Shareholders and stakeholders value creation: an analytic foundation for value creation indicators *

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1. Introduction

The academic debate on the theory of the firm and its value has been monopolized for a long period by the contraposition between the shareholders model (or finance model) and the stakeholders model (Zamagni 2006).

With no externalities and/or monopolies (with all the assets having a price for the different states of the world), the supporters of the first approach argue that social welfare is maximized when all the firms maximize their own market values (not just the value of their equity, but also the value of all the other rights issued such as preference shares, warrants, etc.).

On the other hand, those who invoke the stakeholders model as the most suitable framework for studying the firm claim that the management should take its decisions considering the interests of all the stakeholders (workers, customers, government, regulators etc.). It

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* The first part of the paper has been prepared by Masera (§ 1, 2 and 3), while the model (§ 4) has been developed by Mazzoni. We thank two anonymous referees for their useful comments and suggestions on a first version of the paper. Opinions and ideas expressed in this paper are those of the authors and do not engage in any way their respective institutions.

BNL Quarterly Review, vol. LX, no. 240, March 2007, pp. 3-31.

must be pointed out that the traditional formulations of this approach do not supply a clear trade-off mechanism which allows to order or value the potential conflicts of interests between the different stakeholders. The management does not have any quantitative instrument to use for taking its decisions, being left with a high discretional power that cannot be easily monitored.

Jensen (2001) introduced the concept of "enlightened value maximization", and the correspondent idea of "enlightened stakeholders theory", which consider simultaneously both the shareholders and the stakeholders value maximization. The proposed approach starts from the preliminary consideration that the synthetic (and measurable) objective function to be considered, and maximized, by the management still remains the (long run) market value of the firm. The innovation of this approach comes from the explicit acknowledgment of the importance recognized to the different constituencies of the firm. In this framework value maximization cannot be obtained without considering the interests of all the stakeholders.

Masera (2006a), focusing his analysis on the banking sector and obtaining results that can be easily generalized, developed an approach that can be defined as "a quantitative synthesis of the enlightened stakeholders theory". He supported the idea that the total value of the firm is maximized when the management maximizes both the remuneration of the shareholders and the efficiency and the satisfaction of all the other stakeholders. He advanced his thesis developing an analysis based on the most widely used models and value creation indicators.

Masera argues that a bank (or more generally a firm) focused on total value creation must be characterized by a management of the human resources not only concerned by the reduction of the costs.

A key driver should be represented by the valorisation of both: *i*) the 'human knowledge capital', which is represented by the net worth of all the future cash flows generated by the quality and the organization of human resources (i.e. motivation, ability to innovate, competences, flexibility and productivity) and *ii*) the 'franchise capital', defined as the net present value of extra yields deriving from the investment devoted to the support of the relationships with clients, suppliers, partners and products. Also in this case, we can consider this part of extra yield as an extra value obtained as the result of suitable investment policies implemented over time.

In this framework it is therefore crucial to invest a part of the resources in the relationships with all the stakeholders. This investment can be considered as a prerequisite to guarantee the total value creation of the firm (shareholders included), allowing a durable and renewable growth.

Using these considerations we can infer that it is profitable to invest in the relationships with the stakeholders when their respective marginal returns, measured from the long-run value of the firm, are higher than their respective marginal costs.

In this paper, this thesis will be formalized proposing a model in which we assume a stochastic behaviour for the most relevant value indicators We shall show that the model can be easily extended, allowing the simultaneous analysis of both value creation and capital structure problems.

Before presenting our model (section 4) we shall use the next two sections to underline the most relevant aspects regarding both *i*) the valuation of firms' performance (section 2) and *ii*) the dynamic allocation of capital (section 3).

2. Performance valuation, risks and capital

All the methods currently used to value firms heavily rely on operating profit indicators rather than on book value indices. These approaches are mainly focused on the measurement of the dynamic and statistical relations between capital and risk.

The first index to consider is Nopat (Net Operating Profit After Taxes). Operating profit is an indicator characterized by:

1) the absence of any accountable misrepresentation, being focused on cash flows;

2) the neutrality to the capital structure of the firm (not being affected by the choice between equity capital or debt capital);

3) the absence of any distortion due to taxation, being computed considering only the 'normal' level of taxes paid;

4) the sensitivity to the expected loss of the portfolio at risk for a given time horizon (usually one year).

In statistical terms, for a given frequency distribution of the losses associated to the portfolio, the expected loss, *E(L)*, represents the mean (the first moment of the distribution).

When we consider capital we analyze the liabilities-side of the firm (i.e. the sources of financing), distinguishing between equity capital and debt capital. Nopat is evidently a flow indicator expressed in absolute terms which is not easily comparable. Moreover, Nopat must be appropriately adjusted for taking into account the cost of capital.

On the other hand, the Economic Value Added, EVA, defined as the difference between the Nopat and the total cost of the invested capital, represents an indicator which allows to monitor the efficiency of the investment policies followed by the firm (i.e. the net cash flows generated by the investments which represent the extra value for shareholders):

$$\text{EVA} = \text{Nopat} - \text{invested capital} \times \text{wacc}. \tag{1}$$

Invested capital is represented by all the resources invested in the firm. Economic capital can be considered as the buffer necessary to face all the risks of the firm. We can therefore write: invested capital = economic capital + goodwill + discretional reserves. Where:

$$\text{wacc} = k_e \frac{E}{E+D} + k_d \frac{D}{E+D}.$$

Now we can rewrite 1 as:

$$\text{EVA} = \frac{\text{Nopat}}{\text{invested capital}} \times \text{invested capital} - \text{wacc} \times \text{invested capital} = \tag{2}$$

$$= \text{invested capital} \times (r - \text{wacc});$$

$$\text{with } r \equiv \frac{\text{Nopat}}{\text{invested capital}}.$$

EVA is therefore equal to the operating profits opportunely reduced by the total remuneration of the invested capital.

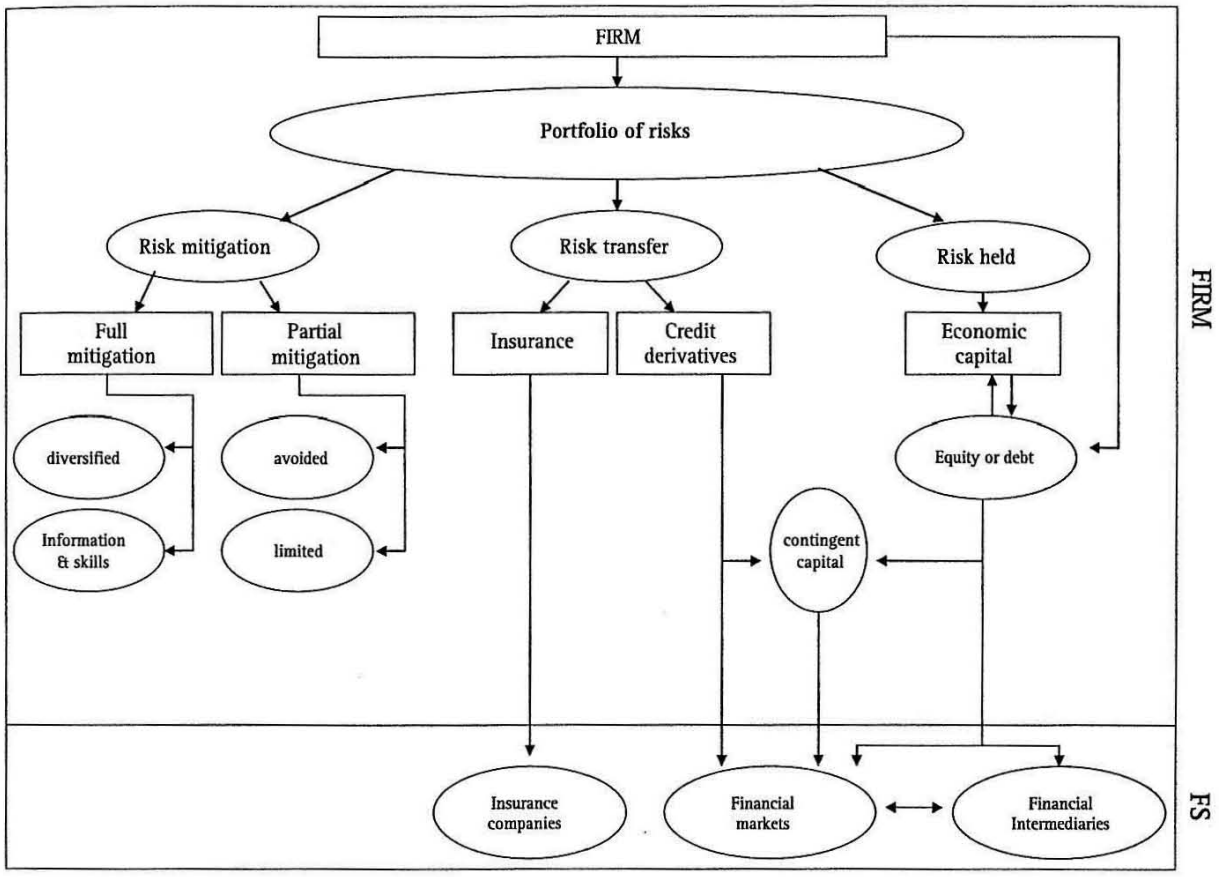
Now we have all the necessary instruments to define the relation between capital and risk. It has been already remembered that the expected loss must be considered as an operating cost. Then the

'true risk' faced by the firm is represented by the volatility of the 'loss function'.

The frequency distribution of the losses faced by the firm, considered as a portfolio of risks, depends from the decisions taken at the level of Enterprise Risk Management, ERM, in terms of risk mitigation, risk transfer and risk held (see Table 1).

TABLE 1

RISK, CAPITAL AND FINANCIAL SYSTEM



The first measure that can be used to quantify this randomness is represented by the standard deviation of the losses associated to a given portfolio:

$$UL_p = \sqrt{E\left[L_p - E(L_p)\right]^2}. \tag{3}$$

If these losses were normally-shaped distribution functions, $E(L_p)$ and $U(L_p)$ would be the only two moments necessary to identify the distribution. Empirical evidence shows non-normal distribu-

tion functions. Moreover, it must be highlighted that standard deviation does not allow to recognize positive returns from negative ones.

However we can consider *U(L)* as a proxy of the risk, recognizing the above mentioned limitations. The economic capital held by the firm is correspondingly defined as the capital (equity or debt) used to face unexpected losses (i.e. the capital necessary to absorb potential losses not identified *ex ante*, but statistically possible).

Risk is therefore a function of volatility of expected results. If we use the standard deviation at portfolio level as a risk measure, we can expand equation 3 obtaining:

$$UL_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} UL_i \times UL_j \times \rho_{ij}}, \qquad (4)$$

where i and j are the indices associated to the assets, while ρ_{ij} represents the correlations between the losses of assets i and j. The variance of losses at the level of portfolio is, by definition, equal to the sum of the covariances between idiosyncratic losses.

Expected loss of the portfolio can be determined as the sum of the expected losses of each asset, while unexpected loss is generally (significantly) smaller than the sum of the individual unexpected losses, being equal only in the limiting case characterized by a perfect correlation between assets.

This simple approach allows to find one of the most relevant results of financial theory: portfolio risk is smaller (usually significantly smaller) and, only in the limit, equal to the sum of individual risks characterizing the assets composing a given portfolio.

We can apply this concept to the firm, considered as a portfolio of risks. Firm and investors accepting idiosyncratic risks try to reduce the unexpected losses (especially those characterized by a low frequency and a high magnitude) which could, in some cases, seriously affect the solvency of the firm.

As a general approach to risk, unexpected loss is an important measure to be considered. However, it is important to note that the current practices reserve a prominent role to the Value at Risk, VaR, as a measure of risk.

VaR, originally defined as risk indicator for markets' risks, is now used as an overall risk measure to monitor risks arising from a

given portfolio. VaR is defined under some strict statistical hypothesis: *i*) prices follow random walks, usually normally distributed; *ii*) variations of prices are serially uncorrelated; *iii*) volatility of the variations is stable over time.

It is therefore clear that VaR allows to monitor the 'tail risk', characterized by a low frequency and a high magnitude.

Formally we can define VaR as as the portfolio's worst loss, for a given confidence interval, α, within a given time horizon:

$$\mathrm{VaR}(\alpha) = \min\left\{ j \middle| P\left(L_p > j\right) \leq (1-\alpha) \right\}. \tag{5}$$

VaR is therefore a measure heavily dependent on both the confidence level and the time horizon chosen. On the other hand, VaR has the advantage of being a measure which immediately allows to quantify the amount of economic capital to be held. The amount of the losses associated to the distribution function is, by definition, equal to the sum of *E(L)*, Economic Capital and VaR. Analytically we can write:

$$EC_\alpha = \mathrm{VaR}_\alpha - E(L). \tag{6}$$

Economic capital can be considered as a buffer of capital to be used for avoiding the insolvency of the firm for a given time horizon (generally one year), and a given confidence level, α. Confidence level α is usually chosen coherently with the default probability defined by the target credit rating. However, it is important to understand that, for rating agencies, capital reserves represent just one of the indicators to be analyzed for assessing the probability of default of the firm.

The axiomatic approach to coherent risk measurement, pioneered by Artzner *et al.* (1999), pointed out the limits of the VaR methodologies by showing that this measure, which represents a standard commonly accepted by market participants, is a problematic (non-coherent) risk indicator. The two main drawbacks highlighted are its 'non smoothness' (i.e. events with probability below the chosen confidence quantile are not considered at all) and its 'non subadditivity' (i.e. VaR of a diversified portfolio could be higher than the sum of idiosyncratic VaR computed for each risk factor). Intuitively, it can therefore be said that VaR does not take into account the entire

lower tail of the P&L distribution, by just picking out one point (the quantile chosen).

To overcome these problems alternative risk measures have been proposed, which consider the lower tail of the returns distributions and satisfy the subadditivity property. Among these new measures the Expected Shortfall (ES) is perhaps the most well-known. In particular, ES measures the expected loss once that the VaR limit is violated. However, it must be recognized that the theoretical attractiveness of this indicator is limited by the problems due to its practical implementation.

After having defined both the risk and the economic capital, we have now all the necessary instruments to analyze simultaneously the EVA, as a performance measure, with the so-called RAPM (Risk Adjusted Performance Measurement) indicators (see table 2).

There are a lot of models and acronyms used to define these indicators (Roraa, Roroa, Rorac, Raroc, Rarorac ...).[1] Despite the risk of being excessively simple, we shall focus our attention on one indicator defined as:

$$\text{Rorac} = \frac{\text{revenues} - \text{costs} - \text{expected loss}}{\text{risk capital}} = \frac{\text{Nopat}}{\text{risk capital}}. \quad (7)$$

At the numerator of the previous formula we have the Nopat opportunely reduced by the operating cost represented by the expected loss, that technically does not represent a risk factor. At the denominator we put the economic capital. Economic Capital (or Risk Capital), as already pointed out, is composed by both equity and debt capital.

If we consider the economic capital and the invested capital as equivalent, this framework allows to link EVA with Rorac. In fact, using equations 7 and 1 we obtain:

$$\text{EVA\%} = \text{Rorac} - \text{wacc}; \quad (8)$$

with EVA% ≡ EVA / invested capital.

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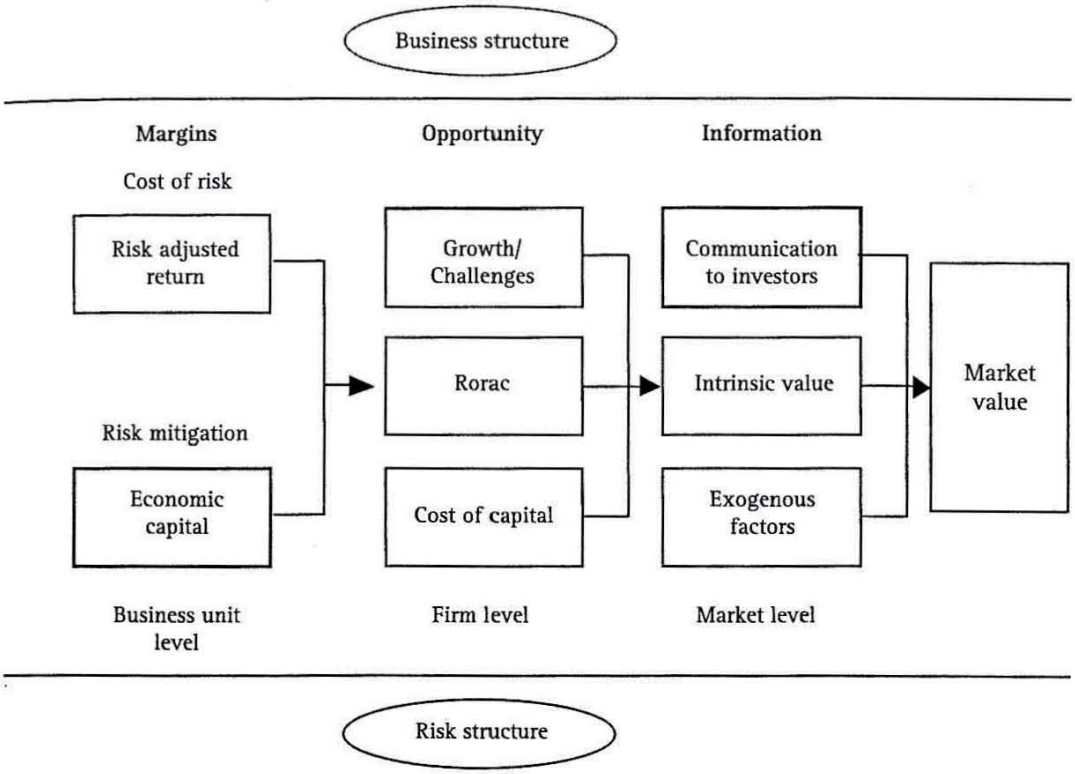
[1] Despite some marginal difference, these indicators share the common features of comparing invested capital with opportunely adjusted revenues, taking into account the risk component.

TABLE 2

RISK, ECONOMIC CAPITAL AND MARKET VALUE



3. Dynamic allocation of capital and value creation

As already pointed out, the performance of the firm is intimately related to both the growth and the profitability of the invested capital, after considering the randomness of the expected results. Profitability adjusted for both the expected loss and the sustainable growth is strictly related to the risk profile of the assets, the capital resources held against unexpected losses and, as we shall see, human and relational capital.

An efficient allocation of capital represents a key variable to monitor and foster value creation over time.

The attention accorded to risk is coherent with the importance attributed to the sustainability of value creation. To obtain this result we must leave the concept of short run value creation for accepting the broader idea of total value creation focused on the creation of value for all the stakeholders.

In this framework the dynamic allocation of capital and the implementation of corporate governance rules (which allows to foster total value creation) represent two sides of the same coin.

Value creation is intimately related to the expected performance. On the other hand, it is clear that markets' expectations are heavily affected by the historical ability of the management to achieve the announced results. The credibility of the business plans presented depends on both the announcements of profits and the historical track record performed.

It is therefore important to have indicators of results that can be used *i*) to perform the backward testing (for controlling the result achieved) and *ii*) to analyze the expected performance. These measures, and the relative datasets, must be transparent, not biased and easily explainable to the markets. Measures and methodologies previously analyzed are innovative and, for some aspects, complex, especially those referring to the measurement of the risk and the calculation of the economic capital.

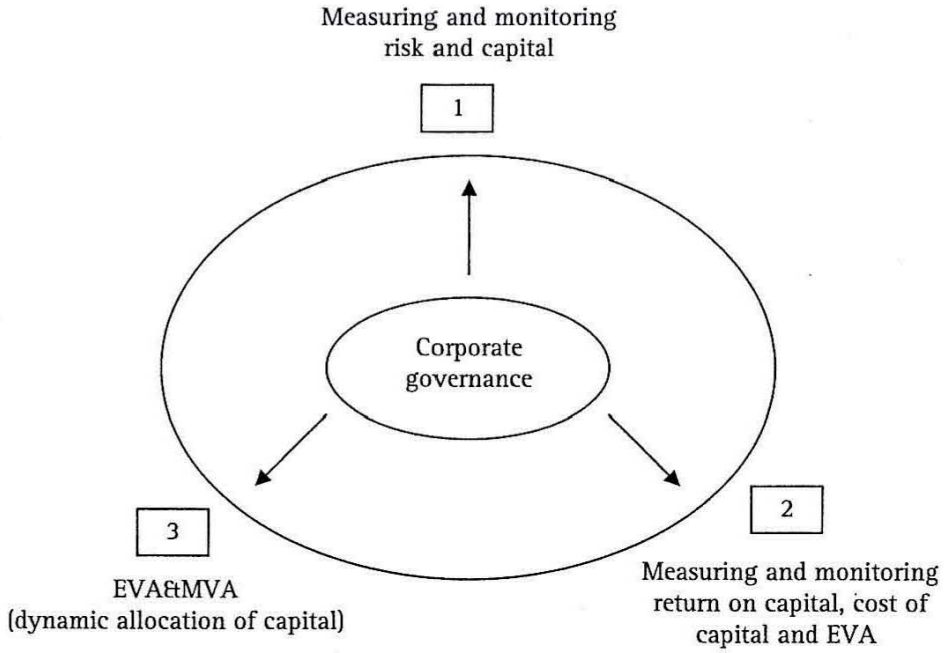
This dynamic relation is represented by the value maximization achieved with the dynamic allocation of capital. This process needs to measure the risk and the adjusted performance (corrected for the expected loss) both at disaggregated (i.e. business unit) and aggregated level (see table 3).

The return adjusted for the level of the expected loss and the capital at risk represent two necessary prerequisites to face the problem of the dynamic measurement of the performance, by allowing to measure the market value of the firm.

In an efficient market the difference between the market value of the capital and its book value represents the value creation recognized by the market to the firm. This amount defined as the MVA (Market Value Added) heavily depends on the expected EVA. With a positive expected EVA we shall have a positive MVA: the firms produce profits higher than the cost of the invested capital. In the opposite case, with a negative MVA the firm will have a market value smaller than its book value. If we have the case of a market loss higher than than the equity capital invested, we are technically in default.

TABLE 3

VALUE CREATION: RISK, RETURN AND COST OF CAPITAL, GROWTH AND CORPORATE GOVERNANCE



$$MVA = \sum_{t=1}^{\infty} \frac{EVA_t}{(1 + wacc)^t}. \tag{9}$$

The relation between the MVA and the book value of the capital is summarized in the following table.[2]

It is important to point out that the valuation of the performance based on the EVA and on the MVA represents an application of the well-known DFCF (Discounted Free Cash Flow) method, usually defined as DCF.

The FCF (Free Cash Flow), produced by the firm after the new investments in capital, is defined as:

$$FCF = Nopat - Net\ investment. \tag{10}$$

[2] On this point see Masera (2006b).

TABLE 4

MVA, BOOK VALUE AND EVA

Market value of firm	MVA Book value of invested capital	EVA & wacc	Operating profits
Price number of shares	Market value added MVA	EVA	Nopat
NpV of loans	Cumulated invested capital equity vs non equity	Cost of capital	

It is not difficult to show that the value of the firm can be computed as the discounted value of both the free cash flows and the EVA.[3] Formally we have:

$$V = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+wacc)^t} = IC + \sum_{t=1}^{\infty} \frac{EVA_t}{(1+wacc)^t}, \qquad (11)$$

where *IC* represents the book value of the invested capital at the end of the previous year.

If we value the firm by considering explicitly a time horizon of 5-10 years (or in general n years), we need to define a continuing value of the firm, CV_n. By assuming a deterministic and constant long-term growth rate for the Nopat, g, we may rewrite equation 11 as:

$$V = IC + \sum_{t=1}^{n} \frac{EVA_t}{(1+wacc)^t} + \frac{EVA_{t+n+1}}{wacc} + \frac{Nopat\left(\frac{g}{Ronic}\right)(Ronic - wacc)}{wacc}, \qquad (12)$$

where the *Ronic* represents the return on new invested capital.

This last formulation easily shows that a firm can 'beat the market', being characterized by both a i) Ronic > wacc and ii) g > growth rate of the market's demand, if, and only if, its investment policies are able to increase the 'return' of the 'relational' capital in-

[3] See Shrives and Wachowicz (2000) and Koller, Goedhart and Wessels (2005).

vested by fostering the relationships with its customers and its human capital.

Therefore, as we previously pointed out, the management should be focused in the valorization of both *i*) the 'human knowledge capital', represented by the net worth of all the future cash flows generated by the quality and the organization of human resources and *ii*) the 'franchise capital', defined as the net present value of extra yields deriving from the investment devoted to the support of the relationships with clients, suppliers, partners and products.

EVA and DCF efficiently summarize the relation between the capital allocation, the profitability of the firm and the risk, allowing to develop a suitable framework for monitoring the realized and expected performance. Moreover, these methodologies represent an important instrument for valuing the investment of the firm and all the different opportunities of growth.

4. The model

We now present a model characterized by a stochastic behaviour for the most relevant value indicators. This model will be used to formalize the above mentioned concept of 'enlightened value maximization'. In this framework we shall show that the management will have to invest in the relationships with the different stakeholders when their respective marginal returns are higher than their respective marginal costs. This is the only way to maximize the value of the firm.

We consider a firm that raises on the market a given amount of resources (debt capital or equity capital)[4] equal to IC (which can be considered as the book value of the invested capital when the firms begins its activity). The management has to decide the allocation of IC between 'non relational' capital, K, that includes the investments in all the assets of the firm (durable and non durable), and 'relational' capital, (or stakeholders capital) H, that can be considered the capital used to foster the relationships with all the stakeholders of the firm

[4] In the last paragraph of this paper we shall analyze the relation between value creation problems and capital structure problems. At this stage of the analysis it is not important to distinguish between equity or debt.

(financial claimants, customers, workers, government, regulators, etc.). We can therefore write: IC = K + H.

4.1. *Prices and costs*

We assume that the firm will compete in perfectly functioning markets with a perfect information. This means that the firm will be price taker on both the market of the only good (or service) produced and on the markets of production factors used (i.e. 'relational' and 'non relational' capital). In particular, we assume that the stochastic differential equations defining *i*) the price of the only good (or service) produced, p_y, *ii*) the cost of the 'non relational' capital, p_k, and *iii*) the cost of the 'relational' capital, p_h are respectively given by:[5]

$$\frac{dp_y}{p_y} = \mu_y dt + \sigma_y dz, \tag{13}$$

$$\frac{dp_k}{p_k} = \mu_k dt + \sigma_k dz, \tag{14}$$

$$\frac{dp_h}{p_h} = \mu_h dt + \sigma_h dz, \tag{15}$$

with $dz \sim N(0, \sqrt{dt})$. In the model we introduce a log-normal behaviour for both the price of the good (or service) produced and for the costs of the different types of capital invested. For a given level of prices/costs at current time t, in the future at time T their respective distribution will be given by:

$$\ln(p_i) \sim \phi\left[\ln(p_i) + \mu_i(T-t) - \frac{\sigma_i^2}{2}(T-t); \sigma_i\sqrt{T-t}\right]; \tag{16}$$

for $i = y, k, h$. ϕ represents the density function for a normal distribution.

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[5] To keep the analysis as simple as possible we considered both the 'non relational' and the 'relational' capital in an aggregate way. However, it must be pointed out that all the results obtained do not depend on this hypothesis and can be easily generalized.

For a given amount of resources K devoted to the 'non relational' capital and resources H invested in the 'relational' capital, the units of the 'non relational' asset, k, and the units of the 'relational' asset, h, will be given respectively by:

$$k \equiv \frac{K}{p_k}, \tag{17}$$

$$h \equiv \frac{H}{p_h}. \tag{18}$$

4.2. *Production function, investment rates and Nopat*

In the 'enlightened stakeholders' framework, output produced by the firms (goods or services) will be a function not only of the units of 'non relational capital' invested, k. Production will be also a function of the units of 'relational capital' invested, h (represented by the relationships with financial claimants, workers, government, regulators, etc.).

Formally we can write:

$$y = f(k,h). \tag{19}$$

To keep the analysis as simple as possible, we assume that the investments in 'non relational' and 'relational' capital will be decided deterministically with investment rates s and δ. Net investments in the two forms of capital for the time horizon, dt, will be given respectively by:

$$\frac{dk}{k} = sdt, \tag{20}$$

$$\frac{dh}{h} = \delta dt. \tag{21}$$

In this framework the behaviour of the production function y will be given by:

$$dy = \frac{\partial y}{\partial k}dk + \frac{\partial y}{\partial h}dh = \frac{\partial y}{\partial k}skdt + \frac{\partial y}{\partial h}\delta hdt. \tag{22}$$

Assuming a Cobb-Douglas specification for the production function:

$$y = k^{\alpha}h^{1-\alpha}, \tag{23}$$

we note that:

$$\frac{\partial y}{\partial k} = \alpha k^{\alpha-1}h^{1-\alpha}, \tag{24}$$

$$\frac{\partial y}{\partial h} = (1-\alpha)k^{\alpha}h^{-\alpha}. \tag{25}$$

We may therefore rewrite 22 as:

$$\frac{dy}{y} = [\alpha s + (1-\alpha)\delta]dt. \tag{26}$$

Now the Nopat, *Y*, of the firm can be defined as:

$$Y \equiv p_y \times y; \tag{27}$$

because of the stochastic behaviour of *y*, the (stochastic) behaviour of *Y* can be derived as:

$$\frac{dY}{Y} = \frac{dp_y}{p_y} + \frac{dy}{y}. \tag{28}$$

Substituting equations 13 and 26 in 28 we obtain:

$$\frac{dY}{Y} = [\mu_y + \alpha s + (1-\alpha)\delta]dt + \sigma_y dz. \tag{29}$$

A first look to the previous equations allows to understand that in our model the management is characterized by a full control of both the output produced and the production factors (having assumed a non stochastic behaviour for both the production function and the investment rates). However the management will have to deal with a stochastic Nopat because we assumed a stochastic behaviour for the exogenous price of the good (or service) produced.

4.3. *Market value of the firm, EVA and MVA*

In our model the FCF generated is given by:

$$FCF = Y - \delta H - sK = Y - \delta p_h h - sp_k k . \tag{30}$$

FCF behaviour will be given by:

$$dFCF = dY - \delta dH - sdK . \tag{31}$$

We note that:

$$\frac{dH}{H} = \frac{dp_h}{p_h} + \frac{dh}{h} \tag{32}$$

and

$$\frac{dK}{K} = \frac{dp_k}{p_k} + \frac{dk}{k} . \tag{33}$$

Using equations 14, 15, 20 and 21 we can respectively rewrite equations 32 and 33 as:

$$\frac{dH}{H} = (\mu_h + \delta)dt + \sigma_h dz, \tag{34}$$

$$\frac{dK}{K} = (\mu_k + s)dt + \sigma_k dz. \tag{35}$$

Using these two last equations and 29 we can write FCF behaviour explicitly as:

$$dFCF = \{(\mu_y Y - s\mu_k K - \delta\mu_h H) + s(\alpha Y - sK) + \delta[(1-\alpha)Y - \delta H]\}dt + \tag{36}$$

$$+ (\sigma_y Y - s\sigma_k K - \delta\sigma_h H)dz.$$

For a growth rate of FCF equal to g and a variance rate σ_{FCF} we can rewrite 36 as

$$dFCF = gFCFdt + \sigma_{FCF} FCFdz, \tag{37}$$

then we can derive:

$$g = \frac{\{(\mu_y Y - s\mu_k K - \delta\mu_h H) + s(\alpha Y - sK) + \delta[(1-\alpha)Y - \delta H]\}}{FCF} = \tag{38}$$

$$= \frac{\{(\mu_y Y - s\mu_k K - \delta\mu_h H) + s(\alpha Y - sK) + \delta[(1-\alpha)Y - \delta H]\}}{Y - \delta H - sK} =$$

$$= \frac{\{(\mu_y p_y y - s\mu_k p_k k - \delta\mu_h p_h h) + s(\alpha p_y y - s p_k k) + \delta[(1-\alpha)p_y y - \delta p_h h]\}}{p_y y - \delta p_h h - s p_k k},$$

$$\sigma_{FCF} = \frac{\sigma_y Y - s\sigma_k K - \delta\sigma_h H}{FCF} = \frac{\sigma_y p_y y - s\sigma_k p_k k - \delta\sigma_h p_h h}{p_y y - \delta p_h h - s p_k k}. \tag{39}$$

In this framework the behaviour followed by the market value of the firm and its *wacc*, *i*, will be characterized by the following relation:

$$\frac{dV}{V} + \frac{FCF}{V}dt = idt + \sigma_V dz. \tag{40}$$

We can assume that V = FCF × G, where *G* is a constant (that we shall derive); the behaviour of *V* will be given by:

$$\frac{dV}{V} = \frac{dFCF}{FCF} = gdt + \sigma_{FCF}dz. \tag{41}$$

Substituting equation 30 and 41 in 40 we obtain:

$$gdt + \sigma_{FCF}\,dz + \frac{[Y - \delta H - sK]}{V}dt = gdt + \sigma_{FCF}\,dz + \tag{42}$$

$$+ \frac{[p_y y - \delta p_h h - s p_h k]}{V}dt = idt + \sigma_V dz.$$

In the previous equation equating the coefficients of the deterministic part and those of the stochastic part we can write:

$$g + \frac{[Y - \delta H - sK]}{V} = g + \frac{[p_y y - \delta p_h h - s p_h k]}{V} = i, \tag{43}$$

$$\sigma_{FCF} = \sigma_V. \tag{44}$$

Few simplifications of equation 43 allow us to derive the following well-known relation:

$$V = \frac{[Y - \delta H - sK]}{i - g} = \frac{[p_y y - \delta p_h h - s p_h k]}{i - g}, \tag{45}$$

from which we can derive $G = 1/(i - g)$.

In this model the MVA can be defined as:

$$MVA = V - IC = \frac{Y + (g - \delta)H + (g - s)K - iIC}{i - g} = \tag{46}$$
$$= \frac{p_y y + (g - \delta)p_h h + (g - s)p_k k - i(p_k k + p_h h)}{i - g}.$$

For $g = s = \delta$ we can rewrite 46 as:

$$MVA = \frac{Y - iIC}{i - g} = \frac{EVA}{i - g}, \tag{47}$$

where $EVA \equiv Y - iIC$.

4.4. *The optimal investment in 'non relational' and 'relational' capital*

MVA can be maximized with respect to the 'non relational' and the 'relational' capital by deriving equation 46 respectively for K and H:

$$\frac{\partial MVA}{\partial K} = \frac{\partial Y}{\partial K} + (g - s) - i = 0, \tag{48}$$

$$\frac{\partial MVA}{\partial H} = \frac{\partial Y}{\partial H} + (g - \delta) - i = 0. \tag{49}$$

We note that

$$\frac{\partial Y}{\partial K} = p_y \times \frac{\partial y}{\partial K}, \tag{50}$$

$$\frac{\partial Y}{\partial H} = p_y \times \frac{\partial y}{\partial H} \tag{50 bis}$$

and then

$$\frac{\partial y}{\partial K} = \frac{\partial y}{\partial k} \Big/ \frac{\partial K}{\partial k} = \frac{\partial y}{\partial k} \Big/ p_k \tag{51}$$

$$\frac{\partial y}{\partial H} = \frac{\partial y}{\partial h} \Big/ \frac{\partial H}{\partial h} = \frac{\partial y}{\partial h} \Big/ p_h. \tag{51 bis}$$

We can rewrite 48 and 49 respectively as:

$$\left(\frac{\partial y}{\partial k} \frac{p_y}{p_k} \right) - (i - g) = s^*, \tag{48 bis}$$

$$\left(\frac{\partial y}{\partial h} \frac{p_y}{p_h} \right) - (i - g) = \delta^*, \tag{49 bis}$$

obtaining the optimal investment rates in 'non relational' and 'relational' capital given respectively by s^* e δ^*.

Investment rates in the two types of capital will be higher than zero when their respective marginal returns in terms of Nopat, and therefore in terms of market value of the firm, are higher than the opportunity cost of the invested capital. In particular, we note that for both the 'non relational' capital and the stakeholders capital their respective optimal investment rates, s^* e δ^*, are higher than zero when their respective marginal productivity for each euro invested, $(\partial y/\partial k) \times (py/pk)$ and $(\partial y/\partial h) \times (py/ph)$, will be higher than the opportunity cost of the invested capital.

Remembering the Cobb-Douglas specification of the production function we can derive the optimal investment rates in 'non relational' and 'relational' capital:

$$s^{*} = \alpha k^{\alpha-1} h^{1-\alpha} \frac{p_y}{p_k} - (i-g), \tag{52}$$

$$\delta^{*} = (1-\alpha) k^{\alpha} h^{-\alpha} \frac{p_y}{p_h} - (i-g). \tag{53}$$

4.5. *Capital structuring with a Merton structural model*

In the model that we have just presented we have completely neglected all the financing problems of the firm. We have focused our attention only on the problems related to the analysis of the relation between the market value of the firm, *V*, and the investments in 'non relational' and stakeholders capital. Now we shall show that the framework that we have presented in this paper can be easily extended to analyze simultaneously investment problems (in 'relational' and 'non relational' capital) and financing policies.

In this paragraph we shall follow the analysis pioneered by Merton (1974) for the development of the so-called structural models. The financial structure of the firms is characterized by *a*) a zero-coupon type debt with maturity *T* and face value *F* (whose market value is *B*) and *b*) equity with market value *E*.

The fundamental assumption behind the Merton model is represented by the independence of the market value of the firm from its financial structure (Modigliani-Miller theorem, M-M). If we maintain this hypothesis, at this stage of the analysis, we can use 45 to write:

$$V = \frac{[Y - \delta H - sK]}{i-g} = \frac{[p_y y - \delta p_h h - s p_h k]}{i-g} = E + B. \tag{54}$$

We immediately understand that in our model the M-M assumptions imply that the costs of the 'non relational' capital, p_k, and that of the stakeholders capital, p_h, are independent from the leverage ratio of the firm (this hypothesis will be removed in the next paragraph).

Applying the Ito's Lemma to 41 and remembering equation 44, we can write:

$$d\ln V = \left(g - \frac{\sigma_V^2}{2}\right)dt + \sigma_V dz; \tag{55}$$

it follows that:

$$\Delta V = V\left[e^{\left(g-\frac{\sigma_V^2}{2}\right)\Delta t + \sigma_V \varepsilon\sqrt{\Delta t}} - 1\right]. \tag{56}$$

We can now define the VaR of the firm as:

$$\text{Prob}\left(-\text{VaR} \le \Delta V\right) = \alpha, \tag{57}$$

where α is the fixed confidence level. Exploiting equation 57 we can compute the VaR for a given time horizon T and a given confidence level as:

$$\text{VaR}\ (V, \alpha, T) = V\left[1 - e^{\left(\mu-\frac{\sigma^2}{2}\right)\Delta t - \sigma\varepsilon_\alpha\sqrt{dt}}\right]; \tag{58}$$

with:

$$\text{Prob}\left(-\varepsilon_\alpha \le \varepsilon\right) = \alpha\,. \tag{59}$$

In the Merton model market values of equity, E, and debt, B, are respectively given by:

$$E = V\,N(d_1) - Fe^{-rT}N(d_2) \equiv \text{call}(V,F), \tag{60}$$

$$B = Fe^{-rT} - Fe^{-rT}\,N(-d_2) - V\,N(-d_1) \equiv B^* - \text{put}(V,F) \tag{61}$$

with:

$$d_1 = \frac{\ln(V/F) + \left(r + \sigma_V^2/2\right)T}{\sigma_V\sqrt{T}};\ \text{e}\ d_2 = d_1 - \sigma_V\sqrt{T}\,.$$

In this simplified framework if we assume that the firm will hold economic capital equal to the VaR level of its assets, we can use 58 to write:

$$E = VaR(V, \alpha, T) = V\left[1 - e^{\left(g - \frac{\sigma_V^2}{2}\right)\Delta t - \sigma_V \varepsilon_\alpha \sqrt{dt}}\right]. \qquad (62)$$

To understand which is the fixed confidence level, α, for the VaR computation, and therefore for the holding of economic capital, we can assume that the firm has a credit rating target which implies a target credit spread, $\bar{\eta}$. Knowing that:

$$B = Fe^{-(r+\eta)T}, \qquad (63)$$

some simplifications of 63 allow to write:

$$\eta = -\ln\left(\frac{B^* - put(V,F)}{B^*}\right)\Big/T. \qquad (64)$$

From the previous equation we can easily infer that the target credit spread, $\bar{\eta}$, is coherent with just one level of debt $\bar{F}$. If we remember that in the Merton model the market value of equity is represented by a call option written on the assets of the firm, we can use equations 58 and 60 obtaining:

$$E = VaR(V, \alpha, T) = call(V, F). \qquad (65)$$

We can therefore infer that the target credit spread, $\bar{\eta}$, and the target level of debt, $\bar{F}$, will determine the target confidence level $\bar{\alpha}$.

4.6. *Value creation and capital structure problems (a first analysis)*

In the previous paragraph we showed that the Merton model can be easily used to analyze capital structure policies (with the definition of the economic capital or with the choice of the leverage ratio coherent with a given level of credit rating and default probability, etc.). Unfortunately this framework cannot be used for the analysis of shareholders and stakeholders problems. As already pointed out in a world

à la M-M, with the behaviour of assets, *V*, exogenously modelled and independent from the leverage ratio, we do not have any chance to analyze simultaneously investment (value creation) problems and financing choices.

However, we have to remember that the M-M theorem relies on the following hypothesis:

- financial markets without frictions (with the absence of transaction costs);
- no taxes;
- absence of asymmetric information for all the economic agents;
- absence of costs of default;
- absence of any moral hazard problem between equity owner, managers and creditors.

It is easy to understand that, if we modify some of the previous assumptions (and especially those referred to the problems of asymmetric information, moral hazard and expected default costs), we have to abandon the hypothesis of independence between the market value of the firm (its cash flows) and the leverage ratio of the firm.

The model of value creation presented in this paper can be easily generalized to overcome these limitations allowing a simultaneous analysis of investment problems (in 'non relational' and 'relational' capital) and financing decisions. This flexibility comes from the possibility to model the cost of the stakeholders capital, p_h^l, as a function of the leverage policies.

In particular, we can assume that p_h^l will be related to p_h as follows:

$$p_h^l = p_h \times l, \tag{66}$$

where the multiplier l represents, in some sense, the extra cost demanded by the *stakeholders* of the leveraged firm.

Coherently with the modeling of the other prices/costs of our model, we assume that the multiplier l follows a stochastic log-normal process. We can therefore rewrite:

$$\frac{dl}{l} = \mu_l(F)\,dt + \sigma_l dz, \tag{67}$$

with

$$\frac{\partial \mu_l(F)}{\partial F} > 0. \tag{68}$$

We are basically assuming that the drift rate of the stochastic process followed by the cost multiplier, *l*, demanded by the stakeholders of the leveraged firm, is positively affected by the level of debt, *F*.

In this new framework we can write the FCF of the leveraged firm, FCF^l, as:

$$FCF^l = Y - \delta p_h^l h - s p_k k = Y - \delta p_h l h - s p_k k. \tag{69}$$

Following an approach similar to that developed in paragraph 4.3, we can write:

$$dFCF^l = g^l FCF^l dt + \sigma_{FCF}{}^l FCF^l dz. \tag{70}$$

It follows that:

$$g = \frac{\{[\mu_y Y - s\mu_k K - \delta(\mu_h + \mu_l + \sigma_l \sigma_h)H^l] + s(\alpha Y - sK) + \delta[(1-\alpha)Y - \delta H^l]\}}{FCF} = \tag{71}$$

$$= \frac{\{[\mu_y Y - s\mu_k K - \delta(\mu_h + \mu_l + \sigma_l \sigma_h)H^l] + s(\alpha Y - sK) + \delta[(1-\alpha)Y - \delta H^l]\}}{Y - \delta H^l - sK} =$$

$$= \frac{\{[\mu_y p_y y - s\mu_k p_k k - \delta(\mu_h + \mu_l + \sigma_l \sigma_h)p_h^l h] + s(\alpha p_y y - s p_k k) + \delta[(1-\alpha)p_y y - \delta p_h^l h]\}}{p_y y - \delta p_h^l h - s p_k k},$$

$$\sigma_{FCF}{}^l = \frac{\sigma_y Y - s\sigma_k K - \delta(\sigma_h + \sigma_l)H^l}{FCF^l} = \frac{\sigma_y p_y y - s\sigma_k p_k k - \delta(\sigma_h + \sigma_l)p_h^l h}{p_y y - \delta p_h^l h - s p_k k}. \tag{72}$$

The market value of the leveraged firm, V^l, and its *wacc*, Vi^l, are related by the following equation:

$$\frac{dV^l}{V^l} + \frac{FCF^l}{V^l} dt = i^l dt + \sigma_V^l dz. \tag{73}$$

Few simplifications, similar to those of paragraph 4.3, allow to write the market value of the leveraged firm, V^l, and its MVA,

MVA^{l}, respectively as:

$$V^{l}=\frac{[Y-\delta H^{l}-sK]}{i^{l}-g^{l}}=\frac{[p_{y}y-\delta p_{h}^{l}h-sp_{h}k]}{i^{l}-g^{l}}=\frac{[p_{y}y-\delta p_{h}lh-sp_{h}k]}{i^{l}-g^{l}} \quad (74)$$

and

$$MVA^{l}=V^{l}-IC=\frac{Y+(g^{l}-\delta)H^{l}+(g^{l}-s)K-iIC}{i^{l}-g^{l}}= \quad (75)$$

$$=\frac{p_{y}y+(g^{l}-\delta)p_{h}^{l}h+(g^{l}-s)p_{k}k-i^{l}(p_{k}k+p_{h}^{l}h)}{i^{l}-g^{l}}=$$

$$=\frac{p_{y}y+(g^{l}-\delta)p_{h}lh+(g^{l}-s)p_{k}k-i^{l}(p_{k}k+p_{h}lh)}{i^{l}-g^{l}}.$$

For $g^{l}=s=\delta$ we can rewrite 75 as:

$$MVA=\frac{Y-i^{l}IC}{i^{l}-g^{l}}=\frac{EVA^{l}}{i^{l}-g^{l}}, \quad (76)$$

where $EVA^{l}\equiv Y-i^{l}IC$.

As we did for the unleveraged firm, we can now derive the optimal investment rates in 'non relational' and stakeholders capital for the leveraged firm:

$$\left(\frac{\partial y}{\partial k}\frac{p_{y}}{p_{k}}\right)-(i^{l}-g^{l})=s^{l*}; \quad (77)$$

$$\left(\frac{\partial y}{\partial h}\frac{p_{y}}{p_{h}^{l}}\right)-(i^{l}-g^{l})=\left(\frac{\partial y}{\partial h}\frac{p_{y}}{p_{h}l}\right)-(i^{l}-g^{l})=\delta^{l*}. \quad (78)$$

If we derive the optimal investment rate in stakeholders capital for the leveraged firm, we obtain:

$$\frac{\partial \delta^{l*}}{\partial F} = -\frac{\partial y}{\partial h}\frac{p_y}{p_h l^2}\frac{\partial l}{\partial F} - \left(\frac{\partial i^l}{\partial F} - \frac{\partial g^l}{\partial F}\right). \tag{79}$$

We know that:[6]

$$\frac{\partial i^l}{\partial F} > 0, \tag{80}$$

$$\frac{\partial g^l}{\partial F} < 0, \tag{81}$$

$$\frac{\partial l}{\partial F} = \frac{\partial l}{\partial \mu_l}\frac{\partial \mu_l}{\partial F} > 0. \tag{82}$$

We can therefore conclude that:

$$\frac{\partial \delta^{l*}}{\partial F} < 0. \tag{83}$$

Moreover, if we look equation 77 we note that $(i^l - g^l) > (i - g)$. We can therefore infer that:

$$\frac{\partial s^{l*}}{\partial F} < 0. \tag{84}$$

In our model a higher level of leverage implies a reduction in the net investment of both the 'non relational' and the 'relational' capital. The reduction of the net investments associated with the simultaneous increase of the opportunity cost of capital (due to the increase of the *wacc*) will produce a reduction of the market value of the firm, and therefore a reduction of the market values of both equity and bond.

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[6] A higher level of leverage will increase the expected value of the costs of default faced by the firm producing, in this way, a higher *wacc*, i^l, demanded by the investors of the leveraged firm. We have therefore a positive relation between the *wacc* of the leveraged firm and the level of debt *F*.

5. Conclusions

In this paper we presented a stochastic model for the most relevant value indicators. In this framework we showed that the supposed contradiction between shareholders value creation and stakeholders value creation seems to be a misleading problem.

We proved that the optimal investment rates in both 'relational' and 'non relational' capital are higher than zero when their respective marginal returns, in terms of market value of the firm, are higher than their respective marginal costs.

We then showed that our model can be easily generalized to allow a simultaneous analysis of both investment problems and value creation policies.

A first extension of the model could address the problems of shareholders and stakeholders capital at the level of business units. Another possible generalization could investigate the relations between the investments problems and the financing policies with contingent capital opportunities. These topics are left for future research.

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