# An index for sustainable development

## ENRICO CASADIO TARABUSI and PAOLO PALAZZI

## 1. Introduction

International and intertemporal comparison of levels of development is a notoriously hard task. Use of the per capita income parameter, in both the standard version and the version with Purchasing Power Parity (PPP), is by now widely considered an unreliable proxy of the measure of the development level of a country. In fact, development is commonly regarded as a multidimensional variable, of which income is only one - albeit important - component that, nevertheless, needs to be taken along with various aspects of human life other than mere consumption and control of goods and services. Of the many attempts to construct more complex indicators taking into account the manifold aspects of development, the best known is the institutional Human Development Index (HDI), proposed in several versions by the United Nations Development Programme, which, although much criticized, had the merit of introducing the concept of multidimensional development into the institutional and media world.

In the present work several development indicators – some making use of the statistical technique of principal components - are computed with the purpose of proposing not so much new measurements of development as new methods of incorporating the concept of sustainability into them. In the non-specialized and scientific literature the expression *sustainable development* customarily refers to the definition applied in the famous Brundtland Report (World Commission on

<sup>□</sup> Università degli Studi di Roma "La Sapienza", Dipartimento di Matematica

<sup>&</sup>quot;G. Castelnuovo", Roma (Italy); e-mail: casadio@mat.uniroma1.it; Università degli Studi di Roma "La Sapienza", Dipartimento di Scienze Economiche, Roma (Italy); e-mail: paolo.palazzi@uniroma1.it.

Environment and Development 1987) known as *Our Common Future*, which identifies it with the capacity to satisfy the needs of the present generation without jeopardizing the possibility of doing so for future generations. Sustainability level measurements had been attempted through re-computation of income, taking into consideration several negative aspects (from the environmental, social and life quality standpoints) linked to the growth process; see the pioneering works of Nordhaus and Tobin (1973) and Daly and Cobb (1989) and the recent survey by Lawn (2003). Some variables, whose values are available for a large enough number of countries, will be included in our measurements of development in order to represent some of the aspects of environmental and social sustainability. We see the introduction of this set of variables as important because some of them show negative correlation with the variables commonly used to measure economic development.

Indeed, the concept of sustainable development has since its introduction combined purely environmental aspects with matters of social equity, along with fight against poverty, and education and health for all, and these are exactly the variables that make up the HDI besides per capita income. The concept of sustainability that we adopt in this work is that of 'sustainable society', introducing in the measurement those features of development that describe the possibility for it to be stable. What we propose here is to assess the possibility of introducing a way to evaluate sustainability that enables modification, or correction, of the measurement of development level by penalizing those situations for which the social sustainability is lower. In our opinion sustainability is necessarily linked to a theoretical definition of dynamic balance among the various aspects (positive or negative) of development: the level of sustainability is maximal along a certain equilibrium locus in the variable space, and becomes smaller as the distance from such a locus increases.

In dealing with sustainable society we need to define what we take to be a balanced social situation. In general we shall say that a social situation is balanced if all the aspects characterizing development have reached similar levels. This general definition derives from the obvious observation that, since human development consists of several aspects, each of which is non-negligible, it is impractical to assume complete substitutability among them: hence a concept of balance is introduced for practical purposes. In a preceding paper

Palazzi and Lauri (1998), starting from criticism of the assumptions implicit in the construction of HDI, such as those of complete substitutability and equal weighing of the three aspects chosen for development, proposed a development level computation method to recompute the development index through identification of a balance locus, including a penalty that increases with distance from that locus. The present work develops these concepts further and proposes a more general and sophisticated computation method for a balanced social situation level for a large number of countries. In this work the aspects of development that we shall take into consideration are economic, environmental and social.

The plan of the work is: in section 2 we begin by introducing the variables we chose to describe the three aspects – economic, environmental and social – and the countries for which they are all available. We then go on to describe how to transform each of them so that its scale will be uniform throughout its range of values. For each variable we:

- 1) take logarithms in an appropriate way, depending on the classification of the variable into one of a small number of categories according to its *ex ante* nature;
- 2) change the sign if the expected correlation with its relevant aspect of development is negative;
- 3) (globally optionally) transform it non linearly with a function from a given toolbox in order to minimize the least-square distance of its quantile function from affine (i.e., straight line) ones.

The values thus transformed will be used as input for each of the syntheses described in the sequel.

In section 3 we describe how two kinds of synthetic indices of the development level can be constructed. The first (subsection 3.1) is obtained using the statistical method of principal components in two steps:

1) first we take the principal component separately within the group of variables relative to each of the three aspects of development taken into consideration. We shall regard the value of the first component as a proxy of the level that each country has achieved in that aspect;

2) on the three synthetic values thus obtained we repeat the principal component analysis, and again take the first component as the indicator of the overall development level of each country.

A similar procedure was proposed by Palazzi (1997). A different kind of indicator (subsection 3.2) can be obtained with a method able to identify a locus of sustainable development in the space of variables, and to penalize the development level of each country according to its lack of sustainability. This is achieved by:

- 1) rescaling each of the variables, as obtained in section 2, to a neighbourhood of the real interval [0,1];
- 2) further transforming it with a suitable concave function whose parameters can be calibrated;
- 3) assigning a weight to each of the three groups of variables, as well as a weight to each variable relative to its group;
  - 4) then taking the weighted arithmetic mean of the results.

In view of all the steps listed above for this procedure, and in particular of the rescaling to (a neighbourhood of) the same interval [0,1], the sustainability locus is taken to be simply the diagonal line in the space of variables. This procedure satisfies a minimal set of axioms that improves on the one proposed by Chakravarty (2003). As we shall see, the ranking of countries according to their level of development shows significant differences from the ranking achieved with methods that do not take balanced development into account.

### 2. Variables

The N=39 variables used in this paper (of which 10 economic, 7 environmental and 22 social), whose values refer to the year 1998, are listed in Table A.1, while the n=126 countries appear as the leftmost column in Table A.2, both in the Appendix. As is well known, when it comes to analysing developed and underdeveloped countries simultaneously, the choice of variables and countries is constrained by data availability. Since our main goal is to illustrate a computation method

that introduces sustainability into the evaluation of development, we chose to consider a large number of countries, which means having fewer variables available. This may entail an incomplete representation of the three aspects of development chosen, even if, given the computation method, the inclusion of additional variables would hardly yield significant changes in the results.

All the synthesis methods we describe include the following sequence of common preliminary transformations on each single raw variable  $x_j$ , where j = 1, ..., n indicates the country.

- 1) We first transform the variable so as to improve its metric homogeneity, or in other words so that like additive increments of the transformed values, independently of their initial level, have like significances for the aspect described by the variable. We distinguish three cases:
- a) if the variable (e.g., the per capita income) can only take values greater than a fixed constant l (usually l=0) but otherwise unrestricted, then we subtract l and take the decimal logarithm of the result. (In fact, if the variable does take the extreme value, in order to avoid infinite values in a homogeneous fashion we preliminarily add to every  $x_j$  the difference d between its second smallest value and l itself.) Thus, as is well known, equal differences of the logarithms correspond to equal ratios of  $x_j l$ ;
- b) if the variable can only take values between two fixed constants l and L (a percentage is usually of this kind, with l=0 and L=100), then we take the decimal logarithm of the ratio  $(x_j-l)/(L-x_j)$ . (As before: add the same d as above to the numerator if the variable takes the value l; add D, the difference between L and the second largest value of the variable, to the denominator if the variable takes the value L.) In this way, like differences of these logarithms correspond to like ratios of  $x_j-l$  for values of  $x_j$  close to l, or ratios of  $l-x_j$  for values of l close to l;
- c) if the variable (unlike all those used in this article) can take unlimited positive and negative values, then we leave it untouched.
- 2) On the result of the above operation we perform an inversion of sign (i.e., multiply by -1) if the variable (such as the infant mortality rate) is deemed to have an *ex ante* negative correlation with

balanced and sustainable development. No sign inversion will be applied if the *ex ante* correlation is positive.

3) At this stage we can perform a (globally) optional transformation whose purpose is to 'straighten' the quantile function of the variable, in order to mitigate its two tails and the effect of outliers on the forthcoming synthesis processes. Among the functions within a given toolbox (we chose  $f_{c,t_0}(t) = (1/c)$ asinh  $c(t-t_0)$ , with  $c \ge 0$  and  $t_0$  real numbers, with the limit case  $f_{c,t_0}(t) = t-t_0$ ) we apply to the variable (transformed according to the preceding instructions) one that makes the quantile function of the result fit best with an affine (i.e., straight line) function, or in other words that minimizes the ratio of the residual sum of squares (the smallest sum of squares of the differences between such quantile function and a line) over the regression sum of squares (the sum of squares of differences of the quantile function with its average). (The reason for the denominator is to make the ratio scale-invariant. We set 0/0=0 in the limit case of a constant variable.)

Step 3 will be skipped unless otherwise stated.

### 3. Indices

# 3.1. Principal components

Use of the principal components method to analyze phenomena with multidimensional features is very common in social sciences, although somewhat less so in economic sciences, where preference often goes to quantitative analysis tools based on the verification of causal relation assumptions. In our opinion, in some cases, including development measurement, descriptive methods are better suited to analyze a complex phenomenon like development without the need for hypotheses of causal relations.

The method used here to obtain a development index yielding a meaningful ranking of countries is to perform a principal component analysis for each of the three groups of variables separately. The component – denoted by P:E, P:S, P:N – along the first axis can be regarded respectively as a proxy of the level of economic or social devel-

opment, or the degree of environmental sustainability of each country, and is therefore used here as a synthetic variable for the pertaining aspect (economic, social, or environmental) of development. (The ensuing variances amount to 40.3%, 54.7%, 60.0%, respectively.)

A further principal component analysis was performed on the three synthetic variables obtained above, again taking the component along the first axis, which will be used as a synthetic index, denoted by P:G (for 'general'), of the overall development for the countries under consideration. The results are listed in Table A.2 in the Appendix. (The variance is 87.2%, a high value due to the small number of synthetic variables used.) Also shown are the values of the overall development index, labeled P-s:G, obtained with the same kind of analysis on data that were subjected to the optional straightening of the quantile functions described as Step 3 in section 2.

# 3.2. Concave average

As explained in the Introduction, the main goal of this work is to take balance or sustainability into account in the measurement of the development level. We elaborate on previous attempts by Palazzi and Lauri (1998) and by Gentili (2001) to propose here a more complex, but at the same time sounder and more complete method able to perform this task organically and automatically.

The development sustainability considered here stems from the assumption of non replaceability among various aspects of development, in that only development that takes place with harmony among them can be considered sustainable in the course of time, and hence of a higher quality. Our attempt is to measure such sustainability implicitly, giving it a lower value in correspondence to a greater distance of the country point from a locus of ideal balance in the variable space; and to incorporate this measurement in the development index by penalizing the latter with the lack of sustainability. To this end the choice of variables is crucial, and we therefore included in the proposed method the possibility of differential weighing of variables, thereby enabling the researcher to evaluate exogenously and subjectively the relevance of single variables in their capability to describe development.

We first assign positive weights to the three groups of variables considered to reflect development (economic, environmental and social), in order to calibrate the respective influence on the global index: for the sake of simplicity we choose 1 for all three. We also assign a positive weight to each single variable relative to the group it belongs to; differential weighing within a group may be used to reflect the importance that some variables are deemed to have with respect to others, or (as is done here) to acknowledge a certain degree of redundancy of some variables without the need to discard them. Finally, the weight  $w_i$  of a variable relative to the set of all variables is the product of the above two weights divided by the sum of weights of the variables in the same group. (Note that further division of each  $w_i$  by a positive constant, such as the sum of all  $w_i$ 's, would not affect the results.)

The next step is to rescale the variable, as obtained in section 2, to a neighbourhood of the real interval [0,1], in order to have an absolute range for comparison of variables. To this end we apply the affine function f(t) = (t-m)/(M-m), where M and m are the maximum and minimum value, respectively, of the straight line function that best fits – as explained in Step 3 of section 2 – the quantile function of the variable. Taking these instead of the extreme values of the variable itself has the advantage of mitigating the effect of outliers on rescaling, although some of the resulting values may lie outside the interval [0,1].

With the above considerations on balanced and sustainable development in mind, we then postulate that the ideal balance among the N variables occurs when they are all equal; in other words, the ideal balance locus is the diagonal straight line passing through the points (0, ..., 0) and (1, ..., 1) in the N-dimensional real variable space  $\mathbb{R}^N$ .

We now decrease (or 'penalize') each value  $y_j$  of each variable, as transformed according to the procedure described in the previous section, by  $ae^{-by_j}$  where a and b are positive reals that need not be equal for all variables (although we chose a=b=1 for all variables in our calculations). We finally set the value of the index for a certain country to be the weighted average of these decreased values over all the available variables. The function  $f(t)=t-ae^{-bt}$  (i.e. the one we apply to each variable, possibly with different a and b) is defined and smooth on the whole real axis, strictly increasing, strictly concave and asymptotic to t as  $t\to\infty$ . Therefore the function of N variables that returns the proposed synthetic index in terms of the individual transformed variables is given by the weighted average

$$F(t_1, ..., t_N) = \sum_{i=1}^{N} w_i f(t_i) / \sum_{i=1}^{N} w_i$$

and is defined and smooth on the whole  $\mathbf{R}^N$ , strictly increasing with respect to each variable separately, strictly concave, and asymptotic to  $\sum_{i=1}^N w_i t_i / \sum_{i=1}^N w_i$  for large  $t_1, ..., t_N$ . Indeed each of these properties can be given an axiomatic justification:

- the function F must be defined for every n-tuple of variables (not only those with entries between 0 and 1), because, due to the rescaling previously described, there is no ex ante upper or lower bound on the possible values of outliers, although most of the transformed values do lie in the interval [0,1];
- the function F must be strictly increasing with respect to each variable separately, because if country A has a greater value for a given variable than country B, but A has the same value as B for all the remaining variables, then A has a strictly higher level of development than B;
- the strict concavity of *F* reflects the non replaceability of variables and penalizes progressively their unbalances, and also accounts for a more-than-proportional penalization for low values of the variables;
- the higher the values of all the variables, the smaller (in fact, asymptotically vanishing) the total penalization.

The values of parameters a and b in the above formulas determine the difference of marginal penalization for smaller versus larger values, respectively the penalization of unbalances, and can be calibrated accordingly. Note that, besides the final weighted average, all manipulations of variables for this method occur individually – which makes the computational tasks significantly easier than joint elaborations, especially for large numbers of variables – yet the method incorporates non replaceability and penalization of unbalances. On the other hand, the range of variables after penalization, as well as that of the final index, is not determined ex ante. As for the principal components method, the results are listed in Table A.2 in the Appendix, with a coding similar to that for the principal component method: C:E, C:N, C:S will stand for the indices relative to the economic, environmental and social variables, respectively, whereas C:G will refer to all variables globally and C-s:G will also include the straightening of the

quantile functions. A similar set of axioms was proposed by Chakravarty (2003) for a synthetic index obtained from a given set of individual variables, although with shortcomings, such as the need for every variable to have an upper and a lower bound (if they are to be the maximum and minimum, then the procedure is overly dependent on these two particular values, that are often outliers) and the lack of assumption of extensibility and smoothness of the indicator function beyond [0,1] (which loses robustness around the extreme values).

## 4. Results

By introducing sustainability we can more accurately adjust the development indices obtained from a group of variables aiming to describe various aspects of development. A way to assess the results of this operation is to compare the rankings that arise from different measurements of the degree of development of countries. Table 1 (refer to Table A.3 in the Appendix for abbreviations) shows the comparison among the rankings obtained (on the basis of data from year 1998) respectively from: per-capita GDP; HDI; each of the four indices (P:E, P:N, P:S, P:G) obtained via the principal component method; each of the four indices (C:E, C:N, C:S, C:G) obtained with the concave average method; each of the two global indices (P-s:G, C-s:G) obtained with the optional straightening of the quantile function.

TABLE 1

# RANKING

Countries	lGDP	HDI	P:E	P:N	P:S	P:G	P-s:G	C:E	C:N	C:S	C:G	C-s:G
United States	1	3	11	111	110	9	120	5	114	13	15	19
Norway	2	2	16	92	120	13	118	10	87	10	6	7
Switzerland	3	13	3	49	116	20	100	2	62	16	1	1
Iceland	4	5	6	89	114	12	110	4	96	1	3	3
Denmark	5	15	14	99	122	11	116	14	125	6	21	20
Canada	6	1	9	109	124	4	123	7	111	2	7	5
Japan	7	9	1	78	115	1	124	1	79	12	2	2
Belgium	8	7	7	116	109	3	113	20	126	21	56	49
Austria	9	16	12	70	113	19	108	8	76	20	5	6
Australia	10	4	15	114	118	6	119	9	124	8	19	11
Netherlands	11	8	5	107	123	2	122	13	120	7	18	18
Germany	12	14	2	100	111	5	121	3	116	15	12	13
Ireland	13	17	22	88	93	27	102	19	104	34	26	25
France	14	12	21	73	119	23	101	24	102	14	22	26
Finland	15	11	24	83	126	14	112	22	82	4	8	8
Sweden	16	6	13	67	125	18	105	16	81	3	4	4
Italy	17	18	18	76	107	21	107	18	84	28	14	17
United Kingdom	18	10	8	105	121	7	117	6	122	9	16	14
Israel	19	20	19	113	92	17	109	21	119	33	35	35
Spain	20	19	10	82	112	15	114	12	105	17	13	16
Portugal	21	22	17	71	98	29	98	17	73	31	9	9
Slovenia	22	23	25	80	106	24	104	25	75	18	10	10
Greece	23	21	30	87	101	28	103	28	91	37	27	30
Korea, Rep.	24	25	4	110	97	10	125	11	108	38	25	24
Czech Rep.	25	26	20	120	117	8	126	26	121	11	28	34
Argentina	26	27	31	79	81	44	83	38	100	51	69	56
Barbados	27	24	53	56	91	50	77	52	65	5	11	12
Hungary	28	31	45	93	105	32	96	51	109	32	62	70
Saudi Arabia	29	53	28	122	64	35	89	15	101	77	60	55
Slovak Republic	30	30	33	96	108	25	106	33	89	19	24	22
Chile	31	28	27	90	87	33	94	27	92	43	33	33
Uruguay	32	29	37	61	85	48	78	39	97	49	63	59
South Africa	33	73	55	94	59	54	72	58	78	60	76	77
Mauritius	34	50	39	38	79	60	65	42	67	54	38	36
Malaysia	35	42	26	98	76	38	87	31	83	56	44	54
Mexico	36	38	54	84	66	55	75	49	86	63	78	75
Estonia	37	33	32	124	103	16	115	36	113	24	51	42
Poland	38	32	41	121	100	22	111	35	118	25	57	57
Trinidad & Tobago	39	36	36	126	84	26	99	29	110	39	45	51
Croatia	40	35	44	86	96	39	92	41	93	36	41	53
Brazil	41	52	35	60	77	52	74	37	80	52	53	52
Russian Federation	42	43	50	118	94	30	97	46	107	42	72	62
Lithuania	43	37	43	97	99	34	95	59	112	26	65	58
Turkey	44	62	34	85	63	49	82	43	98	74	88	98
Gabon	45	88	60	42	37	82	49	34	33	99	47	66

TABLE 1 (cont.)

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Countries	lGDP	HDI	P:E	P:N	P:S	P:G	P-s:G	C:E	C:N	C:S	C:G	C-s:G
Botswana	46	87	73	37	38	86	41	44	26	91	30	27
Colombia	47	47	40	64	70	57	70	55	59	57	52	61
Costa Rica	48	34	59	44	80	64	61	66	54	44	39	31
Venezuela, RB	49	46	42	123	65	43	85	50	106	66	92	96
Latvia	50	44	49	81	102	37	91	60	88	22	40	45
Romania	51	45	70	103	88	45	81	81	95	46	90	95
Thailand	52	54	29	66	74	46	79	32	56	47	23	23
Tunisia	53	71	52	63	68	61	67	47	69	50	46	47
Panama	54	40	58	72	75	58	69	64	58	48	50	40
Iran, Islamic Rep.	55	68	63	104	61	56	73	79	85	65	104	91
Bulgaria	56	41	66	119	95	36	90	102	117	40	113	116
Algeria	57	76	78	77	48	72	55	71	50	80	77	71
Dominican Rep.	58	63	38	68	52	62	66	30	72	78	58	48
Belize	59	39	65	47	67	69	57	62	48	61	54	39
Kazakhstan	60	51	81	112	83	51	76	88	94	45	94	82
Lebanon	61	59	46	117	82	40	88	54	115	55	96	100
Paraguay	62	58	74	35	46	83	42	69	40	84	67	60
Peru	63	57	48	43	60	66	62	45	49	71	49	44
Macedonia, FYR	64	48	57	115	86	42	86	56	103	35	66	67
El Salvador	65	74	64	39	58	75	51	67	53	69	68	69
Swaziland	66	80	76	21	40	91	32	61	22	92	36	32
Philippines	67	55	51	52	54	67	60	53	57	59	55	50
Guatemala	68	86	84	36	41	87	39	82	45	101	108	106
Guyana	69	67	62	41	55	76	50	57	41	67	42	37
Jamaica	70	60	56	106	78	47	80	48	77	30	29	28
Georgia	71	49	80	58	90	63	64	93	55	27	43	41
Jordan	72	65	72	95	62	59	68	76	71	79	97	83
Morocco	73	89	69	55	53	73	52	68	63	73	75	72
Cape Verde	74	75	67	7	42	96	37	65	6	87	17	21
Ukraine	75	56	75	125	104	31	93	75	123	29	99	84
China	76	70	23	91	73	41	84	23	74	41	20	15
Egypt, Arab Rep.	77	85	68	75	47	70	58	73	52	85	87	90
Ecuador	78	64	61	74	57	65	63	70	66	72	80	76
Sri Lanka	79	61	82	30	72	81	47	74	32	58	32	38
Syrian Arab Rep.	80	79	83	101	45	71	59	97	90	98	126	125
Albania	81	66	102	31	71	89	40	87	31	62	48	43
Zimbabwe	82	93	91	65	34	88	38	91	51	90	106	110
Indonesia	83	78	47	48	50	68	56	40	44	70	34	29
Honduras	84	81	77	46	43	78	48	86	61	82	103	102
Papua New Guinea	85	94	100	26	31	99	28	89	10	88	31	63

TABLE 1 (cont.)

Countries	lGDP	HDI	P:E	P:N	P:S	P:G	P-s:G	C:E	C:N	C:S	C:G	C-s:G
Kyrgyz Rep.	86	69	112	54	69	84	44	120	42	53	98	93
Bolivia	87	82	71	57	51	77	53	72	38	75	59	68
Nicaragua	88	83	86	45	44	80	43	84	64	76	91	97
India	89	91	79	69	39	79	45	80	70	81	100	103
Moldova	90	72	87	102	89	53	71	103	99	23	86	88
Angola	91	115	93	24	12	102	24	78	13	118	74	85
Guinea	92	117	111	8	10	117	9	95	8	111	61	65
Ghana	93	92	109	25	36	100	26	111	25	83	81	80
Lao PDR	94	99	101	6	30	112	14	92	9	114	64	64
Pakistan	95	96	90	51	33	90	36	106	60	100	125	126
Vietnam	96	77	94	40	56	85	46	105	43	68	89	92
Lesotho	97	90	88	16	28	101	25	63	20	94	37	46
Cote d'Ivoire	98	110	97	50	23	95	30	98	47	103	116	121
Mauritania	99	104	92	53	16	92	35	83	29	116	105	112
Mongolia	100	84	96	108	49	74	54	115	68	64	115	115
Cameroon	101	95	106	22	24	108	18	99	28	107	102	107
Gambia, The	102	116	107	19	32	106	22	112	21	93	85	81
Haiti	103	106	113	23	15	110	16	110	24	113	112	111
Togo	104	102	108	27	21	105	21	104	37	115	120	123
Bangladesh	105	103	95	29	35	98	31	77	39	86	79	74
Senegal	106	111	89	34	27	97	29	85	35	106	101	108
Cambodia	107	97	119	4	29	121	8	108	7	102	70	79
Nepal	108	101	98	12	13	111	19	100	18	96	71	73
Uganda	109	104	117	2	8	122	4	116	2	112	73	101
Kenya	110	98	110	32	17	103	23	114	27	89	95	94
Burkina Faso	111	125	105	14	3	116	13	94	19	120	93	87
Benin	112	113	118	28	25	109	17	117	34	105	123	118
Chad	113	120	122	1	2	126	1	119	4	126	111	113
Congo, Rep.	114	108	85	15	19	104	27	90	16	117	83	86
Nigeria	115	107	99	59	22	93	33	101	46	110	122	124
Mozambique	116	121	104	10	7	115	12	96	12	119	82	78
Madagascar	117	100	123	11	18	118	6	124	11	109	110	104
Niger	118	126	125	9	1	124	2	125	5	123	119	122
Yemen, Rep.	119	105	103	62	26	94	34	107	30	108	114	109
Zambia	120	109	114	33	14	107	20	118	36	97	118	120
Rwanda	121	119	120	5	4	123	5	113	1	124	84	89
Guinea-Bissau	122	122	116	20	9	113	15	109	15	125	117	119
Ethiopia	123	124	115	13	5	119	10	121	14	121	121	114
Burundi	124	123	124	3	6	125	3	123	3	122	107	105
Malawi	125	118	126	17	11	120	7	126	23	95	124	117
Tanzania	126	112	121	18	20	114	11	122	17	104	109	99

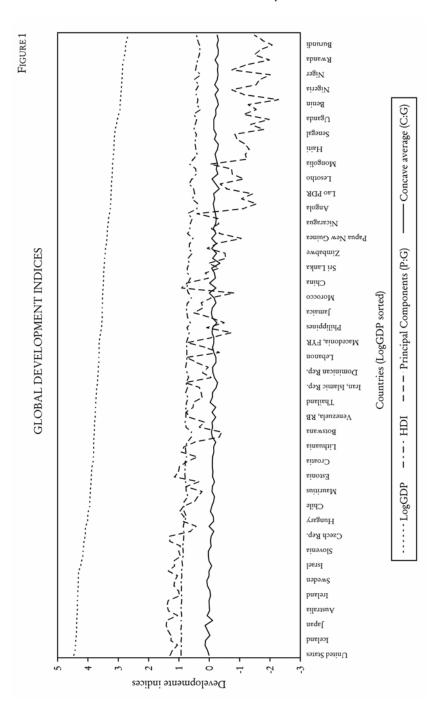
TABLE 2
PEARSON PRODUCT MOMENT CORRELATION

	lGDP	HDI	P:E	P:N	P:S	P:G	P-s:G	C:E	C:N	C:S	C:G	C-s:G
lGDP	1.00											
HDI	0.94	1.00										
P:E	0.94	0.91	1.00									
P:N	-0.71	-0.77	-0.74	1.00								
P:S	-0.91	-0.96	-0.88	0.80	1.00							
P:G	0.91	0.95	0.94	-0.91	-0.96	1.00						
P-s:G	-0.92	-0.94	-0.94	0.89	0.96	-1.00	1.00					
C:E	0.93	0.85	0.97	-0.64	-0.80	0.86	-0.86	1.00				
C:N	-0.79	-0.84	-0.81	0.95	0.87	-0.94	0.93	-0.70	1.00			
C:S	0.88	0.94	0.85	-0.77	-0.97	0.93	-0.93	0.78	-0.84	1.00		
C:G	0.80	0.74	0.80	-0.32	-0.70	0.65	-0.66	0.86	-0.38	0.75	1.00	
C-s:G	0.80	0.77	0.80	-0.36	-0.73	0.68	-0.68	0.85	-0.42	0.77	0.98	1.00

TABLE 3

## SPEARMAN RANK CORRELATION

	lGDP	HDI	P:E	P:N	P:S	P:G	P-s:G	C:E	C:N	C:S	C:G	C-s:G
lGDP	1.00											
HDI	0.95	1.00										
P:E	0.94	0.92	1.00									
P:N	-0.70	-0.73	-0.72	1.00								
P:S	-0.91	-0.96	-0.88	0.77	1.00							
P:G	0.93	0.95	0.94	-0.86	-0.96	1.00						
P-s:G	-0.92	-0.95	-0.94	0.87	0.96	-1.00	1.00					
C:E	0.93	0.87	0.97	-0.63	-0.82	0.88	-0.88	1.00				
C:N	-0.80	-0.83	-0.81	0.94	0.86	-0.92	0.92	-0.72	1.00			
C:S	0.87	0.94	0.85	-0.75	-0.98	0.93	-0.93	0.79	-0.83	1.00		
C:G	0.78	0.76	0.79	-0.31	-0.72	0.68	-0.68	0.85	-0.41	0.74	1.00	
C-s:G	0.79	0.79	0.80	-0.35	-0.75	0.71	-0.71	0.85	-0.44	0.77	0.98	1.00



A more immediate view of the differences between measurements can be obtained via graphic representation of development indices. The graph in Figure 1 compares the four synthetic (global) indicators: the logarithm of per capita GDP, the HDI, the index obtained via principal components and the index obtained with the concave average. Two aspects are brought out by graphic representation: in the first place we find confirmation of the many changes in ranking and significant variations of relative values of indices obtained with the various methods of measurement; secondly, the values for HDI and the new method have similar variation ranges (both being obtained essentially by averaging variables that were equally normalized), and the same holds for the remaining two indices (based mainly on logarithms of unbounded variables). Another interesting phenomenon is that the introduction of environmental variables, negatively correlated with economic and social variables in the two indices proposed here, entails notable changes in ranking with respect to GDP or HDI within highly developed countries, although it does not significantly modify the gap between these and the underdeveloped countries.

### 5. Conclusions

The aim of this work was to propose a development indicator that can structurally take into account the balance in the various aspects of development and thereby weigh any unbalance negatively. Here equilibrium is taken as a proxy of the concept of development socially sustainable in time, that is, development that can minimize internal unbalances between the various aspects of development. A method of computation (concave average) of a development indicator is proposed here that penalizes the development level of countries showing greater unbalance among the various aspects. The method is easy to implement even if the number of variables is large.

In application of the method, three groups of variables were chosen referring to the economic, environmental and social aspects of the development of a country. The choice was influenced by the availability of data, and these variables are therefore not necessarily able to describe the various aspects of development accurately; however, our objective here was to compare the results with those obtained with other indicators and methods.

Finally, we compared the results of the concave average method with the results yielded by the standard methods (logarithm of per capita income and Human Development Index) and those obtained via the method of principal components. From this comparison it clearly emerges that the different development indicators (each taking values in its natural range) yield significantly different country rankings.

# APPENDIX A

TABLE A.1
VARIABLES, WEIGHTS AND SIGNS
(E-xx: economic; N-xx: environmental; S-xx: social)

Identifier	Description	Weight within group	Sign
E-01	GDP per capita (PPP)	1	+
E-02	Gross fixed capital formation (% of GDP)	1	+
E-03	Industry, value added (% of GDP)	1	+
E-04	Credit to private sector (% of GDP)	1	+
E-05	Imports of goods and services (% of GDP)	1	_
E-06	Total debt service (% of exports of goods and services)	1	_
E-07	Labor force activity rate, male (% of male population		
	ages 15-64)	1	+
E-08	Rural population (% of total population)	1	_
E-09	International telecom, outgoing traffic (minutes per		
	subscriber)	1	+
E-10	Telephone mainlines per employee	1	+
N-01	CO <sub>2</sub> emissions, industrial (metric tons per capita)	1/3	_
N-02	CO <sub>2</sub> emissions, industrial (kg per 1995 US\$ of GDP)	1/3	_
N-03	CO <sub>2</sub> emissions, industrial (kg per PPP \$ of GDP)	1/3	_
N-04	Fertilizer consumption (100 grams per hectare of arable		
	land)	1	_
N-05	Land use, arable land (% of land area)	1/2	-
N-06	Land use, arable land (hectares per person)	1/2	_
N-07	Urban population (% of total)	1	_
S-01	General government final consumption expenditure (% of GDP)	1	+
S-02	Fertility rate, total (births per woman)	1/2	_
S-03	Birth rate, crude (per 1,000 people)	1/2	_
S-03	Life expectancy at birth, total (years)	1/2	+
S-05	Mortality rate, infant (per 1,000 live births)	1/2	
S-06	Mortality rate, under 5 (per 1,000 live births)	1/2	_
0.00	intortune, ruce, under 5 (per 1,000 rive briting)	1/2	

TABLE A.1 (cont.)

Identifier	Description	Weight within group	Sign
S-07	Immunization, DPT (% of children under 12 months)	1/2	+
S-08	Immunization, measles (% of children under 12 months)	1	+
S-09	Labor force, children 10-14 (% of age group)	1/2	_
S-10	Labor force, female (% of total labor force)	1/2	+
S-11	Labor force activity rate, female (% of female population ages 15-64)	1	+
S-12	Life expectancy differential (absolute between genders)	1	_
S-13	Population ages 65 and above (% of total)	1/2	+
S-14	Death rate, crude (per 1,000 people)	1	-
S-15	Military personnel (% of total labor force)	1	-
S-16	Television sets (per 1,000 people)	1/2	+
S-17	Radios (per 1,000 people)	1/2	+
S-18	Telephone mainlines (per 1,000 people)	1/2	+
S-19	Mobile phones (per 1,000 people)	1/2	+
S-20	Adult literacy rate	1/2	+
S-21	Combined primary and tertiary gross enrollment ratio	1/2	+
S-22	Age dependency ratio (dependents to working-age population)	1/2	_

TABLE A.2

# INDICES

Countries	lGDP	HDI	P:E	P:N	P:S	P:G	P-s:G	C:E	C:N	C:S	C:G	C-s:G
Albania	3.45	0.71	-1.01	0.74	-0.21	-0.54	0.60	-0.28	0.09	-0.12	-0.11	-0.09
Algeria	3.68	0.68	-0.31	-0.44	0.26	-0.05	0.06	-0.18	-0.11	-0.22	-0.17	-0.15
Angola	3.26	0.41	-0.77	1.01	1.39	-1.13	1.20	-0.24	0.32	-0.56	-0.16	-0.19
Argentina	4.08	0.84	0.80	-0.45	-0.46	0.61	-0.59	0.00	-0.40	-0.05	-0.15	-0.12
Australia	4.35	0.93	1.35	-1.09	-1.32	1.34	-1.35	0.27	-0.58	0.28	-0.01	0.02
Austria	4.36	0.91	1.38	-0.28	-1.26	1.05	-1.10	0.27	-0.26	0.20	0.07	0.06
Bangladesh	3.13	0.46	-0.82	0.86	0.89	-0.92	0.88	-0.24	-0.01	-0.27	-0.17	-0.16
Barbados	4.08	0.86	0.28	-0.08	-0.87	0.44	-0.42	-0.08	-0.18	0.29	0.01	0.01
Belgium	4.37	0.93	1.51	-1.18	-1.21	1.40	-1.22	0.19	-0.72	0.18	-0.11	-0.10
Belize	3.66	0.78	0.00	0.04	-0.16	0.04	0.03	-0.13	-0.09	-0.12	-0.11	-0.08
Benin	2.94	0.41	-1.53	0.86	1.20	-1.29	1.37	-0.51	0.05	-0.45	-0.30	-0.26
Bolivia	3.36	0.64	-0.09	-0.10	0.23	-0.08	0.12	-0.19	0.00	-0.19	-0.13	-0.15
Botswana	3.79	0.59	-0.11	0.31	0.70	-0.41	0.52	-0.04	0.12	-0.30	-0.07	-0.07
Brazil	3.82	0.75	0.67	-0.15	-0.30	0.40	-0.39	0.00	-0.28	-0.05	-0.11	-0.11
Bulgaria	3.68	0.77	-0.01	-1.25	-0.96	0.78	-0.82	-0.35	-0.50	0.06	-0.27	-0.26
Burkina Faso	2.94	0.30	-1.13	1.54	1.65	-1.54	1.53	-0.31	0.25	-0.58	-0.21	-0.20
Burundi	2.76	0.32	-1.82	2.40	1.59	-2.07	1.80	-0.62	0.52	-0.64	-0.25	-0.24

TABLE A.2 (cont.)

0	ICDD	LIDI	DE DN DC DC D C CE CN CC CC C
Countries	IGDP		
Cambodia	3.10	0.51	
Cameroon	3.17	0.53	
Canada	4.37	0.94	
Cape Verde	3.51	0.69	
Chad	2.93	0.37	
Chile	3.94	0.83	
China	3.49	0.71	
Colombia	3.78	0.76	
Congo, Rep.	2.91	0.43	
Costa Rica	3.78	0.80	0.12 0.11 - 0.44 0.16 - 0.07   -0.14 - 0.14 0.00 - 0.09 - 0.07
Cote d'Ivoire	3.20	0.42	-0.89 -0.02
Croatia	3.83	0.80	0.43 - 0.67 - 0.99   0.75 - 0.85   -0.02 - 0.35   0.08 - 0.10 - 0.11
Czech Rep.	4.09	0.84	1.13 - 1.26 - 1.27
Denmark	4.38	0.91	1.35 - 0.82 - 1.35   1.26 - 1.26   0.24 - 0.59   0.29 - 0.02 - 0.04
Dominican Rep.	3.66	0.73	0.55 - 0.24  0.18  0.21 - 0.22    0.08 - 0.22 - 0.21 - 0.12 - 0.10
Ecuador	3.48	0.72	0.07 - 0.36  0.06  0.13 - 0.12
Egypt, Arab Rep.	3.48	0.62	-0.04 -0.41  0.26  0.03  0.02  -0.21 -0.13 -0.26 -0.20 -0.20
El Salvador	3.61	0.70	0.00 0.19 0.01 - 0.07 0.15   -0.14 - 0.13 - 0.16 - 0.15 - 0.15
Estonia	3.89	0.80	0.72 - 1.35 - 1.16
Ethiopia	2.76	0.31	-1.43 1.58 1.64 - 1.66 1.58 - 0.58 0.30 - 0.63 - 0.30 - 0.26
Finland	4.32	0.92	1.07 - 0.57 - 1.62
France	4.33	0.92	1.12 - 0.36 - 1.33
Gabon	3.80	0.59	0.09 0.13 0.83 - 0.32 0.26 0.04 0.06 - 0.41 - 0.10 - 0.14
Gambia, The	3.16	0.40	-1.13 1.23 0.96 -1.18 1.22 -0.45 0.21 -0.35 -0.19 -0.18
Georgia	3.53	0.76	-0.38 -0.10 -0.85
Germany	4.35	0.91	1.70 - 0.83 - 1.24   1.35 - 1.39   0.33 - 0.50   0.21   0.01   0.00
Ghana	3.24	0.56	-1.16 0.99 0.85 -1.07 1.18 -0.44 0.15 -0.25 -0.18 -0.18
Greece	4.14	0.88	0.87 - 0.67 - 1.12  0.95 - 1.02  0.11 - 0.35  0.07 - 0.06 - 0.07
Guatemala	3.54	0.62	-0.47 0.35 0.55 -0.49 0.62 -0.25 -0.07 -0.44 -0.25 -0.24
Guinea	3.25	0.39	-1.17 1.70 1.48 -1.55 1.58 -0.32 0.41 -0.49 -0.13 -0.14
Guinea-Bissau	2.79	0.33	-1.46 1.19 1.54 -1.50 1.42 -0.44 0.30 -0.72 -0.29 -0.27
Guyana	3.53	0.71	0.04 0.14 0.10 - 0.07 0.19 - 0.10 - 0.04 - 0.16 - 0.10 - 0.08
Haiti	3.14	0.44	
Honduras	3.39	0.65	
Hungary	4.01	0.82	
Iceland	4.40	0.93	
India	3.32	0.56	
Indonesia	3.42	0.67	
Iran, Islamic Rep.	3.71	0.71	
Ireland	4.33	0.91	
Israel	4.24	0.88	
Italy	4.31	0.90	
Iamaica	3.53	0.74	
Jamaica	5.55	0./4	0.22 - 0.07 - 0.31

TABLE A.2 (cont.)

Countries	lGDP	HDI	P:E	P:N	P:S	P·G	P-s-G	C:E	C·N	C:S	C:G	C-s:G
Japan	4.37	0.92			-1.27		-1.42		-0.27			
Jordan	3.52	0.72			-0.07							-0.18
Kazakhstan	3.64	0.75			-0.55							-0.18
Kenya	2.99	0.51	-1.16	0.72				-0.48				-0.21
Korea, Rep.	4.13	0.85			-1.00		-1.44		-0.43			-0.06
Kyrgyz Rep.	3.36	0.71										-0.20
Lao PDR	3.24	0.48	-0.96	1.99				-0.30				-0.14
Latvia	3.76	0.77			-1.15			-0.11				-0.09
Lebanon	3.64	0.74			-0.47							-0.22
Lesotho	3.21	0.57	-0.58	1.46		-1.13		-0.13				-0.09
Lithuania	3.81	0.79			-1.07			-0.11				-0.12
Macedonia, FYR	3.63	0.76			-0.68			-0.10				-0.14
Madagascar	2.88	0.48	-1.79	1.60		-1.66		-0.63				-0.23
Malawi	2.72	0.39	-2.16	1.28		-1.74		-0.71				-0.26
Malaysia	3.91	0.77			-0.29		-0.77					-0.11
Mauritania	3.19	0.45	-0.75					-0.26				-0.25
Mauritius	3.92	0.76	0.54		-0.34							-0.08
Mexico	3.89	0.78	0.27	-0.62	-0.11							-0.17
Moldova	3.29	0.70			-0.83							-0.20
Mongolia	3.19	0.63	-0.82									-0.26
Morocco	3.52	0.59	-0.05	-0.07								-0.16
Mozambique	2.89	0.34	-1.09	1.61		-1.53						-0.18
Nepal	3.06	0.47	-0.90	1.59	1.36	-1.37	1.29	-0.34	0.26	-0.38	-0.15	-0.16
Netherlands	4.35	0.93	1.60	-0.93	-1.39	1.40	-1.39	0.24	-0.52	0.28	0.00	-0.01
Nicaragua	3.33	0.63	-0.49	0.05	0.31	-0.31	0.42	-0.27	0.17	-0.20	-0.21	-0.22
Niger	2.87	0.29	-2.09	1.70	1.85	-2.01	1.85	-0.63	0.44	-0.67	-0.29	-0.30
Nigeria	2.90	0.44	-0.93	-0.11	1.23	-0.75	0.79	-0.35	0.07	-0.49	-0.30	-0.30
Norway	4.42	0.93	1.31	-0.74	-1.33	1.21	-1.29	0.26	-0.32	0.25	0.06	0.06
Pakistan	3.23	0.52	-0.68	-0.05	0.94	-0.57	0.73	-0.41	-0.16	-0.42	-0.33	-0.35
Panama	3.72	0.78	0.15	-0.32	-0.29	0.27	-0.29	-0.13	-0.16	-0.03	-0.11	-0.08
Papua New Guinea	3.37	0.54	-0.94	0.94	1.06	-1.05	1.07	-0.29	0.36	-0.29	0.08	-0.13
Paraguay	3.63	0.74	-0.14	0.52	0.27	-0.33	0.44	-0.15	-0.03	-0.26	-0.15	-0.12
Peru	3.63	0.74	0.40	0.11	-0.05	0.12	-0.09	-0.06	-0.09	-0.17	-0.11	-0.09
Philippines	3.55	0.74	0.31	-0.05	0.13	0.08	-0.07	-0.09	-0.16	-0.09	-0.11	-0.10
Poland	3.88	0.81	0.51	-1.26	-1.10	1.02	-1.16	0.01	-0.52	0.15	-0.12	-0.12
Portugal	4.17	0.86	1.31	-0.29	-1.04	0.95	-1.00	0.21	-0.24	0.12	0.03	0.03
Romania	3.75	0.77	-0.06	-0.84	-0.75	0.54	-0.54	-0.25	-0.36	-0.02	-0.21	-0.21
Russian Fed.	3.81	0.77	0.39	-1.24	-0.96	0.92	-0.96	-0.06	-0.42	0.02	-0.15	-0.13
Rwanda	2.82	0.38	-1.58	2.34	1.64	-1.98	1.74	-0.47	0.57	-0.68	-0.19	-0.20
Saudi Arabia	4.01	0.75	0.97	-1.32	-0.07	0.83	-0.78	0.22	-0.40	-0.20	-0.13	-0.11

TABLE A.2 (cont.)

Countries	lGDP	HDI	P:E	P:N	P:S	P:G	P-s:G	C:E	C:N	C:S	C:G	C-s:G
Senegal	3.12	0.42	-0.66	0.55	1.14	-0.84	1.02	-0.27	0.05	-0.46	-0.23	-0.24
Slovak Republic	3.99	0.83	0.71	-0.80	-1.21	0.98	-1.07	0.04	-0.34	0.20	-0.04	-0.04
Slovenia	4.16	0.86	1.03	-0.56	-1.21	1.01	-1.03	0.13	-0.25	0.20	0.03	0.02
South Africa	3.93	0.70	0.22	-0.78	-0.04	0.37	-0.36	-0.11	-0.27	-0.12	<b>-</b> 0.17	<b>-0.17</b>
Spain	4.21	0.90	1.39	-0.57	-1.25	1.15	-1.22	0.25	-0.42	0.20	0.01	0.00
Sri Lanka	3.47	0.73	-0.41	0.76	-0.26	-0.31	0.36	-0.23	0.07	-0.08	-0.08	-0.08
Swaziland	3.58	0.66	-0.24	1.16	0.62	-0.72	0.83	-0.12	0.18	-0.31	-0.09	-0.07
Sweden	4.32	0.93	1.35	-0.24	-1.56	1.14	-1.07	0.21	-0.31	0.35	0.08	0.08
Switzerland	4.41	0.92	1.64	0.01	-1.27	1.05	-1.01	0.39	-0.17	0.21	0.14	0.11
Syrian Arab Rep.	3.46	0.66	-0.46	-0.83	0.30	0.01	-0.01	-0.32	-0.34	-0.38	-0.35	-0.32
Tanzania	2.68	0.42	-1.71	1.25	1.25	-1.50	1.55	-0.60	0.28	-0.44	-0.26	-0.22
Thailand	3.74	0.75	0.90	-0.23	-0.28	0.51	-0.50	0.08	-0.16	-0.02	-0.03	-0.05
Togo	3.14	0.47	-1.14	0.93	1.24	-1.18	1.27	-0.40	0.03	-0.52	-0.30	-0.30
Trinidad & Tobago	3.87	0.79	0.60	-1.62	-0.55	0.98	-1.00	0.08	-0.45	0.06	-0.10	<b>-0.11</b>
Tunisia	3.73	0.70	0.30	-0.19	-0.17	0.23	-0.22	-0.06	-0.21	-0.05	-0.10	-0.09
Turkey	3.81	0.73	0.67	-0.63	-0.07	0.48	-0.56	-0.03	-0.40	-0.19	-0.21	-0.22
Uganda	3.03	0.41	-1.50	2.47	1.57	-1.97	1.79	-0.50	0.54	-0.50	<b>-0.15</b>	-0.23
Ukraine	3.50	0.74	-0.15	-1.50	-1.17	0.90	-0.91	-0.23	-0.56	0.12	-0.22	-0.19
United Kingdom	4.31	0.92	1.51	-0.89	-1.33	1.34	-1.28	0.28	-0.53	0.27	0.01	0.00
United States	4.47	0.93	1.38	-1.02	-1.23	1.30	-1.38	0.30	-0.49	0.22	0.01	-0.03
Uruguay	3.94	0.83	0.57	-0.17	-0.65	0.50	-0.46	0.00	-0.38	-0.03	<b>-</b> 0.14	-0.12
Venezuela, RB	3.76	0.77	0.45	-1.33	-0.09	0.65	-0.68	-0.06	-0.42	-0.15	<b>-</b> 0.21	-0.21
Vietnam	3.23	0.67	-0.80	0.16	0.06	-0.37	0.37	-0.41	-0.06	-0.16	-0.21	-0.20
Yemen, Rep.	2.86	0.45	-1.04	-0.17	1.20	-0.75	0.79	-0.42	0.09	-0.47	-0.27	-0.24
Zambia	2.86	0.42	-1.43	0.59	1.36	-1.21	1.28	-0.52	0.04	-0.38	-0.29	-0.29
Zimbabwe	3.43	0.56	-0.71	-0.23	0.93	-0.52	0.64	-0.30	-0.11	-0.29	-0.24	-0.24

#### KEY TO ABBREVIATIONS

Abbreviation	Description
GDP	Gross Domestic Product
lGDP	Decimal logarithm of Gross Domestic Product
HDI	Human Development Index
P:x	Index obtained with the Principal Component method
C:x	Index obtained with the Concave Average method
<i>x</i> -s: <i>x</i>	With optional straightening of quantile function
x:E	Index relative to economic variables
x:N	Index relative to environmental variables
x:S	Index relative to social variables
x:G	Index relative to all variables globally

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