PSL Quarterly Review, vol. 67 n. 269 (2014), 131-167

On the stability of an Islamic financial system

HOSSEIN ASKARI, NOUREDDINE KRICHENE and
ABBAS MIRAKHOR*

1. Introduction

In November 2013 David Cameron, Prime Minister of the United Kingdom, announced the launch of a new British Islamic Market Index and the first ever Islamic bond, or *sukuk*, issued by a non-Muslim country. He called London "the biggest centre for Islamic finance outside the Islamic world". Cameron went on to add that the U.K.'s ambition is to boost its reputation among Islamic investors with these forays into Islamic finance, unprecedented for a non-Muslim country like Britain. "I want London to stand alongside Dubai and Kuala Lumpur as one of the great capitals of Islamic finance anywhere in the world", he told the audience.[1] While Islamic finance, or more accurately the development and marketing of financial instruments that comply with Islamic teachings (sharia compliant), has received much attention in recent years, there has been much less consideration afforded to the properties of an Islamic financial system. In this paper, we address one of the overriding features of any financial system, its stability characteristics.

The Islamic financial system is based on Islamic economics teachings, where the essential function of the financial sector is to serve and support the operation of the real sector. There are no interest rate

* Hossein Askari: The George Washington University, Washington (DC), email: hgaskari@aol.com; Noureddine Krichene: freelance economist, email: nnkrichene@hotmail.com; Abbas Mirakhor: INCEIF, Malaysia, email: abbas@mirakhor.com. This paper is part of a three-article series on Islamic finance; the first instalment was published in the past issue of *PSL Quarterly Review* (Askari and Krichene, 2014). The authors are grateful for helpful comments from two referees at *PSL Quarterly Review*, but remain responsible for any and all errors.

[1] "Cameron: London can be a world capital for Islamic finance", accessed 17 November 2013, available at http://america.aljazeera.com/articles/2013/10/29/cameron-london-canbethenextdubaiofislamicfinance.html.

based debt instruments and all financial transactions are based on sharing risk and return (Askari *et al*., 2012).[2] Hence, all financial claims are contingent claims. In such a financial system, deposit-taking institutions operate on a 100 per cent reserve system (as opposed to the predominant fractional reserve system); and non-deposit-taking institutions operate on risk and return sharing and their liabilities are not publically guaranteed (akin to a mutual fund). Proposals along these lines for the commercial banking structure are not new. Financial systems in some such form or other have been practiced throughout recorded history. Recently, such an approach was recommended in the Chicago plan. The plan was formulated in a memorandum written in 1933 by a group of renowned Chicago professors (including Henry Simons, Frank Knight, Aaron Director, Garfield Cox, Lloyd Mints, Henry Schultz, Paul Douglas and A.G. Hart) and was forcefully advocated and supported by the noted Yale University professor Irving Fisher in his book *100% Money*. Noting the fundamental monetary cause underlying the severe financial crises of 1837, 1873, 1907 and 1929-1934, the Chicago plan calls for the government to enjoy a full monopoly in the issuance of currency and prohibits banks from creating any money or near money by establishing 100 per cent reserves against checking deposits. Investment banks that play the role of brokers between savers and borrowers were to undertake financial intermediation. Hence, the inverted credit pyramid, highly leveraged financial schemes (e.g. hedge funds) and the monetisation of credit instruments (e.g. securitisation) are excluded. As stated by Irving Fisher (1936):

> "[t]he essence of the 100% plan is to make money independent of loans; that is to divorce the process of creating and destroying money from the business of banking. A purely incidental result would be to make banking safer and more profitable; but by far the most important result would be the prevention of great booms and depressions by ending chronic inflations and deflations which have ever been the great economic curse of mankind and which have sprung largely from banking."

[2] The absence of debt means the economy is highly efficient and saves considerable resources that are involved with the cost of issuing, administrating, recovering and litigating loans (Carroll, 1965).

According to Fisher, the creation of money depends on the coincidence of the double will of borrowers to borrow and of banks to lend. Keynes deplored this double want coincidence as a source of large swings in the circulating medium.[3] Why? In times of recession, borrowers are over indebted and see narrower profit prospects, they become less willing to borrow; banks are saddled with impaired assets and are less willing to lend. Jointly, they cause a contraction of money and, in turn, an aggravation of the downturn in the economic cycle. Irving Fisher wrote: "I have come to believe that that plan is incomparably the best proposal ever offered for speedily and permanently solving the problem of depressions; for it would remove the chief cause of both booms and depressions." More recent than the Chicago plan, Laurence Kotlikoff (2010) has made a proposal along similar lines, terming his approach 'Limited Purpose Banking'. Freixas and Rochet (2008) have observed that the conventional banking sector was recurrently undermined by crises and that bank runs and bank panics were inherent to the nature of conventional banking, and more specifically to the nature of the fractional reserve system. Indeed, bank deposit contracts usually allow depositors to withdraw the nominal value of their deposits on demand. As soon as a fraction of these deposits is used for financing illiquid and risky loans or investments, a liquidity crisis becomes a possibility.

Given that all financial assets in an Islamic economy are contingent claims and there are no debt instruments with fixed predetermined rates of return, Abbas Mirakhor (1993) showed that a fundamental principle that emerges from theoretical studies of such a system is that the returns to financial assets are primarily determined by the endogenous rate of return in the real sector that could replace the monetary interest rate. Thus the rate of return to capital is the mechanism through which the demand and supply of loanable funds is equilibrated because the source of profit in such an economy is the addition to total output, and once

[3] The separation of the banking system into 100% depository banking and equity-based banking does not reduce the profitability of banks, nor does it curtail their short-term liquidity. It makes banking immune to crises and ties the financial sector to the real sector; investment is financed by savings and not by fictitious credit.

labour is paid its distributive share, the residual is divided between the entrepreneur and the saver. He showed that the rate of return to capital equilibrates savings and investment, that the differential between the domestic and foreign rates of return to equity determines the direction of capital flows, and that under a fixed exchange rate system, adjustments are channelled through the asset accounts; all without a fixed and predetermined rate of interest. For the most general case of an open economy with trade in goods and equity shares, the direction of capital flows depends on the differential between the domestic and foreign rates of return to equity shares and, ultimately, on differentials in the marginal product of capital. The main conclusions are two: equilibrium in the absence of interest-bearing assets and system stability against debt shocks. In an Islamic financial system, the rate of return to capital is neither a purely monetary phenomenon determined in the money market by the demand and supply of money, as in a Keynesian model, nor is it purely determined by the real demand for and supply of real savings, as in the Classical model. Instead, the rate of return to capital is determined by the rate of return to ownership position (equity) related to marginal product of capital as well as to the portfolio balance equilibrium.

The Islamic economy model as envisioned in this paper has two markets: a market for assets that include equity (common stocks) and financial assets (money and foreign *sukuk*), and a market for commodities.[4] The labour force is assumed to be constant and the labour market clears at full-employment. As in general equilibrium analysis, the forces of demand and supply apply freely to clear markets. Equilibrium in the asset market is studied by examining the portfolio balance conditions. More precisely, the stocks of physical capital (or the equivalent number of equity shares) and of nominal money are fixed at a given point in time. Although individual wealth holders are able to change the composition of their portfolios, the community as a whole holds the existing stocks of

[4] A *sukuk* (a short form or abbreviation of *sukuk al-ijara*) is an Islamic bond. It is a negotiable financial instrument issued on the basis of an asset to be leased. Investors provide funds to a lessor, such as a bank. The lessor acquires an asset and leases it out if it is not already leased out. The investors become owners of the leased asset in proportion to their investment. The holders of these bonds are entitled to collect rental payments directly from the lessee. These instruments can be made tradable on a stock exchange.

assets. Hence, the rate of return on equity has to adjust until the desired portfolio composition is equal to the actual composition of assets in the economy. The condition of equilibrium in commodity markets is similar to that in the asset market. Aggregate demand for commodities has to be equal to a fixed real output. More precisely, the rate of return of capital and the price level have both to adjust in order to clear the commodity markets. The Islamic economic model establishes short-run equilibrium prices that clear all markets, where prices are an endogenous rate of return on equity and endogenous commodity prices. The model describes the dynamics of the economy and its ability to converge to a steady state where all prices and quantities are at long-run equilibrium.

The Islamic macroeconomic wealth model is formalised along the lines of the models in Metzler (1951), Frenkel and Rodriguez (1975), Dornbusch (1975), Dornbusch and Fischer (1980) and Mirakhor (1993). In addition to money as a form of wealth, these models imbed the securities market, mainly common stocks, and directly relate the rate of return on equity shares to the production sector and to the marginal product of capital.[5] Hence, the rate of return is no longer an interest rate on bonds but a rate of return on common shares that depends on productivity and thrift (savings) in the economy as well as on portfolio balance conditions. The market price of shares enters directly into the wealth equation; it values the capital stock not at its historical cost but at its market value and thus allows for capital gains and losses. Wealth models stress the interaction of flow and stock variables and study both short-run and long-term (steady state) equilibrium. Short-run equilibrium determines flow variables for given stocks of wealth variables. The equilibrium flow variables such as savings, investment and balance of

---

[5] In conventional finance the rate of return on equities is influenced by abundance of credit as illustrated by the South Sea Company in 1720, the Great Depression in 1929 and recent stock market experience (2000-2013). For instance, the marginal product of capital may be 4% per year; however, the yield on stock may be 25% per year. It is speculation fuelled by low interest rates and massive credit that weakens the link between equity returns and marginal product of capital (Holden, 1907). Speculators (e.g. hedge funds, equity funds, brokers, etc.) reap large gains from cheap money. Inevitably, distorted stock markets reach a tipping point beyond which they crash, with disastrous consequences. These elements of instability do not exist in Islamic finance where equity returns cannot be distorted by credit and interest.

payments feed into stock of wealth variables such as real capital stock, money and foreign *sukuk*, and move the economy to a new short-run equilibrium. Eventually, the economy reaches a stationary state where flow variables reach values that no longer affect stock variables. For instance, savings reach zero and no longer have an impact on wealth, or the external current account reaches zero and has no effect on the money stock or foreign *sukuk*.

In section 2, we formalise a closed Islamic economy model in order to specify the basic features of an Islamic economy, derive the short-run general equilibrium rate of return and price level in a simplified framework, and to study its long-run dynamics. In section 3, the model is extended to an open economy with a fixed exchange rate that trades in commodities only and faces a fixed foreign price for its imports. This feature of the economy is akin to Hume's classical open economy (1752). The trade balance is settled along the lines of the Hume mechanism by a flow of specie. The trade balance directly affects the stock of money, a trade surplus increases money and a trade deficit reduces it. Since the economy is open, the general price level becomes akin to the terms of trade. The aim is twofold: to establish a general equilibrium rate of return and price level (terms of trade) that clear assets and commodity markets, and to study the dynamics that lead to a steady state. In section 4, the model is extended to the trade in commodities and assets. To maintain the endogenous rate of return on equities, it is assumed that domestic equities are imperfect substitutes for foreign *sukuk*; we also assume that foreigners do not hold domestic securities. Instead, we assume the existence of sharia compliant *sukuk*, such as *sukuk al-ijara* that pay an income at a fixed annual yield rate, and which are used to settle the current account balance.[6] The features of this *sukuk* (ratings, risk, return, etc.) are defined by international capital markets. We note that for an economy that trades in assets, the external current account replaces the trade balance since there are income flows related to the holding of assets; furthermore, under a fixed exchange rate, wealth holders do not hold foreign currency that pays no income when they instead have the

[6] We could also simply assume that the rate of return on the international equity markets is constant.

opportunity to hold an income paying foreign *sukuk*. As in previous versions of the model (see Mirakhor, 1993), the aim is not only to establish a general equilibrium rate of return and price level that clear assets and commodity markets in both the short run and long run, but additionally to demonstrate the model's stability.

2. A wealth model for a closed Islamic economy

We begin with a closed model of an Islamic economy, later to be extended to an open economy. The economy has two sectors: a real sector and a financial sector. The real sector is described by production technology, consumption, and investment functions. The assumptions are: (i) the economy is assumed to produce an aggregate real output Y, which may be consumed or invested; (ii) the labor force is constant; (iii) wages and prices are flexible; and (iv) full employment prevails. The production technology is represented by a homogenous production function, i.e. constant returns to scale, as follows:

$$Y = F(K) \tag{1}$$

Where Y is real aggregate output per unit of time; K is the physical stock of capital that may include buildings, machinery and inventories; a number of shares or titles of property of equal value are represented by q. The production technology exhibits positive marginal capital product: $\frac{\Delta F}{\Delta K} = F_K > 0$ and decreasing marginal returns $\frac{\Delta^2 F}{\Delta K^2} < 0$

The prices in the economy are the price of output in terms of money, P, the rate of return on capital assets r, and the real price of each security q. These prices are endogenous.[7] We assume each additional unit of capital has a nominal income flow per unit of time measured by $P \cdot F_K$. The rate of profit is defined as:[8]

[7] This assumption is contrary to the Keynesian view, which assumes fixed prices and wages, and variable real output.

[8] This formula is the typical stock valuation formula; F_K is independent of r; and knowledge of r provides q. There is no circularity in the formula.

$$r = \frac{P \cdot F_K}{P \cdot q} = F_K / q \tag{2}$$

Note that the rate of profit is a decreasing function of capital K, since $\frac{dr}{dK} = \frac{\Delta^2 F}{\Delta K^2} / q < 0$. This definition of the rate of return is an essential feature of an Islamic economy. It relates the rate of profit to the marginal product of capital and the market value of common stocks. Most importantly, it rules out the existence of two rates of return that characterise conventional models; namely, the interest rate determined in the money market and the natural rate of interest determined by the rate of profit. The difference between these two rates was recognised to be a source of instability as illustrated in the writings of Thornton (1802) and Wicksell (1898).[9] For a given profit rate r the nominal price of a share is $P * F_K / r$. The real price of an equity share is:

$$q = \frac{P \cdot F_K}{P \cdot r} = F_K / r, \qquad \frac{dq}{dr} = -\frac{F_K}{r^2} < 0 \tag{3}$$

Although real capital stock is fixed, it is represented by a number of tradable securities, i.e. property rights to the capital, equal to K. The real price of a unit of real capital, i.e. of a share, q, is defined as the present value of an expected future flow of real income discounted by a rate of return (i.e. a discount rate) r. Real wealth is defined as:

$$w = qK + \frac{M}{P} = k + m \; w_q > 0, \qquad w_r < 0 \tag{4}$$

where P, the price level, is the money price of output, $k = qK$ is the real market value of old shares; the real physical stock is thus marked to the market and consequently is exposed to capital gains or losses. The notation $m = M/P$ defines real money balances; the assumption $w_r < 0$ implies real wealth declines when the rate of return increases.

The model is a wealth model with real income determined by full-employment of capital and labour; hence, it is not an income

[9] When the money rate is less than the profit rate, there is a rapid multiplication of fictitious credit and a cumulative inflationary process. When real working capital becomes too scarce, the cost of inputs becomes too high (e.g. high energy prices); the profit rate drops below the money rate and debt default and bankruptcy result (Carroll, 1965).

determination model. Accordingly, real consumption C is formulated in terms of real wealth w that is endogenous:[10]

$$C = C(w), \qquad C_w > 0 \tag{5}$$

The investment flow is expressed in terms of increase in real capital stock and also in terms of the market value of the new securities issued. The increase in real capital per unit of time is:

$$\dot{K} = \dot{K}(r, K), \qquad \dot{K}_r < 0 \text{ and } \dot{K}_K < 0 \tag{6}$$

Specifically, as the cost of capital r rises, firms reduce their investment demand; hence, $\dot{K}_r < 0$.[11] The assumption $\dot{K}_K < 0$ is attributed to the Penrose effect. There is fixed managerial and entrepreneurship capacity; as the size of installed capital K increases, the ability to place new investment diminishes (Mirakhor, 1993; Uzawa, 1969). New shares, floated on stock exchanges and acquired by households directly or through investment banks, finance the increase in real capital. The market value of these shares is:[12]

$$I = q\dot{K}(r, K) = I(r, K), \qquad I_r < 0 \text{ and } I_K < 0 \tag{7}$$

If we set $\dot{K}(r, K) = 0$, then there is an implicit long-run relationship between $r\ and\ K$ such that the demand for new investment is zero. We express this relation as:

$$K = \overline{K}(r), \qquad \overline{K}_r < 0 \tag{8}$$

Savings is defined as:

---

[10] Since wealth is a function of the rate of return, C is an implicit function of r with $C_r < 0$. As shown in the production function, wealth and real output may be related. Real income is a return on wealth. Including both variables in the consumption function may lead to multi-collinearity. Nonetheless, a consumption function of the form $C(y, m)$ may overcome multi-collinearity since y and m are independent variables.

[11] The marginal efficiency of capital is assumed to be decreasing; each additional investment has a lower return.

[12] If $I > \dot{K}$, we have the Penrose effect, meaning that there is an installation cost of new real capital formation. For instance, an investment of \$100 may result only in \$80 in additional real capital formation (Uzawa, 1969). This formulation of the investment function is similar to the formulation in Frenkel and Rodriguez (1975): where $I = h(p_k)$, where p_k is the price of one unit of physical capital in terms of consumption goods.

$$S = Y - C(w) = S(K, w), \qquad S_K < 0, \qquad S_w < 0 \tag{9}$$

The assumption $S_K < 0$ means that an increase in K increases production; it also increases consumption via the wealth effect. For stability reasons, we require that the increase in consumption exceeds the increase in real output so that total real wealth will rise at a diminishing rate to a stationary level in the long-run. Without this condition, real wealth will have no upper bound. The assumption $S_w < 0$ is a standard assumption formulated in wealth models according to which consumption rises with wealth and savings diminishes. Since wealth is inversely related to the rate of return, savings is positively related to the rate of return, i.e. $S_r > 0$. The equilibrium in the commodity markets (*IS*) is defined as:

$$Y = F(K) = C(w) + I(r, K) \tag{10}$$

Or equivalently:

$$S(K, w) = I(r, K) \tag{11}$$

The financial sector has two assets: outside nominal money M and real physical capital stock K measured in units of real output.[13] We assume that at a given instant of time K and M are fixed.[14] The demand for money in real terms is inversely related to the rate of return r, which is the opportunity cost of holding money. As the rate of return on shares rises, wealth owners hold less cash and more securities. The demand for money is expressed as:

$$m_d = \alpha(r)w, \qquad \alpha_r < 0 \tag{12}$$

---

[13] The money sector operates along the principles formulated in the Chicago plan (Phillips, 1994) as well as the principles of Islamic finance. It is 100% reserve based with an equity market. There is no credit creation and no money creation and destruction arising from debt. We deal only with the asset component of the financial sector; we omit the role of the banking sector in facilitating the exchange and production activities of the economy.

[14] In an Islamic economy, assuming 100% reserve banking is in place, money is outside money; credit plays no role in money expansion or contraction. Recalling the metallic and the convertible paper systems, money is created or destroyed with gold and silver discoveries, conversion from or to non-monetary uses and external balance of payments.

The demand for securities increases when the rate of return rises and can be expressed as:

$$k_d = \beta(r)w, \qquad \beta_r > 0 \tag{13}$$

From the definition of wealth and the demand for assets we have:

$$\alpha(r) + \beta(r) = 1 \tag{14}$$

Hence, only one equation of the two asset demand functions is independent. An excess demand for securities $(k_d - k)$ corresponds to an excess supply of money $(m - m_d)$ and vice versa. The equilibrium of the asset market is achieved when one of the two asset markets is in equilibrium. Equilibrium in the money market implies equilibrium in the securities market and vice versa. Since at equilibrium $m_d = m$ and $k_d = k$, the portfolio balance condition may be stated as:

$$\frac{m_d}{k_d} = \frac{m}{k} = \frac{\alpha(r)w}{\beta(r)w} = \ell(r) \tag{15}$$

where $\ell(r) = \frac{\alpha(r)}{\beta(r)}$ and $\frac{d\ell}{dr} < 0$. When the rate of return falls, money is substituted to securities and vice versa. The asset market equilibrium condition (*LM*) may be written as:

$$m = \ell(r)k \tag{16}$$

It states that the rate of return must be such that the community is willing to hold the actual stock of existing assets, i.e., the desired composition of portfolio is equal to its actual composition at the economy level. An attempt by the community to substitute shares to money will only drive the yield rate down and will not change the actual stocks of assets K and M.

2.1 Short-run equilibrium of a closed Islamic economy

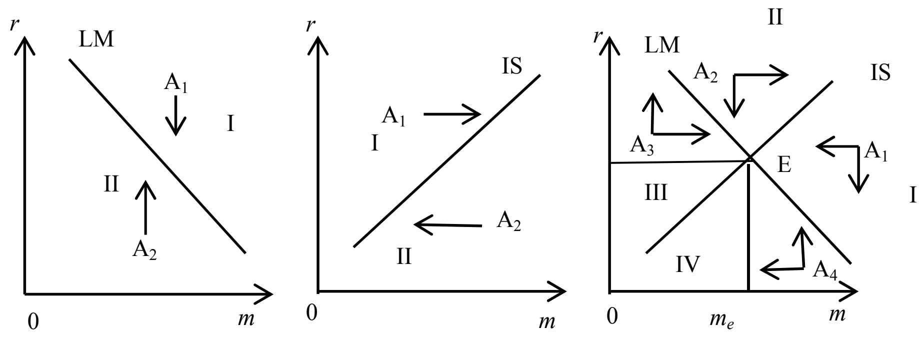
The model is a wealth model where full-employment income is fixed and the stock variables are fixed at K and M. The economy has full flexibility of wages and prices and does not admit Keynesian assumptions of rigid wages and underemployment. It does not have a liquidity trap, i.e. an absolute preference for liquidity, either since the rate of return is not only a monetary phenomenon; the real-wealth effect may be powerful in eliminating a deflationary gap and reducing the demand for money (Siegfried, 1906; Patinkin, 1947). The purpose of the model is to establish a short-run equilibrium in the form of a rate of return and a price level at which full-employment saving is equal to full-employment investment with no inflationary or deflationary gap. We solve for P and r, the prices that equate supply and demand in both assets and commodities markets. Since $m = M/P$ and M is fixed, this is tantamount to solving for m and r. We consider the asset market equilibrium condition: $m = \ell(r)k$. Total differentiation yields:

$$dm = k\frac{d\ell}{dr}dr + \ell(r)K\frac{dq}{dr}dr = \left(k\frac{d\ell}{dr} + \ell(r)K\frac{dq}{dr}\right)dr \quad (17)$$

The slope of the *LM* curve is negative:

$$\frac{dm}{dr} = \left(k\frac{d\ell}{dr} + \ell(r)K\frac{dq}{dr}\right) < 0 \quad (18)$$

Figure 1 – *Equilibrium of money, securities, and commodities markets in an Islamic model*



Panel 1.a. Equilibrium in the money and securities markets

Panel 1.b. Equilibrium in the goods' market

Panel 1.c. General equilibrium

The liquidity preference schedule that is consistent with the equilibrium in the money and securities markets is denoted by LM as shown in figure 1, panel 1.a. The LM is plotted as a downward sloping curve in the space (m, r). By definition, for a given stock of securities K, the LM curve associates with each r the stock of real money balances that establishes the equilibrium in the asset market. Stated differently, for a given stock of securities and a given rate of return the LM computes the stock of real money that establishes the portfolio balance condition. The LM curve defines two regions: I and II. A point in region I, e.g. point A_1, corresponds to a high yield rate and excess demand for securities. Households attempt to reduce their holdings of real cash balances and drive securities prices upward. This adjustment will lower the yield rate and push securities prices upward. Equilibrium is achieved when the yield rate has declined and the value of securities has risen so as to make the desired ratio of real money balances to the value of securities equal to the actual ratio of these two assets. Inversely, a point in region II, such as point A_2, corresponds to a low yield rate and an excess supply of securities. Households attempt to increase their holdings of real cash balances and push security prices downward. This adjustment will increase the yield rate and push security prices downward. Equilibrium is achieved when the yield rate has risen and the value of securities has declined so as to make the desired ratio of real money balances to the value of securities equal to the actual ratio of these two assets.

Total differentiation of the commodity equilibrium condition (IS) yields:

$$\frac{\partial S}{\partial w}dw = \frac{\partial S}{\partial w}(dk + dm) = \frac{\partial S}{\partial w}\left(K\frac{dq}{dr}dr + dm\right) = \frac{dI}{dr}dr \tag{19}$$

Rearranging terms, we obtain:

$$\frac{\partial S}{\partial w}dm = \frac{dI}{dr}dr - K\frac{\partial S}{\partial w}\cdot\frac{dq}{dr}dr = \left(\frac{dI}{dr} - K\frac{\partial S}{\partial w}\cdot\frac{dq}{dr}\right)dr \tag{20}$$

The slope of the IS curve is:

$$\frac{dm}{dr} = \frac{\left(\frac{dI}{dr} - K\frac{\partial S}{\partial w}\cdot\frac{dq}{dr}\right)}{\frac{\partial S}{\partial w}} > 0 \tag{21}$$

Its sign is unambiguously positive. The IS line is drawn as an upward sloping curve in the space (m, r) in figure 1, panel 1.b. For a given stock of securities K, the IS curve associates with each r the stock of real money m, or equivalently the price level P, that clears the commodities market.

The line IS divides the space (m, r) into two regions: I and II. Region I is characterised by a high yield rate for a given amount of real money balances. It is a region with a deflationary gap. Inversely, region II is characterised by a low yield rate for a given amount of real balances. It is a region with an inflationary gap. At point A_1 in region I there is low aggregate demand in relation to full-employment output. The yield rate has to decline; investment and consumption have to increase. Because of the deflationary gap, prices decline and real money balances increase. Equilibrium is achieved when the yield rate and real money balances reach a combination for which aggregate demand is equal to full-employment output. Inversely, at point A_2 in region II there is excess demand in relation to full-employment output. For a given yield rate, real balances have to be curtailed to increase saving and reduce consumption. Or, for given real money balances, yield rates have to rise to reduce investment and consumption. Because of the inflationary gap, prices increase and real money balances are reduced. Equilibrium is achieved when the yield rate and real money balances in combination lead to aggregate demand equalling full-employment output.

The economy reaches a short-run equilibrium (m_e, r_e), which satisfies both the asset and the commodity markets equilibrium; it is provided by the intersection of the *IS* and *LM* curves (figure 1, panel 1.c). From (m_e, r_e) we obtain equilibrium values for the price level P_e, the price of shares q_e and wealth w_e. This equilibrium is stable. If the combination (r, m) is different from (m_e, r_e), then assuming flexibility of wages and prices the economic system will gravitate toward (m_e, r_e). We may note that the yield rate r_e reconciles both the classical theory according to which yield rate is solely determined by time-preferences and the marginal productivity of capital, and the Keynesian theory that holds that yield rate is determined solely in the money market. In an Islamic economy model, the yield rate is determined simultaneously by real forces as well as by adjustments in the money and securities markets. Changes in the demand for money affect the yield rate.[15]

The reduced form for the equilibrium values of m, r, and q are:

$$m = g(K, M) \tag{22}$$

$$r = h(K, M) \tag{23}$$

$$q = z(K, M) \tag{24}$$

The advantage of the reduced forms is that they enable the computation of the multiplier effect of the fixed variables K and M. The short-run multipliers are:

$$\frac{\partial m}{\partial K} = \frac{\partial g}{\partial K}; \ \frac{\partial m}{\partial M} = \frac{\partial g}{\partial M}; \ \frac{\partial r}{\partial K} = \frac{\partial h}{\partial K}; \ \frac{\partial r}{\partial M} = \frac{\partial h}{\partial M}; \ \frac{\partial q}{\partial K} = \frac{\partial z}{\partial K}; \ \frac{\partial q}{\partial M} = \frac{\partial z}{\partial M} \tag{25}$$

2.2. The dynamics of a closed Islamic economy

The short-term flow variables affect the stock wealth variables. Namely, investment increases the capital stock K and savings increases real wealth. The increment of real capital stock per unit of time is defined as:

$$\frac{dK}{dt} = \dot{K} = \dot{K}(r, K) \tag{26}$$

The increment of wealth per unit of time is defined as:

---

[15] A model that operates along the lines of the Chicago plan principles of 100% reserve banking and equity-based banking would attain the same equilibrium as that in an Islamic economy since both models share the same principles of non-interest based debt and 100% money. A model that operates with fractional reserves is inherently unstable as noted by Thornton (1802), Wicksell (1898), Holden (1907), Minsky (1986), Siegfried (1906) and shown by recurrent financial crises. Under a fractional reserve system there are boom-bust cycles with no equilibrium. Graziani (2003) dealt with credit money, which is bank money; banks advance loans to firms to start the production cycle, hire labour and buy machinery and raw materials; credit is extinguished when firms pay back loans. This is a modern version of the XIX century British Banking School and the concept of elastic endogenous money. Ahead of the electronic cashless economy (e.g. credit cards), Wicksell (1898) showed that an economy may eventually operate only without money where all transactions are settled in the books of banks through clearing systems.

$$\frac{dw}{dt} = \dot{w} = S(K,w) \tag{27}$$

The interaction of the flow and stock variables can be described as follows:

$$(K,m,w) \rightarrow (I,S) \rightarrow (K,m,w) \rightarrow (I,S) \rightarrow \ldots \rightarrow (\bar{K},\bar{m},\bar{w}) \tag{28}$$

For the economy to converge to a steady state the time-derivatives $\frac{dK}{dt}$ and $\frac{dw}{dt}$ have to both decrease with time. Hence, the economy reaches a long-run equilibrium at $(\bar{K},\bar{m},\bar{w})$ when $\dot{K} = 0$, $\dot{m} = 0$, and $\dot{w} = 0$. Accordingly, the long-run equilibrium can be stated as:

$$\dot{K} = \dot{K}(r,K) = 0, \qquad \dot{K}_r < 0 \text{ and } \dot{K}_K < 0 \tag{29}$$

$$\dot{w} = S(K,w) = 0, \qquad S_K < 0 \text{ and } S_w < 0 \tag{30}$$

Note that r and w are equilibrium values for which short-run equilibrium is achieved in both the commodity and asset markets. Equations (29) and (30) contain three variables (r,K,w) and do not form a system of differential equations in two variables.[16] We need, therefore, to transform these two equations into a system of two variables. Since K appears in both equations it may therefore be kept. We notice that from equations $w = k + m$ and $m = \ell(r)k$ the variables r and w can be expressed in terms of K and m. Equations (29) and (30) may therefore be transformed into a system of two differential equations in the variables K and m. From equation $m = \ell(r)$ the$k$ the equilibrium yield on capital can be expressed as:

$$\tilde{r} = \tilde{r}(K,m), \qquad \tilde{r}_m < 0 \quad \text{and} \quad \tilde{r}_K > 0 \tag{31}$$

An increase in m is a movement along the *LM* curve and causes a drop in $\tilde{r}$ (see figure 1.a); hence $\tilde{r}_m < 0$. An increase in K moves the *LM* curve to the right; hence $\tilde{r}_K > 0$. The yield $\tilde{r}$ in turn can be substituted in $\dot{K} = \dot{K}(r,K)$ to obtain a reduced form for the equilibrium rate of capital formation:

$$\dot{K} = \dot{K}(\tilde{r},K) \equiv \sigma(K,m), \qquad \sigma_K < 0 \quad \text{and} \quad \sigma_m > 0 \tag{32}$$

---

[16] A system of two differential equations has the form of $\frac{dx}{dt} = f(x,y)$ and $\frac{dy}{dt} = g(x,y)$.

The sign of the partial effect $\sigma_m > 0$ originates from the drop in $\tilde{r}$ caused by an increase in m. The wealth definition yields the following time-derivative:

$$\dot{w} = \dot{m} + Kdq + q\dot{K} = S(K, m + qK) \tag{33}$$

Equation (33) may be written as:

$$\dot{m} = S(K, m + qK) - Kdq - q\dot{K} \tag{34}$$

At the equilibrium in the asset markets the real price of a share is:

$$\tilde{q} = \frac{F'(K)}{\tilde{r}} = \frac{F'(K)}{\tilde{r}(K,m)} \tag{35}$$

Inserting $\tilde{q}$ into equation (34), we obtain a reduced form expression:

$$\dot{m} \equiv \eta(K, m), \qquad \eta_K < 0 \quad \text{and} \quad \eta_m < 0 \tag{36}$$

Equations (32) and (36) form a system of differential equations in (K, m). In equation (32) an increase in K reduces $\dot{K}$, hence $\sigma_K < 0$; to restore $\dot{K} = 0$, the rate of return has to decline; this is equivalent to increasing m; hence $\sigma_m > 0$. In figure 2, the line $\dot{K} = 0$ is drawn as an upward sloping curve. It splits the space (K, m) into two regions. To the right of the line, $\dot{K} < 0$; as time evolves, K decreases until it hits the line $\dot{K}(r, K) = 0$. To the left of the line, $\dot{K} > 0$; as time moves on, K rises until it hits the line $\dot{K}(r, K) = 0$. In equation (36), an increase in K increases both output Y and consumption; we assume the consumption effect outweighs the output effect so $\eta_K < 0$. An increase in m increases real wealth w and therefore reduces savings, hence $\eta_m < 0$. In figure 2, the line $\dot{m} = 0$ is drawn as a downward sloping curve. It splits the space (K, m) into two regions. To the right of the curve, $\dot{m} < 0$; as time evolves m decreases until it hits the line $\dot{m} = 0$. To the left of the line $\dot{m} = 0$, $\dot{m} > 0$; as time evolves m rises until it hits the line $\dot{m} = 0$.

The intersection of the lines $\dot{K} = 0$ and $\dot{m} = 0$ determines the long-run equilibrium, or the steady state, to which an Islamic economy converges. The values at the long-run equilibrium are $\bar{K}$ and $\bar{m}$. Inserted into the equation $m = \ell(r)k$, these values determine $\bar{r}$. They also determine the values of $\bar{w}$, $\bar{P}$, and $\bar{Y}$. The stability of the differential equation system can be studied by considering the matrix A of the first derivatives:

$$A = \begin{bmatrix} \frac{\partial \dot{K}}{\partial K} & \frac{\partial \dot{K}}{\partial m} \\ \frac{\partial \eta}{\partial K} & \frac{\partial \eta}{\partial m} \end{bmatrix} = \begin{bmatrix} \sigma_K & \sigma_m \\ \eta_K & \eta_m \end{bmatrix} = \begin{bmatrix} - & + \\ - & - \end{bmatrix} \quad (37)$$

Necessary and sufficient conditions for stability are that the trace of the matrix A is negative and its determinant is positive. These two conditions are verified:

$$Tr(A) = \sigma_K + \eta_m < 0 \quad (38)$$

The determinant of the matrix is:

$$Det(A) = \sigma_K \, \eta_m - \sigma_m \eta_K > 0 \quad (39)$$

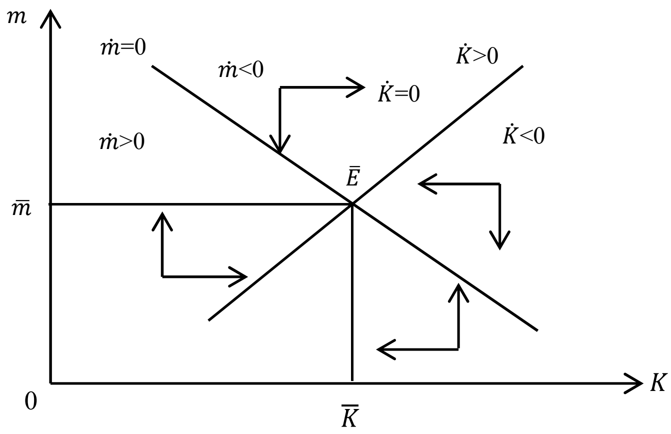
In the phase diagram shown in figure 2, the arrows point to the evolution over time of the pair (K, m). Starting from any point, the dynamics bring the system to a stable long-term equilibrium: $\bar{E} = (\bar{K}, \bar{m})$.

3. The wealth model for an open Islamic economy: trade in commodities only

We assume a small open Islamic economy with only commodity trade taking place. The exchange rate is fixed at e and the foreign price level at P^*. We consider domestic prices to be flexible and that the real exchange rate, $\lambda = eP^*/P$ is to be determined. We assume that households hold two assets, which are non-interest bearing money m and domestic securities K, with the latter not held by foreigners. The trade surplus or deficit translates into an equivalent inflow or outflow of money. According to the Hume (1752) mechanism, an inflow of gold would lead to higher money supply compared to money demand.[17]

[17] To be precise, in Hume's open economy model the asset market is absent; the channel of adjustment was the relative prices of exports and imports. The channel of adjustment

Figure 2 – *Long-term stability of an equity-based economy*



Money holders would shift from money to securities; the price of securities rises, the yield declines; investment rises, consumption rises, and the trade surplus declines. An outflow of money translates into a loss of money balances. Portfolio holders increase their demand for money, sell off securities and drive securities prices downward; the yield rate rises, investment declines; wealth declines and so does consumption. At a given moment in time the stocks of domestic securities K and nominal money M are fixed. The flow variables are savings, investment and the trade balance. At equilibrium of the commodity markets the national income identity requires that the difference between savings and investment be equal to the trade balance. The consumption and investment functions are now reformulated to include the demand for imports. Accordingly, we have:

$$C = C(w, \lambda), \qquad C_w > 0,\ C_\lambda > 0 \tag{40}$$

$$S = S(K, w, \lambda) = F(K) - C(w, \lambda), \quad S_K < 0,\ S_w < 0,\ S_\lambda < 0 \tag{41}$$

via yield rates and the securities market became popular in the second half of the 19th century with the development of capital markets among trading countries.

$$I = I(r, K, \lambda), \qquad I_K < 0, \quad I_r < 0 \text{ and } \quad I_\lambda > 0 \tag{42}$$

The trade balance represents net exports. Since imports are assumed in the consumption and investment functions, net exports depend only on foreign demand for home goods: [18]

$$X = X(\lambda), \qquad X_\lambda > 0 \tag{43}$$

An increase in wealth raises consumption spending, part of which falls on domestic output thus raising demand, $C_w > 0$. An increase in the relative price of foreign goods (a rise in λ) is assumed to shift demand toward domestic goods $C_\lambda > 0$ and $I_\lambda > 0$; consequently, $S_\lambda < 0$. Foreign demand for home goods, X, is just a function of the terms of trade. When domestic prices drop, the terms of trade drop, exports become cheaper and the trade balance increases, hence $X_\lambda > 0$.

The complete open Islamic economy model can be summarised as:

Production function: $Y = F(K)$
Asset market equilibrium (*LM*): $m = \ell(r)k$
Commodity market equilibrium (*IS*): $Y - C(K, w, \lambda) = S(K, w, \lambda) = I(r, K, \lambda) + X(\lambda)$
Definition of real money: $m = M/P$
Definition of real wealth: $w = k + m = qK + m$
Definition of security prices: $q = F_K/r$
Definition of the terms of trade: $\lambda = eP^*/P$
Definition of the increase in nominal money:[19] $\Delta M = P \cdot X(\lambda)$

3.1. The short-run equilibrium of the open economy

In the short-run we assume that the stock variables are fixed; these are nominal money M and the number of old securities K. We also

[18] Some models formulate trade balance as a residual between savings and investment. Although this simplifies the model, it neglects the fact that behavioural relations reflecting both external and internal forces drive foreign trade, in the same fashion as investment and savings.
[19] The increase in real money is $\Delta m = \Delta(M/P) = (P\Delta M - M\Delta P)/P^2$.

assume real output Y is fixed. The asset market equilibrium is illustrated in the space (r, m) by the *LM* curve, which associates with each rate of return r an equilibrium real balances m. Besides r, the commodity equilibrium condition (*IS*) involves w and λ. However, the *IS* relation can easily be represented in the space (r, m). From the definition of real wealth we have $w = w(m, r)$ with $w_m > 0$ and $w_r < 0$.[20] From the definition of the terms of trade and of the domestic price level we have: $\lambda = \frac{eP^*}{P} = \frac{eP^*m}{M}$; hence $\lambda = \lambda(m)$ with $\lambda_m > 0$. The *IS* relation can be restated as:

$$S\big(K, w(m, r), \lambda(m)\big) = I\big(r, K, \lambda(m)\big) + X\big(\lambda(m)\big) \qquad (44)$$

The *IS* relation associates with each m the rate of return r that establishes equality of demand and supply in the commodity markets. In the appendix, the slope of the *IS* line is shown to be positive. The intersection of the *LM* and *IS* lines provides a general full-employment equilibrium (m_e, r_e) for an open Islamic economy as shown in figure 1, panel 1.c. From (m_e, r_e) we deduce equilibrium values for the price level P_e, the terms of trade λ_e, the price of shares q_e and wealth w_e. We also obtain a short-run equilibrium value for the trade balance $X(\lambda_e)$.

3.2. The dynamics of an open Islamic economy

The trade balance translates into a change in the money supply; investment translates into a new issue of securities. The increase in real money supply arises from two sources: change in nominal supply ΔM from foreign trade and change in the price level ΔP. The adjustment in trade balance is affected through the price-specie flow mechanism. A trade surplus causes money to rise, price levels to rise and terms of trade to fall. The trade surplus will diminish. The domestic absorption effect is embedded in the consumption and investment functions. A rise in wealth

[20] We have: $w = m + qK = \ell(r)qK + qK = qK\big(1 + \ell(r)\big) = \frac{F'(K)}{r}K\big(1 + \ell(r)\big)$. Clearly an increase in m increases w. Moreover: $\frac{dw}{dr} = -\frac{F'(K)K(1+\ell(r))}{r^2} + \frac{d\ell}{dr} \cdot \frac{F'(K)}{r}K < 0$.

and/or a drop in the terms of trade will boost imports and narrow the trade surplus. An economy cannot persistently accumulate trade deficits or trade surpluses under the assumptions of an Islamic open model. There is no offsetting credit to the loss or gain of money and the exchange rate is fixed as the case might be under the gold standard. Since the economy has no rigidities, it may be operating near a long-term equilibrium. A steady state is reached when the change in wealth is equal to zero and the change in the capital stock is zero. It is a situation of zero trade balance, i.e. exports are equal to imports, zero investment and zero savings, and it is also a steady state where real output is equal to consumption. Hence the long-run equilibrium can be stated as:

$$\dot{K} = \dot{K}(r, K, \lambda) = 0, \quad \dot{K}_r < 0, \quad \dot{K}_K < 0 \ \text{ and } \ \dot{K}_\lambda > 0 \tag{45}$$

$$\dot{w} = S(K, w, \lambda) = 0, \quad S_K < 0, \ S_w < 0 \ \text{ and } \ S_\lambda < 0 \tag{46}$$

Note that r, λ and w are equilibrium values for which both short-run commodity and asset markets are in equilibrium. We notice that from equations $w = k + m$ and $m = \ell(r)k$ the variables r and w can be expressed in terms of K and m. Moreover, $\lambda = \lambda(m)$. Equations (45) and (46) may therefore be transformed into a system of two differential equations in the variables K and m. From equation $m = \ell(r)k$ the equilibrium yield rate on capital can be expressed as:

$$\tilde{r} = \tilde{r}(K, m) \tag{47}$$

This yield rate in turn can be substituted in (45) to obtain a reduced form for the equilibrium rate of capital formation:

$$\dot{K} = \dot{K}(\tilde{r}, K) \equiv \sigma(K, m), \quad \sigma_K < 0 \ \text{ and } \ \sigma_m > 0 \tag{48}$$

The wealth definition yields the following time-derivative:

$$\dot{w} = \dot{m} + Kdq + q\dot{K} = S(K, m + qK, \lambda) \tag{49}$$

At the equilibrium of the assets market the real price of a share is:

$$\tilde{q} = \frac{F'(K)}{\tilde{r}} \tag{50}$$

Equation (49) may be written as:

$$\dot{m} = S(K, m + qK, \lambda) - Kdq - q\dot{K} \tag{51}$$

Its reduced form expression is:

$$\dot{m} \equiv \eta(K, m), \qquad \eta_K < 0 \quad \text{and} \quad \eta_m < 0 \tag{52}$$

Equations (48) and (52) form a system of differential equations in (K, m). In equation (48) an increase in K reduces $\dot{K}$, hence $\sigma_K < 0$; to restore $\dot{K} = 0$, the rate of return has to decline; this is equivalent to increasing m; hence $\sigma_m > 0$. In figure 2, the line $\dot{K} = 0$ is drawn as an upward sloping curve. It splits the space (K, m) into two regions. To the right of the line, $\dot{K} < 0$; as time evolves, K decreases until it hits the line $\dot{K}(r, K) = 0$. To the left of the line, $\dot{K} > 0$; as time moves on, K rises until it hits the line $\dot{K}(r, K) = 0$. In equation (52), an increase in K increases both output Y and consumption; we assume the consumption effect outweighs the output effect so $\eta_K < 0$. An increase in m increases real wealth w and reduces saving, hence $\eta_m < 0$. In figure 2, the line $\dot{m} = 0$ is drawn as a downward sloping curve. It splits the space (K, m) into two regions. To the right of the curve, $\dot{m} < 0$; as time evolves m decreases until it hits the line $\dot{m} = 0$. To the left of the line $\dot{m} = 0$, $\dot{m} > 0$; as time evolves m rises until it hits the line $\dot{m} = 0$.

The intersection of the lines $\dot{K} = 0$ and $\dot{m} = 0$, shown in figure 2, determines the long-run equilibrium (or the steady state) to which an Islamic economy converges. The values at the long-run equilibrium are $\bar{K}$ and $\bar{m}$. Inserted into the equation $m = \ell(r)k$, these values determine $\bar{r}$. They also determine the values of $\bar{w}$, $\bar{P}$, $\bar{\lambda}$, and $\bar{Y}$. The stability of the steady state equilibrium follows the same reasoning as for the closed Islamic economy. An open Islamic economy evolves from any initial point to a stable long-run equilibrium owing to the full-flexibility of prices and wages and the absence of credit creation and contraction.

4. An Islamic open economy: trade in commodities and assets

With the development of capital markets, capital flows become of great importance in the theory of international payments. The settlement of external current balances via securities has reduced the size of gold flow in international payments. A creditor country may acquire income-bearing securities instead of gold. There are many approaches for modelling capital flows in an open economy. One approach assumes that domestic and foreign securities are perfect substitutes and the rate of interest is fixed in the international markets; the rate of return r is equal to the world-market rate i^*, $r = i^*$. In settling international current balances, domestic residents acquire or dispose of foreign assets that are perfect substitutes for domestic securities. Another approach assumes imperfect substitutability between domestic and foreign assets. In this case, the rate of return on domestic securities differs from the rate on foreign securities $r \neq i^*$. In settling international current balances, domestic residents acquire or dispose of foreign assets that are imperfect substitutes for domestic securities. They may also elect to settle in gold. The balance of payments constraint may be stated as:

current account balance + capital account balance = change in foreign reserves = change in M

How much of the current account is settled in capital transactions and in gold depends on the degree of capital markets integration and capital mobility, risks and returns, exchange systems, exchange rate expectations and a number of other factors. The monetary approach to the balance of payments suggests that an excess demand of money would exert influence on the current account and the capital account to generate the surplus needed for bolstering the cash position. In contrast, excess supply of money would tend to create a deficit in current and/or capital accounts that absorb excess cash. Here, we are interested in proving the existence of a short-run general equilibrium (r, P) and studying the long-run dynamics of the economy. We make the assumption that capital flows balance the current account and do not contribute to depleting or replenishing gold reserves, i.e. $\Delta M = 0$.

An open Islamic economy that trades in assets may settle its current account balance through selling or buying foreign assets. Since foreign assets pay an income, the economy would not hold foreign currency, which earns no income. We may, therefore, assume that the external current transactions are balanced by capital account transactions and that the change in nominal money is zero. To circumvent the issue of interest payments, we assume that there are foreign *sukuk* that pay an income at a fixed annual yield rate i^*; domestic wealth holders will therefore hold these income-bearing *sukuk* instead of foreign currency. The trade in assets affects the definition of the trade account; it now becomes an external current balance, which is the sum of the trade balance and income balance. Domestic wealth holders accumulate foreign assets in case of an external current account surplus and deplete foreign assets in case of a trade deficit. If they are net debtors, they have to issue asset-backed *sukuk* that are identical to the foreign *sukuk* in terms of return and risks rating.

Let the value in domestic currency of outstanding foreign *sukuk* be denoted as eB, where e is the fixed exchange rate and B is the value of *sukuk* held in foreign currency. If B is positive the home country is a creditor and earns income from abroad. If B is negative the country is a debtor; it pays income abroad. The real value of foreign *sukuk* is defined as:

$$b = eB/P \tag{53}$$

The external current account Z can be defined as:

$$Z(\lambda, b) = X(\lambda) + i^* b \tag{54}$$

The relation between the stock of foreign *sukuk* and the external current account can be formalised as:

$$e\dot{B} = e\frac{dB}{dt} = P \cdot X(\lambda) + i^* eB \tag{55}$$

Foreign *sukuk* are increased in case of a positive external current account; and diminished in case of a negative current account. We make further simplifying assumptions that foreigners do not hold domestic securities. This assumption enables us to avoid fixing the rate of return on domestic securities at a given international rate; hence the rate of return on domestic equities remains endogenous.[21] Our goal is to find a general

[21] In Frenkel and Rodriguez (1975) and Mirakhor and Zaidi (1988) the rate of return of an open economy was fixed at the international level i^*. The domestic asset holdings in money and domestic securities are instantly traded upon opening the economy to comply

equilibrium, i.e. an endogenous rate of return on domestic securities r, and a price level P that establish equilibrium in both assets and commodities markets. In analysing short-run equilibrium of the open Islamic economy, we assume that the stock variables are fixed; these are nominal money M, the number of old securities K, and the stock of foreign *sukuk* eB. We assume also real output Y is fixed.

4.1. The short-run equilibrium of the open economy

The wealth portfolio is now composed of three types of assets, domestic money M, domestic securities qPK, and foreign *sukuk* eB. Nominal wealth is expressed as:

$$W = M + qPK + eB \qquad (56)$$

and real wealth is:

$$w = \frac{W}{P} = \frac{M}{P} + \frac{qPK}{P} + \frac{eB}{P} = m + k + b \qquad (57)$$

where: $m = M/P$, $k = qK$ and $b = eB/P$.

Since we are interested in determining an endogenous rate of return r and a price level, it would be helpful to distinguish between real equity wealth which is valued using q and real money and foreign *sukuk* wealth which are valued using P. Consequently, we introduce a new composite financial wealth a defined as the sum of real money and real foreign *sukuk*:

$$a = m + b \qquad (58)$$

The definition of wealth becomes:

---

with the liquidity ratio $\ell(i^*)$; namely the new holdings verify $m' = \ell(i^*)k'$, where m' is new holding of real balances and k' new holding of real capital. The long-run dynamics of the economy were determined by changes in the capital stock and in real wealth. The long-run equilibrium capital stock is established from $\dot{K} = K(i^*, K) = 0$. The change in wealth is defined as a function of wealth and capital stock: $\dot{w} = \dot{w}(w, K)$. The long-run steady state is given by the intersection of the phase lines: $\dot{K} = K(i^*, K) = 0$ and $\dot{w} = \dot{w}(w, K)$. Dornbusch and Fischer (1980) considered the rate of return as given at the international level i^*. Their endogenous variables were the terms of trade and real foreign assets. In their model, the long-run dynamics move the economy to a steady state where the commodities market is in equilibrium and the change in foreign assets is zero.

$$w = a + k \tag{59}$$

The demand for securities increases when the rate of return rises; it is expressed as:

$$k_d = \beta(r, i^*)w, \qquad \beta_r > 0 \tag{60}$$

The demand for financial assets drops when the rate of return rises; it is expressed as:

$$a_d = a(r, i^*)w, \qquad a_r < 0 \tag{61}$$

From the definition of wealth and the demand for assets we have:[22]

$$a(r, i^*) + \beta(r, i^*) = 1 \tag{62}$$

Hence, only one equation of the two asset demand functions is independent. An excess of demand for securities $(k_d - k)$ corresponds to an excess supply of financial assets $(a - a_d)$. The equilibrium of the asset market is achieved when one of the two assets markets is in equilibrium. Equilibrium in the financial market implies equilibrium in the securities market and vice versa. Since at equilibrium $a_d = a$ and $k_d = k$, the portfolio balance condition may be stated as:

$$\frac{a_d}{k_d} = \frac{a(r,i^*)w}{\beta(r,i^*)w} = \frac{a}{k} \tag{63}$$

Rearranging this, we obtain:

$$a = \frac{a(r,i^*)}{\beta(r,i^*)}k = \ell(r, i^*)k \tag{64}$$

Where: $\ell(r, i^*) = \frac{a(r,i^*)}{\beta(r,i^*)}$ and $\ell_r < 0$. The equilibrium condition in the assets market (*LM*) can be written as:

[22] The simple forms of the asset demand function were chosen to simplify the analysis. It is possible to emphasize the imperfect asset substitutability between domestic and foreign assets by including a risk premium, namely domestic asset holders would require a risk premium γ to hold foreign sukuks. In this case the demand functions can be re-written as: $k_d = \beta(r, i^* + \gamma)w$ and $a_d = a(r, i^* + \gamma)w$. The risk premium may be expressed as an increasing function of the stock of foreign assets b. In this case, the demand functions become: $k_d = \beta\big(r, i^* + \gamma(b)\big)w$ and $a_d = a\big(r, i^* + \gamma(b)\big)w$. Since b is already included in w and the risk premium has no direct role in the commodities market, we may settle for simple asset demand functions.

$$a = \ell(r, i^*)qK \tag{65}$$

For a given stock of securities K, and given a, we may find the rate r that establishes equilibrium in the assets market. From equation (65) we have:

$$\frac{da}{dr} = \frac{d\ell}{dr} + K\ell\frac{dq}{dr} < 0 \tag{66}$$

For a given K, the *LM* curve is a downward sloping curve as shown in figure 3 for the standard reason: an increase in r increases the opportunity cost of holding non-equity assets, and therefore reduces demand for these assets.

Since the economy is earning or paying income to the rest of the world, in an amount of i^*eB, or i^*b in real terms, the equilibrium condition in the commodities market can be written as:

$$Y + i^*b = C(\lambda, w) + I(\lambda, r, K) + Z(\lambda, b) \tag{67}$$

The external current account Z can be defined as:

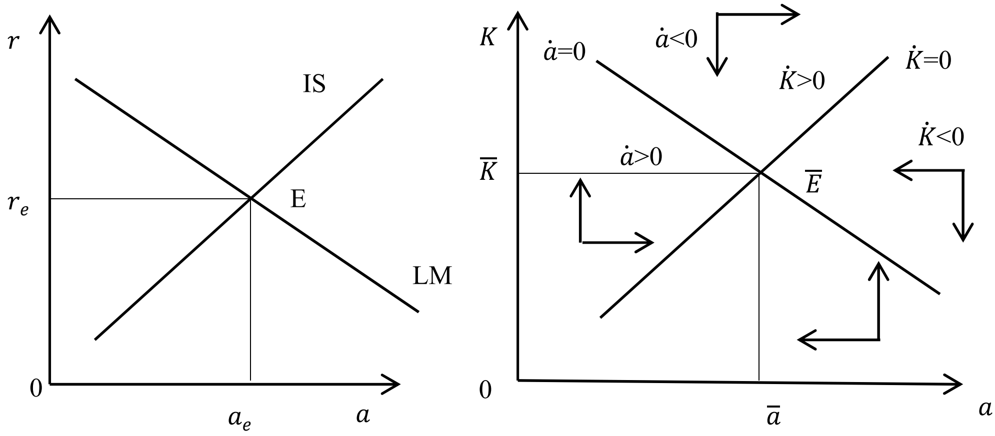
$$Z(\lambda, b) = X(\lambda) + i^*b \tag{68}$$

It is now influenced by the terms of trade and income payments from the stock of foreign *sukuk*, with $Z_\lambda = X_\lambda > 0$ and $Z_b = i^* > 0$. The higher the stock of foreign *sukuk* the higher is the income from *sukuk* and ceteris paribus the higher the external current account. We note that the economy cannot accumulate foreign assets indefinitely since real wealth increases domestic demand faster than real product and leads to depletion of foreign assets. Likewise, the economy cannot remain net debtor indefinitely since real wealth decreases and leads to a replenishment of foreign assets. There is no credit mechanism that offsets the effect on domestic demand of a loss or gain in foreign assets. Even though securities replace direct gold settlement, the economy continues to operate along the classical principles of the gold standard.

As in the asset market equilibrium, we want to express the commodity market equilibrium in the space (r, a). Besides r the commodity equilibrium condition (*IS*) involves b, w and λ. We show that the *IS* relation can be represented in the space (r, a). We note from the definition of real wealth, $w = qK + a = w(a, r)$ with $w_a > 0$ and

$w_r < 0$. The domestic price level may be defined as $P = (M + eB)/a$. From the definitions of the terms of trade and the domestic price level we have: $\lambda = \frac{eP^*}{P} = \frac{eP^*a}{(M+eB)}$; hence $\lambda = \lambda(a)$ with $\lambda_a > 0$. Moreover $b = \frac{eB}{P} = \frac{eB}{(M+eB)}a = b(a)$, with $b_a > 0$.

Figure 3 – *Equilibrium of an Islamic open economy*



Panel a. Short-run equilibrium Panel b. Long-run equilibrium

We define national savings as:

$$S = F(K) + i^*b - C(\lambda, w) = S\big(K, w(a,r), b(a), \lambda(a)\big) \quad (69)$$

The *IS* equilibrium relation can be restated as:

$$S\big(K, w(a,r), b(a), \lambda(a)\big) = I\big(r, K, \lambda(a)\big) + Z\big(\lambda(a), b(a)\big) \quad (70)$$

For a given K, equation (70) is represented by the *IS* schedule (figure 3, panel a) which is associated with the level of financial wealth and a rate of return that establishes equilibrium in the commodity markets, i.e. it equates national savings to investment and the external current account. In the appendix we show that *IS* has a positive slope. As in ordinary *IS*-

LM analysis, the general short-run equilibrium is established by the intersection of the IS and LM curves $E = E(r_e, a_e)$. We have a rate of return r_e, a real financial wealth a_e, and implicitly an equilibrium level P_e and terms of trade λ_e. At this short-term equilibrium the external current account is not necessarily zero and is equal to $Z(\lambda_e, b_e)$.

4.2. The dynamics of the open economy

The external current account balance translates into a change in the foreign *sukuk* holdings; investment in physical capital translates into a new issue of securities. The increase in real financial wealth arises from two sources: a change in the nominal value of foreign held *sukuk* reflected in the external current account balance and a change in the price level. The nominal money M is fixed. A steady-state general equilibrium in the economy is reached when the change in real wealth is equal to zero and the change in the physical capital stock is zero. It is a situation of zero external current account, zero investment and zero saving. It is also an equilibrium where national income is equal to consumption and where the income from foreign assets is offset by the trade balance: $F(K) + i^*b = C\ (w, \lambda)$.[23]

Hence the long-run equilibrium can be stated as:

$$\dot{K} = \dot{K}(r, K, \lambda) = 0, \qquad \dot{K}_r < 0, \quad \dot{K}_K < 0 \quad \text{and} \quad \dot{K}_\lambda > 0 \tag{71}$$

$$\dot{w} = S(K, b, w, \lambda) = 0, \quad S_K < 0, \quad S_b < 0, \quad S_w < 0 \quad \text{and} \quad S_\lambda < 0 \tag{72}$$

Note that r, λ, b, and w are equilibrium values for which both short-run commodities and assets market equilibria obtain. The differential equations (71) and (72) are expressed in terms of K and a. From the equilibrium condition of the assets market the yield rate on capital can be expressed as:

$$\hat{r} = \hat{r}(K, a) \tag{73}$$

[23] Note that a situation of a zero current account is not necessarily a long-term equilibrium; it only means that national savings is equal to investment. Long-term equilibrium requires both savings and investment to be zero.

This yield rate in turn can be substituted in equation (71) to obtain a reduced form for the equilibrium rate of capital formation:

$$\dot{K} = \dot{K}(\hat{r}, K, \lambda) \equiv \sigma(K, a), \qquad \sigma_K < 0 \quad \text{and} \quad \sigma_a > 0 \tag{74}$$

As the size of physical capital rises, the demand for new investment declines; hence $\sigma_K < 0$; furthermore, a decline in the general price level increases the terms of trade λ; hence $\sigma_a > 0$.

The wealth equation yields the following time derivative:

$$\dot{w} = \dot{a} + Kdq + q\dot{K} = S(K, a + qK, b, \lambda) \tag{75}$$

Under asset market equilibrium, the real price of a share is:

$$\hat{q} = \frac{F'(K)}{\hat{r}} \tag{76}$$

Equation (75) may be written as:

$$\dot{a} = S(K, a + qK, b, \lambda) - Kdq - q\dot{K} \tag{77}$$

Its reduced form expression is:

$$\dot{a} = \theta(K, a), \qquad \theta_K < 0 \quad \text{and} \quad \theta_a < 0 \tag{78}$$

It is assumed that an increase in K reduces savings, implying that the rise in consumption from the wealth effect exceeds the rise in the output induced by an increase in K; hence $\theta_K < 0$. An increase in a reduces saving via the wealth effect; hence $\theta_a < 0$. We have a system of differential equations in (K, a).

$$\dot{K} = \sigma(K, a) \tag{79}$$

$$\dot{a} = \theta(K, a) \tag{80}$$

In equation (79) an increase in K reduces $\dot{K}$, hence $\sigma_K < 0$; to restore $\dot{K} = 0$, the rate of return has to decline; this is equivalent to increasing a; hence $\sigma_a > 0$. In figure 3, panel b, the line $\dot{K} = 0$ is drawn as an upward sloping curve. It splits the space (K, a) into two regions. To the right of the line, $\dot{K} < 0$; as time evolves, K decreases until it hits the line $\sigma(K, a) = 0$. To the left of the line, $\dot{K} > 0$; as time moves forward, K rises until it hits the line $\sigma(K, a) = 0$.

In equation (80), an increase in K increases both output Y and consumption; we assume the consumption effect outweighs the output effect so that $\theta_K < 0$. An increase in a increases real wealth w and therefore reduces savings, hence $\theta_a < 0$. In figure 3, the line $\dot{a} = 0$ is drawn as a downward sloping curve. It splits the space (K, a) into two regions. To the right of the curve, $\dot{a} < 0$; as time evolves a decreases until it hits the line $\dot{a} = 0$. To the left of the line $\dot{a} = 0$, $\dot{a} > 0$; as time passes a rises until it hits the line $\dot{a} = 0$. The intersection of the lines $\dot{K} = 0$ and $\dot{a} = 0$ provides the long-term equilibrium $\bar{E}(\bar{a}, \bar{K})$ to which an Islamic open economy converges. Inserted into the equation $m = \ell(r)k$, these values determine $\bar{r}$. They also determine the values of $\bar{w}$, $\bar{P}$, $\bar{\lambda}$ and $\bar{Y}$.

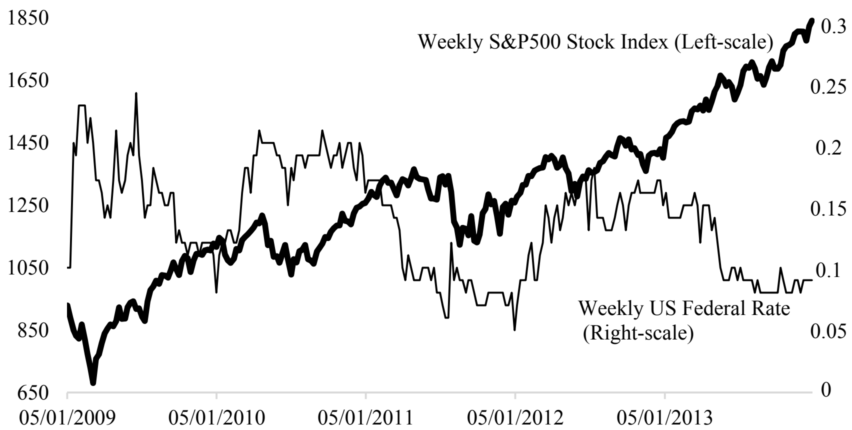
To analyse the stability condition we consider the matrix of the partial derivatives:

$$A = \begin{bmatrix} \sigma_K & \sigma_a \\ \theta_K & \theta_a \end{bmatrix}$$

The stability condition requires that the trace of A be negative and its determinant be positive. In fact $Trace\ A = \sigma_K + \theta_a < 0$ and $Det\ A = \sigma_K\theta_a - \sigma_a\theta_K > 0$. The long-run equilibrium $\bar{E}(\bar{a}, \bar{K})$ is, therefore, stable. From any initial point, an Islamic economy moves toward this steady state equilibrium.

The theoretical stability of an Islamic model may be contrasted to the empirical instability of a conventional model. In figure 4, we portray the S&P 500 stock market index and the US federal funds rate during the period January 2009-December 2013. We observe that the Federal Reserve fixed the interest rate at the near-zero level of about 0.1%; it has been largely negative in real terms; hence, there has been no market-determined interest rate. The central bank forced a near-zero interest rate and strictly denied rewards to fixed-income assets and particularly to savings accounts. Helped with unrestrained liquidity, near-zero interest rates, and speculation, the S&P 500 stock market index has risen about 34% per year during the period 2009-2013, despite sluggish economic growth of about 2% per year and high unemployment. Since the stock yields far exceeded the dividends rate, most of the gains came from a redistribution of wealth in favour of speculators and stockholders at the expense of creditors, workers, and fixed-income recipients. The stock prices index showed no tendency to stabilise around some equilibrium level. Historically stock prices have kept on rising independent of the real economy, mainly propelled by cheap money, until they reach a crisis point. Mirakhor *et al*. (2012) analysed the severe instability of conventional financial markets and argued that Islamic finance, with its core characteristic of risk sharing, may well be a viable alternative to the present interest-based debt financing regime.

Figure 4 – *Weekly S&P 500 Stock Index and US Federal Funds Rate, January 2009-December 2013*



Source: US Federal Reserve and Yahoo! Finance.

5. Conclusion

In an Islamic economy, interest and interest-rate debt are prohibited. It is a 100% reserve banking system and capital is financed by equity. The rate of return is not a fixed interest rate on a loan contract irrespective of actual profits or losses in the venture. It is determined by actual profits and losses. The economy operates along the principle of Say's law; savings are real and are invested on a profit-and-loss sharing basis through equity. Money is not influenced by interest-based credit.

Besides the interdiction of interest, the economy precludes forces that interfere with the flexibility of prices and wages. The economy moves from short-run equilibrium to a stable long-run equilibrium from any initial point as a result of its two important properties: the absence of interest and debt, and the absence of rigidity.

The Islamic financial system moves to equilibrium and has strong stability features. The economy is immune to financial crises that are caused by interest and the interest-based credit system. The financial system facilitates and serves real sector activities. Authors of the Chicago plan (1933) considered a 100% reserve based economy the only system capable of securing financial stability and steady full-employment. The 2008 financial crisis indicates that an interest-based system is unstable (Askari *et al.*, 2010; Minsky, 1986). A number of economists have advocated the abolishment of the credit system and establishing an equity-based system with 100% reserve banking (Walker, 1873; von Mises, 1953; Carrol, 1965; Rothbard, 1994). The theoretical properties of 100% reserve cum equity-based financing and its ability to operate efficiently at full-employment have been assessed. It has been demonstrated that households earn returns from equity investments. There is no wealth redistribution from creditors to debtors as in conventional finance. Wealth is accumulated from savings and not from debt. Based on this paper, it could be claimed that an Islamic equity-based system, if shocked, moves towards equilibrium without powerful fiscal and monetary policy, is not vulnerable to boom-bust cycles, would have strong stability features and would be conducive to promoting international trade, financial stability and economic growth.

REFERENCES

ASKARI H., IQBAL Z., KRICHENE N. and MIRAKHOR A. (2012), *Risk Sharing in Finance: The Islamic Finance Alternative*, Singapore: John Wiley & Sons.

——— (2010), *The Stability of Islamic Finance*, Singapore: John Wiley & Sons.

ASKARI H. and KRICHENE N. (2014),"Islamic Finance: An Alternative Financial System for Stability, Equity, and Growth", *PSL Quarterly Review*, vol. 67 n. 268, pp. 9-54.

CARROLL C.H. (1965), *Organization of Debt into Currency and Other Papers*, Princeton (NJ): D. Van Nostrand.

DORNBUSCH R. (1975), "A Portfolio Balance Model of the Open Economy", *Journal of Monetary Economics*, vol. 1 n. 1, pp. 3-20.

DORNBUSCH R. and FISCHER S. (1980), "Exchange Rates and the Current Account", *The American Economic Review*, vol. 70 n. 5, pp. 960-971.

FISHER I. (1936), *100% Money*, New York: Adelphi Company.

FREIXAS X. and ROCHET J.-C. (2008), *Microeconomics of Banking*, Cambridge (MA): MIT Press.

FRENKEL J.A. and RODRIGUEZ C.A. (1975), "Portfolio Equilibrium and the Balance of Payments: A Monetary Approach", *The American Economic Review*, vol. 65 n. 4, pp. 674-688.

GRAZIANI A. (2003), *The Monetary Theory of Production*, Cambridge: Cambridge University Press.

HOLDEN E.H. (1907), *Lecture on the Depreciation of Securities in Relation to Gold*, London: Blades,

East & Blades.

HUME D. ([1752] 1955), "Of the Balance of Trade", in Rotwein E. (ed.), *Writings on Economics*, London: Nelson.

KOTLIKOFF L.J. (2010), *Jimmy Stewart Is Dead: Ending the World's Ongoing Financial Plague with Limited Purpose Banking*, Hoboken (NJ): John Wiley & Sons.

METZLER L.A. (1951), "Wealth, Saving, and the Rate of Interest", *The Journal of Political Economy*, vol. 59 n. 2, pp. 93-116.

MINSKY H. (1986), *Stabilizing an Unstable Economy: A Twentieth Century Fund Report*, London and New Haven (CT): Yale University Press.

MIRAKHOR A. (1993), "Equilibrium in a Non-Interest Open Economy", *Journal of King Abdulaziz University: Islamic Economics*, vol. 5 n. 1, pp. 3-23.

MIRAKHOR A. and IQBAL Z. (1988), "Stabilization and Growth in an Open Islamic Economy", *IMF Working Papers*, n. 88/22, Washington (DC).

MIRAKHOR A., KRICHENE N. and SHAUKAT M. (2012), "Unsustainability of the Regime of Interest-Based Debt Financing", *ISRA International Journal of Islamic Finance*, vol. 4 n. 2, pp. 25-52.

VON MISES L. (1953), *The Theory of Money and Credit*, New Haven (CT): Yale University Press.

PATINKIN D. (1965), *Money, Interest, and Prices*, New York: Harper & Row.

PHILLIPS R. (1994), *The Chicago Plan and the New Deal Banking Reform*, Armonk (NY): M.E. Sharpe.

ROTHBARD M.N. (1994), *The Case Against the Fed*, Auburn (AL): Ludwig von Mises Institute.
SIEGFRIED J. (1906), "L'alternance des crises commerciales et des périodes de prospérité", *Revue des Deux Mondes*, vol. 36, pp. 823-841.
THORNTON H. ([1802] 1939), *An Inquiry into the Nature and Effects of the Paper Credit of Great Britain*, New York: Rinehart.
UZAWA H. (1969), "Time Preference and the Penrose Effect in a Two-Class Model of Economic Growth", *Journal of Political Economy*, vol. 77 n. 4, pp. 628-652.
WALKER A. (1873), *The Science of Wealth*, Philadelphia (PA): J.B. Lippincott & Co.
WICKSELL K. ([1898] 1936), *Interest and Prices*, London: Macmillan.

Appendix

The following demonstrates that the IS line of an open Islamic economy has a positive slope. Total differentiation of Equation (44) yields:

$$\frac{\partial S}{\partial w}dw + \frac{\partial S}{\partial \lambda}d\lambda = \frac{\partial I}{\partial r}dr + \frac{\partial I}{\partial \lambda}d\lambda + \frac{\partial X}{\partial \lambda}d\lambda$$

$$\frac{\partial S}{\partial w}\left(\frac{\partial w}{\partial m}dm + \frac{\partial w}{\partial r}dr\right) + \frac{\partial S}{\partial \lambda}\frac{d\lambda}{dm}dm = \frac{\partial I}{\partial r}dr + \frac{\partial I}{\partial \lambda}\frac{d\lambda}{dm}dm + \frac{\partial X}{\partial \lambda}\frac{d\lambda}{dm}dm$$

$$\frac{\partial S}{\partial w}\frac{\partial w}{\partial m}dm + \frac{\partial S}{\partial \lambda}\frac{d\lambda}{dm}dm - \frac{\partial I}{\partial \lambda}\frac{d\lambda}{dm}dm - \frac{\partial X}{\partial \lambda}\frac{d\lambda}{dm}dm$$
$$= \frac{\partial I}{\partial r}dr - \frac{\partial S}{\partial w}\frac{\partial w}{\partial r}dr$$

$$\left(\frac{\partial S}{\partial w}\frac{\partial w}{\partial m} + \frac{\partial S}{\partial \lambda}\frac{d\lambda}{dm} - \frac{\partial I}{\partial \lambda}\frac{d\lambda}{dm} - \frac{\partial X}{\partial \lambda}\frac{d\lambda}{dm}\right)dm = \left(\frac{\partial I}{\partial r} - \frac{\partial S}{\partial w}\frac{\partial w}{\partial r}\right)dr$$

The slope of the IS line is:

$$\frac{dm}{dr} = \frac{\left(\frac{\partial I}{\partial r} - \frac{\partial S}{\partial w}\frac{\partial w}{\partial r}\right)}{\frac{\partial S}{\partial w}\frac{\partial w}{\partial m} + \frac{\partial S}{\partial \lambda}\frac{d\lambda}{dm} - \frac{\partial I}{\partial \lambda}\frac{d\lambda}{dm} - \frac{\partial X}{\partial \lambda}\frac{d\lambda}{dm}}$$

It is unambiguously positive.

We show that the IS line of an Islamic open economy that trades in commodities and assets has a positive slope. Total differentiation of equation (74) yields:

$$\frac{\partial S}{\partial w}dw+\frac{\partial S}{\partial b}db+\frac{\partial S}{\partial \lambda}d\lambda=\frac{\partial I}{\partial r}dr+\frac{\partial I}{\partial \lambda}d\lambda+\frac{\partial Z}{\partial \lambda}d\lambda+\frac{\partial Z}{\partial b}db$$

$$\frac{\partial S}{\partial w}\left(\frac{\partial w}{\partial a}da+\frac{\partial w}{\partial r}dr\right)+\frac{\partial S}{\partial b}\frac{db}{da}da+\frac{\partial S}{\partial \lambda}\frac{d\lambda}{da}da$$

$$=\frac{\partial I}{\partial r}dr+\frac{\partial I}{\partial \lambda}\frac{d\lambda}{da}da+\frac{\partial Z}{\partial \lambda}\frac{d\lambda}{da}da$$

$$\frac{\partial S}{\partial w}\frac{\partial w}{\partial a}da+\frac{\partial S}{\partial b}\frac{db}{da}da+\frac{\partial S}{\partial \lambda}\frac{d\lambda}{da}da-\frac{\partial I}{\partial \lambda}\frac{d\lambda}{da}da-\frac{\partial Z}{\partial \lambda}\frac{d\lambda}{da}da-\frac{\partial Z}{\partial b}\frac{db}{da}da$$
$$=\frac{\partial I}{\partial r}dr-\frac{\partial S}{\partial w}\frac{\partial w}{\partial r}dr$$

$$\left(\frac{\partial S}{\partial w}\frac{\partial w}{\partial a}+\frac{\partial S}{\partial b}\frac{db}{da}+\frac{\partial S}{\partial \lambda}\frac{d\lambda}{da}-\frac{\partial I}{\partial \lambda}\frac{d\lambda}{da}-\frac{\partial Z}{\partial \lambda}\frac{d\lambda}{da}-\frac{\partial Z}{\partial b}\frac{db}{da}\right)da$$
$$=\left(\frac{\partial I}{\partial r}-\frac{\partial S}{\partial w}\frac{\partial w}{\partial r}\right)dr$$

The slope of the IS line is:

$$\frac{da}{dr}=\frac{\left(\frac{\partial I}{\partial r}-\frac{\partial S}{\partial w}\frac{\partial w}{\partial r}\right)}{\left(\frac{\partial S}{\partial w}\frac{\partial w}{\partial a}+\frac{\partial S}{\partial b}\frac{db}{da}+\frac{\partial S}{\partial \lambda}\frac{d\lambda}{da}-\frac{\partial I}{\partial \lambda}\frac{d\lambda}{da}-\frac{\partial Z}{\partial \lambda}\frac{d\lambda}{da}-\frac{\partial Z}{\partial b}\frac{db}{da}\right)}=\frac{(-)}{(-)}>0$$

It is unambiguously positive.