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Special issue on the Solow-Pasinetti debate on productivity measurement

The Solow-Pasinetti debate on the measurement of productivity in the light of modern growth theory

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Abstract:

This article aims to study the Solow-Pasinetti debate on the aggregate production function and technical progress as an event that anticipated a divide in the subsequent theories of economic growth and technical change. Ultimately, the debate manifests two completely different approaches, not only to the particular question of technical change but also to political economy tout court. Pasinetti gave paramount importance to the fact that the factors of production could not be treated as symmetrical, as the neoclassical theory treated them. Related to this is Pasinetti's dismissal of the distinction neoclassical economics made between shifts in the production function to evaluate technical change and movements along it. In fact, he dismissed the neoclassical production function altogether.

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Luigi Pasinetti's works cover various topics, such as economic growth, income distribution, structural change, and international economic relations.[1] In particular, he was a great critic of neoclassical economics, which led him to build an alternative system to understand the main problems of industrial economies. This criticism of neoclassical economic theory, simultaneously with his own alternative theoretical construction, led him to propose a dynamic model for an industrial economic system in *Structural Change and Economic Growth* (Pasinetti, 1981).

In his book *Keynes and the Cambridge Keynesians,* Pasinetti affirmed that all economic theories since Adam Smith could be associated with two main paradigms. The marginalism and neoclassical theories could be associated with the paradigm focusing on exchange, utility and subjective value, and the Classical and Keynesian approaches could be associated with a paradigm focusing on production, labour and objective value (Pasinetti, 2007, pp.18-21).

Pasinetti associated these two paradigms with two different phases of economic history. The phase of commercial capitalism, when countries begin to exchange available products and thus

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[1] For a general survey of Pasinetti's trajectory, see Halevi (2016).

increase their material well-being, can be associated with the "exchange paradigm" and the emergence of mercantilist economic theories. The phase of industrial capitalism, when wealth increases progressively with technological and organisational improvements in production, can be associated with the emergence of the "production paradigm"; this began with the Physiocrats, followed by Petty and then Classical Political Economy. Industry and its continuous and cumulative technological changes become the permanent cause of the increase in society's wealth.

According to Pasinetti, the concept of trade is static by nature: it is about allocating existing resources in the best possible way. When the economy moves from a phase in which countries do not trade with each other to a new phase in which they exchange the goods they possess, each country experiences a once and for all improvement in its welfare. Mathematically, the problem is about maximising a function under certain restrictions. The concept of industry, on the contrary, is dynamic by its nature, since it is implicit that new methods are discovered through industrial production. Thus, there is a continuous learning process. In this case, any mathematical formulation must be a function of time, since the learning process is cumulative and involves a rate of change.

Pasinetti stressed his differences with the neoclassical theory of growth on several occasions (Pasinetti, 1981, 2000, 2007) but, at the same time, he admitted that neoclassical theory had a unifying principle that the alternative theories did not achieve. The theories belonging to the production paradigm – among which Pasinetti placed the Keynesian and Kaleckian short-run theories, Leontief's input-output analysis, Sraffa's production of commodities scheme, the theories of the behaviour of the firm,[2] the Harrod-Domar and other post-Keynesian theories of growth and distribution – all presented their contributions independently of one another. They lacked a unifying principle analogous to the marginal principle. To this, Pasinetti added that post-Keynesian economics did not fight back hard enough at the first attempts to absorb Keynes's approach into mainstream theory. Although the Cambridge Keynesians contributed significantly in the 1950s and 1960s in several fields of study, the spread of the Cambridge School of post-Keynesian economics came to a halt. One of the causes, as stated by Pasinetti, was that there was much criticism of mainstream economics but the amount of work necessary to build an alternative paradigm of a monetary theory of production had been underestimated. Additionally, the Cambridge post-Keynesians failed to leave a younger generation of academics following in their tracks. In this way, a concrete possibility of bringing back the production paradigm failed.

In the modern theory of economic growth, the divide described by Pasinetti started as soon as Solow and Swan absorbed the seminal contributions by Harrod and Domar into neoclassical economics. Harrod presented his dynamic theory as "a marriage of the 'acceleration principle' and the 'multiplier' theory" (Harrod, 1939, p. 14). Harrod's warranted rate of growth, $G_W = {}^{s}/_{v}$, is determined by the propensity to save, s, and the capital/output ratio, v. This line traced by G_W "represents the one level of output at which producers will feel in the upshot that they have done the right thing, and which will induce them to continue in the same line of advance" (Harrod, 1939, p. 22). Harrod also introduced the notion of the natural rate of growth, G_N, which is the maximum rate of growth allowed by the increase in population and technological improvement. Harrod showed that, when the rate of growth actually produced is different from the warranted rate, disequilibrium is self-aggravating.

Additionally, there is no inherent tendency for warranted and natural rates to coincide. If the warranted rate is above the natural rate, the economy will tend to depression and unemployment,

[2] Pasinetti refers to the theories of the firm developed after Sraffa's criticism of Marshall, followed by Robinson's and Chamberlin's theories of imperfect competition, the formulation of the full-cost principle and managerial theories of the 1960s.

and, in the opposite case, it will tend to exhibit successions of profit booms. Thus, the capitalist economy is highly unstable. Mainstream economics incorporated Harrod's contribution in the framework of neoclassical production functions, interpreting Harrod's case as the particular case of production with fixed coefficients and proposing production functions where the technological magnitude, v, was variable. The economy can adapt to the natural growth rate through the perfect substitutability of capital and labour. Conversely, post-Keynesians built theories in which the propensity to save varies and the distributive shares, P/Y and W/Y, are determined sequentially.

The Solow-Pasinetti debate on the aggregate production function that is the object of this article took place in the *Review of Economics and Statistics* between 1957, when Solow's article "Technical change and the aggregate production function" was published, and 1959, when Pasinetti responded with his article "On concepts and measures of changes in productivity". Pasinetti's response motivated a rejoinder from Solow and, in turn, a new response from Pasinetti (Pasinetti, 1959; Solow, 1957, 1959). Forty years later, in 1998, the publication of an article by Richard Stone triggered another exchange between the two authors, showing that the disagreements expressed in the fifties persisted (Stone, 1998; Solow, 1998; Pasinetti, 1998). This is not surprising since, in the end, both theories are embedded in different paradigms, i.e., neoclassical theory is embedded in what Pasinetti called the exchange paradigm and Pasinetti's theory is embedded in the production paradigm.

This article presents the Solow-Pasinetti debate of the end of the fifties as an episode that, for the first time, manifests the two different directions the theory of economic growth was beginning to take. Later, the most known manifestation of this divide was the *Two Cambridges controversy*. However, the germ of this difference can already be found in the Pasinetti-Solow debate, which was not given much importance in the literature.[3] The debate manifests two completely different approaches, not only to the particular question of technical change but also to political economy *tout court*.

In section 1, we present the Solow-Pasinetti debate on the measurement of productivity, which anticipates already the main differences between neoclassical and alternative theories of economic growth. Here, we find a first outline of what will later be Pasinetti's main criticisms of the neoclassical theories of growth and technological change, as well as some elements that will play an important role in constructing his theory of structural change and economic growth.

In the following sections, we present two aspects that condense the main points of the divide between post-Keynesian theories and neoclassical theories of economic growth and their implications. In section 2, we discuss the question of the asymmetry of the factors of production. Pasinetti contended that the theories of classical inspiration were well aware of the different natures of land, labour and capital. Neoclassical economics, on the other hand, treated all factors of production as symmetrical, which has consequences on the explanation of income distribution as well.

Section 3 presents the debate over the relevance (or not) of distinguishing between movements along the production function and shifts in the production function when discussing technical change. Pasinetti considered this distinction irrelevant. On the other hand, this distinction is the core of the explanation of technological change for neoclassical economics.

Finally, in section 4, we draw some conclusions.

[3] For a review of the debate and a reformulation of Pasinetti's formula, see Garbellini and Wirkierman (2023).

1. The Solow-Pasinetti debate on the measurement of productivity

Solow's 1957 article aimed to describe "an elementary way of segregating variations in output per head due to technical change from those due to changes in the availability of capital per head" (Solow, 1957, p. 312). This was equivalent to distinguishing between shifts in the production function and movements along it. In order to do this, Solow assumed that factors of production are paid their marginal products and there is neutral technical change, defined as a shift in the production function that leaves the marginal rates of substitution unchanged.[4] He defines the production function as:

$$Q = A(t)f(K, L) \tag{1}$$

where Q represents output, K and L represent capital and labour in physical units, and t appears to allow for technical change. The function $A(t)$ measures the accumulated effects of shifts over time.

Solow proceeded to show that, if all the factors of production can be classified as capital or labour, the sum of the relative shares of capital and labour, w_K and w_L, is equal to one. Assuming that f is homogeneous of degree one and defining $q = {}^{Q}/_{L}$, $k = {}^{K}/_{L}$, Solow obtained the equation:

$$\frac{\dot{q}}{q} = \frac{\dot{A}}{A} + w_K \frac{\dot{k}}{k} \tag{2}$$

In this case, from the series of output per man-hour, capital per man-hour, and the share of capital, it is possible to estimate the technical change index, $A(t)$.[5] Solow presented his estimate of technical change for the US economy for the period 1909-1949. Setting $A(1909) = 1$, he obtained $A(1949) = 1.809$. From this and the time series utilised, he concluded that productivity per man-hour almost doubled, and that 12.5% of that change corresponded to increased capital per man-hour[6] (K/L) and the remainder 87.5% to technical change.

In his response, Pasinetti stated that this type of analysis "although satisfactory for land and labor, is still incomplete with respect to capital, because it does not take into account the characteristic that capital is not something which comes from heaven, or from outside, but from the production process itself: it is reproducible" (Pasinetti, 1959, p. 272). This might seem an obvious assertion, but it has consequences because it implies that capital production is also subject to technical change.

To show why Solow's treatment is misleading, Pasinetti began treating the problem along traditional lines. He assumed at time t the production function $Q = f(L, K)$ and at time $t + \phi$ the production function $Q = \phi(L, K)$, where:

[4] This is the concept of neutrality proposed by Hicks: technical change is neutral if it increases the output attainable from given inputs, leaving the marginal rates of substitution unchanged. Pasinetti uses, instead, neutrality in the sense of Harrod, which considers technical change neutral if labour productivity in the consumption goods sector changes in the same proportion as overall capital productivity does. Pasinetti does not use neoclassical production functions, so the concept of the marginal rate of substitution makes no sense to him in this context.

[5] Solow also considers the case of non-neutral technical change but arrives at equivalent conclusions. In this article, we will limit the study to the simpler cases, since it does not affect the essence of the discussions.

[6] Solow calls the capital per man-hour capital intensity. Instead, Pasinetti calls this the degree of mechanisation and defines capital intensity as the capital-output ratio. This is important, because capital intensity and the degree of mechanisation do not necessarily have to vary in the same direction. Hence, variations in the degree of mechanisation are not, in general, an indicator of the variation of capital intensity (for more details, see Pasinetti, 1981, pp. 180-184).

Q: number of pieces (one piece being the quantity which was sold in the base year at $1 million)

L: man-years

K: tons of steel (one ton being equal to the quantity that was bought in the base year at $1 million)

As mentioned above, Pasinetti remarked that Solow's analysis was incomplete because, besides the production function of the consumption goods industry, it is necessary to consider the production function of the capital goods industry. Technical change might take place in either of the two sectors. If there is technical change in the capital goods industry, the same quantity of tons of steel of capital does not have the same meaning in time t or in time $t + \theta$, if at $t + \theta$ the same capital can be obtained from smaller quantities of capital or labour. This implies that, for the purpose of evaluating changes in productivity, measuring capital in physical units is misleading. Additionally, Pasinetti stresses that it is important to consider that both industries are not symmetrical, because whatever form technical change takes in the capital goods industry (neutral, labour-saving or capital-saving), its effect is always capital-saving for the consumption goods industry.

In the first simplified case he presented, Pasinetti assumed that the production of capital uses only labour and that capital lasts forever.

Now, to compare productivity between two different equilibrium situations, it is necessary to consider, besides Q/K and Q/L, the changes in the productivity of the capital goods industry, that is, K/N (where N is the quantity of labour that would be necessary to reproduce the stock of capital). To avoid the difficulties associated with the fact that the conditions of production of K in the past may have been different from the conditions at the present time, Pasinetti chose to redefine capital, K, in terms of its capacity, C.[7] Now, N is the labour required to reproduce the existing productive capacity with the technique available when observations are made. The three variables that must be explained are Q/L, Q/C and C/N. In the case considered by Pasinetti at this stage, in a neo-classical framework with full employment, the output, Q, coincides with the productive capacity of capital, C, since, in equilibrium, capital is fully employed. In this case, $Q = C$ and only Q/L and C/N are left. The first represents labour productivity in the consumption goods sector, and the second is overall capital productivity. Pasinetti presented the following directional index of change:

$$\frac{Q/L}{C/N} \tag{3}$$

The index will remain constant in time if labour productivity in the consumption goods sector changes in the same proportion as overall capital productivity. If the index increases, this means that labour productivity has increased more than overall capital productivity and technical change has been labour-saving; if the index decreases, it means that overall capital productivity has increased more than labour productivity, so the change has been capital-saving.[8] It is interesting to note that, when $C = Q$, the index (3) becomes N/L, that is, the ratio between the quantity of labour needed to reproduce the stock of capital and the quantity of labour used to produce consumption goods.

[7] This will later be at the base of his analysis in terms of vertically integrated sectors.
[8] This is neutrality in the sense of Harrod (see footnote 4, above).

After this simplified presentation, Pasinetti introduced a *numéraire* in order to obtain a way to be able to estimate technical change. The price of output is set at $P_q = 1$. Supposing equilibrium[9] from the production function $Q = f(L, K)$ and calling:

$r = \frac{\partial Q}{\partial K} K \frac{1}{P_c C}$ the rate of profit, where P_C is the price of one unit of capacity,

$a_q = {}^{L}/_{Q}$ the input of labour in one unit of output,

$a_k = {}^{N}/_{C}$ the input of labour in one unit of capacity,

Pasinetti obtained the ratio between capital and output:[10]

$$\beta = \frac{a_k}{r a_k + a_q} \tag{4}$$

The term $r a_k$ is the only difference with equation (3), and, since r tends to be constant over time, $r a_k$ does not modify the direction of change. As in the case of equation (3), technical change is neutral if β remains constant, labour-saving if β increases, and capital-saving if β decreases.

Using the index β, Pasinetti extended his analysis to the same period used by Solow and reached opposite results. While, to Solow capital intensity had increased by 12.5%, to Pasinetti capital intensity had decreased by 20%. Note that capital intensity is measured as the capital-output ratio and not as the capital-labour ratio, as in Solow. The capital-labour ratio can be considered an indicator of technical change only if C/N (overall capital productivity) changes in the same proportion (Pasinetti, 1959, p. 273). Changes in productivity cannot be measured by changes in the productivity of labour alone, since capital also intervenes in production.

Solow's comment on Pasinetti's critique and Pasinetti's response shows that they had completely different views on the relevant factors to consider when dealing with the problem of technical change.

To Solow, the traditional treatment of technical change is based on the distinction between shifts in the production functions and movements along it. Dismissing this distinction means abandoning the possibility of dealing with technical change. The notion of a production function "is meant to express the fact (if it is a fact) that any time a firm or industry or economic system faces a range of technological alternatives anyone of which might be chosen, and that given enough time[11] any previously made choice can be changed without appealing to new knowledge" (Solow, 1959, p. 283). To Solow, Pasinetti's coefficient gives accurate results only in the case of fixed proportions because he abandoned the possibility of substituting labour for capital at a given technical knowledge.

Solow took Pasinetti's proposal of considering the production of capital goods as a mere question of less aggregation.

> This doubling the number of commodities in the model increases its realism by 100 per cent. [...] It is not quite true to say that by failing to distinguish between consumables and capital goods I ignore

[9] This means that the cost of production of capital is equal to its market evaluation.
[10] For details, see Pasinetti (1959, pp. 273-274).
[11] The expression "given enough time" will be discussed by Pasinetti when he will examine the problems of choice of techniques and change of techniques.

> that capital is in fact produced. At best, we are dealing with simplified models (Solow, 1959, pp. 283-84).

Pasinetti stressed that the matter is not one of more or less aggregation but that, at any level of aggregation, evaluation of productivity cannot be made independently of the technical changes in the production of capital. Both types of technical change, in the consumption or capital goods, have different impacts on the economic system.

In his response, Solow made some examples to show that Pasinetti's measurement of technical change can vary even if there is no technical change. The first of these examples is a case where consumer goods are produced by robots alone and the robots are produced by labour alone. If, with a new method, the same quantity of robots can be produced more cheaply, then there is a labour-saving invention in the capital sector, but it is not clear why this represents a capital-saving improvement in the consumer sector. It does not save robots; it just saves abstract "waiting".[12] Solow also drew attention to the fact that Pasinetti's coefficient will change with a change in the rate of profit or with changes in savings or demand, even if there has been no technical change. Pasinetti recognised that this was true, but he started his reasoning by appealing to the fact that statistics show that the rate of profit in capitalistic economies has remained almost constant. Apart from short-run fluctuations, the largest part of changes in productivity over time are due to technical changes and not to changes in income distribution.

To Solow, the difference between both approaches was not related to capital being produced or not but to the worthiness of distinguishing between reversible substitution of the factors of production given a state of technology and shifts in technology.

Pasinetti claimed that he was not denying the existence of different possible production techniques but that, once the choice has been made, the options not chosen are irrelevant and cannot be observed.[13]

> To constrain the alternative bygone possibilities in a function that cannot but be arbitrary because non-observable is of no use for our purpose if, "given enough time [for] any previously made choice to be changed", the firm or the industry or the economic system will be faced with the same rate of interest but with an entirely new set of technical choices (Pasinetti, 1959, p. 285).

In summary, Pasinetti stated, "The contention has been that, since capital comes from the production process itself, on which technical change operates, it cannot be dealt with in the same way as labor and land" (Pasinetti, 1959, p. 281). Technological change cannot be measured by labour productivity alone, neglecting the other factors of production. Moreover, the attempts to evaluate technological change with models based on the production function of neo-classical inspiration need a physical or real notion of capital. However, the same physical unit of capital can have different meanings, according to the technical knowledge that prevails. That is the reason for redefining capital in units of productive capacity. In this way, the two ratios, Q/L and C/N, can give a complete evaluation of the nature of technical change.

[12] To Pasinetti, this is a change in N, so that the ratio of labour locked up in the production of capital to the labour used in the production of consumption goods decreases, that is, there is an overall increase in overall capital productivity. To Pasinetti, there is nothing abstract about the increase in productivity; it is a physical notion.

[13] This will be treated in section 3.

2. The treatment of the factors of production

One of the big differences between growth models of classical or Keynesian inspiration and neoclassical models is their treatment of production factors and the explanation of distribution derived from them. The first kind of growth model takes into account the different nature of the factors of production, which allows an explanation of the distribution related to the explanation of growth. On the contrary, neoclassical models consider production factors symmetrically, as variables of production functions with certain mathematical properties. The distribution ends up being explained with the resolution of some system of simultaneous equations rather than as a process.

The importance of distinguishing between the different nature of the factors of production was clear since Ricardo:

> The produce of the earth – all that is derived from its surface by the united application of labour, machinery, and capital, is divided among three classes of the community, namely, the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated.
>
> […]
>
> To determine the laws which regulate this distribution, is the principal problem in Political Economy […] (Ricardo, 1981, p. 5).

In Ricardo's explanation, capitalists save and invest out of their profits, and, in so doing, they set up a process of capital accumulation and increased demand for workers. In the long run, as a consequence of diminishing marginal returns to land, the rate of profit will decrease until there are no more savings or investments. The way to avoid this outcome is international trade (which allows food to be imported without having to resort to less fertile lands) or a technological improvement that increases the productivity of the land. In this explanation, the marginal principle is applied only to land, a non-produced factor of production. Labour and capital, instead, were associated with different social classes: wages had an explanation and profits a different one, related to the role of workers and capitalists in society. Moreover, the explanation of the growth of the economy was deeply linked to the theory of distribution.

Marginalism in the 1870s, by focusing on the problems of demand, brought back the exchange paradigm to economic theory. The basic model presupposes a set of utility-maximising individuals endowed with a set of given resources and finds the prices that lead to an optimum allocation of resources through exchange. The marginal utility approach was then extended to production, considering land, labour and capital on the same footing. The marginal utility scheme was duplicated to treat production, introducing the well-behaved production function, linear and homogeneous, with marginal productivities equating to the factor prices.

The extension of the marginal principle to capital implies that capital is formally treated like a non-produced factor: neoclassical theory treats all the factors of production *as if* they were land (Pasinetti, 2000, p. 389). Hence, as an extension of the marginal utility functions, production functions were set out as convex and differentiable. Moreover, marginal productivities had to be such that they could be substitutes for the factor prices, making the production function linear and homogeneous. There is no recognition that the nature of each factor is different and that there is a succession by which the distribution is determined.

Ricardo had thought of diminishing returns on land because it was a scarce, non-produced factor. The main objects of economic theory, as in Adam Smith, were reproducible goods:

> These commodities [non-reproducible], however, form a very small part of the mass of commodities daily exchanged in the market. By far the greatest part of those goods which are the objects of desire, are procured by labour; and they may be multiplied, not in one country alone, but in many, almost without any assignable limit, if we are disposed to bestow the labour necessary to obtain them (Ricardo, 1981, p. 12).

Pasinetti's insistence at the end of the 1950s on the fact that "capital does not come from heaven" (Pasinetti, 1959, p. 272) acquires all its relevance seen in the context of the direction taken by growth theories. Fundamentally, the underlying disagreement is on the object of political economy, which for the classical thinkers had to do with production and the distribution of the product among social classes and for the neoclassicals had to do with allocating scarce resources.

In the context of the exchange paradigm:

> […] demand for the final commodities is transformed into demand of the scarce resources. The two poles – utility on the one hand and resources on the other – had thereby been re-established, with everything in between reduced to irrelevance. For all essential purposes, the model had, so to speak, eliminated the process of production from the analysis […] (Pasinetti, 1981, p. 16).

In the production paradigm, the basis is a pure production model, with division of labour and technological progress. Since labour is specialised, each producer needs to exchange with other producers. In this way, production and exchange are linked. Prices reflect difficulties of production, but they do not carry the information necessary to achieve equilibrium in the system. There is a separation between the price equation and physical quantities systems. This model is inherently sequential in time and looks at society from the angle of the division of labour. There is no inherent mechanism that will lead to the full use of resources: a macroeconomic condition related to effective demand is required.

This was taken into account by post-Keynesian authors. In his seminal article of 1946, Domar tried to find the rate of growth G at which the economy needs to expand to remain in a continuous state of full employment (defined as a state in which productive capacity P equals national income Y). To preserve full employment, the economy must grow: if the labour force grows and productivity grows, then income must grow at a rate that combines both to maintain full employment. However, "an increase in the labor force and its productivity only raises productive capacity and does not by itself generate income (similar to that produced by investment)" (Domar, 1946, p. 138). Labour productivity is not a function of technological progress in the abstract but of technological progress embodied in capital goods. Joan Robinson, in her comment on Kalecki (1947), emphasised this aspect, stating that "to decide the most desirable rate at which to increase the productivity of labour is itself a problem, for, at any moment, a higher rate of increase in productivity requires a higher rate of investment in business capital, and a lower level of consumption or of investment in social amenities" (Robinson, 1945, p. 79). The administrative problem of controlling the rate of investment has to be discussed. To Kaldor, this implied that there should be an investment function in a model of economic growth:

> In order that there should be continued growth, it is necessary therefore to suppose both that, on the one hand, output increases as a result of capital investment and, on the other hand, investment takes place in response to an increase in output (Kaldor, 1957, p. 600).

Once a production function with neoclassical characteristics is assumed (first derivative positive, second derivative negative) and homogeneous of degree one to be able to apply Euler's theorem, then any asymmetry between the factors disappears. The factors can be substituted among themselves according to the marginal rates of substitution. By proposing the production function with the characteristics mentioned above and perfect competition, everything will lead to full

employment, and each factor of production will be remunerated according to its marginal productivity. The most utilised function with these characteristics was the Cobb-Douglas function of the form $Q = aL^{\alpha}K^{\beta}$. In this case, the parameters (which correspond to the distribution between salaries and profits) are constant variables that emerge as technological constants, so they do not change. In other words, the theory stops explaining distribution and relates it to growth.[14]

When Solow proposed his model that, along with Swan's work "Economic Growth and Capital Accumulation" (Swan, 1956), signed the absorption into mainstream economics of Harrod's formulation, he interpreted the equation $G = s/v$ as corresponding to the case of fixed coefficients of production, and he generalised it to the possibility of perfect substitution of the factors of production:

> The bulk of this paper is devoted to a model of long-run growth which accepts all the Harrod-Domar assumptions except that of fixed proportions. Instead I suppose that the single composite commodity is produced by labor and capital under the standard neoclassical conditions (Solow, 1956, p. 66).

In Solow's model, the capital-output ratio v can vary so that the actual rate of growth and the warranted rate of growth adapt to the natural rate of growth. Solow assumes a production function homogeneous of first degree, that the propensity to save s is given and the population grows at rate n. He shows that, in the long period, if there is not technical progress, the warranted rate of growth G_w coincides with the rate of growth of the population n. The market forces will also bring the actual rate of growth of the economy G to coincide with G_w. If the actual rate of growth G is higher than G_w, this means that the increase in the demand for labour is higher than the increase in its supply, causing an increase in real wages. In response, capitalists, looking to maximise their profits, will adopt more capital-intensive techniques. In that case, v will increase and the relation $G = s/v$ will decrease. The converse happens if G is lower than G_w.

This reasoning depends on two crucial assumptions. First, that there is a monotonical inverse relationship between factor prices and their physical quantities. Second, that producers can adopt techniques with arbitrarily low or arbitrarily high capital/labour proportions, so that, for any initial endowment of L and K, they can always find a technique that maximises their profits, that guarantees the full employment of resources and the convergence to a stable long-run dynamic path.[15] In this model, there is symmetry and perfect substitutability of factors, and their remunerations are determined simultaneously in a system of equations.

The alternative theories of economic growth also build on the Harrod-Domar model. Kaldor proposed a model where the magnitude that varied to adjust G to the equilibrium value was s (savings/income ratio). This was a weighted average of the propensities to save of workers and of capitalists (s_w and s_p). In the simplest case, where workers do not save, there is only one equilibrium rate of profit, $P/K = \pi = g/s_p$. This has become known as the Cambridge equation. Here, entrepreneurs decide the accumulation rate; in this way, profits increase to generate the savings necessary to finance the new investment. Pasinetti (1962) extended this equation to the case where workers' savings are positive. In both cases, profits are determined first, and wages are the residual. The order of the determination is the opposite of Ricardo's. However, the important point for our argument is that factors of production are not treated as symmetrical and that distribution is explained as a sequential process. Different social classes are considered, and

[14] For a comprehensive criticism of the interpretation of the Cobb-Douglas function, see Sylos Labini (1995).
[15] In the next section, we show that, to Pasinetti, both assumptions are incorrect.

their share in income is determined by different factors. First, entrepreneurs decide the level of investment and thus the rate of accumulation. This, together with their propensity to save s_p ,determines the rate of profit and their share in income, while wages are determined as a residual. The remunerations of capital and labour do not have to do with their marginal productivity but are determined sequentially in an economy with different social classes.

Kalecki also advanced this idea, considering the case of an economy where workers do not save. He put forward the equation, "gross profits = gross private investments + capitalists' consumption":

> What is the proper meaning of this equation? Does it mean that profits in a certain period determine capitalists' consumption and investment or the other way round? The answer to this question depends on which of these quantities is directly subject to the decisions of capitalists. Now, it is clear that they may decide to consume or to invest more in a certain short period than in the preceding period, but they cannot decide to earn more. It is therefore their investment and consumption decisions which determine profits, and not the other way round (Kalecki, 1942, p. 259).

As in the previous case, the propensity to save of capitalists is given. Capitalists decide the rate of capital accumulation and this determines the utilisation of productive capacity. While, in the previous case, the changes in the rate of profits and the social propensity to save drive the system to an equilibrium rate of growth, in this case the variation in the utilisation of productive capacity is what drives the system to the equilibrium path, not necessarily with full capacity utilisation. As in Kaldor and Pasinetti, income distribution is seen as a sequential process that begins with entrepreneurs' decisions.

Even with differences, all the authors that Pasinetti included in the production paradigm treat factors as asymmetrical and the relative determination of distributive shares as a process. Pasinetti (2000, p. 403) stated that the fact that capital and labour are not symmetrical makes the neoclassical production function inconsistent. Labour can be expressed in physical quantities (hours of labour) to which its remuneration is referred (wage per hour). Capital can be expressed in physical quantities, but its remuneration (rate of profits) is commensurate not to its physical quantity but to its value. This means that, if K is expressed in physical quantities, the partial derivative $\partial Y/\partial K$ represents the rental of capital and has to be multiplied by the price of the capital good; or, if K represents current capital value, the derivative has two terms: the variation in the physical quantity of capital and the variation in the price of capital. However, the price cannot be determined without knowing the rate of profits.

What is the sense of the concept of "marginal productivity of capital"? This was a question at the basis of the controversies in the theory of capital of the 1950s and 1960s. Pasinetti remarks that this is not an index number problem; it is a conceptual discussion on the fact that an aggregate expressed in value terms (capital) was treated in the same way as aggregates expressed in physical terms (land and labour).

> The two types of aggregate quantities do not belong to the same logical class, and can thus neither be placed on the same level nor be inserted symmetrically in the same function. This is no longer a matter of a more or less degree of approximation. It becomes fundamental and indeed abyssal conceptual diversity concerning the "factors" labour and land on the one hand, and the "factor" capital on the other (Pasinetti, 2000, p. 405).

3. The production function: shifts and factor substitution

In the debate with Solow, Pasinetti proposed not making assumptions about the form a production function could take, since, in any case, the only observable thing is a combination of factors with a resulting product at a certain point in time. To Solow, instead, this issue was fundamental when dealing with technical progress:

> To sum up, when the difference between Pasinetti's view and my more traditional one is stripped down to essentials it turns out to have nothing to do with whether capital goods are produced or not. The basic question is this: is it worth distinguishing between autonomous shifts in technical possibilities and induced reversible changes in factor proportions? (Solow, 1959, p. 285).

The same criticism towards the distinction between shifts in the production function and movements along it will be put forward by Kaldor:

> The use of more capital per worker (whether measured in terms of the value of capital at constant prices, in terms of tons of weight of the equipment, mechanical power, etc.) inevitably entails the introduction of superior techniques which require "inventiveness" of some kind, though these need not necessarily represent the application of basically new principles or ideas. On the other hand, most, though not all, technical innovations which are capable of raising the productivity of labour require the use of more capital per man – more elaborate equipment and/or more mechanical power (Kaldor, 1957, p. 595).

From this, Kaldor concluded that distinctions between movements along the production function and shifts in the production function are arbitrary and artificial. In the end, the speed at which society can develop new techniques and effectively adopt them will determine the pace of capital accumulation. In turn, the ability to accumulate capital will limit the rate at which society can exploit new techniques. As an alternative, Kaldor proposed a single relationship between capital growth and productivity growth that incorporates the influence of both factors (new techniques and an increase of capital per man-hour).

Kaldor stated that the recognition of this relationship shows the futility of regarding the capital-output ratio as depending on the technical character of the invention as neutral, labour-saving or capital-saving. What matters is the relationship between the flow of new ideas and the readiness to incorporate them into the production process.

In his book of 1981, Pasinetti dealt largely with this subject. First, Pasinetti made clear that, if the economic system is not stationary, the set of techniques relevant to the problem of the choice of techniques is different from the set of techniques that will be important to the problem of the change of technique. The problem of the choice of techniques arises only for a small part of the productive system: new investments and replacement. The techniques of the rest of the productive system have been already chosen in the past. On the other side, the change of technique is a process that takes time and does not have to do with the problem of choice. The change of technique implies a movement to a whole new set of possible techniques to choose from; these are different from the possibilities available before, because many techniques have become obsolete and new techniques have become available.

When Solow postulated his neoclassical production function, he merged both problems:

> The notion of a production function is not simply a matter of formality or convenience. It is meant to express the fact (if it is a fact) that at any time a firm or industry or economic system is faced with a range of technological alternatives any one of which might be chosen, and that given enough time any previously made choice can be changed without appealing to new knowledge (Solow, 1998, p. 283).

According to Pasinetti, the point is that, when that time comes, the set of choices available has changed, and then the problem is a new problem of choice among a different set of possibilities.

Pasinetti, in his 1981 book, treated the matter in the framework of vertically integrated sectors. The decisions are taken in each single production unit at a single point in time. At this point, there is a set of possible techniques, each one using a certain quantity of machines associated with quantities of certain inputs and man-hours. At this stage, all the prices, the wage rate and the profit rate are given. For each production unit, it will be rational to choose the method that entails the minimum cost of production. This is, to Pasinetti, a problem of minimisation.[16]

Pasinetti presents the minimisation problem as:

$$Cost\ of\ the\ chosen\ method = Min \begin{cases} p_{k_j}^{\alpha} K_j^{\alpha} + x_{n_j}^{\alpha} w \\ \vdots \\ p_{k_j}^{\omega} K_j^{\omega} + x_{n_j}^{\omega} w \end{cases}$$

$$j = 1, \dots n - 1 \qquad (5)$$

Where there are $n - 1$ sectors indicated by the subindex j, K_j^k stands for the vector of all the inputs of physical machines and intermediate commodities of technique k for vertically integrated sector j to produce the physical quantity $\bar{X}_j$; $x_{n_j}^k$ is the physical quantity of labour required in sector j with technique k, and $p_{k_j}^{\alpha}$ is the vector of prices of inputs of K_j^{α}. Pasinetti shows that each price p_{k_j} is a multiple of the wage rate w,[17] which means that the wage rate w multiplies each term of the equations. This means that the wage rate does not influence the choice of technique, since the wage rate will change the cost of each alternative method in the same proportion. The rate of profit, instead, affects only that part of labour that is indirect, and the proportion of direct to indirect labour is different in each technique. Consequently, *ceteris paribus*, a change in the wage rate will not affect the choice of technique, while a change in the rate of profit will.

This is in contrast with traditional theory, where this question has been seen as a problem of choice among different proportions of capital and labour, determined by their prices, that is, the rate of profit and the wage rate. As a consequence of this vision, a monotonic relation was postulated between the rate of profit and the proportion of capital to labour. This means that a change in the rate of profit will generate a change in the opposite sense in capital intensity. Nevertheless, Pasinetti argues that there is no way of predicting the direction of the change of technique when the rate of profit changes. A technical method α that emerges as less costly than a technical method β at the rate of profit π_1 can be more costly at a rate of profit π_2, and then α could become, again, less costly at a rate of profit π_3. These conclusions are derived from the debate on the "reswitching of techniques".

> The theoretical implications of these findings, [...] are devastating for the neoclassical theory of income distribution, since they deprive of any general applicability that relationship – assumed (let it be remembered) through the extension and analogy rather than prompted by observation! – between the price of the 'factor' capital and the corresponding quantity, whatever way is chosen to measure it. This is a property that has been taken, since the beginning, as general and characteristic of all relations of traditional economic theory (Pasinetti, 2000, p. 406).

[16] In section 13 of his chapter on "The Accumulation of Capital", Pasinetti shows that, if the rate of profit is the natural rate, then the problem is equivalent to choosing the technique that requires the minimum input of physical quantities of labour.

[17] We cannot go into detail of how Pasinetti derives this formula; for details, please see Pasinetti (1981, pp. 189-191).

To this, Pasinetti added that, in the long run, the rate of profit cannot vary by much, and neoclassical theory exaggerated its importance.

After considering the problem of the choice of technique, Pasinetti considered the question of the change in technique (what the neoclassicals call a shift in the production function). Here, Pasinetti assumed a set of "genuine production functions" (one for each vertically integrated sector) representing the actual production structure. Each of these functions expresses the production of any particular commodity as a whole series of physical inputs of labour, capital goods and intermediate commodities.

In each vertically integrated sector i, the production functions appear as follows:

$$X_i = \frac{1}{a_{n_i}} x_{ni} \tag{6}$$

where a_{ni} expresses the labour input in each physical unit of X_i and x_{ni} expresses the physical quantity of labour employed in sector i; each production function presupposes the existence of a stock of capital K_i. The set of functions X_i presupposes the existence of a series of stock of capital goods:

$$[K_1, K_2, \dots, K_{n-1}] \tag{7}$$

For these production functions, which represent the techniques that are actually in operation, all the other technically available techniques that have not been chosen are irrelevant. That, says Pasinetti, does not mean to deny that several techniques are available at each point in time. However, this is relevant only for the choice of technique function (described earlier), when each productive unit has to choose a technique for new investments or replacements. Once techniques have been chosen, they are represented in the set of production functions, and the ones that have not been chosen become irrelevant. This also means that, at any point in time, the production structure cannot be changed. It will be changed when replacement and new investments are undertaken and new techniques are chosen.

The construction of "genuine production functions" implies that, at any point in time, factors of production are complementary.[18] Neoclassical economists express the production function of sector i as:

$$X_i = F_i(K_i, x_{ni}) \tag{8}$$

where each physical quantity X_i is produced with a physical stock of capital K_i and a physical amount of labour x_{ni}. In Pasinetti's case, the partial derivatives of the function F_i would be 0 since, once the technique has been chosen, factors are complementary. In neoclassical economics, instead, the partial derivatives coincide with prices:

$$\frac{\partial X_i}{\partial K_i} = p_{k_i} \quad and \quad \frac{\partial X_i}{\partial x_{ni}} = w \tag{9}$$

At this point, the case where partial derivatives equal 0 is considered the case of perfect complementarity and the other is the case of perfect substitutability. To Pasinetti, here the analysis becomes incorrect. This is because the meaning of this function is not an alternative way to represent the genuine production functions (6) and (7), that is, the production functions that

[18] As we have shown, this does not mean that only one technique is known.

express the technology actually in use. Instead, it is interpreted as representing other techniques that can be used as alternatives to the one that has already been chosen. Pasinetti remarks that, in this way, the techniques available at a certain point in time (the choice of techniques) come to coincide with the techniques that are relevant to the process of change.

This also makes the concept of marginal productivity meaningless, even if we consider different possible interpretations for function (8).

First, if function (8) is interpreted as representing the productive structure at one point in time, the derivatives are 0. This is not a particular case, as textbooks say, but the general case. Once technology has been chosen, it cannot be changed. It does not make sense to talk about substitutability at a given point in time.

Second, if function (8) is represented as a function for the choice of technique at a certain point in time, the derivatives might not exist. The different techniques have completely different requirements of physical outputs, generating discontinuities. Besides, there is no reason to expect that a change in prices should go in different directions than the changes in quantities.[19]

So, in the end, the process of technical change comes forth from the interplay of the production functions and the choice of technique functions. Over time, each choice of technique function is enriched with new options available, with new technical methods, while others can be eliminated because they become obsolete. This does not per se change anything in the production functions. When new techniques are actually incorporated into production in the corresponding economic sector, part of the production process of this sector will change. Consequently, the technical coefficients will change, representing the new methods continually being incorporated.

4. Concluding remarks

In this article, we showed that the Solow-Pasinetti debate anticipated many aspects of the different directions taken by the theories of growth and technical change after the foundational works of Harrod and Domar. Economic growth and technical change were largely discussed in the following years, and the Cambridge capital controversy may be the most salient debate. In the following decades, Pasinetti developed his own alternative theory, incorporating vertically integrated sectors and elements from Sraffa and Leontief. Even if, in his critique of Solow, the matter was treated along traditional lines, Pasinetti made it clear that:

> As a matter of fact, it has been the purpose of the last section to frame the analysis in such a way that it can be interpreted also, perhaps much better, in other theoretical frameworks, such as the Leontief models or the dynamic growth models which pay more attention to fixed coefficients and to idle capacity. […] This is a useful property especially for comparisons of single industries, where the occurrence of fixed coefficients and the notion of capacity acquire much more precise content than for the economy as a whole and can be explicitly accounted for (Pasinetti, 1959, p. 275).

In 1998, on the occasion of the publication of an article by Richard Stone called "Changes in productivity", Pasinetti and Solow had another exchange in the journal *Structural Change and Economic Dynamics*. In this posthumous article, Stone combined Pasinetti's β coefficient with input-output analysis to obtain a coefficient for the classification of technical progress "based on data which are actually being provided by input-output analysts without any reference to the form of the production functions except at the specific points of time under comparison" (Stone, 1998, p. 231).

[19] See also Pasinetti (1977, chapter 6).

Pasinetti commented on this short article by repeating some of his concerns with the neoclassical theory of technical change (Pasinetti, 1998). The fact that technical progress takes place not only in the production of final goods but also in the production of the capital goods necessary to make those final goods has important consequences for evaluating technical progress. Thus, it is incorrect to consider both the capital-output ratio and the capital-labour ratio as indices of capital intensity. As we have shown, Pasinetti calls capital intensity the capital-output ratio, and he calls the degree of mechanisation the capital-labour ratio. This distinction was essential. The ratios will move in the same direction only in the absence of technical progress, but they might move in opposite directions as soon as technical progress comes into the picture. Pasinetti also repeated the importance he placed, in his formulation, on giving up the distinction between shifts in the production function and movements along it and on renouncing assumptions about the production functions that are not observable.

Solow's response again raised this issue:

> I found it astonishing then and find it astonishing now. [...] I would have thought that economics very often proceeds by thinking about the decisions – optimising or not – made by agents like households and firms. We cannot understand a decision without considering the range of choices that were available, or perceived to be available, to the agent when the decision was made (Solow, 1998, p. 238).

This assertion reflects a more profound disagreement. To Solow, economics had to understand the decisions made by agents among a range of available choices. This is related to the conception of economics as defined by Lionel Robbins as "the science that studies human behaviour as a relationship between ends and scarce means which have alternative uses" (Robbins, 1932, p. 15). Pasinetti, instead, defended the classical conception of economics as the study of how nations grow and distribute their income.

In his last book, *Keynes and the Cambridge Keynesians: A 'Revolution in Economics'*, Pasinetti stressed that it was necessary to go back to classical political economy and the Keynesian school, also including some authors less influenced by Keynes, like Sraffa and Kalecki. The aim had to be to build an alternative paradigm from all these contributions, a unifying theoretical scheme under the production paradigm that accounts for growth and distribution.

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