Is the natural rate of growth exogenous? A comment*

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In a recent paper (Leon-Ledesma and Thirlwall 2000 – from now on LLT) an interesting issue is raised concerning the notion of the natural rate of growth, first proposed by Sir Roy Harrod (1939).

The usual definition of natural rate of growth is: "that rate of growth of national income that equals the sum of the rate of growth of the labour force and the rate of growth of labour productivity" (Definition 1). Traditionally, these growth rates have been conceived of as constant: but what happens if the rate of growth of the labour force and/or the rate of growth of labour productivity are – as is argued in LLT[1] – increasing functions of the actual rate of growth of national income? Does the natural rate become endogenous, i.e. dependent on the actual rate of growth of national income?

In LLT this last question receives a positive answer.[2] The view underlying this position seems to be that, since the rate of growth of the labour force and the rate of growth of labour productivity are increasing functions of the actual rate of growth of national income and the natural rate is the sum of the two, the natural rate is also an increasing function of the actual growth rate.[3]

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1 On this we agree with the authors of the paper.

2 For instance on p. 442 "[...] the natural rate of growth is not exogenously given, but is very responsive to demand conditions in the economy".

3 This view emerges in particular on pp. 437-38.

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For reasons that we shall discuss below, we do not agree with these conclusions.

To see why, let us first of all consider a second definition of natural rate: “the natural rate of growth is the actual rate of growth that keeps the rate of unemployment constant” (Definition 2).

We shall show that when the rate of growth of the labour force and the rate of growth of labour productivity are both constant, these two definitions are equivalent, whilst when these growth rates depend on the actual rate of growth of national income Definition 1 must be modified as follows: “a natural rate g_n is a rate of growth of national income which equals the sum of the rate of growth of the labour force and the rate of growth of labour productivity taken at the values corresponding to g_n itself” (Definition 1bis). We shall also show that the values the natural rate can take are exogenously given and the case of more than one value must be regarded as unlikely. The normal case, therefore, is when the natural rate is not only exogenous, but also unique.

Let us now consider in detail the arguments supporting our position.

First of all, we want to clarify the connections between the definitions of natural rate given above. To this end, let us denote the actual rate of growth of national income, the rate of growth of the labour force and the rate of growth of labour productivity by respectively g, n and λ and consider *the case when n and λ are constant*. Since the level of demand for labour is equal to the level of national income divided by the productivity of labour, its rate of growth is $(g - \lambda)$. Therefore, $(g - \lambda - n)$ is the difference between the growth rate of labour demand and that of labour supply. If and only if this difference is nil, i.e.

$$(g - \lambda - n) = 0, \qquad (1)$$

the ratio between labour demand and supply remains constant and as a consequence the rate of unemployment[4] is also constant. Thus, if and only if equation 1, embodying Definition 1 of the natural rate of growth, is fulfilled, then Definition 2 is also fulfilled.

However, when the rate of growth of the labour force and/or the rate of growth of labour productivity are increasing functions of

[4] Defined as unemployment divided by labour supply.

the actual rate of growth of national income, Definition 1, if we want it to be still equivalent to Definition 2, must be modified as follows: "a natural rate g_n is a rate of growth of national income which equals the sum of the rate of growth of the labour force and the rate of growth of labour productivity taken at the values corresponding to g_n itself" (Definition 1bis).

This modification is crucial, because Definition 2 embodies essential properties of a natural rate: in particular, that an economy at full employment at time t_0 remains in such condition for any time interval (t_0, t_1) – where $t_1 > t_0$ – if and only if during that time interval the actual rate of growth of national income equals the natural rate.[5]

Let us denote the increasing function representing the relationship between $(n + l)$ and g by f, so that

$$(n + \lambda) = f(g).$$

To appreciate the importance of the restriction we have just added to Definition 1 of the natural rate, consider Figure 1, where a possible shape of function f is represented.

By following the line of the argument by which we proved that equation 1, when n and l are constant, is a necessary and sufficient condition for having a constant rate of unemployment, we can see that in this different context a necessary and sufficient condition for having a constant rate of unemployment is

$$g = f(g). \tag{2}$$

In other words, the only value of g that fulfils Definition 2 of the natural rate of growth is the abscissa of the point where f crosses the 45° line, i.e. is the solution for g of equation 2.

Suppose instead that the actual rate is g_0. Then the sum of the rate of growth of the labour force and the rate of growth of labour productivity associated with g_0, $(n_0 + l_0)$, is equal to $f(g_0)$. Since[6] $g_0 < f(g_0) = (n_0 + l_0)$, the rate of unemployment will increase: hence the actual rate of growth of national income, g_0, *is not* a natural rate of

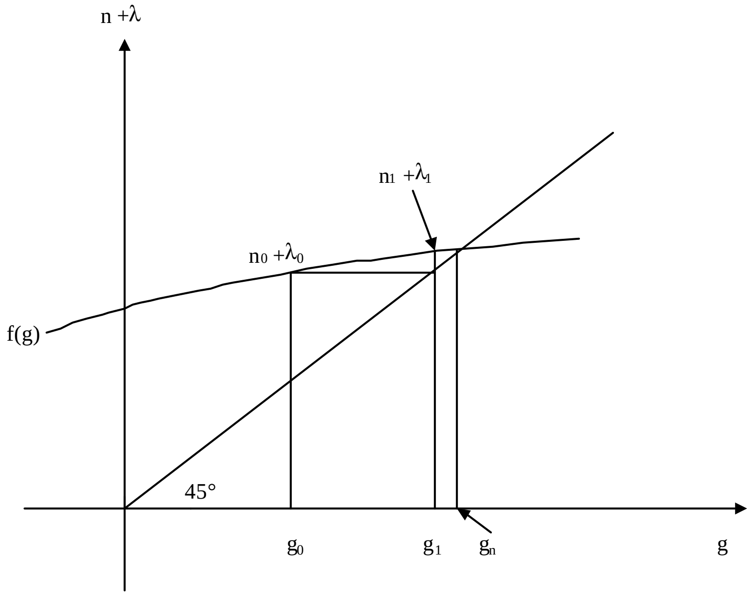
[5] That is, if the rate of unemployment, u, is zero at time t_0 and Definition 2 is fulfilled during the time interval (t_0, t_1), then $u = 0$ for all the interval (t_0, t_1).

Notice also that the empirical work contained in LLT is based on a definition of the natural rate very similar to our Definition 2: "the measured natural rate must be that rate of growth that keeps the unemployment rate constant" (LLT, p. 437).

[6] See Figure 1.

growth. On the other hand, *$(n_0 + l_0)$ is not a natural rate either*: for if g were equal to g_1 º $(n_0 + l_0)$, then $(n_1 + l_1) = f(g_1) > (g_1)$ and again the rate of unemployment will increase.

FIGURE 1



The above argument can be generalised to very different forms of function: for instance to functions – not necessarily continuous – having more than one intersection with the 45° line, step-wise functions included. In this case, of course, we may have *more than one*[7] natural rate of growth.

In general, we can conclude with the following re-wording of Definition 1bis: "a natural rate of growth is a solution for g of equation 2".

The most important consequence for our present purposes is that, although the rate of growth of the labour force and/or the rate of growth of labour productivity are increasing functions of the actual rate of growth, *the values the natural rate can take are not endogenous.* For, if the shape of function f is exogenously given, the same is also true of the solutions of equation 2: whatever happens to the actual

[7] Although this *in principle* is certainly true, we shall see below that the existence of a function making it possible seems most unlikely.

growth rate, the values the natural rate can take are already fixed and no movement of the actual growth rate can shift them!

If the above propositions are correct, what meaning can be given to the statistical results obtained in LLT?

A first explanation of these results is that they point to a multiplicity of solutions of equation 2. This possibility however raises serious difficulties: if *f* is continuous, it is necessary to explain why the effect of *g* on $(n + l)$ is less than one-to-one (i.e. a slope of *f* less than one) for certain intervals of *g* and larger than one-to-one (i.e. a slope of *f* larger than one) for certain other intervals.[8]

These difficulties are further aggravated if – as in LLT (pp. 439-40) – the validity of 'Verdoorn's Law' is accepted. This is because this relationship, linking the rate of growth of labour productivity to the rate of growth of output, usually takes the form of a straight-line function, and even if it takes a non-linear form its slope can never be larger than one.

An alternative explanation of the results obtained in LLT could simply be an incorrect use of the statistical tools. Since in LLT the details of the statistical work are not presented,[9] we do not feel it appropriate to enter here into a full discussion of this point. However, we would like to point out the serious distortions of the estimates that are likely to derive from the use of a dummy like that appearing in equation 3 of LLT.

In the Appendix at the end of this paper we develop in detail two elements of proof of this assertion:

1) We show that in 100,000 simulations with random variables the dummy appears to be almost always significant, in spite of the fact that the generating process adopted by construction does not include the dummy. We do not see any other underlying reason for this result, except a systematic bias connected with this type of dummy.

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[8] If *f* is (bounded but) not continuous the difficulty is greater: one needs to explain why $(n + l)$ undergoes a vertical jump as *g* moves from a point to the left of a discontinuity point of *f* to a point to the right of it. The case of a step-wise function raises the most serious difficulties, because it is also necessary to explain why there is no effect of *g* on $(n + l)$ outside the discontinuity points.

[9] However, thanks to the kind indication of one of the authors, they were made available to us.

2) We believe that we have discovered the likely source of this bias. For a dummy of this type, if the true regression coefficient is zero, it is most likely – as those 100,000 simulations show – that the dummy and the disturbance term are correlated and the estimate of the regression coefficient of the dummy is upward biased. This means that, although the usual tests may indicate the opposite, there is a substantial probability that the dummy coefficient in equation 3 of LLT is not significantly different from zero.

APPENDIX

1)

Let us consider the following model: the variables *z* and *x* and a disturbance term *y* are linked by the relationship

$$z = A + x + y \qquad (1A)$$

where *z* and *y* are random variables of 35 numbers, having mean and variance predetermined: for *z*, mean = 3.47, variance = 5.9, equal to those of *g* in the case of Italy for the years 1961-95; for *y* mean = 0 , variance = 4.4, equal to that of the residuals of the regression actually fitted, for the case just mentioned, by the authors of LLT.[1] *A* is equal to the mean of *z* and *x* is obtained by difference from equation 1A.

100,000 regressions, each with different random variables fulfilling the above restrictions, of the (vector) equation

$$z = a + bx \qquad (2A)$$

give, as expected, estimates of *a* @ *A* and estimates of *b* @ *1*; those of the equation

$$z = a + bx + dD \qquad (3A)$$

where $D = (D_i)$, $D_i = 1$ if $z_i > A$, $D_i = 0$ if z_i £ A, $i = 1, 2, \ldots 35$, give a frequency distribution of the estimates of *d* as in Figure 1A and a frequency distribution of the associated Student's *t* as in Figure 2A (the distributions for *a* and *b*, omitted for reason of space, have an approximate bell shape with modal values lower than those of equation 2A).

[1] OLS estimate of *g* = *a* – *b* *(D%u)* + *g* AR(1). Notice that without AR(1) the variance of residuals would be higher, hence the likelihood of simulation results favourable to the criticisms raised in this Appendix would increase.

FIGURE 1A

ESTIMATES OF *d.*
FREQUENCY DISTRIBUTION OVER 100,000 SIMULATIONS REPRESENTED BY MEANS OF KERNEL DENSITY (EPANECHNIKOV)

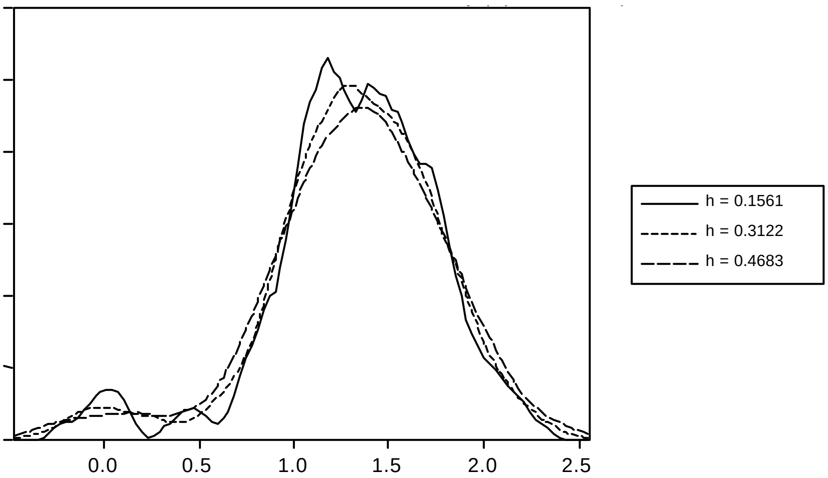
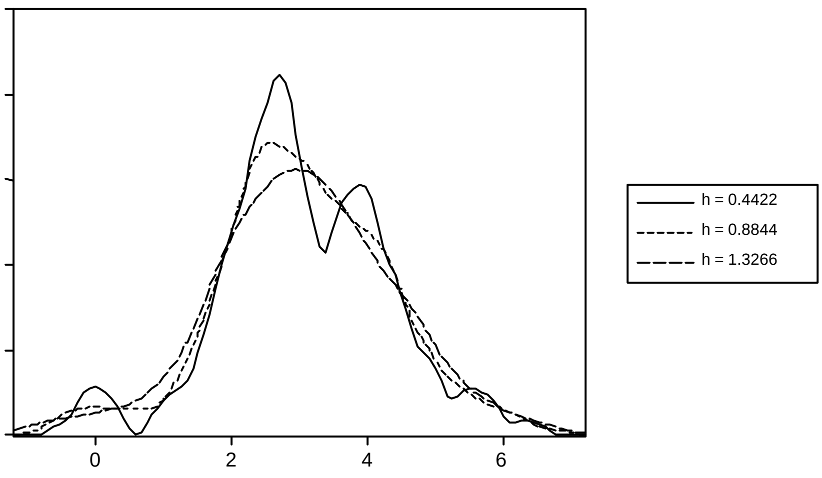


FIGURE 2A

STUDENT'S *t* ASSOCIATED WITH THE ESTIMATE OF *d.*
FREQUENCY DISTRIBUTION OVER 100,000 SIMULATIONS REPRESENTED BY MEANS OF KERNEL DENSITY (EPANECHNIKOV)



2)

Let us now consider equations 2 and 3 of LLT and add a disturbance term:

$$g = a_1 - b_1 \upsilon + e_1 \tag{4A}$$

$$g = a_2 - b_2 D - c_2 \upsilon + e_2 \tag{5A}$$

where e_1 and e_2 are the disturbance term such that $E(e_i) = 0$, $i = 1,2$ and, for notational convenience, we have set $\upsilon \equiv (\Delta\%u)$, the change in percentage unemployment rate.

Let us now assume that 4A is the correct representation of the determination of g.

Then, following the same procedure as in LLT, the dummy is formed as follows:[2]

$$D = 1 \text{ if } g > a_1,\ D = 0 \text{ if } g \leq a_1$$

Since

$$g > a_1 \Leftrightarrow a_1 - b_1 \upsilon + e_1 > a_1 \Leftrightarrow -b_1 \upsilon + e_1 > 0$$

and

$$g \leq a_1 \Leftrightarrow -b_1 \upsilon + e_1 \leq 0$$

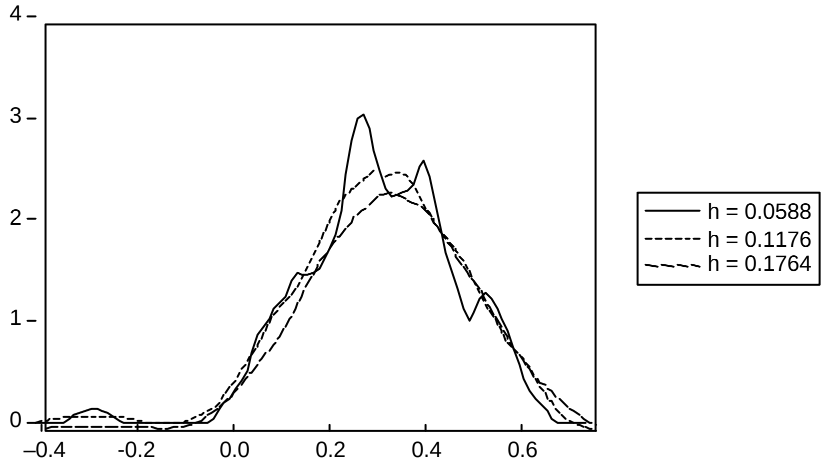
the presumption that a dummy formed in this way is most often correlated with e_1 looks strong. This presumption is confirmed by 100,000 simulations, based on the model of point 1) above and equation 3A. Figure 3A summarises the results.

The above argument is based on the assumption that equation 4A is the correct model generating g. This may not be true and equation 5A may be the correct model. The point is that there is no way of establishing, in spite of an apparently significant estimate of b_2, which one is the true model.

[2] Notice that equations 4A and 5A for notational convenience are written as scalar equations.

FIGURE 3A

CORRELATION COEFFICIENT BETWEEN DUMMY AND DISTURBANCE TERM.
FREQUENCY DISTRIBUTION OVER 100,000 SIMULATIONS REPRESENTED BY MEANS OF KERNEL DENSITY (EPANECHNIKOV)



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