frac{ah}{D}}, \text{ given that } -ah < 0.$$

7. Neither in the case of a tax on goods nor in that of a tax on income is it normally possible for consumer prices to remain stable if both the quantity of money and its transactions velocity are constant. Price stability requires in the case of an income tax that it be levied at the source and that government spending accurately replace private spending also in its timing; and in the case of a tax on goods, that the public monetary circuit have the same number of payments stages as the private one.

Otherwise, since  $MV' \neq MV_r$ , the price level of consumer goods will always be different after the imposition of the tax from what it was before. After an income tax prices to the consumer always fall except in the unlikely case that  $f=0$ ; after a tax on goods they rise except in the unlikely case that  $D \leq h$ . The post-tax price level of consumer goods will be the same under the two forms of taxation, giving an identical money yield (besides the identical real yield in terms of consumer goods) only in the case where (as is shown in the appendix)  $D=h-f$ ; this implies that  $f$  must not be greater than unity so that  $D=h-1$ . The tax on goods would have to be levied on exchanges further removed from consumption spending, and, since it does not enter into private incomes, the government would have to spend the tax revenue not directly but pay it out to others, whereby it would appear on the market as other people's income before consumption spending; on the other hand the yield of the income tax would have to be spent directly by the government. The government's financial process, which is different on the revenue side, would have to be different also on the expenditure side. Only on this condition could we have the situation which Rolph considers as normal, namely the same price level for both types of taxes — with the difference, however, that the prices are not as before tax, but lower (14).

(14) As is shown in the appendix, the final price levels of consumer goods could be equal in other circumstances too. But then the values of  $D$ ,  $h$  and  $f$  would have to be different for the two kinds of taxes and this means that we would be comparing different taxes in different situations.

INCOME TAX  
(money units)

TABLE I

Time unit	Private income	Tax (rate 10%)	Public expenditure	Value of sales
1	100	10	—	90
2	100	10	10	100
3	100	10	10	100
4	100	10	10	100
5	90	9	10	91
6	100	10	9	99
7	100	10	10	100
8	100	10	10	100
9	91	9.1	10.0	91.9
10	99	9.9	9.1	98.2
11	100	10.0	9.9	99.9
12	100	10.0	10.0	100
57	97.25	9.72	9.91	97.44
58	96.29	9.63	9.72	96.39
59	97.55	9.75	9.63	97.42
60	99.00	9.90	9.75	98.86
(n)	97.56	9.76	9.76	97.56

SALES TAX  
(money units)

TABLE II

Time unit	Tax (rate 10%)	Private income	Public expenditure	Value of sales
1	10	100	—	100
2	10	100	10	110
3	11	100	10	110
4	11	90	11	101
5	10.1	90	11	101
6	10.1	99	10.1	109.1
7	10.91	99	10.1	109.1
8	10.91	90.9	10.91	101.81
9	10.18	90.9	10.91	101.81
10	10.18	98.19	10.18	108.37
11	10.84	98.19	10.18	108.37
12	10.84	91.63	10.84	102.47
57	10.50	94.43	10.56	104.99
58	10.50	95.01	10.50	105.51
59	10.55	95.01	10.50	105.51
60	10.55	94.50	10.55	105.04
(n)	10.53	94.74	10.53	105.26

Finally, even when a tax on goods depresses the price level of consumer goods, the new price level is always higher than that which would result from an alternative income tax, provided only that  $D > h - f$ .

8. The above results represent the final position arrived at under the condition of *ceteris paribus*, the tax being the only new element. They can be seen and understood even more clearly by means of an explicit period analysis, such as is set out in the two tables. Table I illustrates the process of adjustment of consumer prices after an income tax, and Table II the same process after a tax on goods. For the sake of simplicity the following numerical values have been assumed:  $D=4$ ,  $h=2$ ,  $f=1$ ,  $a=0.10$  and  $M=400$ . The market is one in which the process of production, distribution and spending of private income comprises only three stages, the government makes direct purchases of consumer goods and services in the market and the indirect tax is a tax on retail sales.

The tables confirm our formulae. The income tax is shown to reduce prices by 2.4 per cent, and the tax on goods to raise them by 5.26 per cent, always assuming unchanged real income. If we assume that the circular velocity of circulation of money is three in a one-year period (that is that the income period, or the period within which the incomes spent equal the money supply, is four months), then the time units 1-12 show the situation during the first year and the time units 57-60 that of the last four months of the fifth year.

In both tables tax collection is the first move of the public financial activity and precedes public expenditure, and the latter begins before the deflationary process set in motion by the tax has run its course. We would get similarly differentiated results if we made the opposite assumption that public expenditure precedes tax collection. But this would mean violating the postulated monetary constraint, because we would have to assume initial credit or an increase in the money supply.

9. As has been noted earlier, we have to consider the effects of the different kinds of taxes on prices in conditions of a monetary constraint under two aspects: the effects on the prices of consumer goods, and those on the general price level of all goods and services, including the remuneration of factors

of production, *i.e.* incomes. The strict distinction we have drawn between the two price levels has enabled us to avoid the common error of first arguing in terms of the general price level and then drawing conclusions regarding the distribution of the tax load which are implicitly connected with the price level of consumer goods.

The necessity for making this methodological distinction arises particularly when we are considering a tax on goods and comparing it with an income tax. A tax on incomes has the same effect on the prices of consumer goods and on those of the remaining goods and hence also on incomes.

When a tax is imposed on income, the amount of money which at any moment still serves to finance the exchange of goods and services and the payment of private incomes becomes  $\frac{D}{D+fa} M$ ; the remainder equal to  $fa \frac{D}{D+fa} \frac{M}{D} = \frac{fa}{D+fa} M$  provides for the purely financial public transactions. The money required for these latter transactions is drawn, in our model, from all types of transactions at an even proportional rate. It therefore spreads a uniform degree of "deflation" over the payments for goods and services in every stage of production and distribution. It bears equally on the stage of production and distribution. It bears equally on the stage of sales of final consumer goods and on the earlier stages. And, as we saw above, the prices of final consumer goods fall by less than the amount of the tax, and this gap is the greater the higher is the degree of differentiation of the economic process. Since the quantity of goods and services at each and every stage in the private sector is, by hypothesis, unchanged, the coefficient of variation of the prices of consumer goods  $\frac{D}{D+fa}$  is, in our abstract model, also the coefficient of variation of the prices of the other goods and services.

If  $P'_g$  is the new general price level, we have

$$P'_g = P_g \frac{D}{D+fa} \quad (5)$$

10. The introduction of a tax on goods has, in the conditions of our monetary postulate, two distinct effects. First, as we have seen, it raises prices to the consumer except when  $h \geq D$ ; secondly, it depresses the remaining prices, the price fall being

the greater the higher is the tax. The higher price level of consumer goods due to a tax on goods is laid upon the general price fall for other goods. In this case too the monetary requirements due to the new tax and public expenditure affect all stages of the chain of production and distribution of national income. On the one hand there is a higher price level for consumer goods, inclusive of tax; and on the other hand a lower level for the prices of untaxed intermediate goods of earlier stages of production and for the prices, net of tax, of taxed goods (or those onto which the tax is shifted) including consumer goods.

As regards the general price level, three concepts may be considered in the analysis: (a) the general level of all prices net of tax, whether the latter affect the respective goods directly or be levied at preceding stages; (b) the general level of all prices inclusive of tax; and (c) the general level of all prices inclusive of tax, but excluding remunerations to factors of production, *i.e.* incomes.

(a) The price level of untaxed goods, or the level of prices net of tax, falls uniformly at all stages, just as in our abstract model the withdrawal of money from every exchange is uniform. Again the monetary flow serves two distinct tasks. First it serves the exchanges of the private sector, which have not changed either in amount or in nature; all tax payments or payments connected with the tax are excluded. Secondly it serves the payments of the entire public circuit, from tax collection through tax shifting along successive exchanges to the final purchase of goods and services either by the government or by those to whom the government pays the tax revenue. The amount of money involved at any moment in

the second task is equivalent to  $ha \left[ \frac{1}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D}$ , that is

the amount of the tax multiplied by the number of payments stages in the public sector. If  $P'_{ge}$  is the new general price level after a tax on goods and net of it, we have

$$P'_{ge} = P_g \frac{1}{1 + \frac{ah}{D(1-a)}} \quad (6)$$



A tax on goods always depresses the general price level of all goods and the level of incomes in the private sector — understood net of tax, when market prices are inclusive of tax. The reduction is the greater the further back in the process of income formation and distribution the tax is levied; it is at the same time an inverse function of the degree of market differentiation. The reduction also occurs when, as is normal, consumer prices inclusive of tax rise above their previous level, since, as we have noted earlier, the “wedge” of the tax is never entirely absorbed by such a rise. The new general price level is lower than the old one in proportion to the amount of money which is taken away, to satisfy the needs of the public authorities, from each payments stage in the private sector.

(b) A tax on goods normally also depresses the general level of all prices inclusive of tax, that is the full prices of taxed goods and those onto which the tax has been shifted.

In the monetary conditions we have postulated, both a direct and an indirect tax always depress the general level of all prices (barring the exceptions enumerated) and they do so by virtue of the absorption of money into the public sector. The quantity of money absorbed for purely public payments which are not parallel to private exchanges, and which therefore does not enter into the private sector's exchanges of goods and services, is limited to the amount needed for the actual tax payment and for civil service salaries and other transfer payments, when the government does not make direct purchases on the market. This quantity of money corresponds at any given moment to  $fa \left[ \frac{1}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D}$ , where  $f$

again indicates public transactions of a pure public finance character. In the case of a tax on goods  $f$  (which, as we have noted, is between 1 and 2) must also be smaller than  $h$ . If  $f=h$  no public transaction would be parallel to private exchanges: the tax would not enter into any price and would cease to have the characteristics of a tax on goods and assume those of an income tax. If  $P''_{go}$  is the new general price level, we can write

$$P''_{go} = P_g \left[ 1 - \frac{fa}{D} \frac{1}{(1-a) + \frac{ah}{D}} \right] \quad (7)$$

The average price fall equals the quantity of money drawn off into the public circuit for pure public finance transactions and distributed among the separate groups of payments which occur between the separate stages of the circular flow of private income. Other things remaining equal, the general price level falls with a reduction in  $h$ : a reduction in  $h$  in fact implies a higher level of prices including tax and consequently a greater withdrawal of money for fiscal purposes.

(c) So far we have considered all prices including incomes. If we take instead the more usual concept of the price level and consider only the prices of goods and services without factor incomes, we arrive at a new price level which is higher than the two preceding ones. Of the public money requirements for pure public finance transactions, which we have seen to equal

$$fa \left[ \frac{1}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D}, \text{ an amount of } \frac{ah}{D} \left[ \frac{1}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D}$$

comes from the money used to make income payments, which are not now considered in calculating the price level. Payments are reduced to  $MV_i \left( \frac{D-1}{D} \right)$ .

If  $P'''_{go}$  is the general price level of goods and services without incomes, we can write

$$P'''_{go} = P_g \left[ D - \frac{1+a(f-1)}{(1-a) + \frac{ah}{D}} \right] : (D-1) \quad (8)$$

This general price level, which is the one usually considered, is normally also lower after the tax than before it. The condition for the price fall is  $f > \frac{h}{D}$ , which is normally the case since  $h$  is normally smaller than  $D$ . For the maximum value  $h = D + 1$ , we have  $f > 1 + \frac{1}{D}$ .

II. The general price level “at factor cost”, that is net of tax, is always lower than the general level of all prices, inclusive of tax

and of incomes; this latter price level is, in turn, lower than the general level of prices, inclusive of tax, of goods only, that is without incomes. These various price levels are different when  $h > f$ , which is always the case. If  $h = f$ , all the price levels would be equal; but in that case the tax under consideration would no longer have the characteristics of an indirect tax on goods.

While the price level of consumer goods is normally higher after a tax on goods than after an alternative income tax, provided that  $D > h - f$ , the general level of all prices inclusive of tax, and including also incomes, is higher in the case of an income tax. The condition is the same as for consumer goods:  $D > h - f$ . In other words: if the number of payments stages in the private circuit is larger than the number of those public stages which are "parallel" to private ones, then the prices of goods inclusive of an indirect tax are higher than the prices due to an income tax; on the other hand, since for equal real yield in terms of consumer goods, a tax on goods draws more money into the public circuit, it also exerts more deflationary pressure on the whole of private payments taken together.

*A fortiori*, the general price level of all goods at factor cost is lower after a tax on goods than it would be after an income tax of equal amount in real terms. The condition is that  $h > f(1 - a)$ , a condition which is always satisfied.

The relations between the prices due to the two types of tax become less certain and tend to be opposite when income payments are excluded and only the general price level of all goods is considered. It is then more likely than an indirect tax will lead to the higher price level, or in other words depress prices less.

If  $f = 1$  this happens when  $D < h + \frac{h-1}{a}$ , and if  $f = 2$  when  $D < (h-1) + \frac{h-2}{2a}$ . The value of  $h$  must be relatively high in relation to  $f$  and  $D$ , that is the tax must enter into the prices of goods at several stages, and the tax rate  $a$  must remain within the usual limits, so that the excess amount of money drawn into the public circuit by the indirect tax be not very large.

12. The difference in the effects which a tax on goods has on the price level of consumer goods and on the general price

level was to some extent recognized by a number of adherents of the quantity theory and also by Brown.

However, the former failed to stress the government's additional cash requirements, or in other words the new monetary transactions involved in the additional government finance. What they did was to contrast the higher price level of consumer goods with the fall in remaining prices within an unchanged general price level. They thereby confused constant general price level with constant money flow, in spite of the greater volume of transactions, or else they implicitly assumed an expansion in the quantity of money or an increase in the transactions velocity  $V_t$ .

For Brown, the contrast between the two effects of a tax on goods is no more than a qualification of the results of an incidence theory constructed and accepted on the basis of stable, or negligibly rising prices to the consumer (15).

Rolph in his turn stated that a general and uniform system of excise taxes does not change prices to the consumer. These are the only prices which are relevant to his incidence theory. Hence his model tacitly implies that the fall in the price of the goods at the stage preceding that where the tax is levied — a fall which equals the amount of the tax — is accompanied by price falls at the earlier stages of the production and distribution of national income.

13. The assumptions set out at the beginning of this study include the postulate that all taxes are general. We introduced this limiting condition for reasons of simplicity and also because the effects of a special income tax on those prices which feel its influence are similar to the effects of a tax on goods.

However, the limitation is not needed and the results which have been deduced both for the general price level and for the price level of consumer goods can be extended also to the case of special taxes. Differences in the effects of general and special taxes (of equal monetary yield) appear only in price relations. The limits of the changes in the price level of consumer goods are always, and irrespective of whether the tax is general or special, determined by the values of  $D$ ,  $h$ ,  $f$ , and the tax rate  $a$ , the latter being a percentage of the total value of sales or of aggregate national income, depending on the type of tax.

(15) H. G. BROWN, *op. cit.*, p. 260.

Within this price level, however, price relations are different in the two cases and hence tax incidence varies according as the tax is general or special.

In the case of a special income tax there are, within the general price fall, wide divergences between the prices of goods produced with the help of taxed factors of production and the prices of other goods. The gross remuneration of the taxed factors of production increases and this raises the price of the goods they produce. Within the general price level determined by the absolute amount of the tax and by the values of  $D$  and  $f$ , this leads to deflationary pressure on other prices, both of final and intermediate goods. This pressure is heavier than that which would have been exerted by a tax equally applied to all incomes.

A special tax on goods causes differences between the prices of the taxed goods and of other goods. The former presumably rise more than they would have risen if the tax had been general and uniformly applied to all goods; consequently the prices of the remaining goods are depressed even more.

With respect to this latter type of tax it should of course be remembered that whenever the tax is levied at one production or exchange stage rather than at another, the extent of the tax base changes. In the preceding analysis it has been implicitly assumed that the value of transactions, before tax, is equal at all stages. In reality this value rises as the goods approach the stage of consumption, so that the tax base changes. But the results do not change, provided we always consider taxes of equal absolute amount, and so modify the tax rate  $a$  correspondingly.

14. The foregoing analysis implicitly assumed uniformity of the money used for consumer spending and for payments by firms. This can happen either if only one kind of money is in circulation, say coin, or if the several kinds of money are interchangeable and used indiscriminately, say credit money alongside coins and bank notes.

In the case of several kinds of money with limited interchangeability and distinct use for different forms of payment, the velocity of circulation of money would be reduced, with deflationary consequences.

Furthermore, money has been considered only as a means of exchange, as in the Brown-Rolph model, that is to say the cash

holdings of firms, government and consumers were assumed to be strictly limited to technical payment needs. If all the functions of money were to be considered, its function of value reserve together with the exchange function, the close interdependence between prices and volume of money would be weakened. The rise in the prices of consumer goods due to tax on goods would be accelerated, and the price fall due to an income tax would be restrained. Reduced sales receipts would not force firms to reduce their purchases and production immediately and in the same proportion, nor would individuals be forced by the curtailment of their income to cut down their consumption at once and to the same extent. There is only one case in which the amplitude of price changes at the end of the adjustment process would be equal to that described above, and that is the case when liquidity preference, profit rate and other market conditions cause the community to replenish its idle funds to the same absolute level as obtained before the introduction of the tax. In all other cases price changes will be influenced by the reduction or increase in the quantity of money used for exchange. If we suppose that market conditions remain unchanged, in particular also as regards the assumption of constant production, and that at the end of the adjustment process individuals reconstitute their reserves so as to represent the same purchasing power in terms of consumer goods as before the tax, then we can say that by and large the ultimate effect of the price changes will be that the indirect tax causes an increase in idle funds and the direct tax their decrease. In either case the flows and counterflows between idle and active money at first facilitates price changes and then contains them.

15. Credit (or a change in the quantity of money) is not a necessary condition of the difference in the price changes due to one or the other tax. The price rise of consumer goods which follows upon the introduction of a tax on goods does not depend on credit; and similarly, even without credit restriction, the price fall due to an income tax is less than the tax. Generally speaking, credit raises the final price level above what it would have been without credit. Private borrowing enables firms to avoid reducing their production and consumers to limit the curtailment of their consumption.

Borrowing by the government, provided no money is withdrawn from the private sector and hence the creation of new money promoted (16), leads to the case when public expenditure precedes tax receipts. This links up with Rolph's assumption regarding a tax imposed in the course of an inflationary process, in so far as public expenditure exceeds tax receipts. After adjustment, prices will clearly be higher than they would have been had fiscal receipts preceded the new expenditure; in all cases the price level will be different according as the new tax is direct or indirect.

Let us assume that within the chosen period the community's money stock completes one full cycle of formation, distribution and spending of income, so that  $Rm = M$ . The period is divided into four time units, as in Tables I and II:  $D = 4$ . Now suppose that, pending the introduction and collection of a new tax at the rate  $a$ , the government proceeds during the whole period to an uncovered expenditure (on direct purchases)  $aM$ . The new tax is then introduced at the end of the period, so that its yield may be available at the very beginning of the succeeding period. Given an interval of one time unit between collection and spending (what comes in today is spent tomorrow), this means that in our example tax collection must begin with time unit four of the first period. The final price level of consumer goods (as of all goods) will be higher by  $a$  than it was in our previous analysis, and this will be so irrespective of whether the new tax is an indirect tax on goods or a direct tax on incomes. In our tables, where  $a =$  ten per cent, the price level of consumer goods which was 100 before the new tax, and at constant volume and velocity of money had become 105.3 after a tax on goods and 97.6 after an income tax, will now become 115.8 and 107.3 respectively.

Let us now suppose that the budget is underbalanced not during the whole period, but only for so long as is necessary to avoid withdrawing funds from the private sector by tax collection. In that case the final consumer prices would rise by the money value of any tax on goods, and would remain unchanged if income tax were imposed. But the budget remains underbalanced for longer in the first case than in the second. With an indirect tax on goods,

(16) The case of loans from the private sector, when individuals do not in their turn take up credits, and do not possess or do not wish to use idle funds, is in our simplified model equivalent to the case of direct income tax.

$h$  assumes values of 2 or more, and the tax requires additional cash both for the government (and possibly for government employees) and for the retail traders who have to pay the tax (and, when the tax is levied at a stage preceding sale to the consumer, for the firms at those earlier stages). Or else, when the collection of a tax on goods begins at once when uncovered expenditure is incurred, so that the yield of the tax can be spent in the time unit immediately following its collection, the sellers (and those onto whom the tax is shifted) must have direct recourse to credit to meet their additional cash requirements. In the case of a tax on income, when  $1 \leq f \leq 2$ , the tax causes additional cash requirements only for the government (or possibly its employees).

But the difference in the quantity of money and in its velocity of circulation in the two cases is not a necessary and sufficient condition for the two taxes to cause different price levels. If the price levels do in fact differ, this is due to the different technical nature of the two kinds of taxes; credit accompanies, accentuates and consolidates the divergence in price movements, but does not determine it.

However, when it is said that an income tax tends to leave consumer prices unchanged while a tax on goods raises consumer prices by its own amount, this statement often rests on the implicit or explicit assumption that the money supply and its velocity of circulation are not constant but adapt themselves to the requirements of the market.

Thus, if in previous papers I have reached the conclusion that even with an income tax, provided it is a special one, the general price level tends to rise as a result of the increase in the prices of the goods produced by the industries affected by the tax, this conclusion too remains strictly subject to the presence of either or both of these conditions: expansion of the volume of money, and/or increase in its velocity of circulation (17).

16. In our extremely simplified model, we have assumed a stationary economy with a government which merely consumes goods and services withdrawn from the private sector without any counterpart.

(17) "Imposte indirette etc.", *loc. cit.*, pp. 303-4.

The results of the analysis depend upon this assumption and are not applicable to the real world without appropriate qualification. In the first place the government must be seen in its double function of consumer and contributor to private economic activity, and secondly the assumption of a stationary economy must be replaced by the more realistic one of a dynamic economy.

The government normally takes part in the country's economic activity either as a producer of goods and services which are usually provided free of charge to individuals, or as a factor of production together with others in the private sector. In a stationary economy with constant resources this implies that the government thereby in either case takes the place of individuals, that the new public activity involves a corresponding contraction in individual productive activity (18). It follows, and this is the relevant point for the question here under discussion, that the new monetary transactions arising from public finance are additional not to unchanged private transactions, as has always been supposed, but to private transactions which have become smaller in number and amount.

When the government undertakes activities which are at the service of the private sector, merely replacing those which that sector previously performed itself, while the supply of consumer goods and services remains unchanged, the new public money transactions are compensated by a diminution of money transactions in the early stages of private production. The increased public cash requirements find their counterpart in the private sector's reduced cash requirements; in the case of a tax on goods the creation of the public monetary circuit is facilitated by the shortening of the private one, and in the case of a tax on incomes the insertion of the government into the private circuit of formation, distribution and spending of income replaces private transactions which have disappeared.

If the government — still replacing similar existing private activity — enters the field of consumer goods and services and supplies them free or below cost, the prices of the remaining consumer goods will increase even more. Indeed, a reduction in the supply on the market then reinforces the rise in the prices to consumers.

(18) Cf. my monograph: "La finanza pubblica nel calcolo del reddito nazionale", Giuffrè, 1957.

In a stationary economy the link between the money supply and any extension or intensification of public spending is in fact looser than it has here been represented; deflationary pressure on private transactions is weaker or absent, and the price level of consumer goods is higher than the formulae indicate.

The whole picture changes if we abandon the hypothesis of a stationary economy and assume that public activity generates an increase in national income. Then the new public cash requirements meet unchanged or increased requirements by the private sector: unchanged when the public activity remains closed within itself and the demand exercised by the factors of production employed by the government does not call forth additional output by the private sector, and increased in the opposite case. In general, therefore, the deflationary pressure due to the lengthening of the course which the same quantity of money has to traverse is reinforced by an expansion in the supply of goods and services. Greater employment of the factors of production and more intensive productive activity accompany the falling prices. The situation described above for a stationary economy is conceivable only for the hypothetical limiting case in which the government produces goods and services which it supplies free of charge to the consumers and this activity leaves private economic activity quantitatively and qualitatively unchanged.

17. But the assumption of a dynamic economy, while it opens the way to a broader and more realistic approach, leads us to the third question and to the general analysis of the relationship between money and economic development. This is outside the scope of this paper, which seeks only to clarify the effects of various types of tax on prices under the condition of monetary restraint.

Even on the abstract assumption of constant money supply and constant transactions velocity, indirect and direct taxes have different consequences for the price level of consumer goods. While an indirect tax on goods generally raises these prices above their preceding level (except for the hypothetical case in which the public monetary circuit is as long or longer as the private one), an income tax depresses prices to the consumer. The results are independent of whether the tax is general or not, provided the monetary yield is the same.

The theory of tax incidence which rests on the assumption of stable prices for consumer goods and which attributes the effects of all taxes to a reduction in factor incomes, uses a premise which is inconsistent with the assumptions of constant quantity of money and constant velocity of circulation. These assumptions are not compatible with a stable price level for consumer goods. Rolph arrived at his result of price stability because he implicitly changed his assumption in passing from the case of an income tax to that of a tax on goods. In the case of a tax on income, the prices to consumers remain unchanged on the assumption that public expenditure coincides with tax collection or takes place when the same cash funds would otherwise have been spent, so that the public cash requirements are compensated by a diminution of private ones. In the case of a tax on goods, on the other hand, the prices of consumer goods can remain unchanged only on the assumption that tax collection precedes public expenditure, and that the latter takes place after a fall in private money incomes due to the government's absorption of cash for its own requirements and the resulting contraction in effective demand. If the two kinds of taxes are considered on identical assumptions about timing, be it that public expenditure is assumed to precede, coincide with or follow tax collection, the results are no longer the same for the two cases.

Price stability would require an assumption not of neutral monetary policy, in the sense of constant  $M$  and  $V_r$ , but of positive government action designed precisely to ensure such price stability. On the other hand, even if we accept as a working hypothesis — representing an extreme case — that the price level of consumer goods remains stable irrespective of the behaviour of the monetary variables, this does not allow of any deductions with respect to the relative effects of the incidence of direct and indirect taxes and to the possibility of shifting them backwards as a whole. These effects are now determined by two factors: the tax and monetary policy. The assumption of a policy of price stabilization for consumer goods is not more "neutral" than the usual implicit assumption of incidence theories that the money supply covers also the new demand arising from the additional cash requirements of the government and of individuals having to pay the tax. The hypothesis of stable consumer goods prices implies full satisfaction of the new cash requirements connected with an income tax and, in the case of a tax on goods, a deflationary policy when  $h < D$  and an

inflationary policy when  $h > D$ , or else it involves different timing of the sequence of tax collection and expenditure for the two kinds of taxes; the other, more usual hypothesis implies a uniform monetary policy in all cases and one which is in all cases designed fully to cover the additional demand for money due to the tax.

The traditional theory's conclusion about the variety of possibilities with respect to the shifting and incidence of the two kinds of taxes are in fact not disproved. In analysing the consequences of various kinds of taxes in conditions of a monetary constraint we have reached, as a first approximation, results which agree with the traditional theory, and the latter fits well into the monetary approach followed in the present article. However, having stated this general conclusion, we must stress once more that any monetary analysis in macro-economic terms can be no more than an introduction to the study of tax incidence. We should not stop at these general first results and should not attribute to them a validity justifying definitive conclusions. The forward and backward shifting of taxes has no link with the general price level, but neither is it necessarily linked with the level of prices to the consumer; tax incidence affects relative price changes: changes as between various incomes, as between incomes and prices of goods, as between the prices of various consumer goods, and as between the prices of consumer goods and investment goods.

Rome

GIANNINO PARRAVICINI

#### APPENDIX

##### Symbols used:

- $M$  Quantity of money, assumed constant.  
 $V_t$  Average transactions velocity of circulation of money in the time unit, assumed constant.  
 $V_r$  Average circular velocity of circulation of money in the same time unit.  
 $V'_r$  Ditto, after the tax.  
 $D$  Average number of payments stages through which a money unit passes in one circular flow of production, distribution and spending of income:  

$$D = \frac{V_t}{V_r}.$$
 $f$  Average number of pure financial payments stages through which passes the money entering into the public circuit of tax payments and public expenditure; this excludes the stages associated with the exchange of goods and services in the private sector.

- $h$  Average number of all payments stages, financial and other, through which passes the money entering into the public circuit of tax payments and public expenditure when recourse is had to a tax on goods. Since this number includes the stages associated with the exchange of goods and services, we have  $h > f$ . Moreover, because of the indirect nature of the tax, we have  $h \leq D + 1$ .
- $a$  Tax rate as a percentage of income or of sales receipts, according to type of tax.
- $Y_m$  Pre-tax national income in monetary terms, in the time unit under consideration.
- $Y'_m$  Ditto, after the tax.
- $P_c$  Price level of consumer goods before the introduction of a tax.
- $P'_c$  Ditto, after the tax.
- $P_g$  General price level of all goods and services before the introduction of a tax.
- $P'_g$  Ditto, after a direct tax on income.
- $P'_{ge}$  Ditto, after an indirect tax on goods, net of the tax.
- $P''_{ge}$  Ditto, after an indirect tax on goods, inclusive of the tax.
- $P'''_{ge}$  Ditto, after an indirect tax on goods, inclusive of the tax, but taking "goods and services" to exclude incomes.
- $T$  Average number of exchange transactions between any two stages of the circular flow of income, in the time unit under consideration.

*Assumptions:* A stationary economy; constant production of goods and services; all monetary transactions are concerned only with the production, distribution and spending of income.

### I. DIRECT TAX ON INCOME

Before the tax  $MV_i = MV_r D$ , after the tax  $MV_i = MV'_r (D + fa)$ , where  $0 \leq f \leq 2$ .

From this we get for the *price level of consumer goods*:

$$V'_r = V_r \frac{D}{D + fa}; Y'_m = Y_m \frac{D}{D + fa}; P'_c = P_c \frac{D}{D + fa}$$

and

$$aY_m \frac{D}{D + fa} \geq Y_m \left( 1 - \frac{D}{D + fa} \right) \text{ when } D \geq f;$$

also

$$P'_c < P_c; aP'_c \geq P_c - P'_c.$$

Before the tax  $MV_i = P_g TD$ , after the tax  $MV_i = P'_{ge} TD + \frac{fa}{D + fa} M$ , be-

cause the pure public finance transactions in the public monetary circuit absorb an amount of money  $fa \left( \frac{D}{D + fa} \right) \frac{M}{D}$ ; if prices remained unchanged and the transactions varied, we would have  $MV_i = P_g TD \frac{D}{D + fa} + \frac{fa}{D + fa} M$ .

From this we get for the *general price level of all goods and services*:

$$P'_g = P_g \frac{D}{D + fa}; P'_g < P_g.$$

### II. INDIRECT TAX ON GOODS

Before the tax  $MV_i = MV_r D$ , after the tax  $MV_i = V'_r [(1-a) MD + aMh]$ , where  $h \geq 2$ .

From this we get for the *price level of consumer goods*:

$$V'_r = V_r \frac{1}{(1-a) + \frac{ah}{D}}; Y'_m = Y_m \frac{1}{(1-a) + \frac{ah}{D}}; P'_c = P_c \frac{1}{(1-a) + \frac{ah}{D}};$$

so that

$$\left. \begin{array}{l} V'_r = V_r \\ P'_c = P_c \end{array} \right\} \text{ when } h = D; \left. \begin{array}{l} V'_r > V_r \\ P'_c > P_c \end{array} \right\} \text{ when } h < D; \left. \begin{array}{l} V'_r < V_r \\ P'_c < P_c \end{array} \right\} \text{ when } h > D.$$

For  $h < D$ , we have:

$$aY_m \frac{1}{(1-a) + \frac{ah}{D}} > Y_m \left[ \frac{1}{(1-a) + \frac{ah}{D}} - 1 \right], \text{ where } -ah < 0.$$

Before the tax,  $MV_i = P_g TD$ ; after the tax, depending on how we define the general price level, we have:

(a) *General level of all prices net of tax:* The transactions of the public sector absorb a total amount of money  $ha \left[ \frac{1}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D}$ , so that we can

write

$$MV_i = P'_{ge} TD + \frac{ah}{D} \left[ \frac{1}{(1-a) + \frac{ah}{D}} \right] M$$

and if prices remained unchanged and the transactions varied:

$$MV_t = P_g TD \left[ 1 - \frac{ah}{D} \frac{I}{(1-a) + \frac{ah}{D}} \right] + \frac{ah}{D} \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] M.$$

From these two equalities it follows that

$$P'_{go} = P_g \frac{I}{1 + \frac{ah}{D(1-a)}}$$

(b) *General level of all prices inclusive of tax*: The pure public finance transactions of the public sector absorb an amount of money

$$fa \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D},$$

so that we can write

$$MV_t = P''_{go} TD + \frac{fa}{D} \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] M$$

and if prices remained unchanged and the transactions varied:

$$MV_t = P_g TD \left[ 1 - \frac{fa}{D} \frac{I}{(1-a) + \frac{ah}{D}} \right] + \frac{fa}{D} \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] M.$$

From these two equalities it follows that

$$P''_{go} = P_g \left[ 1 - \frac{fa}{D} \frac{I}{(1-a) + \frac{ah}{D}} \right]$$

(c) *General level of prices inclusive of tax, excluding incomes*: The volume of payments is reduced to  $MV \left[ \frac{D-I}{D} \right]$ ; pure public finance transactions in the public monetary circuit still absorb a quantity of money  $fa \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D}$ , of which  $\frac{ah}{D} \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D}$  is money formerly destined for income pay-

ments which are not now considered in calculating the price level. Since

$$\left[ fa - \frac{ah}{D} \right] \frac{I}{(1-a) + \frac{ah}{D}} = a \frac{f - \frac{h}{D}}{(1-a) + \frac{ah}{D}},$$

we can write:

$$\begin{aligned} MV_t \left[ \frac{D-I}{D} \right] + \frac{ah}{D} \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D} &= \\ &= P'''_{go} T (D-I) + \frac{fa}{D} \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] M \end{aligned}$$

and if prices remained unchanged and the transactions varied:

$$\begin{aligned} MV_t \left[ \frac{D-I}{D} \right] + \frac{ah}{D} \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] \frac{M}{D} &= \\ &= P_g T \left[ D - I - a \frac{f - \frac{h}{D}}{(1-a) + \frac{ah}{D}} \right] + \frac{fa}{D} \left[ \frac{I}{(1-a) + \frac{ah}{D}} \right] M \end{aligned}$$

From these two equalities it follows that

$$P'''_{go} = P_g \left[ D - \frac{I + a(f-I)}{(1-a) + \frac{ah}{D}} \right] : (D-I).$$

### III. COMPARISON

*Price level of consumer goods*:

$P'_c$  is higher after a tax on goods than after an income tax, in so far as

$$\frac{I}{(1-a) + \frac{ah}{D}} \geq \frac{D}{D+fa} \text{ and hence } D \geq h-f.$$



*General level of all prices net of tax:*

$P'_{go} \leq P'_g$  in so far as

$$\frac{1}{1 + \frac{ah}{D(1-a)}} \leq \frac{D}{D+fa} \text{ and hence } h \geq f(1-a)$$

$P'_{go} \leq P''_{go}$  in so far as

$$\frac{1}{1 + \frac{ah}{D(1-a)}} \leq 1 - \frac{fa}{D} \frac{1}{(1-a) + \frac{ah}{D}} \text{ and hence } h \geq f.$$

*General level of all prices inclusive of tax:*

$P''_{go} \leq P'_g$  in so far as

$$1 - \frac{fa}{D} \frac{1}{(1-a) + \frac{ah}{D}} \leq \frac{D}{D+fa} \text{ and hence } D \geq h-f$$

*General level of prices inclusive of tax, excluding incomes:*

$P'''_{go} \geq P''_g$  in so far as

$$\left[ D - \frac{1+a(f-1)}{(1-a) + \frac{ah}{D}} \right] : (D-1) \geq \frac{D}{D+fa} \text{ and hence } D \leq (h-f) \left( 1 + \frac{1}{af} \right) + 1$$

$P''_{go} \leq P'_g$  in so far as

$$\left[ D - \frac{1+a(f-1)}{(1-a) + \frac{ah}{D}} \right] : (D-1) \geq 1 - \frac{fa}{D} \frac{1}{(1-a) + \frac{ah}{D}} \text{ and hence } h \geq f.$$