On the Congruence of Exchange Rates under a System of Bilateral Trade

by

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This article by Dr. D'Ippolito arose from a discussion, in which Professor Bresciani-Turroni and Professor Gini took part, about bilateral trade and the congruence (1) of direct and cross exchange rates under that system. We think it advisable in the interest of the reader to sum up the essential points of the dispute.

Prof. Bresciani Turroni (see: The problem of the Cross-Rates of Exchange in « Review of the Economic Conditions in Italy », May 1948. Italian version in « Rivista Bancaria », No. 5-6, May-June 1948), criticised a statement which had the support of the International Monetary Fund. This was that, in order to avoid the unfavourable results arising from the present system of bilateral exchanges, i. e. the arbitrage transactions on goods, to the loss of some countries, due to the discrepancies between cross-rates and official parities, it would be desirable to fix cross-rates corresponding to the official parities which have been reported to the Fund. The importance of the problem arising from the discrepancies between cross-rates and official parities had been accentuated — and therefore called to the attention of the International Monetary Fund — by speculation in the sterling area on pounds purchased on the free markets at cross-rates inferior to the official rate of exchange.

Now, Bresciani-Turroni remarked that the measure recommended by the International Monetary Fund ran counter to economic logic and was contradicted by experience, and he pointed out the following things:

(a) as exchange rates are determined as so many unknown quantities belonging to a system of equations or conditions, an attempt to enforce equality on cross-rates and official parities means taking a number of equations or conditions exceeding the number of the unknown quantities, thus setting up a system which would be overdetermined and insoluble;

(b) experience shows the impossibility of enforcing such equality, other market conditions remaining unchanged. This was clearly shown when Italy agreed to equalise the cross-rate and the official parity, fixing the rate of 4.03 dollars for one pound sterling, with the result that the disparity of the Anglo-Italian bilateral balance of trade was greatly accentuated.

Prof. Gini (see: Bilateral and Multilateral Trade in « Banca Nazionale del Lavoro Quarterly Review », No. 8, 1949, Rome. Italian version in « Moneta e Credito », 4th Quarter, 1948), elaborated views in some respects differing from those of Bresciani-Turroni and on more general lines. Generally speaking, Prof. Gini noted the historical need of bilateral trade and its relative advantages, and maintained that, even under a system of bilateral agreements, it is possible to secure a general balance, though the process may be longer and more laborious than under a system of multilateral trade, yet it is perhaps the only one that circumstances allow of. Incidentally he demonstrated with mathematical exactness that, even under a bilateral system, the congruence between direct rates and cross-rates cannot be excluded, even apart from arbitrage transactions on goods. He thus called attention to what he considered to be a formal inaccuracy on the part of Prof. Bresciani-Turroni.

Prof. Gini went on to remark that when an incongruence in exchange rates first appears, it tends, under free market conditions, to be ironed out by arbitrages transactions on goods.

Replying to this remark of Prof. Gini's, Prof. Bresciani-Turroni wrote (see: The Cross-Rates of Exchange Once More in « Review of Economic Conditions in Italy », September 1949. Italian version in « Rivista Bancaria », No. 7-8, July-August, 1949), with particular reference to an alternative considered by Prof. Gini, i. e. that of the possibility of securing a general equilibrium under a system of bilateral exchanges apart from arbitrage operations on goods, and pointed out that such an equilibrium was possible only in theory.

The article by Dr. D'Ippolito which we are now publishing, and the discussions to which it has given rise, seem, in the present state of the question, to have led to a full and satisfactory formulation of the terms of the problem, at least in the case of a static hypothesis.

In any case, the whole debate deals with the more interesting problem and one nearer to existing conditions, which is fundamental to the whole question and correlative to the several points of view from which it is considered, drawn as they are from different political and economic concepts, i. e. the problem of bilateral or multilateral trade. Prof. Bresciani-Turroni thus advocates the expediency of a rapid return to multilateral trade policies and is inclined to accentuate the losses and the deformities of the bilateral system; Prof. Gini is mostly concerned with existing conditions, with historic realities, and is inclined to call attention to the positive features of the bilateral system, considered as a lesser evil that it would be difficult to do without for the time being.

(Note of the Ed.)

(1) *As concerns exchange rates, Professor Gini distinguishes:* (a) *the divergence that often is observed, between the exchange rate concerning the two countries A and B, on the one hand, and, on the other, the ratio between the purchasing powers of the two moneys on the respective internal markets, and* (b) *the divergence between the exchange rate concerning the two countries A and B, on one hand, and, on the other, the ratio of the exchange rate between the countries B and C. Professor Gini remarks that the two divergences have different causes and bearings and thinks it convenient to distinguish them with different denominations. Divergence* (a) *is usually called* disequilibrium of exchange rates, *and Professor Gini keeps for it this denomination, calling instead* incongruence of the exchange rates *the divergence* (b).

It is well known that differences of opinion on matters of political economy very often arise from much more deeply rooted contrasts of ideas, opinions, philosophical outlooks, views on social questions, and from conflicting moral and spiritual needs and prejudices.

It is for this that the solution of the more important economic and social problems presents difficulties whose bearing transcends the possibilities of those charged with solving them.

But not infrequently the difference of opinion has less remote and complex origins, as it arises from different or conflicting conclusions, reached in the strictly scientific field, about the behaviour of economic phenomena. It is in this more limited and less inaccessible field that differences of opinion are less justified as they are more frequently due to misunderstandings or to some loose arguments, and they are also less justifiable as they may give rise to deeper ideological conflicts. They are less justifiable because by their very nature such differences are susceptible of being ironed out with comparative ease, and their elimination would promote a clearer line of conduct on political-economic matters.

1. — With this as a premise, it may be useful to try to make a contribution that may help to settle the recent difference of opinion that has arisen between Bresciani-Turroni and Gini on the congruence of exchange rates under a system of bilateral trade.

The importance of the question is perhaps greater for its reactions on political economy than on account of the gravity of the scientific problem *per se*. Its reflex action on international monetary policy has already been fully studied by the authors of the controversy themselves in their respective statements of the case. I shall therefore consider it here only from the strictly theoretical standpoint.

I will sum up briefly the two positions referring the reader for fuller information to the respective articles (1).

Bresciani-Turroni maintains that under a system of multilateral trade the exchange rate system between the several countries is determined by the conditions that establish the balance between the imports and exports of each country from and to *the whole body of the other countries*. Obviously these conditions do not generally imply the balance of the imports and exports of each country from and to *each* of the others. Should that be the case, the conditions will be those which suffice to determine, apart from a common proportionality factor, an exchange rate for each country and there can be no discrepancy between direct rates and cross rates.

But — Bresciani-Turroni goes on to say — when it is claimed that the balance between imports and exports must occur separately for each pair of countries, then a number of « non-communicating » markets are set up equal to the possible binary combinations of the countries considered, and on each of these markets an equilibrium rate of exchange will arise which, as a rule, will have nothing to do with the rates arising on the other markets. Hence as a rule the impossibility of congruence between the exchanges rates (2). Gini naturally agrees with what Bresciani-Turroni says about the system of multilateral trade, but he does not concur in the conclusions he draws as regards bilateral trade. Specifically, while agreeing that under a régime of bilateral exchanges as many markets may be formed as there are possible pairs of the countries considered, he cannot agree that those markets will be quite « independent » one of the other, and that therefore the exchange rates also that have been formed for each pair will be independent and then, as a rule, incongruous. On the contrary, he maintains that not only is there no need that these exchange rates should be incongruons, but he believes that as a result rates will *tend to become congruous* and such should therefore be considered to be in a state of static equilibrium.

2. — An analogy taken from hydrostatics will help to make the matter clear. Let us consider three containers, representing a like number of countries bound together by commercial relations, and let us suppose that:

(a) in a first case (system of multilateral trade) these containers have a large hole in the bottom and are partially immersed in the same liquid. Evidently the level of the liquid within each container will be the same and will be equal to the level of the liquid outside them (general balance of international exchange rate);

(b) in a second case (system of bilateral trade) we will suppose that the containers have no hole in the bottom but communicate in couples by means of special pipes, thus A with B, A with C, B with C.

Gini asserts that also in this case *under static conditions*, the liquid will rise to the same level in the three containers (i.e. that *under static conditions* the exchange rates will be congruous).

In terms of this analogy, Bresciani-Turroni would assert, on the contrary, that the liquid « cannot » rise to the same level in the three containers and he would argue much as follows: in the case 1, as the three containers all intercommunicate, if a is the level in A then the levels x and y in B and C will be determined by the following system of two independent equations:

$$x = a$$
$$y = x$$

But in case 2, as there are three pair of intercommunicating containers AB, AC, BC each of which does not communicate with the others, if the level of the liquid is to be the same in each pair of containers we must have:

$$a = x$$
$$x = y$$
$$y = a$$

that is to say three « independent » equations with two unknown quantities: hence the conclusion that the liquid cannot rise to the same level in the three containers. This conclusion would be correct if the three equations just written were really independent. As this analogy clearly shows, the supposition that the three equations are independent is based solely on the corresponding supposition that the three pair of *intercommunicating* containers AB, AC, BC *do not communicate* one with the other. Bresciani-Turroni deduces this assumption, we do not quite see how, from the hypothesis that the containers are not all interconnected, but only two by two; or, to express it in other words, as if this circumstance by itself implied the formation within each container of compartments, separating one from the other its two parts which communicate respectively with the other two containers (3).

Now this is the crux of the controversy. According to Gini, and not according to him only, the hypothesis that trade exchanges take place *under a system of bilateral transactions* and hypothesis that the *individual bilateral markets are not communicating*, are two distinct hypotheses, quite independent the one of the other. He argues and draws his conclusions taking, of course, into account only the first, without considering himself bound to accept the second also. Bresciani-Turroni on the other hand accepts them both, sometimes stating explicitly, or letting it be implicitly understood, that the latter is consequent on the former, and sometimes letting us think that the latter is accepted as an independent hypothesis. I quote below from the first page of his first article:

> Let us take three countries A, B and C, each of which trades with the other two on the basis of bilateral transactions. Now let us suppose that all three countries have a currency system consisting of paper money not convertible into gold and, further, that they have all achieved a certain level of home prices unperturbed by inflation or deflation and that both the quantity and type of the goods imported or exported and the exchange rates for the various cur-

(1) C. Bresciani-Turroni: *The Problem of the Cross-Rates of Exchange*, in « Review of the Economic Conditions in Italy », Banco di Roma, vol. II, No. 3, May 1948; Italian version in « Rivista Bancaria », May-June 1948.

Corrado Gini: *Bilateral and Multilateral Trade*, in « Banca Nazionale del Lavoro Quarterly Review », 1949, No. 8; Italian version in « Moneta e Credito », No. 4, 1948.

C. Bresciani-Turroni: *The Cross-Rates of Exchange Once More*, in « Review of Economic Conditions in Italy », September 1949; Italian version in « Rivista Bancaria », July-August 1949.

(2) The term « congruence » has been introduced advisedly by Gini to indicate the coincidence between direct and cross rates of exchange.

(3) If this conclusion were justified there would be no difficulty in granting not only that the levels of equilibrium in the pair of intercommunicating containers would (as a rule) differ, but also that as a rule there would be within each container two different levels. Therefore the above statement of the problem would be mistaken for another reason, in as much as it would tacitly assume the same level within each container.

rencies are the result of transactions effected freely on the markets.

In view of the foregoing supposition, there are three non-communicating markets: AB, AC and BC.

It would seem from the above that the hypothesis that the three bilateral markets *do not communicate* is substantially derived — one does not see how — from the hypothesis of a system of bilateral markets, the accessory hypotheses being implicitly admitted.

It is only in the last pages of the second article that Bresciani-Turroni returns to the subject of non-communicating markets, but this time in a quite different tone and with a quite different meaning:

> Once divergencies have occurred between the cross-rates and direct exchange rates, in a system of bilateral trade, there is no spontaneous tendency to achieve a general equilibrium of exchange rates *as long as the various markets are separated from one another*. Such a tendency can only occur if the markets are not completely separated, so that arbitration is possible in connection with the goods, provoked precisely by the difference between cross-rates and direct rates.

It is then possible that « in view of the foregoing suppositions » the markets are not quite separate! If this be so, the conclusion that the markets are *non-communicating* does not descend from the hypothesis of the system of bilateral exchanges and from the other accessory hypotheses, but should be considered as an ulterior hypothesis formulated *ex novo*.

The evidence adduced by Bresciani-Turroni in support of his statement is moreover such as to lead us to believe that he has not merely excluded possible arbitrage transactions on goods, but that he has gone as far as to exclude any and every form of interdependence among the several bilateral markets. It would not otherwise be possible to explain how he can make the total value expressed in the currency of A, of the goods exported from A to B (E_{ab}) and the total value expressed in the currency of B of the goods imported into A from B (I_{ba}) depend exclusively on the exchange rate K_{ab} existing between the respective currencies, postulating

$$E_{ab} = f\,(K_{ab}) \qquad I_{ba} = \varphi\,(K_{ab})$$

Does Bresciani-Turroni then think that the presence of country C communicating with B under a system of bilateral trade does not in any way affect the balance that may exist between A and B? That is to say, for instance, that A's demand for goods produced in B remains unaltered when A is offered the possibility of importing from C part of the goods it would otherwise have imported from B, or of exporting to C goods previously imported from B, or goods manufactured by it with raw materials first imported from B? (4). And does he likewise suppose that A's offer to B is not modified when A has the possibility of exporting to C goods it would otherwise have exported to B, or of exporting to B goods previously imported from C or goods manufactured in the country from raw materials imported from C?

For even if it be admissible that in passing from a multilateral to a bilateral system, the behaviour and perhaps even the nature of the reactions that each of the partial markets exercise on the others may be modified, it is quite inconceivable that the reactions themselves should be totally suppressed.

3. — What meaning then can we attribute to the synthetic system of 9 equations in 9 unknown quantities as set forth in the second article (p. 390) by which the writer claims to demonstrate the possibility of equilibrium as a whole provided that as many different exchange rates be introduced as there are possible pairs of countries?

If we exclude — as is obvious — that the writer took into consideration only the quantities and the relations that intervene in international trade, neglecting the quantities and relations on the domestic market which, along with the former, help to determine the configuration of the equilibrium as a whole, we must then admit that in the system in question the writer meant to sum up the whole set of conditions that determine equilibrium.

(4) From a scientific standpoint the case in which A exports to C goods it has manufactured with raw materials imported from B and the case in which A exports to C, without processing them, goods previously imported from B are exactly on the same footing. There is indeed no reasonable ground for considering as goods from A only those in which the contribution made to production by A exceeds a pre-established minimum.

If this be so, we do not see which of these relations must be considered as summed up in the system represented by the equation:

$$E_{ab} = I_{ba} \cdot K_{ab}$$

and by the accessory equation quoted above in which system the letter c does not even implicitly appear. We must indeed exclude the possibility that it can be meant to represent the synthesis of *all* the conditions determining the equilibrium between A and B *in the absence of C*, i.e. that it expresses in synthesis the equality between the demands and the supplies of the several commodities within the economic system formed by the two countries, A and B, for it would be absurd to claim that *all* the forces of country A are simultaneously engaged, because of that system, *exclusively* in securing the balance with B, and that, because of the like system represented by the equation:

$$E_{ac} = I_{ca} \cdot K_{ac}$$

and by the two corresponding accessory equations, they are *exclusively* engaged in securing the balance with C, unless we are to suppose that country A has unfolded itself into two distinct countries having the like fundamental economic characteristics (known data and functions) as A.

As in reality country A is only one, a system thus conceived cannot have any sense unless we are to suppose that a kind of *dichotomy* occurs within country A, dividing its economic forces into two systems working independently one of the other and respectively engaged, one in securing the balance with B, the other in doing likewise with C.

It is obvious that if such a dichotomy is implicit in the very way in which the problem has been stated, this does not mean that it is implicit in or can be identified with the conditions required for securing the equilibrium of the several bilateral balances. For it is one thing to decide which of the economic forces of A are to be included in the former system and which in the latter, and it is quite another thing to postulate that in any case the trade accounts between A and B and between A and C balance.

There are no end of ways in which one may think of dividing the system of the economic forces of A into two partial systems. One may, for instance, suppose that the population of A is divided into two sets and that trade between the individuals of the first set and country B is forbidden and likewise trade between the second set and country C, and between the individuals of the first set and the individuals of the second. One may also suppose that the several commodities are divided into two classes and that only goods of the first class may be exchanged with B and only those of the second with C.

This second hypothesis might be the less unacceptable, for, as a matter of fact, country A will generally exchange certain commodities with B and certain others with C. But it is evident that this circumstance, which may occur also under a system of multilateral trade, can only be the result of the equilibrium, not one of its determining causes.

4. — Bresciani-Turroni explicitly refers to Walras (5) and states that his reasoning is but an extension to international trade of the reasoning applied by Walras to the general case of the exchange of goods. But in the case as stated by Walras, though it be based on the hypothesis of the absence of arbitrage transactions on commodities, the reactions exercised by each partial market on the others are far from neglected, as is shown for instance by the fact that the offer of commodity a in exchange for commodity b is made to depend not only on the ratio of exchange p_{ab} existing in the market of the commodities a and b, but also on the ratio of exchange p_{ac} existing in the market of the commodities a and c. For the sake of consistency, it would at least have been necessary, when dealing with the problem of bilateral exchanges between countries, to make the supply and demand of commodities between the two countries A and B depend not only on the exchange rate K_{ab} existing between those countries, but also, respectively, on the exchange rate existing between countries A and C and on the exchange rate K_{ac} existing between countries B and C, stating:

$$E_{ab} = f\,(K_{ab},\ K_{ac}) \qquad I_{ba} = \varphi\,(K_{ab},\ K_{bc})$$

(5) L. Walras: *Eléments d'économie politique pure*, 1926, p. 120.

We may suppose that Bresciani-Turroni, though having in mind Walras' mode of stating the problem, wished to simplify it, but in so doing he has substantially deformed its nature, even if the conclusions would not have been modified by the appropriate integration above suggested.

Now, the reasoning of Walras is quite unexceptionable, and so are therefore the conclusions to which it gives rise, but subject to the hypotheses and conditions he presumes. Now, in this connection it should be noted, first of all, that in the reasoning referred to, Walras only studies that aspect of economic activity which consists in « exchange », setting aside for the time being the « production » side of the question. Now it is precisely in reference to that limited and partial aspect of the question that the hypothesis of the absence of arbitrage transactions on goods leads to the conclusions drawn by Walras. Thus, dealing with the exchange of goods already existing as such on the market he asserts that, on the supposition that one commodity cannot be exchanged with another through the medium of a third, this gives rise to as many partial markets as there are possible couples of commodities, and shows the « possibility » of an equilibrium in which as many independent and therefore (as a rule) uncongruous ratios of exchange are formed as there are partial individual markets.

It is doubtful whether the reasoning of Walras can be extended *sic et simpliciter*, on the basis of the same hypothesis, to the case of international relations. For in this case on the one hand account is taken not only of trade but also of the economic activity as a whole (inclusive therefore of its productive aspects also) and on the other hand also trade is considered no longer from the standpoint of the exchange *between commodities* but from that of exchanges *between countries*. Indeed, as it would be meaningless to speak of exchanges between countries unless with reference to the *commodities* exchanged, the reasoning used in the case of the exchange of goods cannot be extended to the case of exchanges between countries by simply replacing the expression *country* for the expression *goods*, and therefore, speaking of a bilateral market between countries instead of speaking of a bilateral market between goods, of a rate of exchange between two countries instead of a ratio of exchange between two goods and so forth. The only possibility is that of bringing back the problem of the exchange between countries to the problem of the exchange between goods by applying to the latter, and to the latter only, the reasoning of Walras and his related conclusions, unless these conclusions are interpreted in terms of the first problem. The two aspects of the question can only coincide in the case in which, at the start, each of the goods considered should exist only in one of the countries. This may perhaps occur in some exceptional cases, or perhaps it only occurs in some rudimentary theoretical scheme. Generaly speaking, partial markets between countries will overlap with partial markets between goods, and the incongruence of the exchange rates between countries, if it be still possible to speak of exchange rates, will only reflect the general incongruence of the ratios of exchanges between goods arising from the hypothesis of the absence of arbitrage transactions on commodities *between several countries as well as within each country*.

5. — But if the possibility of arbitrage transactions in goods be allowed within each of the countries, then it is no longer possible to bring the problem back to that discussed by Walras, which supposes the general absence of arbitrage transactions, while on the other hand, as we have already noted, the reasoning cannot be extended by merely replacing the category *country* by the category *goods*.

Thus, in the case considered by Walras it can be said that if certain ratios of exchange p_{ab} and p_{ac} exist for commodity a in terms of commodity b and commodity c respectively, the owner of a certain quantity of commodity a will offer in exchange for commodity b and for commodity c respectively the quantities:

$$D_{ba} = f\ (p_{ba} \cdot p_{ac}) \qquad D_{ca} = \varphi\ (p_{ab} \cdot p_{ac})$$

whereby will be expressed, in function of the hypotetical system of ratios of exchange, the components of demand and supply that the individual we are considering exercises « separately » (but not independently!) in *each of the partial markets ab, ac*. But when the exchange between countries is considered, and

when we suppose that within each country, on the hypothesis that arbitrage transactions are allowed, the unknown ratios of exchange are *unique* — i.e. that they are the same whatever be the origin of the commodities exchanged — and that they are *congruous* one with the other — and can therefore be expressed in terms of one and the same commodity — although it will be possible to say the quality and quantity of the goods that each individual of a certain country will demand and will supply, in function of a given hypothetical internal system of ratios of exchange, it will not be possible, as a rule, to say « towards which of the countries » that demand and supply will be directed. In short, it is not possible to determine *a priori*, in function of the system of inernal ratios of exchange, and therefore, indirectly, in function of the exchange rates between the several countries, the components of demand and supply pertaining to country A on each of the partial markets AB and AC which head up to it; components of demand and supply with which, in a state of equilibrium, the quantities imported and exported from country A respectively to or from countries B and C, should coincide.

This therefore excludes the possibility of expressing « by this means » in function of the exchange rates, the quantities of each commodity that a country should exchange under a condition of equilibrium with each of the other countries, unless restrictions be imposed which allow of determining beforehand the way in which the whole body of the demands and supplies exercised by each country as a result of the ratios of internal exchanges « must » be subdivided between the several partial markets, restrictions which certainly do not arise from the hypothesis of bilateral trade.

It seems to me that this consideration is by itself sufficient to show how, on the hypothesis that arbitrage transactions are allowed within each country, the problem of bilateral trade is not susceptible of being approached on lines similar to those used by Walras in the general case of the exchange of commodities.

On the other hand, the fact that the quantity of each commodity that one country will exchange with each of the others under a condition of equilibrium, can be made to depend in the last resort on the exchange rates *through* the set of conditions that connect the several economic quantities *when the equilibrium system is possible and determined*, does not authorise the presumption of a like possibility when it is precisely the possibility and determinateness of the system of conditions that should determine the equilibrium configuration that is under discussion.

Therefore, even if the conditions which the several economic variables should satisfy on the additional hypothesis of the absence of arbitrage transactions *between the several countries*, allow, as I believe they do, of expressing in function of the exchange rates the quantity of goods a country should exchange with each of the others, this cannot be admitted simply by pleading analogy to the case discussed by Walras but should be deduced from a proper statement of the whole problem on other bases.

6. — All the above refers to the mere exchange of goods already existing as such in the countries considered, *i.e.* within the conditions postulated by Walras.

But in dealing with the problem of international trade we must take into account not only the exchange aspect of the question but economic activity as a whole and therefore from the point of view of production also. To assure the validity of the conclusions that may be drawn in this wider field on the basis of certain hypotheses deemed sufficient when the question was limited to a trade economy the hypotheses themselves must be correspondingly widened so as to take into account the other aspects of economic life which have been neglected in the first schema.

Thus, for instance, the hypothesis of the absence of arbitrage transactions on the sthrength of which, if the question be limited to trade exchanges, the possibility that country A might simultaneously import from B and export to C a same commodity should be excluded, would have to be adequately widened so as to exclude also the possibility that country A should export to C commodities produced with raw materials imported from B.

We should also have likewise to widen the more restrictive hypothesis that might be postulated, which would forbid a country from exchanging a given commodity on more than one bilateral market.

7. — But now the time has come to enquire if it be really necessary to have recourse to more or less elaborate hypotheses for the purpose of demonstrating at all costs that under a system of bilateral trade the exchange rates between the several countries connot as a rule, under a postulated condition of equilibrium, be congruous.

What has been so far discussed shows pretty clearly that the hypothesis of a system of bilateral trade is not sufficient to prove the statement, and that it is therefore necessary to have recourse to a supplementary and quite independent hypothesis, either that of the absence of arbitrage transaction, or else the narrower one implicit in the mathematical statement of the case by Bresciani-Turroni.

What are the considerations therefore that can justify the introduction of such an additional hypothesis, a hypothesis which runs counter to the general condition postulated by Bresciani-Turroni himself that « both the quantity and type of the goods imported or exported and the exchange rates of the various currencies are the result of transactions effected freely on the markets »?

Limiting ourselves to the less restrictive hypothesis of the absence of arbitrage transactions on goods, let us see what are in fact the obstacles that would stand in the way of country A exporting to C a commodity imported from B, or from exporting to C a commodity produced with raw materials imported from B. If the individuals of country C demand a commodity produced by B, why should they give up importing it through the intermedium of A should they be unable to import it directly from B? It is well known that ignorance, frictions, inertia, and all the other dynamic factors and effects that in practice contribute to delay, deform, and even make impossible the theoretically possible static equilibrium, are all aspects which cannot be legitimately appealed to in matters in which static equilibrium is supposed to have been attained at the *outset* on the basis of the « mere » needs of the individual economic operators. Had Bresciani-Turroni wanted to take into account these other aspects of economic dynamics, he would have had to make use of different terms, relinquishing the rudimentary arms of economic statics.

The fundamental defect in the way in which Bresciani-Turroni has stated the question arises, in my opinion, from the fact that he believed that in order to study the problem of economic equilibrium under a system of bilateral trade it was necessary to isolate the components of demand and supply that each of the countries exercises on each of the partial markets that head up to it. This, as we have seen, is not possible without the introduction of arbitrary limitations that the hypothesis of bilateral trade does not require; nor is it necessary.

In this connection it should be noted that Walras did indeed introduce similar limitations into the schema that has afforded Bresciani-Turroni his starting point; but that schema represented one stage in the procedure by which, through a series of approximations, he solved the problem of the equilibrium of the market. But, in a subsequent stage, Walras abandons such restrictions and reaches a solution in keeping with real facts (6). If Bresciani-Turroni had also taken this further step, he must necessarily have reached the same conclusions as Gini.

Given a certain system of inland exchange ratios (unique and congruous), a certain total demand and a certain total supply for each commodity will pertain to each country in its foreign relations as it would in the case of multilateral trade. Therefore, the system of equilibrium, will have to be based on the parity for each individual commodity of demand and supply, *for the countries considered as a whole*. The quality and the quantity of the commodities each country will in practice be able to exchange with each of the others under a system of bilateral trade will have to spring from the solution of the system of equations obtained not by requiring that the imports and exports of a country to and from *all* the others should balance, but by requiring that the several bilateral trade accounts shoulds balance.

What has to be ascertained is whether a sistem of equilibrium thus conceived is compatible with a system of unique and congruous rates of exchange, or not rather with a system of unique but incongruous rates, or lastly wether it is not quite incompatible with any system of unique rates of exchange.

(6) L. Walras: *Eléments d'économie politique pure*, 1900, p. 115.

Now, according to Gini, who bases his arguments on the « sole » hypohtesis of a system of bilateral trade, there is a system of equilibrium with unique and congruous exchanges rates that meets the requirements of the said conditions. And it seems to me that this cannot be disproved. It can indeed be affirmed that if, for the sake of simplicity, we suppose that indirect trade does not entail higher costs of carriage than direct trade — and it is known that in the theoretical discussions of questions of this kind the cost of carriage is generally left out of account — the final equilibrium necessarily coincide with that which would arise in the case of multilateral trade.

This seems to me so obvious that I shall only illustrate it by a schematic example, as an elaborate demonstration of the assumption seems to me superfluous.

We will, as usual, consider the case of three countries, A, B, and C, producing respectively oranges, bananas and persimmons, and we will suppose that under a system of multilateral trade equilibrium is secured by the exchange ratios of 1 orange = 1 banana = 1 persimmon, and that the trade exchanges are as follows:

A exports to B 30 oranges,

B exports to C 30 bananas,

C exports to A 30 persimmons.

If for the sake of simplicity we suppose that in the final balanced situation the internal currency units are respectively equivalent to 1 orange, 1 banana, 1 persimmon, then on the assumed ratios of trade exchange the exchange rates would be $K_{ab} = K_{ac} = K_{bc} = 1$

Now, let us pass on to the hypothesis that the three countries can only trade on the basis of bilateral transactions, and assume for the time being that the trade exchanges take place as shown in the following table which shows in each line the supposed exports of each country and in each colum the supposed imports, then the letters *a*, *b*, *c* placed beside the numbers indicate respectively the commodities, oranges, bananas and persimmons:

Imports to / Exports from	A	B	C	*Total Imports*
A	—	20a	10a + 10b	30a + 10b
B	10b + 10c	—	20b	30b + 10c
C	20c	10a + 10c	—	30c + 10a
Total Exports	30c + 10b	30a + 10c	30b + 10a	40a + 40b + 40c

As is seen, in the last resort, each country exports and imports the same net quantities it would have exported and imported in the case of multilateral trade. Moreover, the imports and exports of any one of the countries from and to each of the others, equal each other if the ratios of trade exchanges are the same as those arising from the equilibrium in the case of multilateral trade, *i.e.* 1 orange = 1 banana = 1 persimmon. There would seem to be no reason why, at the conclusion of the bilateral trade arrangement, these ratios of exchange should not be compatible with the final configuration of equilibrium, considering that each country will be able, after all, to dispose within the country of the same quantities of each individual commodity as it disposed of under a system of multilateral trade. It is therefore obvious that the exchange rates also will consequently be congruous.

It is hardly necessary to note that in order to express in mathematical terms the system of equilibrium, it would be necessary in this case to take into account the supplementary demands and supplies (7) that would arise in each country as a consequence of the possibility of reexporting goods previously imported.

But in any case this would not modify the general conditions that express for each commodity the equality of the total demand and supply of all the countries, nor that of the

(7) The situation does not differ substantially from that which occurs when the demand by B of commodities produced in A leads to a supplementary demand by A for raw materials from C.

balance of imports and exports of each country to and from *all* the others, which would be transformed into that of the balance of the several bilateral accounts by adding to both members some properly selected quantities. Therefore, as far as the ratios of trade and exchange rates go, the system of equilibrium would allow the same solution as in the case of multilateral trade, the only solution that is changed being that relating to the quantities imported and exported by each country.

In short, on the supposition that indirect trade does not entail higher costs for carriage than direct trade does, the system that secures an international equilibrium under a system of multilateral trade is determined *as regards the quantities imported and exported* by each country, only, if the possibility of arbitrage transactions is excluded. Otherwise, it is undetermined and among the infinite number of possible solutions any number can be imagined as would satisfy the condition of the balance of the bilateral accounts, which is after all a case of multilateral trade.

The final conclusion we come to is that made evident in the analogy with which we started, i.e. that the equilibrium level obtained in the three containers is independent of the modalities that characterise the communications between them and of the process by which the equilibrium is obtained.

If we take into account the heavier charges for carriage that indirect exchanges entail as compared to direct ones, the equilibrium is more or less modified, but in each case the trend will be towards a static equilibrium in which the exchange rates will be congruous.

It should be remembered that the complicated procedure that a system of bilateral trade calls for and the possible difficulties from the dynamic standpoint of carrying it out, may contribute to prolong the incongruence of exchanges rates; nevertheless it is ascertained that this incongruence can only be considered as a contingent dynamic disequilibrium.

A Reply

by

Professor C. BRESCIANI-TURRONI

Only a few words so as not to weary the reader.

Prof. Gini and Dr. D'ippolito have shifted the question from the position taken by the International Monetary Fund and in my two articles.

The argument advanced by the International Monetary Fund is the following:

(a) under a system of bilateral exchanges and of inconvertible currencies, discrepancies arise on the free markets of the several countries between the cross rates and the official parities fixed by the Fund;

(b) these discrepancies lead to arbitrage transactions on goods that are injurious to some countries. For instance, traders in countries outside the sterling area buy sterling on the free market, with which they purchase raw materials in the sterling area and export them, thus depriving that area of dollars. It was England herselft that raised the problem of the differences between the cross rates and the official parity for sterling;

(c) to avoid these unfavourable results the discrepancies between cross rates and official parities must be suppressed by the action of the several Governments which should fix cross-rates in keeping with the official parities fixed by the Monetary Fund.

The criticisms contained in my two articles relate to point (c). I show that an attempt to impose the general equilibrium of exchange rates (i.e. concordance between cross rates and official parities) under a system of bilateral trade means placing a number of « conditions » exceeding the number of the « unknown quantities », and that this makes it impossible for all the conditions to be satisfied simultaneously.

This leads to a disequilibrium which manifests itself above all in the bilateral trade balances. We have evidence of this in the very heavy deficit of England's bilateral trade balance with Italy, which arose after the Italian Government agreed to impose on the foreign exchange market the rate of 4.03 dollars for one pound sterling. Should the Governments wish to take action to reestablish the balance of bilateral trade accounts, this action would entail (this was the main contention of my second article) the establishment of a series of burdensome controls which would lead the market ever further away from that commercial and monetary freedom which is one of the purposes the Fund aims at securing.

Prof. Gini and Dr. D'Ippolito should have turned their attention to that which, according to my two articles is the crux the question. But instead of that Prof. Gini, starting from some considerations on « marginal utility », endeavours to show that even under a system of bilateral trade a general equilibrium of exchange rates is quite possible. In my second article I have criticised the demonstration that Gini tries to give; as Dr. D'Ippolito makes no reference to it I suppose he accepts my criticism.

Dr. D'Ippolito supports the same proposition as Gini, but with other arguments. Unfortunately for my two critics *their arguments are contradicted by facts*, which show beyond all shadow of doubt that under a system of bilateral agreements a discrepancy immediately arises between cross-rates and the official parities of the several currencies. The same thing occurred before the second world war, when Germany developed the system of bilateral agreements. The result was that the German mark had a different value on different markets. The most recent and best known example is that of the dollar-sterling rate above mentioned. It is quite useless to try to deny this discrepancy. Do not the criticisms of Prof. Gini and Dr. D'Ippolito remind one of that famous character of Manzoni's who tried to prove by his dialectis that the plague did not exist?

A Rejoinder

by

Dr. G. D'IPPOLITO

If facts suffice to prove a theory it would be enough to quote facts without demonstrating the theory, and it would, therefore, be unnecessary for Bresciani-Turroni to trouble to demonstrate his theory, already so amply proved by facts, and to clinch his demonstration by stating: « *I show that an attempt to impose the general equilibrium of exchange rate (i.e. the concordance between cross rates and official parities) under a system of bilateral trade, means placing a number of "conditions" exceeding the number of "unknown quantities" and that this makes it impossible for all the conditions to be satisfied simultaneously* ».

I do not consider that I am shifting the terms of the problem when I resolve the proposition above quoted into its logical component parts, as follows:

(a) *hypothesis*: a system of bilateral exchanges;

(b) *thesis*: the exchange rates cannot be congruous;

(c) *demonstration*: it follows from the hypothesis that the number of *conditions* exceeds the number of the *unknown quantities* (congruous exchange rates); hence the impossibility that the conditions be satisfied.

Now, I affirm:

(1) that the hypothesis (a) does not give rise to the thesis (b), but to the opposite one;

(2) that the demonstration (c) is a mistaken one, because it is not true that the number of conditions exceeds the number of the unknown quantities; on the contrary, the system of equilibrium, which is determined as regards the