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Article

Debt-led growth and its financial fragility: An investigation into the dynamics of a supermultiplier model

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Abstract:

This paper discusses the financial sustainability of demand-led growth models. We assume a supermultiplier growth model in which household consumption is the autonomous component of demand that drives growth and we discuss the financial sustainability of such dynamics of growth from the perspective of worker households. We show that, for positive rates of growth, the model converges to an equilibrium where worker households are accumulating debt and not wealth. We also show that, when the economy is growing at a rate that is positive but not too high, the model also implies that households will not be able to service their debt at the point of full long-run equilibrium. We then conclude that this household debt-financed consumption pattern of economic growth generates an internal dynamic that leads to financial instability.

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This paper discusses the financial sustainability of demand-led growth models in which household debt-financed consumption is the autonomous component of demand that drives growth. The idea of credit-financed consumption has been incorporated both into a neo-Kaleckian framework (see Dutt, 2005, 2006; Setterfield and Kim, 2016, 2020; and Hein, 2012; among others) as well as under a supermultiplier approach (see Fagundes, 2017; and Pariboni, 2016, among others). The contribution of this paper is twofold. First of all, we develop a type of supermultiplier model where workers' consumption is the autonomous component of demand that drives growth, and we show that it is possible to explain the behavior behind it, instead of just assuming it is exogenously determined. Secondly, we show that demand-led growth models for which credit-financed consumption is the autonomous component of demand that drives growth can produce steady-state dynamics that are not financially sustainable[1] from the perspective of the working households.

The ideas developed in this paper stem from the example of the US economy and its recent trajectory. Cynamon and Fazzari (2008) and Barba and Pivetti (2008) emphasized that the increase in "household indebtedness should be seen principally as a response to stagnant real wages and retrenchments in the welfare state, i.e., as the counterpart of enduring changes in income distribution" (Barba and Pivetti, 2008, p. 114). Both these authors also argue that these dynamics will interfere in the consumption behavior of households and should, therefore, be incorporated in the modeling of consumption functions.

The neo-Kaleckian approach assumes an investment-led economy as it follows the investment function described in Bhaduri and Marglin (1990). In these models, household debt dynamics has been incorporated through the definition of a workers' consumption function that allows them to consume beyond their income as they accumulate debt. However, since this approach assumes an autonomous investment function, all other components of demand, including household debt-financed consumption, must adjust to the rate of growth of capital accumulation.

For example, Dutt (2005, 2006) follows a neo-Kaleckian model and incorporates workers' consumption that is partially determined by their current income and partially financed by new loans. It is then assumed that households accumulate a stock of debt that is given by a desired level of new borrowing, which is then determined by current income and, therefore, capital accumulation. Since the accumulation of new loans is a function of workers' current income, total consumption becomes entirely determined by current income and, consequently, capital accumulation, following a typical Kaleckian investment function as developed by Bhaduri and Marglin (1990).

Another attempt to incorporate household debt-financed consumption into a neo-Kaleckian framework has been developed by Setterfield and Kim (2016). In their model, households are divided into worker and rentier households. As in Dutt (2005), workers' households are then assumed to consume partially from current income and partially from new borrowings. However, in Setterfield and Kim (2016) household borrowing is determined by a targeted level of consumption, which in its turn is determined by an emulation effect – keeping up with the Joneses – multiplied by rentiers' consumption. As a result, household consumption becomes, once again, fully determined by current income.

Palley (2010) and Pariboni (2016) emphasize that both models above require that the stock of capital and the stock of debt grow at the same rate.[2] This means that the pace of total consumption (induced plus credit-financed) must be determined by the rate of capital accumulation. In Dutt (2005, 2006) this is done through the assumption that the desired level of borrowing is determined by current income. In the model of Setterfield and Kim (2016) this is done through assuming an endogenous consumption target.

Following a supermultiplier approach, Pariboni (2016) assumes that workers' consumption has an induced part and a credit-financed part, which is autonomous from current income, and that capitalists' consumption is entirely determined by current income. In this model the autonomous component of demand, which drives economic growth, is

[1] By financially unsustainable, we mean households reaching a level of debt that they can no longer service, as is defined in Setterfield and Kim (2016).
[2] This can also be seen in other neo-Kaleckian models that incorporate credit-financed consumption, such as Hein (2012) and van Treeck (2009).

household debt-financed consumption. It is also possible to show that, in this case, the steady-state rate of growth of the economy will be given by the exogenously determined rate of growth of debt-financed consumption. "[T]his result implies that, given enough time, demand and output will tend to evolve at the rate of growth of the autonomous components of demand; in this case, workers' autonomous consumption" (Pariboni, 2016, p. 224).[3]

Finally, it is also important to mention the work of Brochier and Silva (2018), which suggests a stock flow consistent supermultiplier model where the autonomous component of demand is household consumption out of wealth. In their model, household consumption out of wealth becomes the autonomous expenditure component of demand. However, "[d]espite being autonomous (in relation to current income), it is endogenous to the model, since it depends on household wealth, so we can analyze its dynamic through household wealth dynamics" (Brochier and Silva, 2018, p. 423). In this paper, we develop a supermultiplier model of growth in which workers' consumption, financed through debt or accumulated wealth, becomes the autonomous component of demand that drives growth and we study the financial sustainability of such dynamics.

This paper is composed of two sections, besides this introduction and a conclusion. In the first section of this paper, we present our own model for the consumption function of the working household. We argue that workers' consumption is determined by a wealth target, as in Dutt (2006). However, we also suggest that their wealth target cannot be fully explained by their current income. Consequently, their consumption becomes partially autonomous from current income. We then develop a supermultiplier model of growth in which household consumption, which can be financed through debt or accumulated wealth, becomes the autonomous component of demand that drives growth.

In the second section of this paper, we discuss the long-run dynamics of our model, which also allows us to think about the financial sustainability of these patterns of growth from the perspective of the working households as they accumulate debt to maintain consumption. We find that, for most positive rates of economic growth, the long-run stability of our model requires households to accumulate debt. We also find that, for rates of economic growth that are positive but not very high, the dynamics of our model leads households to accumulate a level of debt that is not financially sustainable to them as their debt servicing becomes higher than their wages.

1. Demand-led growth and household debt-financed consumption

The models previously discussed serve as an important theoretical background to the model that will be developed in this section. In it we suggest a consumption dynamic that is driven by a net worth target, similar to what is suggested in Dutt (2006) and van Treeck (2009). However, we will also suggest that this net worth target cannot be fully explained by current income. In other words, we will assume a consumption function that is partially autonomous from current income. Consequently, in this section we develop a type of supermultiplier model in which consumption becomes partially autonomous from current income. We will also allow for this consumption to be financed through debt or accumulated wealth. However, we will focus on the dynamics where households accumulate debt, as it will allow us to discuss the

[3] That these expenditures represent financial dissaving, and are significantly financed through debt, ties in with the endogenous money approach and the credit-creating powers of banks (Fiebiger and Lavoie, 2019, p. 250).

financial sustainability of such dynamics from the perspective of the worker household, as was done by Setterfield and Kim (2020).

We start the presentation of our model by assuming a closed economy without government, for which we have three sectors: banks, firms and households. The last sector is further divided into worker households and rentier households. While worker households can only earn income from wages, rentier households own both firms and banks and will therefore earn income as a result of the activity going on in both sectors. The two tables below describe the transaction flow and the balance sheet for each sector. As can be seen below, we further assume that both workers and rentiers consume out of their income, wages and distributed profits, respectively. However, the main difference between these two types of households is that, while for rentiers we will always assume positive savings ($S^r > 0$), which is a result of having their consumption (C^r) being fully determined by their current income (FD), we will allow negative savings for workers' households ($S^w < 0$) and, therefore, a consumption (C^W) that will not be fully determined by their current income (W).

Table 1 – *Transaction flow matrix of the model*

			Firms		**Banks**		
	Workers	**Rentiers**	**Current**	**Capital**	**Current**	**Capital**	**Total**
Consumption	$-C^w$	$-C^r$	$+C$				0
Investment			$+I$	$-I$			0
Wages	$+W$		$-W$				0
Firm profits		$+FD$	$-\Pi$	$+FU$			0
Bank profits		$+FB$			$-FB$		
Interest on loans	$-r_l L_{-1}$				$+r_l L_{-1}$		0
Interest on deposits	$+r_m M^w_{-1}$	$+r_m M^r_{-1}$			$-r_m M_{-1}$		0
Subtotal	S^w	S^r		S^f			0
Change in loans	$+\dot{L}$					$-\dot{L}$	0
Deposit flows	$-\dot{M}^w$	$-\dot{M}^r$				$+\dot{M}$	0
Issue of equities		$-e\dot{p}_e$		$+e\dot{p}_e$			0
Sum	0	0	0	0	0	0	0

Table 2 – *Balance sheet matrix*

	Households		Firms	Banks	Total
	Workers	**Rentiers**			
Deposits	M^{w}	M^{r}		$-M$	0
Equities		$+ep_e$	$-ep_e$		0
Loans	$-L$			L	
Capital			K		K
Total	V_h	V_r	$K - ep_e$	0	K

1.1. An autonomous consumption function

As can be seen in the two tables above, we have assumed total consumption, C_t, to be divided into workers' consumption, C_t^W, and rentiers' consumption, C_t^r. Furthermore, we also assume that only workers' consumption has both an induced component and an autonomous component, such that:

$$C_t = C_t^w + C_t^r \tag{1}$$

$$C_t^w = c_w W_t + C_t^a \tag{2}$$

$$C_t^r = c_r \Pi_t \tag{3}$$

where c_w is workers' marginal propensity to consume, c_r is rentiers' marginal propensity to consume and C_t^a is workers' autonomous consumption financed out of endogenous credit. We then find that the autonomous expenditures, Z_t, which are neither financed by income nor affect the production capacity of the capitalist sector, are given in our model by:

$$Z_t = C_t^a \tag{4}$$

Since the focus of this paper is on the financial sustainability of working households in a debt-led growth model we will focus on workers' autonomous consumption as the driver of economic growth.[4] Following previous contributions by Dutt (2006), Pariboni (2016), Brochier and Silva (2018), Fagundes (2017) and Mandarino et al. (2020), we suggest a supermultiplier model in which households' autonomous consumption decisions are determined by a net worth target, such that:

$$C_t^w = W_t + r_m M_{t-1}^w - r_l L_{t-1} - S_t^w \tag{5}$$

with

$$S_t^w = \dot{M}^w - \dot{L} = \dot{H} = \beta(H^T - H_t) \tag{6}$$

and

[4] See, for example, Freitas and Cristianes (2020) for the development of a supermultiplier model where government spending is taken to be the autonomous component of demand that drives growth.

$$V_h = H = M^w - L \quad (7)$$

where S_t^w is households' savings, H_t and V_h are households' net worth, β is the speed of adjustment and H^T is a targeted level of net worth. In other words, we suggest that households are making consumption and saving (or dissaving) decisions so as to adjust their net worth to its targeted level. It is important to observe at this point that this net worth dynamics can easily be translated into a debt dynamic as a majority of workers' households accumulate a negative wealth, more precisely, a negative net worth, or a debt, $D_t = -H_t$.

In this model a few scenarios are then made possible. First of all, if $S_t^w > 0$, that means households' disposable income $(W_t + r_m M_{t-1}^w - r_l L_{t-1})$ is higher than households' consumption, C_t^w. As a result, workers will observe an increase in their net worth ($\dot{H} > 0$). Alternatively, if $S_t^w < 0$, that means households' disposable income $(W_t + r_m M_{t-1}^w - r_l L_{t-1})$ is lower than households' consumption, C_t^w. As a result, workers will observe a decrease in their net worth ($\dot{H} < 0$). This last scenario will result in either households taking out more loans ($\dot{L} > 0$) or consuming from previously accumulated wealth ($\dot{M}_w < 0$). The point of this general model is to admit the possibility that households can have all three types of behavior. However, as we will see in section 3 of this paper, in order to have a positive rate of growth of the economy, this model will most likely require that the majority of households are, on average, consuming beyond their disposable income and therefore accumulating debt (i.e., a negative net worth, $V_h = H < 0$).

In that sense, the consumption dynamics suggested in this paper is very close to what is developed by Dutt (2005, 2006), as we are also assuming that households that do not earn enough income for consumption will finance part of this consumption through loans but will have in mind a certain level of debt with which they are comfortable. However, since we are under a supermultiplier framework, we are not required to endogenize our debt target to current income and we can assume a wealth (or debt) target that is autonomous from current income.[5]

Once again, the idea here is that in this model we can either have households saving and accumulating wealth ($\dot{H} > 0$) and, therefore, targeting a certain level of wealth (H^T), or they can be accumulating debt ($\dot{H} < 0$), financing additional consumption through loans. In this last scenario, households will be targeting debt. It is then possible that in our model some households might be targeting for a certain level of debt, while others will be targeting for a certain level of wealth.[6] However, since this is a supermultiplier model in which the driver of growth will be determined by these dynamics, it is also possible to see that, in order for this economy to have a positive rate of growth, we will need a majority of workers' households to accumulate debt and not wealth, as will be shown in further detail in the next section of this paper.[7]

Nonetheless, one important drawback of simplifying the banking sector, as is done here, is that we have to assume that banks are passively supplying any amount of loans that working households ask for. The only role banks play in this economy is setting the interest rate charged

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[5] The point here is to suggest that households, when formulating their wealth or debt target, might take into account factors much beyond their current income.

[6] The idea here is to recognize that not all working households will accumulate debt, as some might be actually targeting for wealth. One next step might be to further divide working households into two sectors, as is suggested in Szymborska (2022), and have just one of them accumulating debt. However, this possibility is left for future research.

[7] One can also see that this will be a direct consequence of assuming $c_w > c_r$ in a demand-led growth model, which is a standard assumption for these types of models;

on loans and paid on deposits. Furthermoe, since we assume capitalists own banks, we also must assume that capitalists (rentier households) either pay out or take receipt of interest income, depending on whether worker households are net debtors or creditors. However, as can be seen in Caverzasi and Godin (2015), as well as van Treeck (2009), this is a common assumption in this type of stock flow consistency model where the focus is on households' debt dynamics. In these models, banks will have to adjust to working households' consumption behavior to provide a flow of funds if they must get in debt or receive a flow of funds if they are able to accumulate wealth.

To put it differently, capitalists' receipts are boosted by worker savings $\dot{H} > 0$, whereas their outflows are boosted by worker borrowings $\dot{H} < 0$. The reason is that, since banks are not creating money in equilibrium,[8] total savings must fund total spending that isn't directly funded by current income. This means that capitalists' plus workers' savings are being directed into new capital formation (investment) when workers are saving, with workers' savings finding their way into the corporate sector via bank deposits and, subsequently, capitalists. Meanwhile, when workers are borrowing, they are being funded by capitalists' savings – the remainder of capitalists' savings being used to fund investment.

For the purpose of illustration, let us start at an initial point where neither wealth nor debt has been accumulated by working households; in other words, we start with $H_{-1} = 0$.[9] There are then two possible scenarios. Scenario A is a scenario in which workers' consumption is less than what they earn in wages so that they end up with positive savings; in other words, $S^w = \dot{H} > 0$. In this case there will be an influx of money going to banks, which is households' accumulated wealth. However, once again for simplicity of the argument, we are assuming that banks will have only a passive behavior in this case and transfer all of the funds to rentiers' households, which own banks. This is then represented as a transfer of funds from workers' households, which put their money into banks that then "send" it to rentiers' households as deposits ($\dot{M}^r < 0$).[10]

In scenario B, workers' consumption is higher than their income (i.e., $C^w > W_t + r_m M^w_{t-1} - r_l L_{t-1}$). In this case, workers will have to ask for new loans, which will be represented by $\dot{H} < 0$. This second scenario will result in the transfer of funds from rentiers' households to workers' households ($\dot{M}^r > 0$). Furthermore, for the next iteration, under scenario B, workers' households will have accumulated debt (i.e., $H_{-1} < 0$); this means households will be paying interest on loans (i.e., $r_l L_{-1} > 0$) and banks will be paying interest on deposits to rentiers' households, such that $r_m M^r_{-1} > 0$. Now that we have detailed the behavior of workers' consumption, as well as the stock flow consistency explanation behind it, we must look at the other components of demand in order to derive a growth model for our economy.

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8 Once again, this is only a simplifying assumption of our model to allow us to focus on household debt dynamics. Since we are assuming that banks' net worth is equal to zero ($V_b = 0$), then we cannot have banks accumulating wealth or creating money in equilibrium. It is possible to add a more complicated, and realistic, banking sector into our model, but this would only make us diverge from the focus of this paper.

9 Even though the model described here is clearly a continuous time model, this discrete time illustration is suggested at this point to help clarify understanding of the model.

10 Once again, this is a direct consequence of assuming a simplified banking sector, which does not accumulate any net worth directly. In other words, if $V_b = L - M = 0$, then, if working households are able to save and deposit these savings in their banking accounts, rentiers' households, which own banks, will have to "cash out" these deposits ($\dot{M}^r < 0$) so that we can still have $V_b = 0$;

1.2. The investment function and the short-run equilibrium

Since, as mentioned before, we are assuming a closed economy without government, we must have that in equilibrium:

$$Y_t = C_t + I_t \tag{8}$$

Following a supermultiplier approach, we have assumed that workers' consumption is the autonomous component of demand, such that our consumption dynamics were described in the last section by the following set of equations:

$$C_t = C_t^w + C_t^r \tag{9}$$

$$C_t^r = c_r Y_t^r = c_r(FD + FB + r_m M_{t-1}^r) \tag{10}$$

$$C_t^w = \omega Y_t + r_m M_{t-1}^w - r_l L_{t-1} - S_t^w \tag{11}$$

where $\omega = 1 - \pi$ is the share of wages on total income. In order to simplify our calculations, we will also assume that $r_m = r_l = r$, so that banks are not making any profits, $FB = 0$, and that firms are not keeping any profits as undistributed profits, so that $FD = \Pi = \pi Y_t$. As a result, the equations above then become:

$$C_t^r = c_r(\pi Y_t + r M_{t-1}^r) \tag{12}$$

$$C_t^W = \omega Y_t + r H_{t-1} - S_t^W \tag{13}$$

Furthermore, following Freitas and Serrano (2015), we assume that the investment function, I_t, is determined by current income, Y_t, and h_t, the marginal propensity to invest, such that:

$$I_t = h_t Y_t \tag{14}$$

with

$$\dot{h} = h_t \gamma (u_t - u_n) \tag{15}$$

The equations above show that the changes in the marginal propensity to invest are determined by the adjustment of the rate of capacity utilization, u_t, to its normal level, u_n, such that Harrodian instability is not necessarily engendered by this type of demand-led growth model (see Lavoie, 2015, and Hein et al., 2012, for more details on this issue). Finally, if we define $\kappa_t = \frac{S_t^W}{Y_t}$, the ratio of change in wealth (or debt) to income, and $d_t = \frac{H_t}{K_t}$, the ratio of the total stock of wealth (or debt) to total capital, then we can arrive at the following short-run equilibrium solution of our model (see appendix A for further derivation of short-run equilibrium):

$$u_t = \frac{vr(1-c_r)d_t}{[(1-c_r)\pi + \kappa_t - h_t]} \tag{16}$$

$$Y_t = \frac{r(1-c_r)H_t - S_t^W}{\pi(1-c_r) - h_t} \tag{17}$$

where $u_t = \frac{Y_t}{Y_t^P}$ is, by definition, the rate of capacity utilization and $v = \frac{K_t}{Y_t^P}$ is, also by definition, the capital to output ratio. As we can see above, there are then three fundamental variables for the short-run determination of our model: i) the wealth (or debt) to capital ratio, d_t; ii) the new savings (or new loans) to income ratio, κ_t; iii) and the marginal propensity to invest, h_t.

From the Keynesian stability condition, we also have that the savings to income ratio must be higher than the investment to income ratio, which in the case of our model results in the following stability condition[11]:

$$\frac{S_t^W}{Y_t} + (1 - c_r)\pi > h_t \quad (18)$$

which shows that $u > 0$, the short-run equilibrium rate of capacity utilization is positive, as long as $r\frac{H_t}{K_t} > 0$. Given the short-run equilibrium described above, we can also derive a long-run steady-state rate of growth for our model, which is done in the following section.

2. Steady-state equilibrium and its financial stability

Following Serrano (1995a, b), Cesaratto (2015) and Freitas and Serrano (2015), we assume a supermultiplier model, as described in the previous section, such that in steady state the rate of capacity utilization adjusts to a normal level and $u_t = u_n$, $\dot{h} = 0$, $h_t = h$ and $\dot{u} = 0$. As shown in the supermultiplier literature, this will then imply that:

$$\frac{hu_n}{v} = g_K^* = g_Y^* = g_Z \quad (19)$$

$$g_K^* = g_Z = \frac{d/dt[r(1-c_r)H_t - S_t^W]}{r(1-c_r)H_t - S_t^W} \quad (20)$$

As in our model, $Z_t = r(1 - c_r)H_t - S_t^W$.[12] In other words, the steady-state rate of growth of our economy is determined by household consumption decisions. More precisely, if households are on average consuming beyond their income, the driver of growth for our economy will be the accumulation of debt by working households. This is not a surprising result, given that we are assuming a supermultiplier model in which households' credit-financed consumption is the autonomous component of demand that drives economic growth.

Furthermore, as has already been emphasized, the aim of this paper is to look at the financial stability of this model. In order to do so, we must now turn our attention to the two variables defined in the previous section, d_t and κ_t. The first one, $d_t = \frac{H_t}{K_t}$, is the ratio of the stock of net worth over the stock of capital and the second one, $\kappa_t = \frac{S_t^W}{Y_t}$, is the ratio of the flow of savings over the flow of income. This means that in a scenario where worker households are actually accumulating debt, d_t will represent the ratio of the stock of debt over the stock of capital, while κ_t will represent the flow of new loans over the flow of income.

Therefore, in steady state it is interesting to look at the behavior of d_t, which will allow us to do an analysis of the financial stability of the worker households in this model, similar to the analysis in Setterfield and Kim (2016). First, it is worth mentioning that we can arrive at the following steady state relationship between κ_t and d_t[13]:

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[11] From the definition of savings, we must have $\frac{S_t}{Y_t} = \frac{S_t^W}{Y_t} + \frac{S_t^r}{Y_t} = \frac{S_t^W}{Y_t} + (1 - c_r)\pi$;

[12] See the appendix for further derivation of the steady-state rate of growth of the economy.

[13] We arrive at this result by taking $u_t = u_n$ and $h_t = h$ in equation (16) above.

$$d_t = \frac{u_n[(1-c_r)\pi + \kappa_t - h]}{vr(1-c_r)} \tag{21}$$

This is the variable we will be exploring further in the next subsection to study our steady-state equilibrium and household financial stability. However, before we do that, it is important to emphasize that $Z_t = r(1 - c_r)H_t - S_t^W$ is our autonomous component of demand. All of the other growth variables will have to adjust to its rate of growth in the steady state. This means that d_t will have to change as g_Z changes, which allows us to analyze the financial stability of the worker households as is done in the following section. However, we must emphasize a final simplifying assumption that we will be making for the reminder of the paper, which is that at steady state the difference between net worth and its target is given and grows at an exogenously given rate. We do think that an interesting exercise would be to look further into this dynamic and see what would happen if we assumed different specifications of the autonomous component of household consumption. However, we thought this exercise was out of the scope of this article, which aimed at looking at the financial instability of the dynamics, and we left it, therefore, for future research.

Finally, it is interesting to observe what will happen if we assume that households actually reach their wealth target in the long run. In this case, our steady-state position will be described by:

$$\dot{u} = 0 \text{ as } g^* = g_K^*$$

$$\dot{h} = 0 \text{ as } u^* = u_n \tag{22}$$

$$S_t^W = \dot{H} = 0 \text{ as } H_t = H^T$$

Additionally, we will observe a $\kappa_t = 0$ and an economy that has no growth as $Z = \dot{H} = 0$. Also, for this reason, we have decided to just assume that the difference between net worth and its target is given and grows at an exogenously given rate, as is done in the following section to analyze the financial sustainability of our steady-state dynamics.

2.1. Steady-state equilibria and household financial stability

As was previously mentioned in the introduction of this paper, one important aspect of the Setterfield and Kim work (2016, 2020) has been to emphasize the issue of the financial sustainability of their model of growth from the perspective of the worker household.

Given the equations of their model, Setterfield and Kim (2016) emphasize that, in order to determine a full long-run equilibrium, we must take into account the dynamics of the debt to capital ratio, d_t. They argue that, since this ratio will vary endogenously as the economy grows and workers accumulate debt, it then becomes important to understand these debt dynamics and their implications for growth. This is done by analyzing the long-run steady-state behavior of the debt to capital ratio, which in the model developed here has been defined as the ratio of net worth over capital, $d_t = \frac{H_t}{K_t}$.

Setterfield and Kim (2016) then show that from the dynamics of their model we can derive $\dot{d}$ as a function of d_t, which results in a quadratic function, with two steady-state equilibria. If we then define d^{max} to mean a worker's maximum feasible debt servicing payment, it is possible to discuss the relationship between the steady state d_t that guarantees the equilibrium of $\dot{d} = 0$ and d^{max}. This will then help us determine the actual feasibility of the steady state from the financial perspective of the worker household.

Following Setterfield and Kim (2016) we can depart from the definition of $d_t = \frac{H_t}{K_t}$ and look at the steady-state dynamics of our debt to capital ratio. We then depart from the definition of the rate of growth of the ratio of net worth to capital and we get that:

$$\hat{d} = \widehat{H} - \widehat{K} \tag{23}$$

$$\dot{d} = \frac{\dot{H}}{K} - g_K d \tag{24}$$

Replacing in equation (22) variables previously defined, we get that:

$$\dot{d} = \frac{\kappa_t u_n}{v} - g_K d \tag{25}$$

Equation (23)[14] can be represented by a linear function between $\dot{d}$ and d_t. We can then replace the parameters of the model for reasonable values, as determined in table 3 below, and observe the behavior of $\dot{d}$ as a function of d_t. To start the illustration of our argument, we first assume that: i) the profit share is equal to 40%, $\pi = 0.4$; ii) rentiers' marginal propensity to consume is 0.2, $c_r = 0.2$; iii) the annual real interest rate is equal to 2%, $r = 0.02$; iv) a normal rate of capacity utilization at 80%, $u_n = 0.8$; v) a capital to output ratio of 2.5, $v = 2.5$; and vi) a rate of growth of the economy of 3%, $g_Z = 0.03$.

Table 3 – *Values of parameters used for simulations*

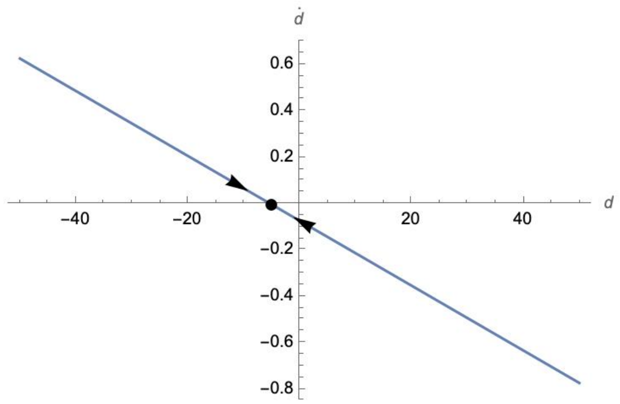
Parameter	**Value**	**Relevant interval**
r (Interest rate)	0.02	0.01 – 0.1
π (Profit share)	0.4	0.4 – 0.6
v (Capital-output ratio)	2.5	1.5 – 4
u_n (Normal rate of capacity utilization)	0.8	0.7 – 0.9
c_r	0.2	0.1 – 0.3

As can be seen in table 3 above, these initial chosen values follow what is commonly found in these types of simulations of demand-led growth models (see, for instance, Setterfield and Kim, 2020; Brochier and Silva, 2018; Fazzari et al., 2020; and Ferri and Tramontana, 2020, on this). In section 3.3 of this paper, we will also present the results of simulations for different values of these parameters within the relevant intervals as suggested in the third column of table 3 above. But first the initial simulations will use the values of parameters as presented in

[14] We also know that, under steady state, we must have $\kappa_t = h - (1 - c_r)\pi + \frac{d_t v(1-c_r)r}{u_n}$, $h^* = \frac{g_Z u_n}{v}$ and $g_K^* = g_Z$. Once again, for simplicity of the argument, we have assumed g_Z to be exogenously given for this financial sustainability presentation.

the second column of table 3 and look at the financial sustainability of our model for the different rates of growth of the economy. Assuming then the fixed values described above, as well as a rate of growth of the economy exogenously given at 3%, we get that the behavior of $\dot{d}$ as a function of d_t will be given by the following dynamics:

Figure 1 – *Dynamics of $\dot{d}$ and d_t when the rate of growth of the economy is equal to 3%*



In figure 1 above, the negative slope of the function tells us that the system will converge to a point of full equilibrium, in other words to the point where $\dot{d} = 0$, if it starts from out of equilibrium. Additionally, we also have that the point where $\dot{d} = 0$, that is, the point of full steady-state equilibrium, is given by $\frac{H_t}{K_t} = -5.17$. This means that the steady state in this case will converge to a long-run point of full equilibrium where households are, in aggregate, accumulating a stock of debt that is significantly larger than the stock of capital.

As a second example, we now assume that $\pi = 0.4$, $c_r = 0.2$, $r = 0.02$, $u_n = 0.8$, and $v = 2.5$, as we did before, but that $g_Z = 0.10$; in other words, that the rate of growth of the autonomous component of demand and, therefore, of the economy is equal to 10%. Under this scenario, we then get that the behavior of $\dot{d}$ as a function of d_t is given by:

It is interesting to observe that, at a higher rate of growth, the relationship is still negative, which means the system converges to equilibrium at $\dot{d} = 0$, which is now the point where the ratio of the stock of wealth to the stock of capital is given by $\frac{H_t}{K_t} = -0.03$. This means that, under this scenario, the long-run equilibrium point still happens with households accumulating debt but at a ratio that is proportionally lower than when the economy was growing at 3%, which gave us $\frac{H_t}{K_t} = -5.17$ at full long-run equilibrium.

Finally, if we assume, once again, $\pi = 0.4$, $c_r = 0.2$, $r = 0.02$, $u_n = 0.8$ and $v = 2.5$ but that $g_Z = -0.05$, a negative rate of growth, then we arrive at the following behavior of $\dot{d}$ as a function of d_t:

Figure 2 – *Dynamics of* $\dot{d}$ *and* d_t *when the rate of growth of the economy is equal to 10%*

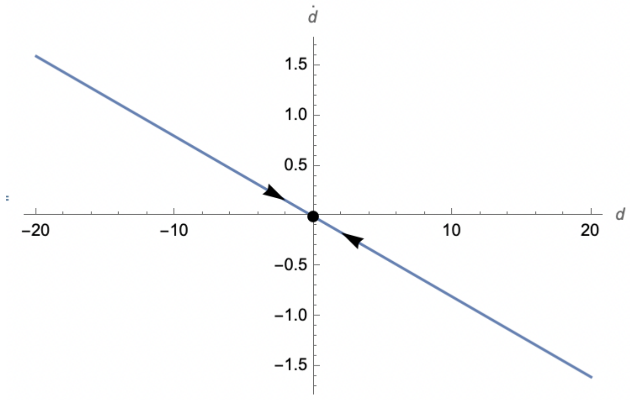
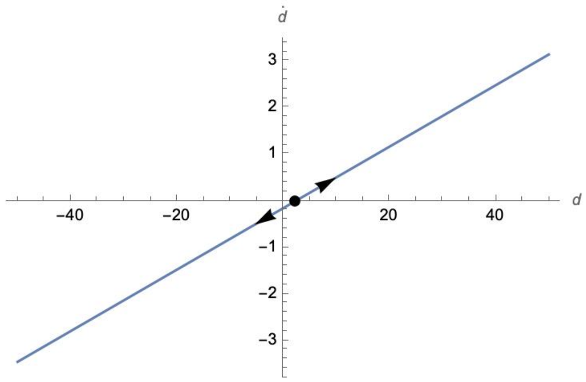


Figure 3 – *Dynamics of* $\dot{d}$ *and* d_t *when the rate of growth of the economy is equal to -5%*

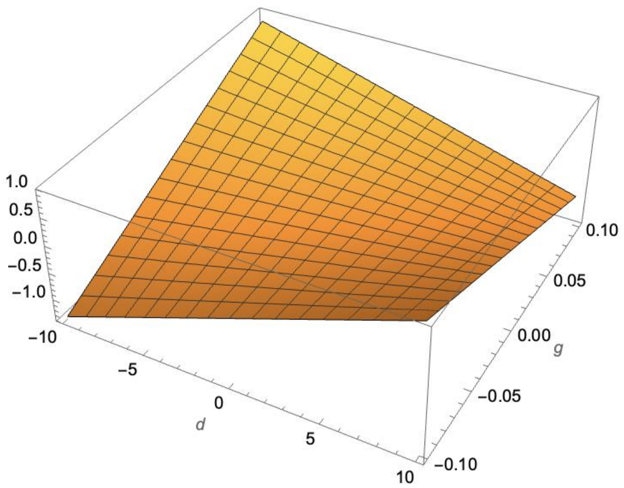


It is interesting to observe that the relationship now becomes positively inclined, meaning that the dynamics is unstable and it will not converge to an equilibrium point where $\dot{d} = 0$. We also arrived at the point of full equilibrium, which is given by a positive d_t = 2.31, meaning households are now accumulating wealth, as opposed to debt, at the point of full equilibrium. However, this is not a stable equilibrium and, as a result, households will not converge to it if they start out of equilibrium.

2.2. Household financial stability under different growth scenarios

As an additional exercise for this section, we present below an analysis of how the dynamics of the ratio $d_t = \frac{H_t}{K_t}$ changes with the different rates of growth. We assume once again that $\pi = 0.4$, $c_r = 0.2$, $r = 0.02$, $u_n = 0.8$, and $v = 2.5$ and we plot below $\dot{d}$ as a function of both d_t and g_Z such that:

Figure 4 – *Dynamics of* $\dot{d}$ *and* d_t *for different rates of growth of the economy*



In the graph above, g_Z varies from -0.10 to 0.10 and d_t varies from -10 to 10. We can also see that, for negative rates of growth, we get a positively inclined relationship between $\dot{d}$ and d_t, which means the dynamic is unstable. Meanwhile, for positive rates of growth – and higher than 0.02 as we will see in figure 5 below – we get a negatively inclined relationship, which means that the dynamics is stable and the system converges to equilibrium. However, as we have seen, the equilibrium is obtained at a negative ratio of d_t, which means households are accumulating debt as opposed to wealth.

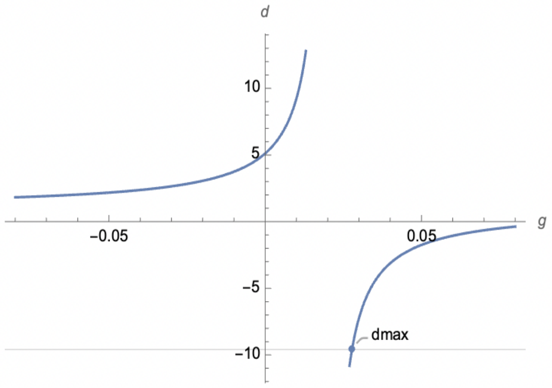
In the final graph below, we present the equilibrium value of d_t for the different rates of growth.[15] At this point it becomes important to know the d^{max}, the maximum amount of debt with which worker households can actually operate. For that we follow Setterfield and Kim (2016) and we turn back to the initial consumption function of the worker household, which was:

$$C_t^w = W_t + rH_t - S_t^w \quad (26)$$

If we assume that workers are not consuming or taking out new loans and are using all their income to service their debt, then we can find a maximum ratio of stock of debt over stock of capital, which is given by[16] $d^{max} = \frac{(1-\pi)u_n}{vr}$ in the case of our model. Assuming, once again, the parameter value as defined in the second column of table 3 above, we can then calculate the value of d^{max} to be given by -9.6. This value is plotted in figure 5 below as a horizontal line representing the financial stability threshold for d, our debt to capital ratio. As can be seen in the figure below, the value estimated for d^{max} is actually above the equilibrium point of $\dot{d} = 0$ for positive rates of growth of Z below 2.4%. This means that, for these rates of growth, the debt to capital ratio dynamics converges to a point of full equilibrium at which households are accumulating a stock of debt that is unsustainable.

[15] The derivation of the function that is plotted in figure 5 can be found in appendix A, section A3.
[16] This is the equation that we obtain assuming that workers' consumption and savings are equal to zero, such that workers' income is all directed towards servicing debt.

Figure 5 – *Equilibrium values of d_t, which is the ratio of the stock of debt over the stock of capital, y-axis, for different rates of growth of the economy, x-axis*



Consequently, we can infer from the discussion above that, if the economy is following the dynamics of growth described above, where household debt-financed consumption is the autonomous component of demand that drives a positive, but not too high, growth, households will not be able to service their debt. In other words, if the economy is following the dynamics of growth described above, then the economy will necessarily tend to a point of full equilibrium at which households cannot even financially sustain debt servicing. It is also interesting to observe that the dynamics becomes unsustainable in its full long-run steady state for rates of growth of the economy that are between 1.6% and 2.4%, which is a significant scenario of growth for different economies in the world.

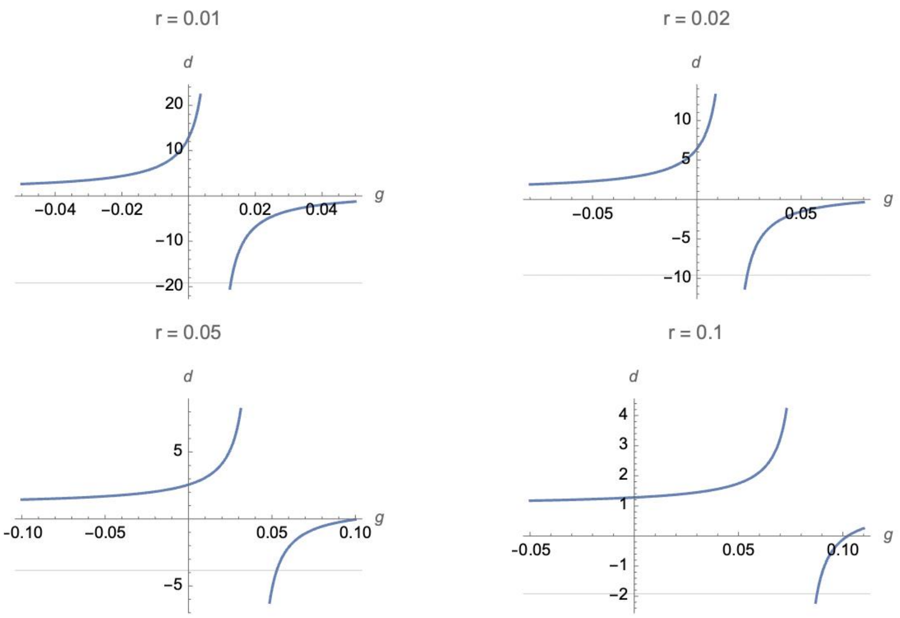
As a final step in our financial instability analysis, the following and final subsection discusses how the results presented above change when the values of the relevant parameters of the model change.

2.3. Household financial stability analysis for different parameter values

Additionally, to show that the analysis presented above was not the result of a specific choice of parameters, in this section we test if these results hold across relevant intervals. In table 3 above we reported the relevant intervals for the different parameters of our models.

We then start this analysis by first looking at the interest rate and how it changes the instability dynamics. In figure 6 below we present the same dynamics described in figure 5 but with different values for the interest rate. In the four graphs of figure 6 we can observe three changes in the dynamics as interest rates increase. First, the rates of growth of the economy for which the point of full steady state still requires households to accumulate wealth, as opposed to debt, increases. For example, we can see that, when the rate of interest equals 10%, the point of full equilibrium is at a positive level of d_t, indicating that households are accumulating wealth and not debt.

Figure 6 – *The interest rate and the change in the debt instability analysis*



A second pattern that we can observe in figure 6 above is that, as the interest rate increases, the rates of economic growth for which the steady-state position requires households to accumulate a level of debt which is unsustainable also increase. Thirdly, we also observe that, as the interest rate increases, d_{max} decreases in absolute terms. All of the three dynamics described above were expected and also show that the main results hold. Despite the increase of the interest rate, full steady-state equilibrium still requires households to accumulate a stock of debt that is not sustainable for households when the economy is growing at positive, but not too high, rates. In appendix B we also report the graphs for changes in all of the other parameters, which will also not significantly affect the results obtained above. Table 4 below points out some interesting results from this illustration exercise.

Table 4 – *Summary of main findings from illustration exercise*

Case	**Scenario description**	$d^* > 0$	$-\infty < d^* < d^{max}$	$0 > d^* > d^{max}$
Baseline	$\pi = 0.4$; $c_r = 0.2$; $r = 0.02$	$g_z < 0.016$	$0.016 < g_z < 0.024$	$g_z > 0.024$
Increase in interest rate	$\pi = 0.4$; $c_r = 0.2$, $r = 0.05$	$g_z < 0.04$	$0.04 < g_z < 0.053$	$g_z > 0.053$
Increase in profit share	$\pi = 0.6$; $c_r = 0.2$; $r = 0.02$	$g_z < 0.016$	$0.016 < g_z < 0.035$	$g_z > 0.035$
Decrease in rentiers consumption	$\pi = 0.6$; $c_r = 0.1$; $r = 0.02$	$g_z < 0.018$	$0.018 < g_z < 0.027$	$g_z > 0.027$

Table 4 above shows the rates of growth consistent with each scenario. For the first baseline scenario, if the economy is growing less[17] than 1.6% the steady-state position of our debt to capital ratio (d^*) is positive, indicating households are accumulating wealth. In a second scenario, if the economy is growing at rates of growth between 1.6% and 2.4%, then households on average will be required to accumulate a stock of debt that is financially unsustainable, as $-\infty < d^* < d^{max}$.

In a third scenario, when the rate of growth of the economy is higher than 2.4%, households are still required to accumulate a certain level of debt, which is nonetheless financially sustainable. Finally, table 4 also illustrates that changes in the parameters might slightly change the intervals of financial instability, but the main result still holds, i.e., that this type of debt-led growth model requires households, on average, to accumulate a level of debt that is financially unsustainable when the economy is growing at rates of growth that are positive but not too high.

4. Conclusion

This paper suggested a demand-led growth model in which household consumption is the autonomous component of demand that drives growth. In our model, worker household consumption becomes partially autonomous from current income, as it can be financed through credit or accumulated wealth. We followed a supermultiplier approach, as it allowed us to think of credit-financed consumption as the autonomous component of demand that drives growth. We also showed that the type of household credit-financed consumption suggested in Dutt (2005, 2006) and van Treeck (2009), in which households adjust their consumption so as to reach a targeted level, is also compatible with a supermultiplier framework. However, in the case of our model, we do not need to endogenize this target, as household consumption can be taken as the autonomous component of demand that drives growth. Finally, this model also allowed us to think about the financial sustainability of these patterns of growth from the perspective of the working households as they accumulate debt in order to maintain a certain level of consumption.

Once we derived our model, we then turned to its long-run equilibrium properties, focusing on the financial sustainability of the model from the perspective of the worker households. We first found that, for most positive rates of economic growth, our model requires that the worker household accumulates debt, and not wealth, in the long run. Secondly, we also find that, for positive rates of economic growth that are positive but not very high, the dynamics of our model leads households to accumulate a level of debt that is not financially sustainable, as their debt servicing becomes higher than their earned wages. As a third and final point, we have also shown that these results also hold for different values of parameters, within the acceptable range defined by the demand-led growth literature.

In conclusion, this paper argues that this household debt-financed consumption pattern of economic growth generates an internal dynamic that leads to financial instability. In our model, positive rates of economic growth require worker households to reach a level of

[17] In this model we are allowing for negative rates of growth in the economy, as can be seen in figures 4 to 6. However, if we were to look at just positive rates of growth, as done in most of the supermultiplier literature, then the interval of growth for which $d^* > 0$ would just be given by $0 < g_z < 0.016$.

indebtedness that is not sustainable, as their wages will eventually become lower than required to service their debt.

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Appendix A – Further derivation of the model

Section A1 – Derivation of the equilibrium rate of capacity utilization, $\boldsymbol{u_t}$

$$\frac{Y_t}{K_t} = \frac{C_t}{K_t} + \frac{I_t}{K_t}$$

$$\frac{Y_t}{K_t} = \frac{C_t^W}{K_t} + \frac{C_t^r}{K_t} + \frac{h_t Y_t}{K_t}$$

$$\frac{u_t}{v} = \frac{W_t}{K_t} + \frac{rH_t}{K_t} - \frac{S_t^W}{K_t} + \frac{c_r \pi Y_t}{K_t} - \frac{c_r r H_t}{K_t} + \frac{h_t u_t}{v}$$

$$\frac{u_t}{v} = \frac{(1-\pi)Y_t}{K_t} + rd_t - \frac{S_t^W}{Y_t}\frac{Y_t}{K_t} + \frac{c_r \pi Y_t}{K_t} + \frac{h_t u_t}{v}$$

$$\frac{u_t}{v} = \frac{(1-\pi)u_t}{v} + r(1-c_r)d_t - \kappa_t \frac{u_t}{v} + \frac{c_r \pi u_t}{v} + \frac{h_t u_t}{v}$$

$$\frac{u_t}{v}[\pi + \kappa_t - c_r\pi - h_t] = rd_t$$

$$\frac{u_t}{v} = \frac{r(1-c_r)d_t}{[\pi(1-c_r) + \kappa_t - h_t]}$$

Section A2 – Derivation of the steady state rate of growth $\boldsymbol{g}$

$$Y_t = C_t^W + C_t^r + I_t$$

$$Y_t = (1-\pi)Y_t + rH_t - S_t^W + c_r \pi Y_t - c_r r H_t + h_t Y_t$$

$$Y_t = \frac{r(1-c_r)H_t - S_t^W}{[\pi(1-c_r) - h_t]}$$

$$\frac{dY}{dt} = [\frac{d(r(1-c_r)H_t - S_t^W)}{dt}][\frac{1}{[\pi(1-c_r) - h_t]}] - [\frac{d[\pi(1-c_r) - h_t]}{dt}][\frac{r(1-c_r)H_t - S_t^W}{[\pi(1-c_r) - h_t]^2}]$$

If we define $Z_t = r(1-c_r)H_t - S_t^W$ and since $\dot{h} = 0$ in steady state, then we have that:

$$\dot{Y} = \frac{\dot{Z}}{[\pi(1-c_r) - h_t]}$$

$$g^* = \frac{\dot{Y}}{Y} = \frac{\dot{Z}}{Y[\pi(1-c_r) - h_t]} = \frac{\dot{Z}}{Z} = g_Z$$

Section A3 – Derivation of a function for $\boldsymbol{d_t}$ when $\dot{\boldsymbol{d}} = \mathbf{0}$:

From $\dot{d} = 0$ in equation (25), we have that:

$$g_k d_t = \frac{\kappa_t u_n}{v}$$

$$d_t = \frac{\kappa_t u_n}{v g_k}$$

$$d_t = \frac{(h - (1 - c_r)\pi) u_n}{v g_k} + \frac{d_t v (1 - c_r) r}{v g_k}$$

$$\frac{d_t (g_k - (1 - c_r) r)}{g_k} = \frac{(h - (1 - c_r)\pi) u_n}{v g_k}$$

$$d_t = \frac{(h - (1 - c_r)\pi) u_n}{(g_k - (1 - c_r) r) v}$$

Appendix B – Further changes in parameters

Figure B.1. – *Change in the debt instability dynamics for different income distribution and rentiers' consumption*

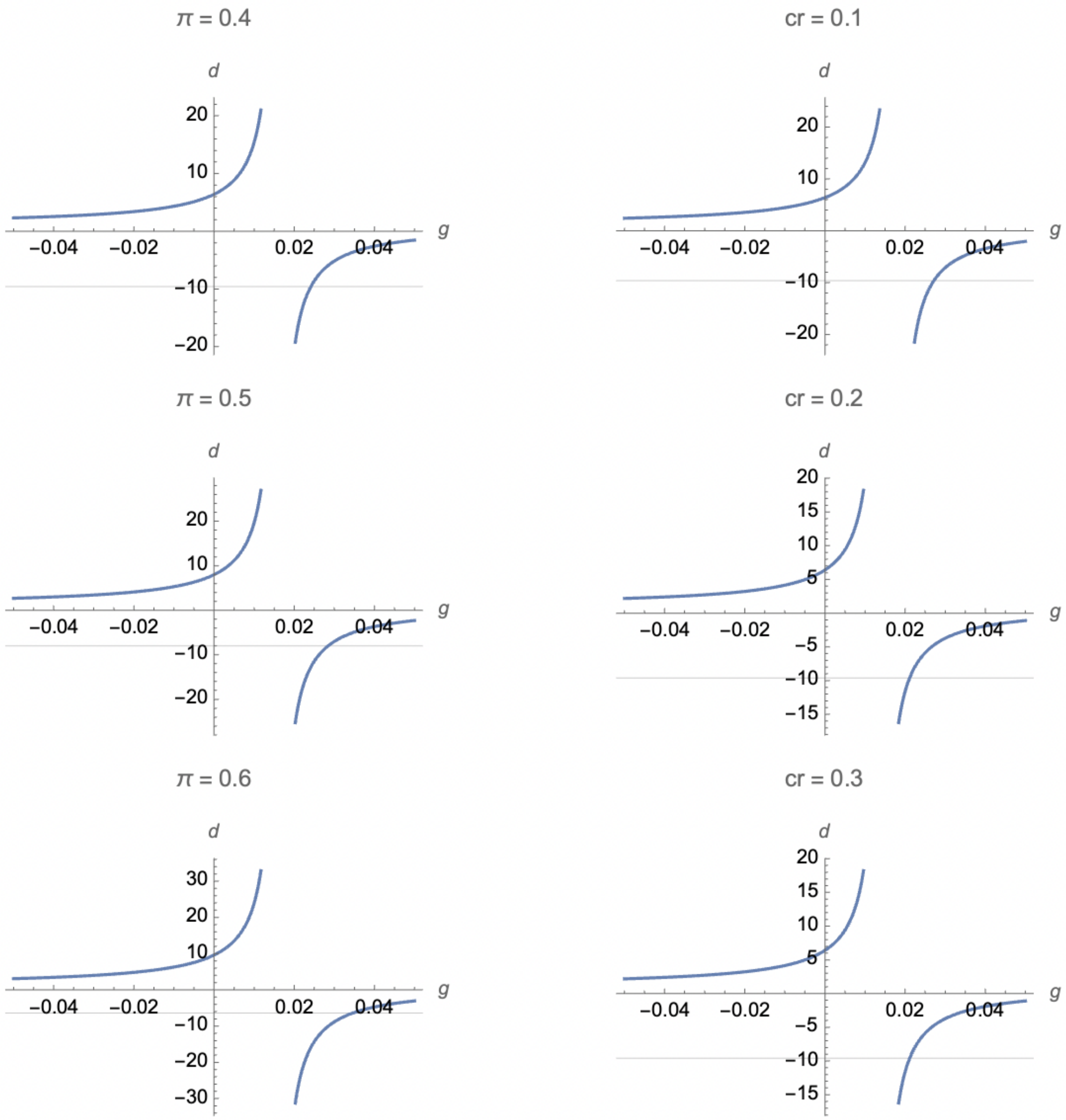

Figure B.2. – *Change in the debt instability dynamics for different normal rates of capacity utilization and capital-output ratio*