

A search for distinctive features of demand-led growth models

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1. Introduction

This paper aims at a critical and constructive assessment of some past and recent efforts to extend Keynes's principle of effective demand to the long period and to the theory of economic growth. A criticism is addressed to a certain causal interpretation of the demand-led growth models and to the notion of normal capacity utilisation adopted in such models. A positive argument tries to achieve a consistent characterisation of the same type of models by stressing certain conditions that allow the autonomous changes in aggregate demand to become *effective (actual)*. The paper is divided into two parts. The first part deals with two main arguments as an extended introduction to the rest of the paper: 1) a basic property of the models of compound (geometric) accumulation and growth, and 2) a pseudo demand-led growth model derived by opening Harrod's model through the inclusion of autonomous demand. The second part presents 1) a generalised notion of normal capacity utilisation and the idea of normal prices without normal quantities; 2) an investment constrained growth model; and 3) suggests an extension of normal capacity utilisation to the financial sector.

The paper is organised into the following sections. Section 2 deals with a property of the compound rate of growth already noticed in the 1950s by Solow and Samuelson in their stability analysis of linear disaggregated models and resurrected in recent debate. Section 3 emphasises some basic non-Keynesian features of Harrod's model when

* Email: sergio.parrinello@uniroma1.it. This paper is a revised version of the working paper published as *Centro Sraffa Working Paper*, n. 2, May 2014. A first draft was presented as a keynote lecture given by the author at *Edward J. Nell's Retirement Conference*, 3 May 2014, The New School, New York. This final version has especially benefited from the insightful comments of a referee.

it is augmented by an autonomous aggregate demand. In such a context, an increase of the foreign demand for exports brings about a negative impact on the equilibrium rate of growth, despite the assumption of an unlimited supply of labour. A numerical example (section 4) serves to describe the impact of autonomous demand on moving equilibrium and the distinction between pseudo, hypothetical and effective multipliers related to virtual and effective changes in demand (sections 5 and 6). The rest of the paper is concerned with some difficulties when reconciling the leading role of demand with normal values and distribution without falling into the trap of a steady growth where such reconciliation seems to fade out. A sequence of related arguments deals with a simple investment-constrained growth model with variable capacity utilisation (sections 7 and 8), the balance-of-payments constrained growth model (section 9), the distinction between demand-led and closed growth models (section 10) and a generalised notion of normal capacity utilisation (sections 11 and 12). The different threads of reasoning mentioned above lead to the conclusion (section 13) that a model of growth confined to the *real* economy is not adequate enough to reconcile a theory of demand-led growth and the theory of normal values.¹ The extension of the *effectiveness* of demand from the short term to the long term should cover two possible sources of flexibility: a *range* of utilisation of productive capacity and a range of access to financial capacity, which cannot be reduced to the mere assumption of endogenous money.

2. A property of compound growth

In their analysis of stability applied to disaggregated models, Solow and Samuelson (1953) have stressed the following property of compound accumulation:

¹ This conclusion shares similar persuasions held by Foley (1986), Foley and Michl (1999), Nell (1998) and Duménil and Lévy (2012).

“[i]n the case where the system is expanding geometrically, one unit of output lost in the distant past could have been the progenitor of an ever-growing quantity of output lost since that time. This capacity is clear if, as suggested earlier, we think of $X_i(t)$ as being the size of a population in the i th age group. Now suppose one mother had been subtracted from the population many generations ago. If the population is expanding geometrically in the Malthusian manner, the number of potential descendants would also be increasing approximately geometrically. The loss in potential population attributable to the past disturbance would be increasing, not decreasing. The steady-growth solution cannot be stable in the absolute sense that changes in initial conditions have effects ultimately damping to zero. What we might expect, however, is that the equilibrium *relative* age distribution might tend to reestablish itself; that the population might tend to resume its geometrical expansion at the rate λ ” (Solow and Samuelson, 1953, pp. 418-419).

This property of compound accumulation suggests the following remarks. Firstly, a small change in the capacity utilisation in one period is a progenitor of cumulated effects in the next periods both in terms of output and capacity expansion or contraction. Garegnani (1992) has put forward this argument to claim that the potential multiplier effect of a change in demand in the long run can be much higher than in the short run and the independence of normal prices and income distribution from the changes in the autonomous demand can hold in the long period, contrary to Kaldor’s earlier theory of growth with his assumption of full employment. This argument will be reconsidered in section 12.

Secondly, the property of compound accumulation sheds doubt on the relevance of the asymptotic, $t \rightarrow \infty$, stability of an equilibrium proved by means of difference-differential equations. It is not only the lack of persistence of the structure of the model at issue, but the fact that a recursive solution, starting from given initial conditions and moving ahead over an indefinite time horizon, may amplify any small error of specification or the omission of an interfering factor at a point in time, through a process of compound accumulation of the same error or omission. This possibility suggests that, at least for the purpose of a policy oriented general theory, the explanatory scope of a dynamic growth model should be confined to the direction of movement of the economy over a finite number of periods.

Thirdly, it has been suggested, in particular by Kaldor in his early writings, that a positive excess of the natural rate of growth over the warranted rate of growth, and a corresponding compound growth of unemployment, cannot persist indefinitely and therefore for a long-period analysis we should rely on full employment growth models. Similarly, Thirlwall ([1979] 2011) has claimed that a persistent geometric growth of the deficit of the balance of payments is not sustainable and therefore balance-of-payments constrained models are better descriptions of a growing economy than those that admit such unlimited imbalance. The property of compound accumulation, combined with the second remark, suggests that the choice of models, based on the real world evidence of the impossibility of unlimited unbalances, should be reconsidered if the growth models are bounded in their time-range.

3. Harrod's equation with autonomous demand: a pseudo demand-led growth model

Let us start from Harrod's equation,

$$g = \frac{s}{v} \quad (1)$$

where s is the proportion of savings out of total income; g is the *rate* of growth;² and v is the capital/output ratio.

Equation (1) derives from the equilibrium equation of the product market $sY_t = v(Y_t - Y_{t-1})$ where the propensity to save is subject to $0 < s < 1$ and Y_t is the total output of period t . Let us interpret the equation $I_t = K_t - K_{t-1} = v(Y_t - Y_{t-1})$ as a simple accelerator mechanism, where v is the *desired* capital/output ratio, $(Y_t - Y_{t-1})$ is the expected *and* realised change in national income; and the capital depreciation is null.³ Equation (1)

² "Change" and "rate of change" in the text mean "rate of change" and "proportional rate of change" respectively.

³ We assume a one-commodity model where the same commodity can be used as a means of production not subject to depreciation or for consumption. The model of an open

corresponds to the steady growth conditions $g = g_k = g_s = g_y$, where g_k , g_s and g_y denote the rates of growth of capital, saving and output, and g is, in Harrod's words, the *warranted* rate of growth.

This model seems to be a natural extension of Keynes' short period analysis to the long period. In the short period the *level* of national income is the adjustment variable between savings and investment; in the long period this role is instead played by the *rate of change* of national income. In both cases the supply of labour is assumed unlimited at a given conventional wage rate and the saving function appears distinct from the investment function; a departure from Say's Law. However, the attribution of a driving role to demand is unsustainable if the coefficient v in the investment equation is interpreted as a *fixed desired and actual* capital/output ratio, associated with the assumption of perfect foresight.⁴ This assertion becomes more transparent if we open the model by way of introducing autonomous demand.

The emphasis of Harrod and subsequent commentators has been centred on the problem of the relative⁵ instability of a moving equilibrium path determined by equation (1) in the *closed* model, where no room is left to the autonomous components of aggregate demand. This instability is attributed to the rigidity of the accelerator mechanism, such that any initial deviation of the capital/output ratio from the equilibrium ratio $\frac{K_0}{Y_0} = v$ brings about a cumulative deviation of the rate of growth from its equilibrium (warranted) value. The focus on relative stability has obscured a more simple property of the open model. The autonomous components of aggregate demand typically consist in private or public expenditure for consumption or investment or in exports. We shall confine the analysis to a model in which autonomous demand does not

economy will deal with a composite commodity made of domestic and imported goods in fixed proportions.

⁴ The assumption of perfect foresight in a deterministic model is equivalent to the assumption of rational expectations.

⁵ Relative stability means the stability of the warranted *rate* of growth as distinct from the stability of the *absolute* growth of income (see Solow and Samuelson, 1953). See section 2.

directly affect the productive capacity of the economy:⁶ the model of an open economy with exogenous exports X_t and endogenous imports M_t . The usual assumptions and conventions are adopted. X_t and M_t are measured in physical units. Imports are proportional to the national product: $M_t = mY_t$ where $0 < m < 1$; the terms of trade are fixed and the units of measure of the traded quantities are chosen so that the price of imports in terms of exports (the terms of trade) is equal to one. In the absence of international capital flows, the difference $X_t - M_t$ is the balance of payments in real terms. We neglect the public sector and therefore m is the only leakage coefficient out of the national income circuit, besides the propensity to save, s . The macroeconomic equilibrium condition is:

$$sY_t + mY_t = v(Y_t - Y_{t-1}) + X_t \quad (2)$$

Equation (1) is generalised accordingly:

$$g_t = \frac{s + m - h_t}{v} \quad \text{with} \quad h_t \equiv \frac{X_t}{Y_t} \quad (3)$$

This formalisation resumes an old generalisation⁷ of Harrod's equation, in which the role of government expenditure and taxation is similar to that of exports and imports in the present model.⁸ If we set $h_t = m$ in equation (3), the balance of payments constraint $X_t - M_t = 0$ is satisfied in each period and the equilibrium rate of growth is determined, like in a closed economy without autonomous demand, by equation (1). Instead, exports in the present model are exogenous and the exports to income ratio, h_t , is endogenous. In both cases an increase of h_t is associated with a *decrease* in the equilibrium rate of growth, g_t . A crucial

⁶ We are assuming a case of demand whose change does not bring about a change in the productive capacity of the economy, as would instead be the case with most autonomous investment. The fact that we deal with a change in an additive component of the aggregate demand instead of a shift of its aggregate function does not seem relevant for the sake of our argument about the *effectiveness* of the change itself.

⁷ See Baumol (1963), chapter 4.

⁸ The same equation has recently been reconsidered by Commendatore *et al.* (2003), who tried a reconstruction of Harrod's ideas spread over different writings (Harrod, 1939; 1948; 1964; 1973). However, in their exegesis the adoption of a special unit of measure of quantities seems to imply that h_t , instead of X , is an exogenous variable.

condition adopted in this open Harrodian model is the equality between the desired and the actual capital/output ratio in each period: $v = \frac{K_t}{Y_t}$, $t = 0, 1, 2 \dots$. Some elementary tools of dynamic analysis are recalled in the appendix and will be applied to the following numerical examples.

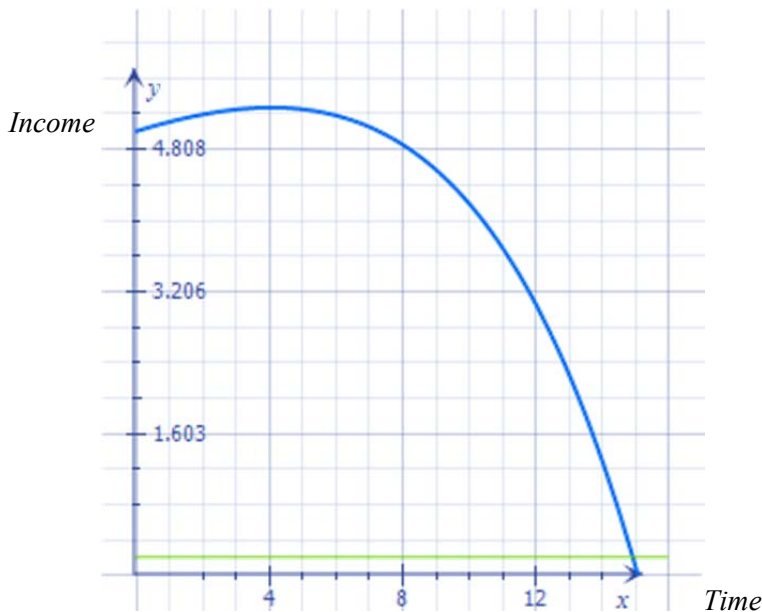
4. A numerical example of moving equilibrium

Even without resuming the instability of the warranted rate of growth in the closed model, the moving *equilibrium* may go off course in the presence of an autonomous component of aggregate demand, exports in our case. Figure 1 describes the model with the parameters $s + m = 0.3$, $v = 5.0$ and $X_t = 1$. The corresponding general solution is $Y(t) = \left(Y_0 - \frac{1}{0.3}\right) \left(\frac{50}{47}\right)^t + \frac{1}{0.3}$ where Y_0 is the initial level of national income. The figure shows that any initial slight deviation of Y_0 from the stationary equilibrium $Y^* = \frac{1}{s+m} = 10/3$ (the horizontal red line) brings about a cumulative decay or growth of Y_t .

A deviation below, with $Y_0 = 3$, will lead the blue line down to the green floor, where $Y_t = X/v = 1/5$. A deviation above, with $Y_0 = 4$, instead leads to the grey line that describes a growth path at a variable positive rate which *tends* to a steady warranted rate of growth, because the influence of the constant foreign demand X becomes more and more negligible relative to the endogenous growth mechanism.

Figure 1 – *Moving equilibrium with constant autonomous demand*

This conclusion can be generalised assuming that the volume of exports, instead of being constant, follows an exponential trend $X_t = X_0 b^t$. The curve in figure 2 describes the case in which the parameters are those of the previous example, with the new parameters $b = 5/4$ and the initial condition $Y_0 = 5$. The curve shows that the negative influence of the exogenous trend of exports is initially dominated by the endogenous growth and then any such influence is reversed leading to the decay of the national product down to the green line limit.

Figure 2 – *Moving equilibrium with exponential autonomous demand*

The examples suggest that the equilibrium dynamics are the combined result of two components: the endogenous and the exogenous or autonomous dynamics. Depending on the values of the parameters, one of the two components can dominate the variable rate of growth, relative to the other. As argued in section 2, a special focus should be addressed to the initial periods, instead of the asymptotic tendency to either an exogenous or endogenous constant rate of growth.

The examples show that exports in the model play either a negative or a negligible role in the growth performance of the economy, as if the benefits highlighted by Keynes through his parable of digging holes into the ground would become damaging in the equilibrium dynamics of Harrod's model with autonomous demand. Despite the fact that the model assumes an unlimited supply of labour, an initial equilibrium path can change only through a *trade-off* between the autonomous demand and the endogenous investment. This result is not surprising: it

depends on the lack of excess capacity of the capital stock and on the assumption of perfect foresight (the accelerator coefficient v is assumed identical to the actual capital-output ratio) that make the investment function $I_t = v(Y_t - Y_{t-1})$ implicit in the other functions, instead of being the result of the independent decisions of the entrepreneurs. This trade-off contrasts with the interpretation of Keynes's short period equilibrium, when the activity level of the economy has reached the maximum capacity: in this case an increase of the autonomous demand brings about an inflationary pressure without necessarily having a substitution effect.

5. Hypothetical and effective multipliers

The question of the changes in demand that may or may not become effective can be introduced by a comparison of different notions of multipliers.

In figure 1 the comparison between the two coloured curves is a comparison of *different* economies, each in its own dynamic equilibrium. They do not represent what will happen to an economy if at time $t = 0$ its exports change, *ceteris paribus*. The *ceteris paribus* clause cannot be applied because the initial difference ΔX has to be associated with the simultaneous differences $\Delta Y = \frac{1}{s+m}$ and $\Delta K = \frac{v}{s+m}\Delta X$. Let us write the macro equilibrium condition (2) in the form:

$$Y_t = \frac{1}{s-v+m}(X_t - vY_{t-1}) \quad (4)$$

and derive

$$\frac{\Delta Y}{\Delta X} = \frac{1}{s-v+m} \quad (5)$$

The fraction $\frac{1}{s-v+m}$ looks like a short period foreign trade multiplier, but it is a *pseudo multiplier*, independently of the fact that it may turn out to equal less than 1. In fact it represents a one-period impact of the change in exports on the change in income. Next, subject to the

stationary equilibrium condition $Y_t = Y_{t-1}$, (see the appendix), the equation (5) becomes

$$\frac{\Delta Y}{\Delta X} = \frac{1}{s+m} \quad (6)$$

The fraction $\frac{1}{s+m}$ looks like the long period foreign trade multiplier. If the changes are attributed to the same economy and the change in exports is said to *cause* a multiplicative change in the national income of the same economy, this is a counterfactual assertion where the initial Δ -occurrence would be like a *miracle* in the sense adopted by philosopher David Lewis (1973) and (6) might be called a *miracle* multiplier. The expression (6) differs from the short period multiplier in Kahn and Keynes. The difference does not rest only on a different autonomous demand and the distinction between comparative dynamics and comparative static. In Kahn and Keynes the autonomous change ΔX is supposed to be *effective* and the induced change in Y becomes actual through a feasible change in the rate of utilisation of the existing capacity and an implicit soft budget constraint attributed to the investors. Instead the changes in equation (6) are *purely hypothetical*.

6. A digression: effective demand and the “effectual” supply of exhaustible natural resources

Let us exploit once more the simple model of compound accumulation of an open economy. Imagine that figure 1 is extended to the left so that a backward sequence of periods $t = -1, -2, \dots$ is also represented on the time axis. Suppose that the economy is in equilibrium before period $t = 0$, with the actual capital/output ratio equal to the desired ratio. Normal and abnormal fluctuations and normal and abnormal accidents impinge both on the demand and the supply side of the economy. Suppose that something *new* happens in $t = 0$: an autonomous change either in demand or in supply. For example, the opening of a foreign market (a case of Schumpeterian innovation) is an

autonomous increase in the foreign demand for exports X . Conversely, the closure of a foreign market is an autonomous change in the foreign supply of imports that is transformed from an unlimited amount to a sort of rationing. In both cases we can calculate the effect of the hypothetical change by counterfactual reasoning, but we wonder what are the conditions under which the hypothetical changes become *effective* for the economy instead of being only potentially so.

We have a ready answer in the case of an increase in exports. Suppose that before $t = 0$ the producers had foreseen that some change might happen in the future, for example a normal seasonal fluctuation or an accident, and some margin of productive capacity exists in period $t = 0$ to satisfy a profitable change in demand. If the autonomous increase of exports is not too large it can become an actual change ΔX_0 without displacing other demand components and therefore it becomes a net increase in national income in period $t = 0$, ΔY_0 , followed by a multiplier effect. The actual autonomous change can be either an impulse that occurs once and for all and with negligible effects on investment; or it can be persistent and affect the investment plans.

By contrast, we do not have a ready answer in the case of a sudden limitation of foreign supply. The structure of the model has to be changed. To avoid this limitation bringing a collapse of the economy for the lack of a necessary imported commodity, let us suppose that the economy possesses a backstop technique that, when needed, may be activated to provide the imported commodity, although at a higher cost. In this case the autonomous change in the foreign market becomes effective through such a substitution effect. However, we cannot at the same time deny that the growth performance of the economy is *led* (constrained) by the supply of imports and admit that it is demand-led in the case of the expansion of exports. There is no decisive reason for assuming a priori that, of the two potential autonomous changes, one in exports and the other in imports, only the former – on the demand side – is effective whereas the latter – on the supply side – remains potential. Another example, even more challenging for the claim of generality attributed to the demand-led models, is offered by what elsewhere I called the *effectual supply* of exhaustible natural resources in the context

of Sraffa's price equations (Parrinello, 2004). Instead of the total *stock* of the resource, say oil in the ground, it is its *flow* of supply that is one of the determinants of the normal prices. This flow depends on the long-term expectations of the owners of oil deposits whose decisions regarding extraction may conform to the Hotelling rule, like the investors in Keynes are supposed to conform to the rule of the marginal efficiency of investment. Once more there is no *logical* reason to deny that an abrupt change in the expectations of the owners of the resource is less effective at causing a decrease in the supply of oil than a change in the expectations of the investors.

We are still looking for some consistent characterisation of demand-led growth models, instead of agreeing with the provocative title of Duménil and Lévy's (1999) article "Being Keynesian in the short term and classical in the long term".

7. An investment constrained growth model with variable capacity utilisation

Not surprisingly we have not found a satisfactory notion of demand-led growth in Harrod's model. Let us cover another route to escape from the *cage* of the steady growth models that satisfy the dual condition: 1) a *constant* and *uniform* rate of growth of quantities, and 2) *constant* relative prices with a *uniform and constant* rate of profit. A step in this direction would be to recognise different kinds of flexibility of an economy that undergoes foreseen and random deviations from a steady growth. A certain flexibility in a model with fixed prices is provided by inventories of storable goods and money, incomplete contracts, bank overdrafts and lines of credit and margins of excess productive capacity. In the recent debate an overwhelming emphasis has been put on the latter, perhaps with undue understatement of the other elements of flexibility. This flexibility can be operative *within certain limits* without relaxing the assumption of fixed prices and, in particular, of a fixed rate of interest underlying the desired capital/output ratio. For the sake of argument we

shall change the initial assumptions and move to a growth model of a closed economy with variable capacity utilisation.

Let us assume a closed economy and the classical saving function: a positive proportion of saving out of profits, s_π , is given, and the propensity, out of wages is zero. The overall propensity to save s is equal to s_π multiplied by the ratio of total profits, to save Π out of the national

income Y . The initial Harrod equation (1) becomes: $g = \frac{s}{v} = \frac{s_\pi \Pi}{Y} \frac{Y}{K} = s_\pi r$

where r is the rate of profit.^{9,10} Let λ and v denote *normal* technical coefficients: respectively the amount of labour and capital per unit of output when the productive capacity is utilised at a *normal* rate. Let us assume that a change in capacity utilisation affects the productivity of labour and capital in the same proportion,¹¹ u , and define the critical rates of capacity utilisation by: $u = 1$: normal capacity, $0 < u < 1$: excess capacity, $1 < u < u_m$: capacity overutilisation, where u_m is a technical maximum. v is the normal capital/output ratio corresponding to $u = 1$ and, $Y = \frac{K}{v}$ is the normal output at a given K , $Y_a = uY$ is the actual output, and $Y_m = u_m Y$ is the full capacity output.

Let w and r denote the real wage and the (net and gross) rate of profit. The general price equation is

$$w\lambda + rv = u \quad (7)$$

⁹ Since the depreciation rate of capital is null, r denotes both the gross and net rate of profits.

¹⁰ The equation $g = s_\pi r$ is the received Cambridge equation, that can be differently interpreted, but not in a certain way. In particular, both g and r can be either *actual/ex-post* or *normal/theoretical* rates of growth and profits respectively, but not a mixture of such attributes. See Garegnani (1992).

¹¹ The same assumption is adopted in Foley and Michl (1999). This section is inspired by chapter 10 of their volume. However, an important difference remains, and it is due to the fact that the classical saving function in the present work is formulated in the usual way, i.e. savings are a given proportion of the total profit flow. Instead, Foley and Michl assume that savings are a given proportion of the total wealth of the capitalists. Such a stock/flow relation seems to be the correct formulation to be adopted in the model extended to the financial sector of the economy (see Foley, 1986).

and $w\lambda + rv = 1$ in the normal state. The desired rate of capital accumulation depends on long term expectations. Instead of the dependence on the expected growth rate of output related to the idea of the accelerator, let us assume, according to the intuitive idea of the *animal spirits* of the entrepreneurs suggested by Joan Robinson, that g_k is an exogenous variable: $g_k = \bar{g}_k$

The equilibrium condition between desired investment and saving is the Cambridge equation:

$$\bar{g}_k = s_\pi r \quad (8)$$

Suppose that the wage rate is fixed at a conventional level:

$$w = \bar{w} \quad (9)$$

The solution to equations (7), (8) and (9) is:

$$r = \frac{\bar{g}_k}{s_\pi} \quad (10)$$

$$u = \bar{w}\lambda + \frac{v}{s_\pi} \bar{g}_k \quad (11)$$

By substitution, the actual national income associated with a given K is

$$Y_a = K \left(\frac{\bar{w}\lambda}{v} + \frac{\bar{g}_k}{s_\pi} \right) \quad (12)$$

and the actual consumption

$$C = Y_a - \bar{g}_k K = k \left[\frac{\bar{w}\lambda}{v} + \bar{g}_k \left(\frac{1 - s_\pi}{s_\pi} \right) \right] \quad (13)$$

The solution (10), (11) and (12), (13) recovers the Keynesian features that have been lost in the initial model of moving equilibrium. In fact, there is no trade off between the *actual* rate of profit and the *actual* real wage on the side of prices, and no trade off between consumption and investment on the side of quantities. In particular, if \bar{g}_k increases,

the actual rate of profit and consumption per capita increase, whereas the conventional wage rate remains fixed. However, how general is such a Keynesian result for a growth theory combined with a long period theory of distribution? As a matter of fact the Keynesian features mentioned above belong to the short period and do not subvert the inverse relation between the normal rate of profit and the real wage, which in this simple model is imposed by the price equation (7), subject to the norm $u = 1$.

8. The reduction of the model to a special case

We cannot pretend to have achieved a dynamic growth theory by means of a sequence of self-contained short period equilibria. The previous investment constrained growth model has indeed such a shortcoming, despite the possibility of determining a sequence of capital stocks K through a recursive solution. Depending on the specification of the investment function, instead of the assumption of an exogenous rate of accumulation $g_k = \overline{g_k}$, the Keynesian properties mentioned above may or may not be preserved. We shall stress two difficulties behind the choice of the investment function.

Firstly, a preliminary problem is the definition, measurement and role of *normal* output and of a normal degree of capacity utilisation, represented in our simple model by the value of the variable u , and extensively discussed in the recent debate about different post-Keynesian theories of growth and income distribution. The appraisal of this problem is postponed to section 12.

Secondly, some relation should be established between the expectations of the entrepreneurs underlying their investment plans and the normal state of the economy that in the model corresponds to the value $u = 1$. A critical limit exists to too strict a relation, beyond which the model is brought back to a steady growth model. Consider two simple investment functions of expected variables, which might be combined into one investment function along with the rate of capacity utilisation to form a more general model.

The intuitive idea of the accelerator can be formalised by the assumption that the desired rate of accumulation g_k is a positive proportion σ of the *expected* rate of growth of national income g_Y^e :

$$g_k = \sigma g_Y^e \quad (14)$$

Similarly, the intuitive idea of the *animal spirits* suggests that g_k is proportional to the expected rate of profit r^e :

$$g_k = \eta r^e \quad (15)^{12}$$

Where η is a positive volatile parameter. (14) and (15) can be interpreted as distinct components of an investment function. Under the assumption of perfect foresight the expected rates g_Y^e, r^e are equal to the rates g_k and r determined in the *normal* state of capacity utilisation, $u = 1$.

Hence $r^e = \frac{1 - \bar{w}\lambda}{v} = r$, and $\sigma = 1$ and $\eta = s_{\pi}$. An interpretation of the

Cambridge equation follows in which the rate of growth, instead of being affected by the independent decisions of the entrepreneurs, is determined by the proportion of saving out of profits and the normal rate of profit: this means a return to a steady growth model where the role of the effective demand fades out.

Two complementary routes have been explored to avoid the assumption of steady growth and at the same time preserve the extension of the principle of effective demand to the long period.

One consists in assuming that some component of aggregate demand has an autonomous trend. In the next section we shall deal with an open economy subject to a balanced trade constraint and autonomous exports. The second route maintains the relation between the normal and expected

¹² The same form of the proportional investment function (22) has been adopted in Foley and Michl (1999) to formalise in a simple way the role of the animal spirits of the entrepreneurs, subject to the assumption, where the current rate r may be different from the expected rate r^e , Marglin and Bhaduri (1990) adopt a more complex investment function where ηr is combined with other explanatory variables.

rate of profits, $r^e = \frac{1 - \bar{w}\lambda}{v}$, but denies that the normal prices have to be associated with normal quantities conceived as steadily growing quantities.

9. Growth with balanced foreign trade

A preliminary notion of the foreign trade multiplier with balanced trade $X = mY$ can be traced back to Harrod¹³ and takes the form:

$$\frac{\Delta Y}{\Delta X} = \frac{1}{m} \quad (16)$$

Harrod was aware that equation (16) is a special case of a more general short period multiplier associated with other autonomous components of demand (consumption, investment and public expenditure). Palumbo (2009; 2011) observes that the same form of the multiplier $\frac{1}{m}$ that is found in Kaldor (1970) does not neglect such additional items in demand, but would be derived from Hicks's (1960) super-multiplier $\frac{1}{s - i + m}$, in which $i \equiv I/Y = vg$ is the propensity to invest induced via the accelerator mechanism. Equation (16) obtains in the special case $s = i$ independently from the level of income, which is a strong departure from Keynes's principle. Furthermore the so-called Thirlwall's Law (Thirlwall, 1979)¹⁴ derives from the equation of balanced trade $X = mY$ and states that a proportional change in exports *drives* a proportional change in income. Since the income elasticity of imports is equal to one, and in the absence of international capital movements, the law in continuous time can simply be written:

¹³ See Harrod (1957), p. 120.

¹⁴ See Thirlwall (2011), McCombie (2011) and Setterfield (2011).

$$\frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} \quad (17)$$

with a dot over a variable meaning the time derivative of the variable.

Different interpretations and derivations of (16) and (17) have been suggested following the evolution of the ideas of Harrod and Kaldor and the development of balance-of-payments constrained growth models. We shall short-circuit such useful exegetical work and confine ourselves to a few analytical considerations.

In principle the condition of macroeconomic equilibrium of the product market can be written as a sum of n excess demands or ‘balance-of-payments’ functions $[D_t(Y) - S_t(Y)]_j$ where the aggregate income, Y , is the only independent variable:

$$[D_{1t}(Y) - S_{1t}(Y)] + \dots + [D_{nt}(Y) - S_{nt}(Y)] = 0 \quad (18)$$

We may assume that the sum of all n excess demands except the first one is always null independently of the level of Y and fix the first demand as exogenous variable, $D_{1t}(Y) = \bar{D}_{1t}$; then derive a multiplier for the given \bar{D}_{1t} and call a growth model which adopts such assumptions a D_1 -led growth model or $[D_1(Y) - S_1(Y)]$ -constraint-growth model.¹⁵ In particular, we may pick the foreign balance of payments $X_t - mY_t$ as such excess demand and derive an export-led growth model. A critical issue, which cannot be settled without empirical evidence, arises if we pretend that such a type of demand-led or constrained growth modelling represents a general theory instead of a special case.

¹⁵ Unfortunately the twin connotation of demand-led and demand-constrained growth reflects the difficulties of the meaning of causation in philosophy: the counterfactual dependence of the effect from the cause as distinct from the primitive notion of production of the effect by the cause; and the general problem of the selection of a cause from the background conditions within the multiplicity of causal conditions. See Parrinello (2013a; 2013b).

10. Demand-led growth and closed models

Relative to the equation of macroeconomic equilibrium (18), we can distinguish two general types of models: 1) models in which some or all addends $[D_i(Y) - S_i(Y)]$ are not null in equilibrium (for example the government budget or the trade balance) and have their own autonomous demand; 2) models in which all components $D_i(Y)$ and $S_i(Y)$ are endogenous variables. The closed models of accumulation, in which *no* demand or supply is autonomous, belong to type 2. Marx's scheme of accumulation and the models of Harrod, von Neumann, Leontief (his closed input-output model), Duesenberry (1958) and Smithies (1957) are examples of closed models of endogenous growth with or without cycles. It might be tempting to call them *supply-led* or supply-constrained growth models by exclusion: they do not possess the demand-led feature due to the autonomous demand. However this does not seem the correct interpretation of closed growth models. In particular it is true that growth at the warranted rate in Harrod's model is not *driven* by a limited supply of labour nor constrained by natural resources and it might seem to be driven or constrained by the 'supply of savings'. Yet, suppose we follow Kaldor and assume that the propensity to save out of profits, s_π , is mainly governed by large corporations who decide the amount of non-distributed profits and how to allocate them for investment. In this case the saving coefficient s_π simultaneously represents the propensity to save *and* to invest and only our parsimony of language induces us to choose one (the former) of the two terms instead of a third one. The only safe intuitive connotation of those closed models is that they are models of growth led by the search for profits and constrained by the technical conditions of the economy. However, 'led' and 'constrained' do not mean 'caused' within the context of steady growth, because such a growth path is like an inertial state that is uncaused in its continuation, although its initiation may have been caused. The same safe feature would apply to the demand-led or demand-constrained growth models in so far as they are driven by the search for profits and constrained in a similar way, but we are looking for a distinctive driving factor.

11. Different exits to avoid the steady growth: normal prices without normal quantities?

According to certain interpretations of the long period states of the economy adopted by the classical school and Sraffa, there is an asymmetry between theoretical prices and quantities.¹⁶ It is maintained that the relative speed of adjustment of market prices towards their normal values is high, whereas the stocks of the capital goods (in particular fixed capital) and the capacity utilisation move slowly or do not tend at all towards normal levels in the presence of a multitude of interfering forces.

There is a sense in which such a distinction between price and quantity adjustment is obvious on the basis of the feature of compound growth (see section 2). In fact, any impulse that changes the productive capacity of the economy at a point in time has a cumulative effect in the following periods on the absolute quantities of capital goods, and therefore a steady growth model is inherently unstable relative to those quantities: it is indeed an elementary case of path dependency. The absolute amounts of the capital stocks cannot play the role of normal quantities. If some gravitation exists on the side of quantities, it has to be confined to the proportions among quantities and to their uniform rate of growth. By contrast, in another sense, the distinction between the different speeds of adjustment of relative prices and quantity proportions is not obvious. It is not always clear whether the arguments advanced by the participants in the debate¹⁷ are confined to the problem of gravitation toward normal values and to the sequence of short periods of the

¹⁶ See Palumbo and Trezzini (2003) and Ciccone (2011). Roncaglia (1978) goes even further and argues that normal prices and quantities in such a theoretical context do not represent attractors or centres of gravitation for market magnitudes.

¹⁷ A useful critical overview of the debate on the role of normal quantities and normal capacity utilisation in demand-led growth models can be found in Palumbo and Trezzini (2003). The debate includes, apart from the early contribution by Steindl (1952), Ciccone (1986; 2011), Duménil and Lévy (1999), Garegnani (1992), Kurz (1986; 1990), Lavoie (1992), Park (2000), Palumbo (2009; 2011), Palumbo and Trezzini (2003), Serrano (1995), Semmler (1990), Trezzini (1998) and Vianello (1985).

convergence process or instead are implying an ‘intermediate period’ analysis between the short period and a long period conceived as a fully adjusted long period equilibrium in terms of both normal prices and capital stocks. The following argument tries to clarify the notion of a sort of *intermediate period* that seems suggested by some contributions,¹⁸ albeit without the help of a formalised disaggregated model.

A basic discrimination concerns the assumptions about the pace of technical change and the proportion between gross investment and the total stock of fixed capital. On the basis of the idea of embodied technical progress adopted in the vintage growth models, the adjustment of prices and outputs equally depends on the amount and allocation of gross investment, i.e. on the new capacity installed in the different sectors, compared with the size of the existing stocks of fixed capital inherited from the past. If the annual flow of gross investment is small compared to the total stock of fixed capital, the diffusion of the best technique embodied in the new fixed capital will be relatively slow as well. The annual flows of outputs can adjust relatively quickly towards the proportions consistent with a uniform rate of profit on the supply price of the non-obsolete capital goods by means of a variable degree of utilisation of the total stocks of fixed capital and a corresponding change in the inputs of circulating capital. Instead the proportions among such *total* stocks, including the obsolete machines that will not be produced any more, and the degree of total capacity utilisation would adjust slowly or may even not adjust at all towards normal proportions in the presence of a non-stop flow of supervening new techniques. This picture describes an ‘intermediate’ period position consistent with the emergence of quasi-rents.¹⁹ The Cambridge equation would still hold under the classical saving function, but the rate of profit in this equation should now be re-interpreted. The rate of profit r cannot be a uniform rate of profit on the

¹⁸ See Ciccone (2011). It need hardly be mentioned that the choice of different lengths of period analysis does not mean a choice of calendar time, but that of the givens within a model, suggested by an implicit theorising about different speeds of adjustment of the economic variables.

¹⁹ See the treatment of obsolete machinery in Sraffa (1960), chapter 11, section 91.

supply price of all capital goods, some of which (the obsolete machines) receive at most quasi-rents. However the condition of a general uniform rate of profit may be preserved if we assume that the value of an obsolete machine is not a price of production, but the value of its own quasi-rent capitalised at the rate of profit that applies to the non-obsolete machines. Of course, this is an additional assumption of a stationary state. More importantly the actual income distribution in such an intermediate period does not coincide with the distribution determined by the subsystem of production formed by the best techniques. By contrast, if the annual flow of gross investment is not small compared to the total stock of fixed capital, and granted a correspondence between the normal prices and the choice of the best/cost-minimising techniques, there seems to be no reason why the composition of the total stocks of fixed capital and the market prices should adjust at different speeds. In particular, the intermediate period analysis suggested above would become useless if the economy should implement only circulating capital, without fixed capital accounted as joint-products.

In general, normal prices cannot be completely decoupled from a normal degree of capacity utilisation as a result of cost-minimising choices. Our argument on this issue is a development of some previous reflections of the author (Parrinello, 1990; 1992), and is related to Kurz (1986; 1990) and Lavoie (1992). In the recent literature it is usual to make the following distinctions between the capital/output ratios, K_t/Y_t : the ‘actual’ ratio, ‘the normal’ ratio, the ‘desired’ ratio and the ‘technically minimum/maximum’ ratio.²⁰ The debate is especially focused on the problematic notions of normal quantities and, in particular, on the role of normal capacity utilisation. Both the normal and desired ratios are problematic. Only the merest trace of information concerning normality is conveyed by a single capital/output ratio. The fraction K_t/Y_t can be *desired* in period t but can be hardly said to be *normal*: only a (either deterministic or probabilistic) *distribution* of

²⁰ In the case of a blast furnace, mentioned below, a minimum degree of capacity is also relevant, below which the plant irreversibly goes out of order.

quantities over a certain time interval can possess such an attribute.²¹ This shortcoming is not overcome if the ratio K/Y applies to a time average of ex-post capital/output ratios. The cases of a power plant or a blast furnace can be useful examples of the problem of the definition and measurement of capacity. What does it mean to measure the degree of utilisation of a power plant by a single percentage, say 50%, in terms of output over a certain period of time? The actual capital/output ratio in a certain period, except for some abnormal peak or trough, cannot be judged abnormal to the extent that the investment plans become affected. The choice of the type of power plant depends on the expected distribution of the demand for kilowatts subject to peaks and troughs over some time horizon. Furthermore the correspondence between ‘normal’ and ‘desired’ is problematic.

“[...] the normal degree of capacity utilization is the one that firms have in mind in taking their investment decisions; i.e. the degree that, if attained, would make them – to use Harrod’s famous expression – ‘content with what they are doing’” (Palumbo and Trezzini, 2003, p. 111).

We wonder what the meaning is of the assertion that investors are “content with” a certain actual K_t/Y_t instead of other ratios. If we want a non-circular answer, we can hardly avoid the conclusion that this preferred ratio, expressed by the word ‘contentment’, is the result of a search for higher profits. In the previous example, the choice of the initial plant and of the capacity utilisation, associated with deterministic and random fluctuations in energy consumption, involves indivisibilities and nonlinearities and therefore cannot be formulated like the problem of the choice of linear techniques used for the criticism of the neoclassical theory of value and distribution.²² Despite such complexities, the choice of the degree of capacity utilisation in the long run, as already stressed by Kurz (1986; 1990), cannot be separated from the cost-minimising choice of techniques behind the investment plans.

²¹ See Parrinello (1990; 1992).

²² See Kurz and Salvadori (1995).

12. A corridor (range) of normal degrees of capacity utilisation

In the Marshallian theory of the firm, the plant is given in the short period and variable degrees of capacity utilisation can be chosen at given prices, one of which is assumed to be profit-maximising. In a demand-led macroeconomic growth model the ‘short’ period means not only a given maximum capacity of the economy but also given long term expectations underlying the investment plans.²³ We may assume²⁴ a corridor of normal degrees of capacity utilisation instead of a single value. The degree of capacity utilisation can change within the corridor and the actual income distribution and market prices may change accordingly, because the demand for some inputs (e.g. circulating capital) will be affected through the cost-minimising choices of the producers. However, the normality of such an occurrence means that the long-term expectations of normal prices and the normal rate of profit will not change and therefore the investment plans will persist. This seems to concede the flexibility required for the consistency between a driving autonomous demand and the normal capacity. Notice that a constant corridor of rates of utilisation is consistent with an ever-expanding corridor of *absolute* quantity changes in the presence of compound growth. More importantly, in general the norm cannot be defined by a range of quantity *levels*, but by a *range of normal distributions* (not necessarily in the statistical sense of a bell-shaped distribution) *of capacity utilisations*. Normal accidents should be kept distinct from abnormal accidents²⁵ and embedded in a loose notion of normal output proportions.²⁶ Despite such a generalisation of the notion of steady growth, normal capacity utilisation, conceived as a range of distributions of utilisation rates, cannot be separated from the choice of techniques that is governed by cost-minimising and profit-maximising behaviour. A

²³ An articulate classification of different ‘period’ analyses in terms of the choices between exogenous/endogenous and fulfilled/unfulfilled expectations is offered by Kregel (1976) in his appraisal of the modelling methods of Keynes and the post-Keynesians.

²⁴ See Lavoie (1992, pp. 327-332 and pp. 417-422). See also Dutt (1990, pp. 58-60).

²⁵ See Hannsgen (2013) for a review of types of heterodox shocks.

²⁶ See Parrinello (1990; 1992).

change of capacity utilisation within the normal range does not leave the costs unaffected; still, it preserves the long-term expectations of investment profitability.

The position of the corridor depends on the past and the expected future utilisation. If for example the actual distribution of utilisation rates falls outside the corridor, it is plausible that the investment plans and the normal prices can be affected and the position of the corridor may shift. As already stressed by Nell (1998, pp. 492-493), a one-off change in the level of the autonomous demand should be kept distinct from a persistent change in the rate of growth of demand. More precisely, we should keep distinct *i*) a one-off impulse like the subtraction of a mother from a population in the example of Solow and Samuelson (1953), *ii*) a persistent change in the absolute level of the autonomous demand; *iii*) a persistent change in the rate of growth of the autonomous demand. The changes *i*) and *ii*) can have a cumulative effect on the growing scale of the economy but they may be consistent with a persistent corridor; instead the latter can also affect such a normal range of values, the investment plans and the income distribution.

Combining the different threads of reasoning in this section, the following conclusion may be derived. A problem of realism should be distinguished from a logical problem of over-determination. The example of the power plant and the idea of a corridor of capacity utilisations, with long-term expectations and normal prices not being affected by different distributions within that range, belong to the former. In such a context a constant normal price of energy may not be attributed to a pure physical commodity (like corn, steel, etc.), but to a bundle of services over time and has to conform to the existing social norms; the 'price' written in a power supply contract is an example of this type of pricing. However, the assumption of a range of normal distributions of utilisation does not remove, but instead only shifts to a different level, the main problem of consistency at issue. In the presence of a persistent autonomous change in the aggregate demand, the normal range of capacity utilisation can be trespassed with fixed prices, but in general such utilisation outside that range cannot be an adjustment variable between the rates of growth of savings and investments *and* at the same time the result of a cost

minimising choice of techniques. A sufficiently large change in the demand can be satisfied by the existing fixed capital only through a change in prices and income distribution and possibly leading to a change in long-term expectations and investment plans.²⁷ In fact, the productive capacity of an economy at a point in time can be flexible outside the normal range, provided that some prices are permitted to deviate from the normal long period values and to approach the prices of an intermediate period equilibrium.²⁸

13. A residual characterisation of demand-led growth and conclusion

A residual distinctive characterisation of growth models is to be found in the conditions that make the autonomous changes in demand *effective* (i.e. feasible, actual). Such conditions are mainly related to the workings of financial institutions and are implicit or neglected in the mathematical growth models confined to the real economy.²⁹ Demand-led growth models formulated only in terms of real variables are not well suited to this task. The source of the asymmetric effectiveness of demand versus supply cannot be reduced only to the existence of margins of

²⁷ The following conclusion in Garegnani's essay intends to limit the reach of his own argument and seems to become close to that presented in the text: "it should finally be stressed that the present argument for which a long-period rise in investment needs not alter distribution in order to generate the corresponding savings, should not be taken to deny the possibility – or indeed the likelihood – of interactions between the real wage and the normal rate of profit on the one hand, and the speed of capital accumulation on the other. We have just mentioned, e.g., the possibility that a rise in the incentive to invest and the consequent improvement in the situation of labor employment might result in a rise of the real wage (rather than a fall which we would expect from the First Keynesian Position [Kaldorian]. What is disputed is only the necessity of the particular effect postulated in the First Keynesian Position" (Garegnani, 1992, p. 64, [Kaldorian] added).

²⁸ The latter statement and the notion of intermediate period analysis is expanded by the author in Parrinello (2014).

²⁹ A similar focus seems to be suggested by the distinction between investment *decisions* and investment *spending* in Nell (1998, chapters 10 and 11), and in the assumption of a propensity of saving out of total wealth (including financial wealth), instead of income (see footnote 12).

excess productive capacity and to the assumption of an endogenous money supply. The financial sector should be explicitly specified, alongside the investment function and the saving function, in order to describe the financial (institutional), not only the real (technical), conditions that allow effective demand to play its driving role in the growth process, leaving the supply to adapt to it.³⁰ In the following concluding remarks we surmise three general warnings and a tentative research agenda.

Firstly, it would not be of much help to claim that demand – in particular the demand for investment – plays a *causal* role, whereas the supply of finance and financial investment is a *background condition*.³¹ For a growth theory it would be more useful to find, on the basis of the empirical evidence, the locus of persistency/normality/long-term and that of volatility/short-term of effective demand. A distinction in this direction is found in Nell (1998), where the plans of investments are supposed to be grounded on persistent expectations and beliefs, whereas the locus of the volatile animal spirits is the timing of investment expenditures, because the latter depend on the vagaries of short-term decisions and on the mutable conditions of the credit market. This suggestive claim should be further developed and supported by empirical evidence. In the textbook *vulgata* of Keynes's theory, productive investment is said to govern savings instead of the other way round. Yet, we should refrain from invoking a one-cause growth model, like demand-led growth models or, by reaction, a *finance-led growth* model, although it is tempting to choose such a spot of distinction.

Secondly, it should be admitted that a certain representation of the leading role of investment, associated with the autonomous decisions of the entrepreneurs, looks rather exaggerated in the present stage of capitalism. It seems to attribute a prevalent role to the desired plans of the heroic figure of a Schumpeterian entrepreneur who would be capable of

³⁰ A recent contribution in the direction suggested in the text is offered by Duménil and Lévy (2012).

³¹ See Parrinello (2013a; 2013b) on causes, background conditions and normal states in economics and, in particular, in macroeconomic models.

transforming inventions into innovations whilst according only a passive role to credit institutions and the financiers with their short-term outlook. This vision should be reconsidered in light of the development of the modern financial institutions that are not simply passive lenders. By extending the metaphor of Joan Robinson, the *spirits* of the corporate financiers should be superimposed on the *animal spirits* of the entrepreneurs in order to explain the growth performances of the modern capitalist economies.

Thirdly, a long-term outlook and volatile factors are not a distinctive feature of real investments. The long term expectations of the consumers for durables can also be volatile, and even the owners of a stock of an exhaustible natural resource decide the flow of its supply in a certain period on the basis of mutable long term expectations. However, important distinctions remain in the conditions that make their plans effective. Consumers who lack financial assets are highly limited by the request for real collateral if they want to have access to bank credit. The flow-supply of oil is limited by its stock in the ground but is unconstrained by financial conditions. Instead the investment plans of the entrepreneurs are subjected to a budget constraint that can be soft to varying extents, according to the institutional setting.³²

We should be agreed on the premise that to become effective, without crowding out effects, a plan of *productive* investment requires not only the existence of spare productive capacity, but also, so to speak, a capacity for spending that derives from access to lines of credit, in addition to money inventories and self-financing out of retained profits. In the textbook Keynesian theory of the multiplier, both (productive and financial) excess capacities are implicit and are supposed to make the multiplier effective. In the invoked extension of that theory to the long run and to the theory of growth, excess expenditure capacity should be made explicit like excess productive capacity. In particular, it should be

³² It might be objected that the softness of the budget constraint of the investors vis-à-vis the consumers is a question of degrees, not a sharp distinction, but since a long time we have learnt that differences in quantity beyond a certain point become differences in quality.

explored whether, by analogy to productive capacity, a corridor of credit distribution exists, such that any change in the realisation of such distributions does not affect normal prices and the income distribution including the long term interest rate; whereas outside that range this influence does materialise and affects investment plans and the path of accumulation. The analogy is not misplaced because both ranges are supposed to depend on cost evaluations, despite the fact that the normal range of productive capacity utilisation must also satisfy technical constraints.

Appendix: The simple model of compound accumulation

The equation (in section 3)

$$sY_t + mY_t = v(Y_t - Y_{t-1}) + X_t \quad (2)$$

is a first order, linear nonhomogeneous difference equation, and can be written in the simple form:

$$Y_{t+1} = aY_t - B_t \quad (a.1)$$

$$\text{with } a \equiv \frac{v}{v - (s + m)} \text{ and } B_t \equiv \frac{X_t}{v - (s + m)}$$

Given the initial conditions: $Y_{t=0} = Y_0$ and $K_{t=0} = vY_0$, the level of income in period t is determined by the sum of two parts, which are the general solution to the homogeneous equation $Y_{t+1} = aY_t$ and a particular solution to the nonhomogeneous equation (a.1). National income in period t can be determined also recursively from the equation (a.1) with the initial conditions:

$$Y(t) = Y_0 a^t - \sum_i^{t-1} a^i B_{t-1-i}$$

and the stock of capital in period t is determined by $v Y_t = K_t$.

If exports have a geometric trend, $X_t = X_0 b^t$, the general solution to (a.1) becomes:

$$Y(t) = Y_0 a^t - B_0 b^{t-1} \left[\frac{\left(\frac{a}{b}\right)^t - 1}{\frac{a}{b} - 1} \right]$$

Where $B_0 \equiv \frac{X_0}{v - (s + m)}$, and $\left[\frac{\left(\frac{a}{b}\right)^t - 1}{\frac{a}{b} - 1} \right] = \sum_t^{t-1} \left(\frac{a}{b}\right)^t$ is the sum of a geometric series.

In the simple case of constant exports, $X_t = X_0$, the general solution is

$$Y(t) = \left(Y_0 - \frac{X_0}{s + m} \right) a^t + \frac{X_0}{s + m} \quad (\text{a.2})$$

and $Y^* = \frac{1}{s + m} X_0$ is a stationary solution to (a.2), a state where *net* investment is zero and $K^* = vY^*$ is the constant capital stock.

Conditions for positive growth

A necessary condition for a positive rate of growth in the closed Harrod model is:

$$\frac{s}{v} > 0$$

In the open model the condition for each t becomes:

$$\frac{s + m - h_t}{v} > 0; \quad h_t \equiv \frac{x_t}{y_t};$$

and is satisfied by the assumption:

$$Y_t > \frac{1}{s + m} X_t \quad (\text{a.3})$$

The autonomous exports in real terms must be less than the amount of product not consumed, so that the capital stock is not eroded during the period. In particular this inequality applies in the initial period:

$$Y_0 > \frac{1}{s+m} X_0$$

It means that growth in the open model requires that the initial level of income is higher than its stationary level determined by the autonomous demand. Furthermore in each period, the conditions for the survival of the economy must be satisfied:

$$K_t > X_t, \quad Y_t > X_t/v \quad (\text{a.4})$$

With $K_0 > X_0$, $Y_0 > X_0/v$ in the initial period. Conditions (a.4) mean that the unproductive autonomous demand, despite it being a source of finance for imports, must not ‘eat the whole cake’, otherwise the economy will cease to exist. It is plausible to assume $v > s + m$, and therefore condition (a.3) implies condition (a.4).

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