Planning the Mechanization of the Bank

1. — In its task of improving procedures of work and introducing new methods in a bank, for more efficient and economical operations, the management is faced with the following alternatives: (a) leaving things as they are, (b) applying a new method or a new combination of methods requiring more modern equipment immediately or gradually in the future, (c) using only such new equipment and in such quantities as will improve efficiency to a certain degree and will pave the way to more advanced equipment which is expected at present but will not be available for some time, reducing to a minimum the cost of adjustment. Decisions as to type, degree, and timing of mechanization are made by the management on the basis of the results of experiments in other banks and of rough estimates of costs, performance, and growth of the institution, taking into account the size of available funds.

It is shown here how this important programming problem can be set up mathematically with all possible details and requirements and can have an exact solution (exact for all practical purposes) minimizing costs over a predetermined period of time. Of course, the solution is only exact if the data fed in the problem are accurate. However, if the estimated errors of the data are not large, under most circumstances the solution may still be relied upon as a working guide for the management in planning. It may be worth stressing that, by requiring certain accurate data and providing an « automatic » solution for the most economical decisions to be made, the mathematical approach outlined here shifts the burden of programming largely from top-level officials to cost analysts and experts of methods, producing in the process a coordinated survey of banking operations, which are both desirable goals in themselves.

This approach is based on the method of linear programming, a very flexible tool which was developed mainly in the United States after the war and has already found a variety of applications to several disparate fields: economics, industrial technology, transportation, logistics, etc. (1). The name itself of this method points out its basic assumption of linearity, which means that all factors are assumed to be connected in a very simple way, such that their relationships may be described by straight lines on ordinary diagrams. To the limitations of linearity, as opposed to schemes more adherent to reality, apply the commonly accepted considerations that in general such assumption does not substantially affect the results and that in many instances, even if a more realistic non-linear approach and a method of solution are found, the small gain in the precision of the results thus obtained would not justify the larger amount of computations involved, or there might be greater chances for errors.

Decisions concerning technical changes in a bank are obviously to be made looking ahead over a period of several years. The reader might object that there is little sense in binding the management for several years to a solution which would probably become inefficient after some time under changing tech-

(1) On this matter see especially:

George B. Dantzig, *Maximization Subject to Inequalities, and other papers*, in T. C. Koopmans, ed., *Activity Analysis of Production and Allocation*, Cowles Commission Mo. 13, New York, John Wiley & Son, 1951.

Marshall K. Wood, George B. Dantzig, *Programming of Interdependent Activities*, in « Econometrica », July-Oct. 1949.

A. Charnes, W. W. Cooper, A. Henderson, *An Introduction to Linear Programming*, New York, John Wiley & Sons, 1953.

R. Dorfman, *Application of Linear Programming to the Theory of the Firm*, Berkeley, University of California Press, 1951.

nological conditions and discrepancies between expected and actual volumes of business, even if it was an optimum solution when it was worked out. However, linear programming — and any form of sound planning in general — should be revised often, to be adjusted to changing conditions, although any single solution covers a period of several years to take into account all possible elements known or foreseen at the time. The management might also find it convenient to do some exploratory work comparing programming solutions — at any one time — under different hypotheses about processes, forecasts of volumes of operations, and available funds.

2. — A practical example is given in the table on pages 158-159 to describe such approach. This form of exposition was chosen for greater clarity and also because it provides a blue-print for any application, with the modifications which the particular case may call for. The figures entered in the table were drawn from thin air and little effort was made to present realistic proportions, beside the characteristics relevant for illustrative purposes. Figures were replaced by asterisks whenever they do not seem to be relevant for the exposition. For simplicity, our imaginary bank has only three functions — checking accounts, commercial credits, and letters of credit — different kinds of machines and labor are grouped into large classes, there are three different kinds of processes, or procedures of work — which may be applied singly or in any combination — and planning is done over a period of only four years. Any practical application, on the other hand, will involve a larger number of functions and also a breakdown of kinds of machines, labor, and office space, and will span a longer period of time, maybe even taking intervals shorter than a year within that period.

On the left of the table are indicated the functions of the bank, factors of production, funds available for technical improvements, and costs; on top of the table are shown all possible activities of the bank: forecasted levels of operations, different processes, purchasing, storing, selling machines, training and letting off employees, etc. The first four columns of the table give initial conditions and requirements which are to be satisfied in the four-year period. Let us assume that planning is done at the end of 1955. Reading from the top down, the first column states that in 1956 there are foreseen checking accounts for an average of $ 853 million, commercial credits for $ 581 million, and letters of credit for $ 124 million. These figures are obtained by market research, projection of trends, and other methods of forecasting. In 1956 there will be available 893 office machines — in a practical application it would be advisable to break down this heterogeneous group into typewriters, accounting machines, etc. — a complete set of punched-card equipment, 292,000 square feet of office space, 2,327 clerks, 187 junior executives, and a reserve fund of $ 6.5 million for improvement of the operations of the bank. The next three columns show the forecasted volumes of business in the subsequent three years and planned additions to the reserve fund, of $ 1.5 million in 1957, etc.

The fifth column shows the amount of factors necessary for each $ 10 million of checking accounts, applying the first process. Thus, for instance, it takes 1,810 square feet of office space and 16.41 clerks. These are the figures in the rows marked « in » (input). The figures in the « out » (output) rows indicate what is left at the end of the year. The reduction of machines from 5 to 4.43 and of office space from 1,810 to 1,600 square feet represents depreciation caused by deterioration and obsolescence of machines and buildings and includes maintenance, repairs —taken as a fraction of the working capacity of the factors — while the reduction of the number of clerks from 16.41 to 13.19 is the result of their turnover — which in this example was taken to be approximately 20%. The bottom figure of $ 92,000 gives the total average salaries paid to clerks ($ 2,400 each) and to junior executives ($ 6,000) administering $ 10 million of checking accounts in a given year. Salaries include the employer's contributions to a pension fund, social insurance, etc. They

Functions, Factors \ Activity Level		Exogenous Activities (conditions and requirements at times 1, 2, 3, 4) $X_0^{(1)}=1$	$X_0^{(2)}=1$	$X_0^{(3)}=1$	$X_0^{(4)}=1$	I Process (t=1, 2, 3, 4) Checking Accounts $X_1^{(t)}$	Commercial Credits $X_2^{(t)}$	Letters of Credit $X_3^{(t)}$	II Process (t=1, 2, 3, 4) Checking Accounts $X_4^{(t)}$	Commercial Credits $X_5^{(t)}$	Letters of Credit $X_6^{(t)}$	III Process (t=3, 4) Checking Accounts $X_7^{(t)}$	Commercial Credits $X_8^{(t)}$	Letters of Credit $X_9^{(t)}$
Checking Accounts ($ 10,000)	in	85,300	89,000	92,200	93,500	− 1,000			− 1,000			− 1,000		
	out													
Commercial Credits ($ 10,000)	in	58,100	60,400	61,900	63,200		− 1,000			− 1,000			− 1,000	
	out													
Letters of Credit ($ 10,000)	in	12,400	13,100	13,700	14,200			− 1,000			− 1,000			− 1,000
	out													
Office Machines	in	− 893				5.00	4.34	9.92	2.68	*	*	*	*	*
	out					4.43	3.84	*		*	*	*	*	*
Punch-Card Equipment (Complete set)	in	− 1				0.0011			0.0033	0.0012	0.0016	0.0020	0.0005	*
	out					0.0010			0.0029	0.0011	0.0014	0.0018	0.00047	*
Electronic Equipment (Complete set)	in											0.00823	0.00345	
	out											0.00741	0.00310	
Office space (1,000 square feet)	in	− 292				1.81	1.21	2.90	*	*	*	*	*	*
	out					1.60	1.12	*	*	*	*	*	*	*
Clerks	in	− 2,327				16.41	9.00	31.37	14.59	*	*	*	*	*
	out					13.19	7.24	25.32	*	*	*	*	*	*
Junior Executives	in	− 187				0.73	1.43	*	0.33	*	*	*	*	*
	out					0.69	1.36	*	*	*	*	*	*	*
Reserve Funds ($ 10,000)	in	− 650	− 150	− 50	− 65									
	out													
Costs (in the process-columns: salaries only) ($ 10,000)	in					9.2	2.9	*	3.7	*	*	*	*	*
	out													

Functions, Factors		Office Machines (t=1, 2, 3, 4) → Procuring $X_{10}^{(t)}$	Storing $X_{11}^{(t)}$	Selling $X_{12}^{(t)}$	Punch-Card Equipment Procuring $X_{13}^{(t)}$	Storing $X_{14}^{(t)}$	Selling $X_{15}^{(t)}$	Electronic Equipment Procuring $X_{16}^{(t)}$	Storing $X_{17}^{(t)}$	Selling $X_{18}^{(t)}$	Office Space Procuring $X_{19}^{(t)}$	Idle $X_{20}^{(t)}$	Selling $X_{21}^{(t)}$	Clerks Training $X_{22}^{(t)}$	Excess of Staff $X_{23}^{(t)}$	Letting off $X_{24}^{(t)}$	Junior Executives Training $X_{25}^{(t)}$	Excess of Staff $X_{26}^{(t)}$	Letting off $X_{27}^{(t)}$	Postponing Expenditures $X_{28}^{(t)}$
Checking Accounts ($ 10,000)	in																			
	out																			
Commercial Credits ($ 10,000)	in																			
	out																			
Letters of Credit ($ 10,000)	in																			
	out																			
Office Machines	in		1	1																
	out	1	0.96																	
Punch-Card Equipment (Complete set)	in					1	1													
	out				1	0.95														
Electronic Equipment (Complete set)	in								1	1										
	out							1	0.95											
Office space (1,000 square feet)	in											1	1							
	out										1	0.98								
Clerks	in													0.20	1	1				
	out													1	0.82					
Junior Executives	in																0.50	1	1	
	out																1	0.96		
Reserve Funds ($ 10,000)	in	0.03			8.2			110			1.8									1
	out			0.005			3.4			42			1.6							1.03
Costs (in the process-columns: salaries only) ($ 10,000)	in	0.03			8.2	0.09		110	0.17		1.8	0.015		0.048	0.24	0.05	0.3	0.6	0.15	
	out			0.005			3.4			42			1.6							0.03

only appear in the row of costs under the columns of the processes, because the costs of all relevant factors with the exception of employees are indicated on the right-hand side of the table. It will be noticed that no mention is made of senior executives and of a number of factors which are essential for the operations of the bank and are important items of costs, such as postage, telephone and telegraph, stationery, travelling, legal actions, etc. This is because it was assumed that the number of senior executives does not change if new processes are applied or the volumes of business of the bank vary within certain rather wide limits, and that the quantities used of the above-mentioned factors are proportional to levels of operations of the bank independently of the processes being selected. Under such assumptions the number and salaries of senior executives and the quantities and costs of these factors are irrelevant for our problem. In general all factors that are not affected by the selection of a particular program should be left out of the analysis. Also advertising and other forms of business promotion are neglected here because they simply affect forecasts of volumes of business, and they are not subject to decisions concerning the technical organisation of the bank.

The subsequent columns under the « process » headings are analogous. The three processes are supposed to be progressively more advanced and the missing figures should reflect this situation: the second process uses a larger proportion of punched-card equipment than the first, and the third one introduces electronic equipment. Probably the second and third processes will use a smaller proportion of clerks, office space, and also office machines. The third process is assumed to be roughly known at present but to be applicable, according to reliable expectations, only in the third year of the plan. It shows how foreseen innovations with the consequent risk of rendering present processes obsolete may affect decisions concerning improvements of operations.

The remaining columns describe the activities of procuring, storing, selling equipment, procuring and selling office space and leaving office space idle, training and letting off employees, postponing expenditures on new equipment and office space. In the third of these columns, for instance, the figure of $ 50.00 represents the average proceeds from the sale of an old typewriter, which are credited to the reserve fund and appear also as negative costs. Similarly, the column under the heading « training clerks » shows that on the average it is estimated that it takes one fifth of the time of an experienced clerk, for one year, to train a new clerk. Obviously the cost of training a clerk will be $ 480.00 which is one fifth of the annual salary of a clerk. The coefficients of the storage activities allow for depreciation, e.g. 2% for buildings. The cost of maintenance of idle office space will be negative — i.e. an income — if this space is rented out. The purchase price of equipment includes costs of installation and of getting the operations under way, which are high for punched-card and electronic equipment. The table can be easily modified to take into account also rented equipment and office space.

3. — The problem of optimum programming is to keep discounted costs to a minimum in the four-year period, satisfying all the — present and forecasted — requirements and conditions stated by the table, including the condition that no level of activity may be negative. Even in the framework of the present oversimplified example the problem is rather complicated and, if it is not trivial, i.e. if the data are not such as to yield a solution by simple inspection, a mathematical method is required for its solution. The solution will determine the level of each column, representing an activity of the bank, with the exception of the first four columns that are fixed. The level of each activity is expressed by a variable $x^{(t)}$ on top of the column, which multiplies each input coefficient in the same column; the same variable taken at the previous interval, $x^{(t-1)}$, multiplies each output coefficient of that column, because the output of last year is the input of this year. Supposing, for instance, that in the optimum solution only the second process is applied to

checking accounts in the second year of the plan, the level of this activity will be given by $x_4^{(t)} = x_4^{(2)} = 89{,}000$. That is, the bank will have a capacity of handling checking accounts for $ 890 million; it will use 2.68 x 89 office machines, 0.0033 x 89 sets of punched-card equipment; 0.33 x 89 junior executives will be appointed to this activity, etc. The salaries paid to employees working on checking accounts will total $ 37,000 x 89, in this second year. The reader with only a general interest in this paper may omit the following technical discussion in fine print.

Mathematically, this problem is the minimization of a linear function of non-negative variables subject to a system of linear difference equations. It can be proved that there is a unique, finite solution. Denoting each column of the input coefficients in the table — leaving out the row of costs — by *a* and each column of the output coefficients by *b*, with the same subscript as *x* on top of the column, the problem is written in compact matrix notation,

$$\Sigma_t e^{-it} (9.2\, x_1^{(t)} + 2.9\, x_2^{(t)} + \ldots + 0.03\, x_{10}^{(t)} + \ldots - 0.03 x_{28}^{(t)}) -$$

$$- e^{-i4} \Sigma_j c_j x_j^{(4)} = \text{Min. subject to the conditions}$$

$$a_0^{(t)} + a_1 x_1^{(t)} + \ldots + a_{28} x_{28}^{(t)} = b_1 x_1^{(t-1)} + \ldots + b_{28} x_{28}^{(t-1)}$$

$$\text{and } x_j^{(t)} \geqq 0 \qquad (j = 1, 2, \ldots 28)\ (t = 1, 2, 3, 4)$$

Where *i* is the rate of discount, $\Sigma\, c_j x_j^{(4)}$ denotes the total costs of buildings and equipment at the end of the planning period — which are a linear function of the activity levels in the last interval of time — $a_0^{(t)}$, for $t=1$, is the first column of the table; for $t=2$, is the second column, etc.; a_1 is the column of the coefficients (— 1,000, 0, 0, 5, 0.0011, 0, 1.81, 16.41, 0.73, 0); b_1 is the column of the coefficients (0, 0, 0, 4.43, 0.0010, 0, 1.60, 13.19, 0.69, 0); etc. The equations state that the output of each function of the bank and factors of production in any interval of time is equal to the input in the next interval plus a constant given by the columns of requirements in the table; and expenditure on new equipment and new buildings must not exceed available funds in any one interval.

Some factors, such as punched-card equipment and electronic computers, have large undivisible units. However, the solution will in general call for fractions of such units. The best way of dealing with this complication is to explore the region in the proximity of the solution at the points marked by all possible combinations of the integers of these factors, which are the nearest to the fractional values determined by the solution. For instance, if the solution quantity of complete sets of punched-card equipment is 2.6, the total costs should be compared assuming we use alternatively two and three sets, while varying levels of activities so as to satisfy all requirements. If the solution determines also a quantity of 0.73 electronic computers, total costs should be compared for the four combinations of two or three sets of punched-card equipment on one hand, and zero or one electronic computer on the other. In general, these combinations to be explored are 2^n, if there are *n* factors with large units. Of course, factors with a large number of undivisible units, such as clerks, can be rounded to the next integer without any significant loss of efficiency.

Geometrically, the conditions of the problem — linear inequalities — define a polyhedron in hyperspace; one vertex of the polyhedron gives minimum costs. Efficient methods of computation aim at following the shortest path to this vertex, moving along the edges of the polyhedron. When factors with large undivisible units are present, the solution will move away from the vertex of minimum costs to a point on the edge of this vertex, which insures minimum costs under the further condition that rules out fractions of these factors.

4. — A general method of solution involving a relatively small amount of computations was devised by Dantzig and is known as the simplex method. The computations are programmed in a routine form with built-in checks. Let $x^{(t)}$ be the vector of levels of activities $x_j^{(t)}$; *A* be the matrix of the a_j-vectors; and *B* be the matrix of the b_j-vectors. The system of linear difference equations may be written

$$\begin{array}{lllll}
A x^{(1)} & & & & = - a^{(1)} \\
- B x^{(1)} & + A x^{(2)} & & & = - a^{(2)} \\
 & - B x^{(2)} & + A x^{(3)} & & = - a^{(3)} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
 & & & - B x^{(T-1)} + A x^{(T)} & = - a^{(T)}
\end{array}$$

where *T* is the last interval of time in the planning period. In our example the A- and B-matrices are of order 10 x 28, but in any practical application they will be larger, maybe of order 15 x 40. Assuming planning extends over a period of six years, the matrix of the above expression will then be 90 x 240 or even larger. The solution by the simple method for a matrix of such size involves a large amount of computations, even considering that these are much simplified by the particular structure of the matrix, with submatrices only along the diagonal.

The computations would probably require over 2,000 man-hours, if carried out with the only aid of a desk computer. It is roughly estimated that the same work could be performed by the newest large electronic digital computers in about four hours. At least one American company, which sells the services of these computers on a time basis, has already set up the instructions to be given the machine for the solution of linear-programming problems and could find the solution to the present problem for a cost of about $ 2,000.00, without the long delay and additional expenses involved in preparatory work for problems never before tackled by the electronic computer. If the data of the problem do not vary much up to the time when the plan is revised, the previous work could be utilized to cut down the number of computations. Then, maybe a desk computer would be adequate.

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