PSL Quarterly Review

vol. 77 n. 308 (March 2024)

Articles

A micro-founded Kaldor-Pasinetti model considering an open economy: An inter-generational cum life-cycle approach

JOÃO GABRIEL DE ARAUJO OLIVEIRA, BEATRIZ ESTULANO VIEIRA, RENATO NOZAKI SUGAHARA*

Abstract:
This paper expands Baranzini's (1991) approach by introducing the assumption of an open economy to a model of capital accumulation in an intergenerational framework. Our results show the importance of government activity and foreign trade interrelations in determining the path of the income distribution and growth processes. Government revenue derives from inheritance taxation, which is used as income transfers. Exports affect capital accumulation negatively, and the inverse result for imports is true. Moreover, the government does not have to incentivize exports; otherwise, capital accumulation will be harmed. Thus, both assumptions influence the determination of the income distribution and growth processes.

de Araujo: Brazilian Institute of Capital Market, Brasília, and University of Brasília (Brazil),
email: joaogabrielaraujooliveira@gmail.com
Estulano: Institute for Applied Economic Research, Brasília, and University of Brasília (Brazil),
email: beatriz.estulano@uel.br
Nozaki: State University of Londrina (Brazil),
email: sugahara@uel.br

How to cite this article:
De Araujo Oliveira J.G., Estulano Vieira B., Karthikeyan S.I. (2024), "A micro-founded Kaldor-Pasinetti model considering an open economy: An inter-generational *cum* life-cycle approach", *PSL Quarterly Review*, 77 (308), pp. 89-104.

DOI: https://doi.org/10.13133/2037-3643/18148

JEL codes:
D15, E12, F41

Keywords:
capital accumulation, open economy, stability conditions, income distribution

Journal homepage:
http: //www.pslquarterlyreview.info

International friendship among economists of different schools of thought is more important than that of the same school, especially in such a difficult period of history when political and economic interests are giving rise to so many conflicts.

Morishima (1977, p. 61)

* The authors are grateful for the helpful comments and suggestions offered in the Webnário PPGECO/UnB, 14 Brazilian Keynesian Association International Meeting, and also made by Dr. Jorge Thompson Araujo, Mr. Daniel Soares Fogo, and the two anonymous referees. They bear no responsibility for either errors or opinions presented here. João Gabriel is grateful for the support of the Applied Studies Center (IBMEC-DF) and also for the Department of Economics at the University of Brasília, where he is a post-doctoral researcher. João is also a Consultant at the World Bank and the views expressed in this article are those of the authors only and do not necessarily coincide with those of the World Bank, its Executive Directors, or the countries they represent.

Baranzini (1991) developed the Wealth and Distribution Theory, whereby he introduced the micro-foundation to the post-Keynesian argument, which is his central contribution to this kind of framework.[1] Wolff's (1988) analysis reinforces the validity of incorporating microeconomic foundations into a two-class fixed savings model. This approach expands Pasinetti's (1962) results regarding interest rates and productivity growth in steady-state equilibrium, where the output is given and investment is exogenous.[2] In this vein, such methodology highlights the importance of the demand side of the economy in understanding wealth distribution, particularly in the context of the life-cycle theory and its implications for different classes within the economy.[3]

This theory shows how the classes accumulate capital from an intergenerational perspective,[4] considering the original Kaldor-Pasinetti proposal of full employment in a competitive market. However, Baranzini barely discussed how the government and international trade affect the model. Teixeira et al. (2002) and Sugahara et al. (2016) add the public sector to the model, expanding the original theory to observe how the government affects both inheritance and capital accumulation. Their framework was focused on observing the impacts of inheritance taxation on capital accumulation, but in a closed economy.

Thus, our contribution here is to expand Teixeira et al. (2002), demonstrating an alternative proposal considering both government activity and international trade. We consider that these assumptions affect capital stock and inheritance. Here, international trade deals with capital exports and imports, information, knowledge, and other subjects. These issues characterize the influences of an open economy, as in Dalziel (1989), Teixeira and Araujo (1997), and de Araujo Oliveira and Teixeira (2020).

There are three main differences between our paper and Baranzini's extensions presented above: (i) none of those models formalized the assumption of an open economy; (ii) we demonstrate how international trade affects capital accumulation; and (iii) we prove that our model is stable by using Olech's criteria.[5] In this vein, our new framework links two out of nine lines generated from Pasinetti's theorem indicated by Baranzini and Mirante (2021, p. 479): "5 The introduction of micro-foundations" and "7 Overlapping generations models cum inter-generational transmission of wealth".

Our work is divided into four sections. This introductory section presents part of the baselines of Baranzini's model, our objectives, and the main differences between our approach and the original model and its previous extensions. Next section revisits the extension of Baranzini's

[1] For more information, see Baranzini and Mirante (2013).

[2] According to Baranzini (1991, p. 6), "[...] a given capital/output ratio; to the circularity of the system according to which it is investment, exogenously determined by employment conditions, which governs overall savings; and to the equality between investment and savings ensured by changes in the distribution of income among classes rather than by changes in the level of economic activity as in the case of the traditional neoclassical model".

[3] According to Wolff (1988), the specification of a life cycle savings model with a two socio-economic class seems to be consistent with Pasinetti's result regarding the rate of interest and productivity growth in steady-state equilibrium. For the worker, the salary is not determined by budgetary constraints and for the capitalist (according to Baranzini, 1991) it is not a function of marginal productivity. So the micro-foundation concerns the intertemporal behaviour of the capitalist and the worker and not a micro-foundation on the supply side.

[4] Following this line, Baranzini (1991) developed a theory of wealth distribution, considering intergenerational theory and a reason for inheritance. His model establishes the microeconomic foundations for the theory of growth and distribution based on the post-Keynesian framework. According to Blecker and Setterfield (2019, p. 9), in post-Keynesian models, growth is fundamentally a demand-led process, as in Baranzini's (1991) model. In this way, even the seemingly supply-determined limits to economic activity at any point in time are, in fact, likely to be influenced by the demand side of the economy. Baranzini's objective would be to justify a better allocation of resources and economic growth.

[5] See section 2 and appendix A1.

model considering bequest taxation. The subsequent one deals with our new hypothesis considering an open economy, designing our new framework, and discussing the main results of the model. The final section presents the concluding remarks.

1. Revisiting Baranzini's model and the extensions of Teixeira, Sugahara and Baranzini

Baranzini (1991) developed an overlapping generations model concerning micro-foundations and introduced the concept of inheritance in a personal income distribution function.[6] He focused on the determination of a "flexible" propensity to save of the groups and their patterns of consumption, savings, and accumulation, via: (a) the interest rate (current and expected); (b) behavioural parameters; (c) institutional parameters; (d) demographic parameters; and (e) technical parameters. To avoid the "razor-edge" on the dynamic system in a long-period perspective, the author had to consider Pasinetti's solution (see Baranzini, 1991, chapters 1 and 5) and, like him, had to assume the Cambridge equation to deal with the long-period equilibrium path. As a result, he derives a personal income distribution, which seeks to maximise the two-class (workers and capitalists) utility function. Baranzini (1991), in Chapter 5, revealed his basic structure, where only capitalists transmit their wealth to their descendants, while workers accumulate only lifecycle savings.

His approach focused on integrating the neoclassical theory, using utility functions and micro-foundations, in a post-Keynesian context, which concerns the demand side of the economy and divides the economy into two social classes. Both solutions were linked by Teixeira et al. (2002), who developed a framework considering government taxation on the bequest to examine government activities' implications from an intergenerational perspective. At the beginning of their work, they show the maximisation of two utility functions. The first one concerns the capitalists, and the second is for the workers. Their model presents a modification of the restrictions by introducing the mentioned tax policy. So:

$$MaxV(C_t; C_{t+1}; B) = Max\,\frac{1}{a}\left[(C_t^c)^a + \frac{C_{t+1}^c}{1+\sigma} + \frac{1+g}{1+b}B_t^a\right] \quad (1)$$

$$\text{s.t. } (1-t_b)(1+r)B_{t-1} = C_t^c + \frac{C_{t+1}^c}{1+r} + (1+g)B_t$$

Equation (1) is the objective function which is a utility function due to the current capitalist's consumption in period t (C_t^c) regarding constant elasticity utility of the V function (α) and the consumption of this class in the next period (C_{t+1}^c) considering the time-preference (σ), the inheritance received from the past generation (B_{t-1}), and the taxation on inheritance (t_b), besides population growth rate g. The restriction of such maximisation is the net capital accumulation represented by the inheritance's gross value. The equation below shows the worker's consumption, C_t^w and C_{t+1}^w in period t and $t+1$:

[6] Following Dafermos and Papatheodorou (2015), the distribution of functional income occurs between the factors of production (labour, interest, distributed profits, and others.) and the distribution of personal income among members of society, that is, the domestic sectors are classified in various groups, which are characterised by different types of income and of balance sheet structures, such as capitalists, workers, and others.

$$MaxV(C_t^w; C_{t+1}^w) = Max\frac{1}{a}\left[(C_t^w)^a + \frac{(C_{t+1}^w)^a}{1-\sigma}\right] \quad (2)$$

$$\text{s.t. } t_b(1+r)B_{t-1} + W_t = C_t^w + \frac{C_{t+1}^w}{1-r}$$

where the profit rate is $r = \frac{P}{K}$, with P being the total profit and K the full capital stock, B_{t-1} is the inheritance bequest in $t-1$, and W_t is total wages.

The workers maximise their intergenerational discounted consumption volume in both periods. This maximisation is restricted by the liquid capital accumulation represented by the wage and taxation's gross value. The authors assume that the government intervenes in the economy by levying a direct tax on the bequest handed forward to each new generation of capitalists. This taxation on wealth is totally and immediately transferred to workers, via the state. As is pointed out in Teixeira and Araujo (1997), the state might impose a special tax in order to achieve, simultaneously, a higher level of economic growth and a better distribution of income between profits and wages. Their model contrasts with the Kaldor-Robinson original and the expanded growth and income distribution models in the sense that there always exists a trade-off between growth and a more equal distribution of income.

After solving these maximisations, one has the results of consumption for each class in two discrete times. Based on these propositions, Teixeira and Araujo analysed the effects caused by the introduction of bequest taxation on the capital stock and the distribution of capital between capitalists and workers. The authors divided the capital stock into the following parts: (i) the capitalist's life cycle savings in period (S_t^c); (ii) the worker's life cycle savings in period t (S_t^w); and (iii) the capitalist's intergenerational heritage, as was initially done in Baranzini (1991). Additionally, the capital accumulation expression in Teixeira et al. (2002) is offered here as equation (3):[7]

$$K_t = B_{t-1} + S_t^c + S_t^w \quad (3)$$

Equation (3) presents the total capital in period (K_t); this function also expresses capitalists' and workers' investments. In equilibrium conditions, total savings and total investment are equal in period t. In this vein, the authors assume that the elasticity and the numbers of workers and capitalists are constantly equal to one. These considerations result in the proper savings function in a closed economy. Thus:

$$S_t^c = B_{t-1}[(1-t_b)r - 1 - b] \quad (4)$$

$$S_t^w = K_t^w = \frac{1}{2+\sigma}[W_t + t_b(1+r)B_{t-1}] \quad (5)$$

Equations (4) and (5) show the income transfer from capitalists to workers, since, in equation (4), we have the inheritance liquid amount represented by $B_{t-1}(1 - t_b)r$ being absorbed by the working class, $t_b(1 + r)B_{t-1}$ in equation (5), where b is the subjective discount rate for the bequest and K_t^w is the workers' capital stock in period t. The workers do not leave inheritance in

[7] According to (Baranzini, 1991, p. xvii): "A first fundamental stage consists in sketching out a set of general conditions that the process of growth must fulfil in order to be considered 'satisfactory'. At this first stage, only basic problems are faced, such as those of compatibility among production sectors, final demand composition and over-all avoidance of waste. This problem concerns what we may call structural efficiency". In this vein, we are not considering depreciation in our model.

this kind of model; from equation 5, both socio-economic classes conclude that their savings are equal to the capital owned by them. This model is essentially substantiated by the Cambridge equation perspective, where savings adjusts itself to the level of investment (in a given value) and that accommodates aggregate demand to a given output. Besides, the investment in full employment is assumed as in Kaldor (1955), Pasinetti (1962), Steedman (1972), de Araujo Oliveira and Teixeira (2020), and so on. According to the post-Keynesian tradition, savings adjusts passively to investment and the equality between these variables is ensured by redistribution between profits and wages, as well as among capitalists and workers. Bearing in mind $B_{t-1} = B_t = B^*$ in equilibrium[8] and applying some mathematical manipulation, they obtain the total capital stock in equilibrium (K^*):

$$K^* = \frac{(2+\sigma)(1-t_b)r^*-(2+\sigma)b+t_b(1+r^*)}{(2+\sigma)-\left(\frac{Y}{K}-r^*\right)} \tag{6}$$

Following Feu (2001), the capital-output ratio — $\left(\frac{K}{Y}\right)$ with Y being total income or GDP and K the capital stock — is usually higher than one, which means that the product-capital ratio is less than one. In this vein, to guarantee a positive value of equation (6) we should make two propositions. The first one is $(2+\sigma)+r^* > \frac{Y}{K}$, meaning that the preference for consumption in the future will depend on the opportunity cost of capital; in other words, future return is important for the increase in the capital stock, i.e., the agent prefers to consume in the future because the efficiency of capital in production is low. The second proposition is $(2+\sigma)(1-t_b)r^* + t_b(1+r^*) > (2+\sigma)b$, the net accumulation of wealth should ideally exceed the importance that capitalists attach to their descendants; the former phenomenon arises from the fact that, if they increase the parameter b, the inclination to bequeath a greater sum to their descendants would correspondingly intensify. Thus, bequest taxation affects the capital stock positively in equilibrium. This result is easy to prove by applying the partial derivative in K^* concerning t_b, showing that an increase in the tax raises the capital stock. From equation (6), it was possible to get the shares of the bequest concerning the capital stock:

$$\left(\frac{B}{K}\right)^* = \frac{(2+\sigma)-\left(\frac{Y}{K}-r^*\right)}{(2+\sigma)r^*+t_b(1+r^*)-(2+\sigma)(t_b r^*+b)} \tag{7}$$

With r^* being the profit rate in equilibrium, the authors had a positive value of equation (7) when imposing the same restrictions as above. These conclusions (equations 6 and 7), led them to affirm the inverse relation of the effects of fiscal policy on capital accumulation bequests. Manipulating equations (6) and (7) gives us the capitalist's capital stock K_c. Thus:

$$\frac{K_c}{K} = 1 + \frac{r^*-\frac{Y}{K}}{2+\sigma} - \frac{t_b(1+r^*)\left(2+\sigma+r-\frac{Y}{K}\right)}{2+\sigma[(2+\sigma)(r^*-t_b r^*-b)+t_b(1+r^*)]} \tag{8}$$

The approach presented above shows the importance of bequest taxation to Baranzini (1991). Since workers do not accumulate capital at the end of their lives, the authors did not consider $\frac{K_w}{K}$ formula. They (Teixeira et al., 2002, p. 20) say: "[…] on the basis of the above argument, we are able to conclude that the main features of the 'Cambridge Equation' are preserved in our extension

[8] In equilibrium in a model with two classes, all variables increase at the same rate. This assumption is reinforced in Baranzini (1991, pp. 125-127) and Vieira et al. (2023).

of Baranzini's approach, to include taxation on capitalists' bequest", preserving the stability of the model in the steady-state.

This issue is still being addressed today. Rehm and Schnetzer (2015) argue that the processes of cumulative causation between wealth and power can intensify wealth inequality, showing that inheritance is the most important factor for wealth inequality. The structural power to shape economic and political institutions is increasingly concentrated. Albis et al. (2017) address the issue empirically from the national transfer accounts (NTA). The authors measure how individuals produce, consume, save and share resources at each age.

Our work is an extension of Baranzini's model concerning an open economy and analyzing international trade effects on capital accumulation. Our extension expands the Teixeira et al. (2002) model. They studied the implications of government activities from an intergenerational perspective. The long-term growth path in a post-Keynesian perspective is led by aggregate demand (demand-led type of models). Considering this framework, when the economy has already carried out its industrialisation process and/or its capitalist revolution, the theoretical models must include the assumption of an open economy. Thus, our model adds such a hypothesis to become more realistic for economic analysis in developed and developing economies. In this vein, our article propose to dialogue with government economic policies adopted to limit the strong presence of the external sector in the current economy and extend the literature.

2. The effects of international trade and government activities on capital accumulation

Baranzini (1991) and Teixeira et al. (2002) presented the capital accumulation function in equilibrium. Here, we treat the function with a slight modification in the national account, which follows Dalziel (1989) and Teixeira and Araujo (1991), as we can see below:

$$C + I + G + X - M = Y = W + P + T \tag{9}$$

According to equation (9), the demand side of GDP is represented as the sum of total consumption (C), total investment (I), government spending (G) and the net exports, being X total exports and M total imports; or we can define the GDP from the income perspective, i.e., the sum of wages, profits and taxes $(W, P, T$, respectively). Baranzini (1991) considers $I = S = S^c + S^w$, where the capitalist savings $S^c = K^c - B_{t-1}$ and worker savings $S^w = K^w$, thus:

$$S = I = K_t^c - B_{t-1} + K_t^w = K_t - B_{t-1}$$

This means that total investment is equal to total capital stock in period t or a capitalist's capital stock in period $t\,(K_t^c)$ minus the inheritance. Here, we are considering a balanced government budget, which means $G = T$ for all t. Substituting the investment definition above in equation (9) gives us:

$$K_t - B_{t-1} + X - M = W + P - C \tag{10}$$

We can approach this substitution based on Keynesian principles since this school considers that investments precede savings. So, in our approach, the equation can be re-written with investment (savings) as the difference between net income $Y - T = W + P$ and consumption. Thus:

$$K_t - B_{t-1} + X_t - M_t = I = S_t \tag{11}$$

As we can see in the above equation, total savings are affected by foreign trade. Therefore, equality for the national savings has to be maintained, since, once effective demand increases, positively affecting the income, savings of both capitalists and workers will rise. This solution is in line with the argument of de Araujo Oliveira and Teixeira (2020), where their "Equation 1" savings of each class is equal to investment plus net exports. They also demonstrated the influence of international trade on each income, which reinforces the argument that national savings will be affected by foreign trade. Considering $M_t = mY_t$, with marginal propensity to import (m) and total income in period t, we have:

$$K_t = S_t + B_{t-1} - X_t + mY_t \tag{11}$$

Unlike equation (3) presented in the previous section, our model introduces export and import assumptions in equation (11). Our solution concludes that M_t has a positive effect on capital accumulation and X_t has a negative impact. Substituting equations (4) and (5) in equation (11), we have:

$$K_t = B_{t-1} + B_{t-1}[(1 - t_b)r - 1 - b] + \frac{1}{2+\sigma}[W_t + t_b(1 + r)B_{t-1}] + X_t - mY_t$$

Therefore, $Y = W + P + T$ and $G = T$, which means $W = Y - P - G$ and that, in equilibrium, $B_{t-1} = B_t = B, \frac{P}{K} = r^*, \frac{X}{K} = x$ export-capital share, $\frac{G}{K} = \tau$ and $\frac{K}{Y} = v$ the technology. Thus:

$$K^* = \frac{B[(1-t_b)(2+\sigma)r+t_b(1+r)-(2+\sigma)b]}{\left\{(2+\sigma)+r^*+\tau-(2+\sigma)x-\frac{1}{v}[1-m(2+\sigma)]\right\}} > 0 \tag{12}$$

From Baranzini (1991), we have $s_w = \frac{1}{2+\sigma}$, so $\frac{1}{s_w}[(1 - t_b)r - b] + t_b(1 + r) = \beta$. The importance that the agent gives to the next generation (b) is $b \leq (1 - t_b)r$, which guarantees a result bigger than zero. The more significant b is, the less will be the importance of leaving an inheritance to the agent in $t + 1$. This means that the amount of the capital stock in equilibrium will be determined by $0 \leq b \leq (1 - t_b)r$, which raises two hypotheses. The first one is that, when $b = (1 - t_b)r$, there is no interest in leaving an inheritance. The second one is that, when $0 \leq b < (1 - t_b)r$, some or full altruism may be in evidence. If agents attach a high value to the future well-being of their descendants and wish to accumulate wealth to pass on to them, they will be able to increase their savings, reducing the interest rate; for this reason, the agent will, at most, fail to inherit their gross income. In this vein, the value of β depends on the preference that the agent gives to the next generation.

Taking an analytic approach, $\frac{1}{s_w} + r^* + \tau + \frac{x}{s_w} - \frac{1}{v}\left[1 + \frac{m}{s_w}\right] > 0$, since $v > 1$ and $0 \leq m \leq 1$. In this case, *ceteris paribus*, the determination of the capital stock amount could depend on the size of m and x.

$$\frac{\partial K^*}{\partial x} = \frac{-1}{s_w\left\{\left[\frac{1}{s_w}+r^*+\tau-\frac{x}{s_w}-\frac{1}{v}\left[1-\frac{m}{s_w}\right]\right]\right\}^2} < 0 \tag{13}$$

$$\frac{\partial K^*}{\partial m} = \frac{1}{s_W v\left\{\left[\frac{1}{s_W}+r^*+\tau-\frac{x}{s_W}-\frac{1}{v}\left[1-\frac{m}{s_W}\right]\right]\right\}^2} > 0 \qquad (14)$$

These results show the importance of international trade to capital accumulation. We expected from both equations (13) and (14) that, from one side, the exports-capital share harms the capital stock. On the other side, we also confirm that the variation in the capital stock about the imports share is positive. Our intention here is to analyse the isolated effects of the variance of the exports-capital share (equation 13) or the imports share (equation 14) as a mechanism of adjustment, all other variables being constant in these particular analyses. These results show that, if the economy sells products to the rest of the world, the capital stock is in equilibrium and the capital stock supply crushes, which confronts the neoclassical literature. Import's derivative is positive, representing the opposite result of the exports. Grossmann (1992) demonstrate that an expansion of the international capital process (an increase in exports) generates leakage of the idle capital and decreases the profit ratio.

The post-Keynesian literature reinforces these results. de Araujo Oliveira and Teixeira (2020) affirm that, in the case of an open economy, one has to restrict the model to the case of a positive current balance account.[9] These results do not match with the export-led literature (like Thirlwall, 1979) but guide a new perspective of the effects of international trade when a micro-foundation is considered in a capital accumulation framework. Boudreaux (2018) says that, if the economy exports more with the sole aim of improving the trade balance, it will cause the internal supply of products to fall. Fewer products on the domestic market imply a direct reduction in the standard of living. It is important to adopt policies that encourage exports and imports; otherwise, you will only enrich others and impoverish yourself.

Since we are dealing with government activity and international trade, equations (15) and (16) show the effects of taxation on imports and exports.

$$\frac{\partial^2 K^*}{\partial x \partial \tau} = \frac{-2}{s_W\left\{\left[\frac{1}{s_W}+r^*+\tau-\frac{x}{s_W}-\frac{1}{v}\left[1-\frac{m}{s_W}\right]\right]\right\}^2} < 0 \qquad (15)$$

$$\frac{\partial^2 K^*}{\partial m \partial \tau} = \frac{2}{s_W v\left\{\left[\frac{1}{s_W}+r^*+\tau-\frac{x}{s_W}-\frac{1}{v}\left[1-\frac{m}{s_W}\right]\right]\right\}^2} > 0 \qquad (16)$$

Equations (15) and (16) show the influence of the external sector concerning government expenditures, reflecting different results to the total capital stock by assuming the export rate level. Such a second derivative gives a full understanding to the impact of the relationship between international trade and government activities on capital accumulation. From equation (15), one has a slowdown effect on the relationship between government activities and exports. Equation (16) expresses the inverse relation considering imports. Both equations (15) and (16) address the impact of an increase (decrease) in government expenditures affecting the exports and causing a negative (positive) influence on the capital stock in equilibrium.

[9] Unlike de Araujo Oliveira and Teixeira (2020), we are not considering the presence of a financial market in our model. However, according to the authors, if the model considers $X \neq M$, we have two situations: $X > M$ surplus and $X < M$ deficit. For them, the first result will provide a permanent positive effect on incomes of both capitalists and workers. In this vein, a positive current account balance is needed to ensure growth of the income and consequently all the variables which depend on this income. Our assumption is also in line with the Nah and Lavoie (2017) model (even considering a different range of perspective); for them, a permanent deficit would not be sustainable, even in the steady-state path.

Contrastingly, the Globalization narrative appears to be the most convincing on theoretical and empirical grounds. It suggests that non-financial corporations of rich economies have been able to capture gains from the dynamism of developing economies and, at the same time, that investment opportunities in the developing world have discouraged domestic investment (Durand and Gueuder, 2018, p. 24).

Equation (12) allows the determination of the bequest share, as seen in equation (17):

$$\frac{B}{K^*} = \frac{\frac{1}{s_W}+r^*+\tau-\frac{x}{s_W}-\frac{1}{v}\left[1-\frac{m}{s_W}\right]}{\beta} > 0 \tag{17}$$

The same results presented by the capital stock are analysed here, since the result is only a reorganisation of equation (12), which represents the importance of consumption to the next generation. Government activities impact the inheritance of bequests as well as international trade. This analysis is different from that of Sugahara et al. (2016), where they look only at the fiscal policy. The impact of international trade in government expenditures on the inheritance is null since the second partial derivative is zero. From the result of equation (12), one can determine the value of the capital accumulation to each class:

$$1 = \frac{K_t^c}{K} + \frac{K_t^w}{K} \tag{18}$$

For the capital shares of capitalists and workers,[10] one has:

$$\frac{K_t^w}{K} = s_W\left[\frac{1}{v} - r^*\right] - \frac{t_b(1+r^*)\left\{1+r^*s_W+\tau s_W-x-\frac{s_W}{v}\left[1-\frac{m}{s_W}\right]\right\}}{\beta} \tag{19}$$

$$\frac{K_t^c}{K} = 1 - s_W\left[\frac{1}{v} - r^*\right] - \frac{t_b(1+r^*)\left\{1+r^*s_W+\tau s_W-x-\frac{s_W}{v}\left[1-\frac{m}{s_W}\right]\right\}}{\beta} \tag{20}$$

Equations (19) and (20) represent the capital share for each class, where s_W is the worker's savings rate. In both cases, the import ratio and the exports-capital share impact these formulas. The capital stock of each class in the period has a negative impact on the import's ratio and a positive effect on the exports-capital share. Equations 19 and 20 agree with Gandjon Fankem and Feyom (2023, p. 12): "Increasing trade openness leads to a reduction in the level of industrialization".

In considering the impact of exports and imports on the intergenerational utility consumption for the worker's and capitalist's capital accumulation, assuming the technology (capital-output ratio) to be bigger than two[11], we concluded that the intertemporal preferences are affected positively by the marginal propensity to import, impacting positively on the worker's capital share; therefore, in the same circumstances, the capitalist's capital-share is affected negatively. In regards to the case of the technology being less than two, we have the inverse results to the exports; the technology does not affect the results and, as to the worker's capital share, the effects

[10] The demonstration of equations (19) and (20) is in appendix A1.

[11] This result is based on the Feu (2001) analysis of the Brazilian and OECD economies. This means that, to produce one unit of output, at least two units of capital are needed. The cited author demonstrates that most of the analysed countries, since 1970, have been demonstrating an increasing need for capital to produce output. In this vein, the author demonstrates that, in the scope of these countries, we have a structural effect reallocating products to more capital-intensive sectors.

of the exports-capital share on the intergenerational preferences are always positive, while the capitalist capital share will always be negative. Therefore, the government expenditure in international trade on the capital accumulation of each class is null, since the second partial derivative is zero (equations 19 and 20). Each capital accumulation is impacted by the consideration of international trade, resulting in effects on the total capital of the economy, which means that the profit rate and profit share will be adopted as the income distribution of our approach.

Comparing our approach with Teixeira et al. (2002), here exports and imports affect the level of preference between the utility consumption today and in the next generation (σ), thus affecting the total capital accumulation, and that accumulated by each class. Another difference is that we are in the presence of government activities in our line, as expressed by τ. Our framework differs from that of de Araujo Oliveira and Teixeira (2020) by considering the impact of exports and imports in specific intervals (partial derivatives), determining the effects of the international trade concerning the income derived from the capital to each class; our model does not consider financial international markets, another difference in the papers. However, Teixeira et al. (2002), found results similar to equation (12) without considering the inheritance.

Besides, it is important to demonstrate that the model converges to a sustainable path and we have to prove in which conditions the stability of the model can be ensured. Thus, appendix A2 proves such conditions incorporating a variable α into the dynamic model: if, in the steady-state, it is equal to one, we have an export-led path and, if, in the steady-state, it is equal to zero, we have an import substitution situation. For our model, the only interesting and efficient solution is the first one; in this vein, the economy (productivity) depends on incentives to national sectors to improve their international competitiveness, since, in our case, we need to export more and more.

According to Myrdal (1957), this concept is known as export-led cumulative causation. Consequently, economic growth, technical progress, international competitiveness, and export success interact in self-reinforcing virtuous circles. In this way, such strategies are an alternative to increasing aggregate demand. This type of development was adopted in countries like Belgium, Germany and China during the pre-2007-crisis financialization period, causing an accumulated increasing surplus in their trade and current account balances (see Hein, 2011). Thus, policies that aim to promote economic growth through expansion of exports are highly effective in promoting economic growth.

3. Conclusion

This article contributes to the current literature in at least three ways. The first is that we dealt with both government activities and an open economy simultaneously in a capital accumulation intergenerational model. It is noteworthy that both imports and exports influence the results of capital stock and inheritance and the formulation of national income and GDP. From the perspective of the capital stock, foreign trade also impacts the Cambridge equation, as in Dalziel (1991) and de Araujo Oliveira and Teixeira (2020). Therefore, our model distinguishes its assumptions from the mentioned papers by analysing the effects of inheritance taxation,[12] which differs from income taxation, since we are taxing wealth, which is similar, albeit not equal, to the model of Teixeira et al. (2002). For us, this is an important issue, as great concentrations of wealth

[12] Wealth taxes are imposed on the wealth stock or, in our case, on inheritance. Income tax is imposed on the flow of wealth stock.

come from bigger inheritances, as pointed out by Piketty (2013) and Saez and Zucman (2014). Additionally, as presented above, such inheritance not only influences domestic income but also has a significant impact on foreign trade.

The second contribution is that we demonstrated the impacts of foreign trade on the total capital stock and then the effects of the government as a decelerator mechanism. It is interesting that, our results diverge from the neo-Classical Growth economic theory, the exports-capital share does not have a positive impact on the total capital stock, and the government presence means that accelerated exports without regulation will compromise the capital stock, as capital will flow proportionally from one country to another. On the other hand, imports have an analogous interpretation, increasing capital (entrance of external capital), and the government seems to accelerate its results by increasing capital.[13] A similar interpretation is demonstrated for the capital stock of each class in equations (19) and (20). Both results seem to agree with Boudreaux (2018).

The third contribution is that we prove the dynamics of each capital accumulation and its stability, as well as the exogenous decision to raise exports or imports, which affects individuals' decisions about inheritance. Applying the Olech theorem, we noticed that there is only one possible value that ensures balance to the model (see appendix A2). Nell (1991) commented on Kaldor's conception of stability: "The economy's path of motion is not necessarily headed toward any stable destination, nor [...] as a path, necessarily stable. It has to be managed by policy". As a control, our model determines one case, where only one interval to the growth rate of exports ($\alpha = 1$) is considered, and when we consider any other value, the model will be unstable or marginally stable. This represents the share of exports in net exports, so when it is equal to 1, our economy will be driven by exports. However, like in Setterfield (1997), the possibility of an analytical, non-equilibrium economics must also be centred on a conception of dynamic processes.

We also concluded that exports impact negatively the capital stock if the level of exports is smaller than the difference between the capital stock and the total worker savings. The influence of the external sector concerning government expenditures reflects different results in the total capital stock by assuming the export's rate level. Besides, we have only one value interval to exports-capital shares, which expresses the level of capital accumulation and inheritance bequest. The level of industrialization is a negative function of market openness and is related to the lowest qualification of human capital, which agrees with Gandjon Fankem and Feyom (2023). Our model looks at it from a different perspective, providing a demand-led alternative solution to capital accumulation (in part, industrialization), which also agrees with the conclusions made by the mentioned authors. We do not raise questions here about the division of profit between classes, taxation for each income, and others.

[13] For both demonstrations, see equations (13), (14), (15) and (16).

Appendix

A.1. Demonstration of equations (19) and (20)

The capital can be rewritten as:

$$\frac{K_t}{K} = 1 = \frac{K_t^c}{K} + \frac{K_t^w}{K}$$

Thus,

$$1 - \frac{K_t^w}{K} = \frac{K_t^c}{K}$$

$$K_t^w = K\left(\frac{Y}{K} - r^*\right)(2+\sigma)^{-1} + [t_b(1+r^*)B_{t-1}](2+\sigma)^{-1}$$

$$1 - \left[\frac{\left(\frac{1}{v} - r^*\right)}{2+\sigma} + \frac{[t_b(1+r^*)B]}{K(2+\sigma)}\right] = \frac{K_t^c}{K}$$

Thus:

$$\frac{K_t^w}{K} = \frac{\frac{1}{v} - r^*}{2+\sigma} - \frac{t_b(1+r^*)B}{K(2+\sigma)}$$

$$\frac{K_t^c}{K} = 1 - \frac{\frac{1}{v} - r^*}{2+\sigma} - \frac{[t_b(1+r^*)]B}{K(2+\sigma)}$$

From $\frac{B}{K}$, we have:

$$\frac{K_t^w}{K} = \frac{\frac{1}{v} - r^*}{2+\sigma} - \frac{t_b(1+r^*)\left\{(2+\sigma) + r^* + \tau - (2+\sigma)x - \frac{1}{v}[1 - m(2+\sigma)]\right\}}{(2+\sigma)\{(1-t_b)(2+\sigma)r^* + t_b(1+r) - (2+\sigma)b\}}$$

$$\frac{K_t^c}{K} = 1 - \frac{\frac{1}{v} - r^*}{2+\sigma} - \frac{t_b(1+r^*)\left\{(2+\sigma) + r^* + \tau - (2+\sigma)x - \frac{1}{v}[1 - m(2+\sigma)]\right\}}{(2+\sigma)\{(1-t_b)(2+\sigma)r^* + t_b(1+r) - (2+\sigma)b\}}$$

A.2. Stable condition

a) Dynamic variables

Assuming $A = X - mY$ and differentiating with respect to t:

$$\dot{A} = \dot{X} - m\dot{Y}$$

$$\frac{\dot{A}}{A} = \frac{\dot{X}}{X - mY} - \frac{m\dot{Y}}{X - mY}$$

where $A = X - mY \leftrightarrow \frac{\dot{A}}{A} = \frac{\dot{X}}{X-mY} - \frac{m\dot{Y}}{X-mY} \leftrightarrow 1 = \alpha + (1-\alpha)$, being the exports' proportion concerning net exports $\alpha = \frac{X}{X-mY}$. Thus:

$$\frac{\dot{A}}{A} = g_x\alpha + g_y(1-\alpha)$$

where g_x is the export growth rate and g_y the output growth rate. Logarithmic α and differentiating with respect to t:

$$\dot{\alpha} = \alpha(1-\alpha)(g_x + g_y)$$

From K_t, we have:

$$K_t = B_{t-1}[(1-t_b)r - 1 - b] + \frac{1}{2+\sigma}[Y_t - P_t - G_t + t_b(1+r)B_{t-1}] - X_t + mY_t$$

$$K_t = B_{t-1}[(1-t_b)r - 1 - b] + \frac{1}{2+\sigma}[Y_t - P_t - G_t + t_b(1+r)B_{t-1}] - (X_t - mY_t)\left(\frac{X_t}{X_t}\right)$$

Differentiating with respect to t. Thus:

$$\frac{\dot{K}}{K} = \frac{1}{K}\left\{\dot{B} + \dot{B}[(1-t_b)r - 1 - b] + \frac{\dot{Y}}{2+\sigma} - \frac{r\dot{K}}{2+\sigma} - \frac{\dot{G}}{2+\sigma} + \frac{\dot{B}t_b(1+r)}{(2+\sigma)} + \dot{X} - m\dot{Y}\right\}$$

Rearranging and considering: $g_b = \frac{\dot{B}}{B}$; $g_y = \frac{\dot{Y}}{Y}$; $g_k = \frac{\dot{K}}{K}$; $g_e = \frac{\dot{G}}{G}$; $\frac{X}{K} = x$; and $\varphi = \frac{B}{K}$. Thus:

$$\frac{\dot{K}}{K} = g_b\varphi[(1-t_b)r - b] + \frac{g_y}{v(2+\sigma)v} - \frac{rg_k}{2+\sigma} - \frac{g_e\tau}{2+\sigma} + \frac{g_b\varphi t_b(1+r)}{(2+\sigma)} + g_x x - \frac{m}{v}g_y$$

Being $\dot{B} = \varphi K$, we have:

$$\frac{\dot{B}}{B} = \frac{\dot{K_w}}{K_w + K_c} + \frac{\dot{K_c}}{K_w + K_c}$$

where: $\frac{\dot{K_w}}{K_w} = g_{K_w}$ and $\frac{\dot{K_c}}{K_c} = g_{K_c}$. Thus:

$$g_b = \frac{g_{K_w}K_w}{(K_w + K_c)} + \frac{g_{K_c}K_c}{(K_w + K_c)}$$

We have that: $\frac{K_w}{K_w + K_c} = \varepsilon$

$$g_b = \varepsilon g_{K_w} + (1-\varepsilon)g_{K_c}$$

$$\frac{\dot{K}}{K} = [\varepsilon g_{K_w} + (1-\varepsilon)g_{K_c}]\varphi[(1-t_b)r - b] + \frac{g_y[1 - m(2+\sigma)]}{v(2+\sigma)} - \frac{rg_k}{2+\sigma} - \frac{g_e\tau}{2+\sigma} + \frac{[\varepsilon g_{K_w} + (1-\varepsilon)g_{K_c}]\varphi t_b(1+r)}{(2+\sigma)} + g_x x$$

Defining:

$$[\varepsilon g_{K_w} + (1-\varepsilon)g_{K_c}]\varphi[(1-t_b)r - b] = \delta$$

$$\frac{rg_k}{2+\sigma} - \frac{g_g\tau}{2+\sigma} + \frac{[\varepsilon g_{K_w} + (1-\varepsilon) g_{K_c}]\varphi t_b(1+r)}{(2+\sigma)} = \theta$$

$$\frac{[1]}{v(2+\sigma)} = \mu$$

$$\frac{dK}{dt} = \left(\delta - \theta + g_y\mu + [\frac{g_x - (1-\alpha)(g_x+g_y)}{\alpha}]x\right)K$$

δ, θ, μ are constants expressions.

b) The stability condition

Here, we apply Olech's theorem (his criteria are presented in appendix A3) to analyse our extension's stability, considering an open economy. Thus, from equation (11), we have:

$$\frac{dK}{dt} = \left\{\delta + \theta + g_y\mu - \left[\frac{g_x - (1-\alpha)(g_x+g_y)}{\alpha}\right]x\right\}K$$

Where $\theta, \mu > 0$, and $A = X_t - M_t$, we have:

$$\dot{\alpha} = \alpha(1-\alpha)(g_x + g_y)$$

From equations (19) and (20) we can analyse the stability condition using the steady-state values presented in the appendix above, considering the Taylor expansion's first term. From this, we are able to determine the matrix system, where we present the Jacobian matrix (J):

$$\begin{bmatrix} \delta + \theta + g_y\mu - \left[\frac{g_x - (1-\alpha)(g_x+g_y)}{\alpha}\right]x & \frac{\alpha(g_x - g_y)xK - [g_x - (1-\alpha)(g_x+g_y)xK]}{\alpha^2} \\ 0 & (1-2\alpha)(g_x+g_y) \end{bmatrix}$$

Considering $\alpha^* = 1$ as a steady-state equilibrium from equation (21) to the analysis, and applying Olech's theorem, we have all the tools to analyse the stability in a matrix 2x2. This is a necessary and sufficient condition if the Jacobian matrix trace $\left(Tr\,(J)\right)$ is negative and the determinant ($|J|$) is positive. Thus:

$$1^{st} - \alpha^* = 1$$

$$Tr(J) = \delta + \theta + g_y\mu - g_x x - (g_x + g_y) < 0$$

If: $\theta < -\delta - g_y\mu + g_x x + (g_x + g_y)$

And:

$$|J| = -(\delta + \theta + g_y\mu - g_x x)(g_y + g_x) > 0$$

To sustain the stability, we need $\theta < -\delta - g_y\mu + g_x x$. In the long-run process, this case sustains the stability condition, by respecting the value of θ.

A.3. The Olech theorem

Olech's theorem (see Garcia, 1972) can guarantee global stability in the plane if the following assumptions are respected. Considering a system with $x = (x_1\ x_2\ ) \in R^2$ and the differential equation is $\dot{x}_n = f_n(x) = \frac{dx_n}{dt}$, $n = 1\ or\ 2$ and in equilibrium $\dot{x}_n = 0$. We can obtain the Jacobian Matrix, as we can see below:

$$J(x) = \left[\frac{\partial f_1}{\partial x_n}\ \frac{\partial f_2}{\partial x_n}\right]$$

An equilibrium point is uniformly, globally and asymptotically stable if it satisfies the following assumptions:

1) The trace of the Jacobian matrix is negative: $(J) < 0$, $\forall\ x \in R^2$.
2) The Jacobian determinant is positive: $|J| > 0$, $\forall x \in R^2$.
3) And further assume: $\frac{\partial f_1}{\partial x_1}\left(\frac{\partial f_2}{\partial x_2}\right) \neq 0$ or $\frac{\partial f_1}{\partial x_2}\left(\frac{\partial f_2}{\partial x_1}\right) \neq 0$

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